A Finite-dimensional Control Design for Parabolic Distributed Parameter Systems

著者	HASHIMOTO Yukio, SATO Hiroki, ABE Yoshikazu, DOTE Yasuhiko
journal or publication title	Memoirs of the Muroran Institute of Technology. Science and engineering
volume	37
page range	135-148
year	1987-11-10
URL	http://hdl.handle.net/10258/727

A Finite-dimensional Control Design for Parabolic Distributed Parameter Systems

橋 本 幸 男・佐 藤 弘 樹・安 部 嘉 一・土 手 康 彦

Yukio HASHIMOTO, Hiroki SATO, Yoshikazu ABE and Yasuhiko DOTE

Abstract

A finite-dimensional discrete-time distribution controller is designed for a class of distributed parameter systems with control inputs in and/or on the body. The systems are described by a partial differntial equation of parabolic type. The measured outputs of the system are assumed to be obtained through a finite number of point sensors located in and/or on the system. The proposed controller is a combination of a low-spillover distribution observer and a linear state feedback law. Sufficient conditions are given for the existence of the output regulation. A practical trade-off measure is also shown between the order of the controller and the sampling interval. The low-spillover distribution observer is realized on the basis of an accurate modeling of the system which is described in discrete time and contains a special feedforward pass. By using standard state variable techniques in the finite-dimensional control theory, it becomes possible for system designers to construct a state feedback distribution observer-regulator without troublesome preparations such as sensor allocation to avoid the observation spillover.

1. Introduction

It is natural to try to design a control system for distributed parameter system on the basis of a reduced order model. Balas has given a modal control design method for the distibution control system, using eigenfunction expansion of the system state to get the reduced order model[1),2)]. Kobayashi has dealt with a construction of finite-dimensional state observer in the same way for parabolic distributed parameter systems[3)]. Furthermore, Kobayashi showed a finite-dimensional servomechanism with state observer for continuous-time parabolic systems[4)] and for discrete-time ones[5)]. However, they all assumed that no observation spillover was present.

For reducing the observation spillover due to the infinite-dimensional nature of distributed parameter systems, at least three ideas have been reported, but not the way of increasing the order of system model. The first idea is to locate sensors at low spillover positions. The second is the reduction by using a lot of sensors to approximate distributed sensors. Both are reported by Balas[2)]. The third is the one proposed by Fujii and Hirai[6)]. They added sensor infuence functions to the usual basis for Galerkin approximation and then produced a new basis by Gramm-Schmidt's orthogonalization. The observation spillover problems have been overcome by the modeling in which the Galerkin approximate solution is sought in terms of this new basis. The last method, however, fails for boundary observation.

This paper solves the problems of spillover from a standpoint of an accurate system modeling. The systems are described by a parabolic type partial differential equation. Calling upon the fact that most of real distributed parmeter systems are controlled through their boundaries, we deal with the systems which are driven by control inputs in and/or on it. In addition, for wide application of the theory, the boundary inputs are assumed to act on the system through mixed-type boundary condition. That is, this paper discusses the control problem of following class of distributed parameter systems.

$$\frac{\partial T(t,x)}{\partial t}=\sum_{j,k=1}^{r}\frac{\partial^2 T(t,x)}{\partial x_j \partial x_k}+\sum_{j=1}^{r} b_j \frac{\partial T(t,x)}{\partial x_j}+cT(t,x)+\sum_{i=1}^{M'} f_i(x)\psi_i(t) \quad \text{in D}$$

with a boundary condition

$$\alpha(\xi)\,T(t,\xi)+(1-\alpha(\xi))\nabla_\nu T(t,\xi)=\sum_{i=M'+1}^{M} g_i(\xi)\psi_i(t) \quad \text{on S} \tag{1. 1}$$

where D is a bounded open domain in Euclidean r-space R^r with piecewise sufficiently smooth surface S, T(t, x) is a scalar valued function of time t and the spatial coordinate vector $x=(x_1, x_2, \ldots, x_r)$ and represents the state of the process, $t>0$, scalar valued functions $f_i(x)$ $(i=1, \ldots, M')$ are all Hölder continuous everywhere the closure of D, control inputs $\psi_i(t)$ $(i=1, \ldots, M')$ are also Hölder continuous in any time interval $[0, t_0]$, $\xi \in S$, ∇_ν denotes the projection of gradient to the outer normal vector ν on ξ, $g_i(\xi) \in C^2(S)$, $\psi_i(t)$ $(i=M'+1, \ldots, M)$ has a continuity that $d\psi_i(t)/dt$ is Hölder continuous in any time interval $[0, t_0]$, and $\alpha(\xi)$, defined on S, belongs to C^2 and is further assumed to be $0 \leqq \alpha(\xi) \leqq 1$. It would be obvious that a particular case of $\alpha(\xi)=1$ corresponds to the Dirichlet-type boundaray problem, and $\alpha(\xi)=0$, the Neumann-type one.

It is possibe to formulate the distributed parameter system by time evolution equation in the Hilbert space $H=L^2(D)$ and then discuss its dynamics, as many workers do[7]. In this paper, however, we restricted ourselves to the classical treatment on the distributed parameter system, for the aim of immediate applications.

Use of a digital computer is inevitable for realizations of distribution control. Whenever a digital computer constitutes a part of a control system, the continuous signal must be discretized in order to be digestible by the computer. Therefore the control design was developed in discrete-time form. As the proposed reduced order model approximates a time evolution behaviour of the distributed parameter system in a good accuracy, not only the observation spillover but also the control spillover can be neglected. By using standard state variable methods, it became possible for system designers to construct a state feedback distribution observer-regulator without difficulty.

In Section 2, the distributed parameter system is transformed to the infinite-dimensional state variable equations by modal expansion method. A discrete-time reduced order model for control design is derived from the result, and simultaneously a practical trade-off measure between the digital sampling inteval and the order of the reduced model is proposed in Section 3. In Section 4, We construct an identity distribution state observer and discuss the estimation error. In Section 5, a distribution regulator with state observer is given and its regularity is proved.

2. System Transformation to State Variable Equations

Let (a_{kj}) be a $(r \times r)$ matrix whose k-j compounent is a_{kj} in Eq. (1. 1). If (a_{kj}) is a symmetric positive definite matrix, then the system is transformed to the following canonical expression[8).

$$\frac{\partial T(t, x)}{\partial t} = \Delta T(t,x) + cT(t, x) + \sum_{i=1}^{M'} f_i(x)\psi_i(t) \qquad \text{in D}$$

$$\alpha(\xi)T(t, \xi) + (1-\alpha(\xi))\nabla_\nu T(t, \xi) = \sum_{i=M'+1}^{M} g_i(\xi)\psi_i(t) \qquad \text{on S} \qquad (2.\ 1)$$

where Δ is a Laplacian; $\Delta = (\partial^2/\partial x_1^2, \partial^2/\partial x_2^2, \ldots, \partial^2/\partial x_r^2)$.

Note that each quantity in Eq. (2. 1) is not same as that in Eq. (1. 1) any longer, although the same expressions are used in both systems. The assumption that (a_{kj}) is a symmetric and positive definite matrix would be justified by many examples of real distributed parameter systems. Therefore, we control the system described by a partial differential equation of Eq. (2. 1). The initial state of the system is assumed to be given by

$$T(0, x) = T_0(x) \qquad \text{in D.} \qquad (1.\ 2)$$

Now, consider a self-adjoint partial differential operator $A \equiv \Delta + c$ with homogeneous mixed-type boundary condition. There exists a following sequence of eigenfunctions and eigenvalues[9).

$$\{\lambda_i, \ \phi_{ij}(x); j=1, 2, \ldots, m_i, i=1, 2, \ldots\ldots\} \qquad (2.\ 3)$$

where

(1) $c = \lambda_1 > \lambda_2 > \ldots > \lambda_i > \ldots\ldots, \ \lim_{i\to\infty} \lambda_i = -\infty$

(2) the sequence of eigenfuncitions $\{\phi_{ij}(x); j=1, 2 \ldots, m_i\ i=1, 2, \ldots\ldots\}$ makes a complete orthonormal basis in $L^2(D)$; m_i represents the degeneracy of the eigenvalue λ_i.

Here let us renumber the subscripts of the eigenfunctions such that $\phi_1(x)$, $\phi_2(x)$, ... and associated eigenvalues λ_1, λ_2, ... for the simple notation, and expand the system Eq. (2. 1) by eigenfunctions in order to get modal decomposition. This procedure gives the following infinite-dimensional sate equations (see Appendix).

$$\dot{e}(t)=A_c e(t)+B_c u(t) \tag{2. 4}$$

$$T(t, x)=C(x)e(t) \tag{2. 5}$$

where

$e(t)=[e_1(t), e_2(t), \ldots]^T\ (\infty\times 1)$; $\quad e_i(0)=\int_D T_o(x)\phi_i(x)\,dx$

$u(t)=[\psi_1(t),\ \psi_2(t), \ldots,\ \psi_M(t)]^T\quad (M\times 1)$

$A_c=\{A_{ij}\}(\infty\times\infty)$; $\quad A_{ij}=\lambda_i\delta_{ij}$

$B_c=\{B_{ij}\}\ (\infty\times M)$; For $1\leqq j\leqq M'$, $B_{ij}=f_{ij}$,

and for $M'+1\leqq j\leqq M$, $B_{ij}=\int_S(\phi_i(\xi)-\nabla_\nu\ \phi_i(\xi))\,g_j(\xi)dS$

$C(x)=[\phi_1(x),\ \phi_2(x), \ldots]\quad (1\times\infty)$

3. Derivation of reduced order model

Let us suppose the system is sampled at every τ seconds and the each control input $\psi_i(t)$ to the system is given by the output of Oth order holder. Then, Eqs. (2. 4) and (2. 5) are rewritten as

$$e(k+1)=Ae(k)+Bu(k) \tag{3. 1}$$

$$T(k, x)=C(x)e(k) \tag{3. 2}$$

where $k=0, 1, 2, \ldots\ldots$.

Here, let us introduce following space decomposition. This corresponds to the orthgonal projection decomposition of the Hilbert space l^2 to invarint subspaces HP and HQ such as $l^2=HP+HQ$.

$$e_P(k+1)=A_P e_P(k)+B_P u(k) \tag{3. 3}$$

$$e_Q(k+1)=A_Q e_Q(k)+B_Q u(k) \tag{3. 4}$$

$$T(k,x)=C_P(x)e_P(k)+C_Q(x)e_Q(k) \tag{3. 5}$$

where N is chosen such that $\lambda_N \neq \lambda_{N+1}$, and

$$e_P(k)=[e_1(k),\ e_2(k),\ \dots,\ e_N(k)]^T$$

$$e_Q(k)=[e_{N+1}(k),\ e_{N+2}(k),\ \dots\dots]^T$$

$$A=\begin{array}{c} \overset{\longleftarrow N \longrightarrow}{} \\ \left[\begin{array}{c|c} A_P & O \\ \hline O & A_Q \end{array}\right] \end{array}\begin{array}{c}\updownarrow N\end{array},\quad B=\begin{array}{c} \overset{\longleftarrow M \longrightarrow}{} \\ \left[\begin{array}{c} B_P \\ \hline B_Q \end{array}\right] \end{array}\begin{array}{c}\updownarrow N\end{array},\quad C(x)=\underset{\longleftarrow N \longrightarrow}{[C_P(x),\ C_Q(x)]}$$

$$A_P=\{a_{ij}\}\ (N\times N);\quad a_{ij}=e^{\lambda_i \tau}\,\delta_{ij}$$

$$B_P=\{b_{ij}\}\ (N\times M);\quad \text{For } 1\leqq j\leqq M',\ b_{ij}=(e^{\lambda_i \tau}-1)\,f_{ij}/\lambda_i$$

and for $M'+1\leqq j\leqq M$, $b_{ij}=(e^{\lambda_i \tau}-1)\int_S(\phi_i-\nabla_\nu\phi_i)g_j dS/\lambda_i$

$$C_P(x)=[\phi_1(x),\ \phi_2(x),\ \dots,\ \phi_N(x)].$$

Many works neglected e_Q in Eqs. (3. 4) and made a reduced order model of the system. In the following discussions, we call the finite-dimensional approximation of this type "truncated modal approximation (TMA)". In our reduced order modeling of the system, we utilize the residual modes, and produce an accurate model of the system in order to overcome the spillover problems.

Throught out following discussions, $\|\cdot\|_{L^2}$ means L^2-norm on D. $\|\cdot\|_{HP}$ and $\|\cdot\|_{HQ}$ give N-dimensional Euclidean and l^2-vector norms, respectively. The norm of matrixes which map vector in normed X-space to one in normed Y-space is the one induced from the vector norm. That is

$$\|A\|_{XY}=\sup_{e_{\text{in}}\mathbf{X}}\{\|Ae\|_Y/\|e\|_X\}.$$

If X=Y, then the subscript of matrix norm is omitted.

Lemma 3. 1　For any given ε, there exist τ and N such that $\|A_Q\|<\varepsilon$.

Proof　Consider the Reileigh quotient[10], then one gets

$$0<e_Q^T A_Q^T A_Q e_Q/e_Q^T e_Q \leqq \exp(2\lambda_{N+1}\tau).$$

This means　$0<\|\mathrm{A}_Q e_Q\|_{HQ}/\|e_Q\|_{HQ} \leqq \exp(\lambda_{N+1}\tau)$.

From the definition of the norm, one obtains

$$\|A_Q\|=\exp(\lambda_{N+1}\tau).$$

Either the increasing τ with N fixed or the increment of N with τ fixed results in monotonous decreasing of $\|A_Q\|$. It is obuious that such N and τ exist. □

Consider the following system;

$$e_P(k+1)=A_P e_P(k)+B_P u(k) \tag{3. 6}$$

$$T'(k,x)=C_P(x)e_P(k)+C_Q(x)B_Q u(k-1). \tag{3. 7}$$

Theorm 3. 1　For any gien ε, there exist N and τ such that

$$\|T(k,x)-T'(k,x)\|_{L^2}<\varepsilon \quad \text{for all } k$$

Proof　Calculate $\|T(k,x)-T'(k,x)\|_{L^2}$ directly, then

$$\|T(k,x)-T'(k,x)\|_{L^2}=\|C_Q(x)e_Q(k)-C_Q(x)B_Q u(k-1)\|_{L^2}$$

$$=\|\mathrm{A}_Q e_Q(k-1)\|_{PQ}$$

$$\leqq\|A_Q\|\ \|e_Q(k-1)\|_{PQ}$$

For bounded inputs, there exists finite Γ such that $\|e_Q(k)\|_{PQ}\leqq\Gamma$.

Hence

$$\|T(k,x)-T'(k,x)\|_{L^2} \leqq \|A_Q\|\Gamma \quad \text{for all } k.$$

Using Lemma 3. 1, it can be shown that for any given ε, N and τ exist such that $\|T(k,x)-T'(k,x)\|_{L^2}<\varepsilon$. □

From Theorem 3. 1, we can conclude that Eqs. (3. 6) and (3. 7) approximate the system behaviour in some accuracy and the degree of accuracy can be improved by choosing appropriate N and τ.

Next, supposing that the system is stable, we will express Eq. (3. 7) in the first N-dimensional vector space. The steady state distribution of the system can be calculated if a constant boundary input U_s is known:

$$T(\infty,x)=C_s(x)U_s.$$

While, denoting (N×N) unit matrix by I_P, Eqs. (3. 6) and (3. 7) yield

$$T(\infty,x)=C_P(x)\,(I_P-A_P)^{-1}B_Pu_s+C_Q(x)B_Qu_s.$$

Hence

$$C_Q(x)B_Q=C_S(x)-C_P(x)\,(I_P-A_P)^{-1}B_P.$$

Consequently, we obtain the relation:

$$e_P(k+1)=A_Pe_P(k)+B_Pu(k) \tag{3. 8}$$

$$T'(k,x)=C_P(x)e_P(k)+\{C_S(x)-C_P(x)\,(I_P-A_P)^{-1}B_P\}u(k-1) \tag{3. 9}$$

This is the reduced order model that we propose. The steady state output of the model, as a matter of course, completely coincides with the true one if time-invariant input is applied. For practical appliations, it would be enough to choose N and τ such that $\lambda_{N+1}\tau \leqq -5$, because all residual modes e_Q damp faster than $\exp(\lambda_{N+1}\tau)$ during the sampling interval τ. In case of omitting the second term in Eq. (3. 9), the resultant output error is estimated as

$$\|T(k,x)-T'(k,x)\|_{L^2} \leqq \Gamma \text{ for all } k.$$

Therefore our reduced order model gives $\|A_Q\|$ times smaller output error than the usual TMA model in a sense of L^2-norm.

4. Distribution state observer

In this section we construct a distribution state observer. The distribution state $T(k, x)$ of the system is measured by L point sensors which are located at positions $x_1, x_2, \ldots, x_L$ in and/or on the system. Then, the observation vector $Y(k)$ is $Y(k) \in R^L$, and represented as

$$Y(k)=Pe_P(k)+Qe_Q(k) \tag{4.1}$$

where P is a $(L \times N)$ matrix with i-j component $\phi_j(x_i)$, and Q, a $(L \times \infty)$ matrix with i-j component $\phi_{j+N}(x_i)$.

Supposition 4. 1 The finite-dimensional subsystem (A_P, P) is observable.

This supposition would be easily checked[11),12)]. Therefore, we assume the system is observable.

Now we estimate the distribution state $T(k,x)$ by using usual identity observer:

$$\tilde{e}_P(k+1)=A_P\tilde{e}_P(k)+B_Pu(k)+K(Y(k)-\tilde{Y}(k)) \tag{4.2}$$

where K is a gain matrix. $\tilde{Y}(k)$ represents an estimated output vector which is given by

$$\tilde{Y}(k)=P\tilde{e}_P(k)+QB_Qu(k-1) \tag{4.3}$$

The estimated distribution state $\tilde{T}(k, x)$ is

$$\tilde{T}(k, x)=C_P(x)\tilde{e}_P(k)+C_Q(x)B_Qu(k-1). \tag{4.4}$$

Theorem 4. 1 For any given ε, there exist N and τ such that

$$\lim_{k\to\infty}\|\tilde{T}(k, x)-T(k, x)\|_{L^2} \leqq \varepsilon$$

in the system Eqs. (4. 1), (4. 2), (4. 3) and (4. 4).

Proof Consider the L^2-norm of $\|\tilde{T}(k,x)-T(k,x)\|_{L2}$, then

$$\|\tilde{T}(k, x)-T(k, x)\|_{L2}^2$$

$$=\|\tilde{e}_P(k)-e_P(k)\|_{HP}^2+\|A_Qe_Q(k-1)\|_{HQ}^2$$

$$\leqq\|A_P-KP\|^{2k}\|\tilde{e}_P(0)-e_P(0)\|_{HP}^2$$

$$+\frac{2\Gamma\|A_P-KP\|^k\|\tilde{e}_P(0)-e_P(0)\|_{HP}\|KQ\|_{HQHP}\|A_Q\|}{1-\|A_P-KP\|}+\|A_Q\|^2\Gamma^2\{1+\frac{\|KQ\|^2_{HQHP}}{(1-\|A_P-KP\|)^2}\}. \tag{4. 5}$$

From Supposition 4. 1, one can set observer gain matrix K such that A_P-KP is stable (for example, by using popular pole allocation technique). The first two terms in Eq. (4. 5) approach to zero as k increases. The third term can be smaller than arbitraily given ε^2 with appropreate N and τ from Lemma 3. 1. Therefore,

$$\lim_{k\to\infty}\|\tilde{T}(k,x)-T(k,x)\|_{L^2}\leqq\varepsilon\ .\ \square$$

This identity observer estimates the distribution state T(k, x) in any accuracy. With a time-invariant input, the observer gives true estimation as time increases. It is also shown, without difficulty, that the estimation error is always smaller than that of the TMA model, because of the factor $\|A_Q\|^2$ in the third term of Eq. (4.5).

5. Distribution state regulator

We construct a state feedback distribution regulator with the obserser discussed in the previous section. The control scheme is given by

$$e(k+1)=Ae(k)-BF\tilde{e}_P(k) \tag{5. 1}$$

$$\tilde{e}_P(k+1)=A_P\tilde{e}_P(k)-B_PF\tilde{e}_P(k)+K(Y(k)-\tilde{Y}(k)) \tag{5. 2}$$

$$\tilde{Y}(k)=P\tilde{e}_P(k)-QB_QF\tilde{e}_P(k-1) \tag{5. 3}$$

$$T(k,x)=C(x)e(k)$$

where F is a gain matrix which satisfies

$$\|K\|\ \|P\|\ \|B_P\|\ \|F\|\sum_{k=0}^{\infty}\|A_P\|^k\sum_{k=0}^{\infty}\|A_P-B_PF-KP\|^k<1 \tag{5. 4}$$

$$\|K\|\ \|P\|\ \|B_P\|\ \|F\|\sum_{k=0}^{\infty}\|A_P\|^k+\|K\|\ \|Q\|\ \|A_Q\|\ \|B_Q\|\ \|F\|\sum_{k=0}^{\infty}\|A_Q\|^k<\{\sum_{k=0}^{\infty}\|A_P-B_PF-KP\|^k\}^{-1} \tag{5. 5}$$

Suppose that desired output distribution $T_d(x)$ is a steady state solution of Eq. (2. 1), and consider the error system of $T_d(x)-T(k,x)$. Then, the regulator problem is transformed to a stability problem around a null-distribution O(x). Assuming that Eqs. (5. 1), (5. 2) and (5. 3) describe the error system, we prove the stability. In this section, we omit subscripts of norms for a simple notation.

Lemma 5. 1 If U(k) represents a state vector: U(k+1)=AU(k)+V(k), then the following inequality holds:

$$\sum_{k=0}^{\infty}\|U(k)\|\leqq\sum_{k=0}^{\infty}\|\Lambda\|^{k}\cdot\{\|U(0)\|+\sum_{k=0}^{\infty}\|V(k)\|\}.$$

Proof Considering that

$$\mathrm{U}(k)=\Lambda^{k}U(0)+\sum_{i=0}^{k-1}\Lambda^{k-i-1}V(i),$$

$$\sum_{k=0}^{\infty}\|U(k)\|\leqq\|U(0)\|\sum_{k=0}^{\infty}\|\Lambda\|^{k}+\sum_{k=0}^{\infty}\sum_{i=0}^{k-1}\|\Lambda\|^{k-i-1}\|V(i)\|$$

$$=\|U(0)\|\sum_{k=0}^{\infty}\|\Lambda\|^{k}+\sum_{k=0}^{\infty}\sum_{i=0}^{k-1}\|\Lambda\|^{i}\|V(k-i-1)\|$$

$$=\|U(0)\|\sum_{k=0}^{\infty}\|\Lambda\|^{k}+\sum_{i=0}^{\infty}\|\Lambda\|^{i}\sum_{k=i+1}^{\infty}\|V(k-i-1)\|$$

$$\leqq\|U(0)\|\sum_{k=0}^{\infty}\|\Lambda\|^{k}+\sum_{i=0}^{\infty}\|\Lambda\|^{i}\sum_{i=0}^{\infty}\|\Lambda^{i}\sum_{i=0}^{\infty}\|V(k)\|$$

$$=\sum_{k=0}^{\infty}\|\Lambda^{k}\cdot\{\|U(0)\|+\sum_{k=0}^{\infty}\|V(k)\|\}\square$$

Theorem 5. 1 If the conditions (5. 4) and (5.6) hold, and (A_P, B_P) is controllable, an output T(k,x) of the system (5.1), (5. 2) and (5.3) always converges to a null-distribution.

Proof From Eqs. (5.1), (5. 2) and (5. 3), and from Lemma 5.1, we have

$$\sum_{k=0}^{\infty}\|e_P(k)\|\leqq\sum_{k=0}^{\infty}\|A_P\|^{k}\{\|e_P(0)\|+\|B_P\|\ \|F\|\sum_{k=0}^{\infty}\|\tilde{e}_P(k)\|\}\tag{5. 6}$$

$$\sum_{k=0}^{\infty}e_Q(k)\|\leqq\sum_{k=0}^{\infty}\|A_Q\|^{k}\{\|e_Q(0)\|+\|B_Q\|\ \|F\|\sum_{k=0}^{\infty}\|\tilde{e}_P(k)\|\}\tag{5.7}$$

$$\sum_{k=0}^{\infty}\|\tilde{e}_P(k)\|\leqq\sum_{k=0}^{\infty}\|A_P-B_PF-KP\|^{k}\{\|\tilde{e}_P(0)\|+\|K\|\ \|P\|\sum_{k=0}^{\infty}\|e_P(k)\|$$

$$+\|K\|\ \|Q\|\ \|A_Q\|\sum_{k=0}^{\infty}\|\mathrm{e}_Q(k)\|\}\tag{5. 8}$$

The substuition of Eq. (5. 6) to Eq. (5.8) and the condition of (5. 4) yield

$$\sum_{k=0}^{\infty}\|\tilde{e}_P(k)\|\leqq\gamma+\delta\sum_{k=0}^{\infty}\|e_Q(k)\|\tag{5. 9}$$

where

$$\gamma = \frac{\{\|\tilde{e}_P(0)\| + \|K\|\ \|P\|\ \|e_P(0)\| \sum_{k=0}^{\infty} \|A_P\|^k\} \sum_{k=0}^{\infty} \|A_P - B_P F - KP\|^k}{1 - \|K\|\ \|P\|\ \|B_P\|\ \|F\| \sum_{k=0}^{\infty} \|A_P\|^k \sum_{k=0}^{\infty} \|A_P - B_P F - KP\|^k}$$

$$\delta = \frac{\|K\|\ \|Q\|\ \|A_Q\| \sum_{k=0}^{\infty} \|A_P - B_P F - KP\|^k}{1 - \|K\|\ \|P\|\ \|B_P\|\ \|F\| \sum_{k=0}^{\infty} \|A_P\|^k \sum_{k=0}^{\infty} \|A_P - B_P F - KP\|^k}$$

Substitute Eq. (5.9) into Eq. (5.7), then one obtains an inequality:

$$\{1 - \delta \|B_Q\|\ \|F\| \sum_{k=0}^{\infty} \|A_Q\|^k\} \sum_{k=0}^{\infty} \|e_Q(k)\| \leqq \{\|e_Q(0)\| + \gamma \|B_Q\|\ \|F\|\} \sum_{k=0}^{\infty} \|A_Q\|^k \tag{5. 10}$$

Here, from the condition of (5. 5).

$$1 - \delta \|B_Q\|\ \|F\| \sum_{k=0}^{\infty} \|A_Q\|^k > 0.$$

Thereby, it is concluded that $\sum_{k=0}^{\infty} \|e_Q(k)\| < \infty.$ (5. 11)

The result (5. 11) means that $\|e_Q(k)\|$ approaches to zero as k increases. $\|\tilde{e}_P(k)\|$ also converges to zero from Eq. (5. 9), and then $\|e_P(k)\|$ does from Eq. (5. 6). Thus, one gets Theorem 5.1. □

Theorem 5.1 guarantees the output regulation of our distribution controller. Next we appreciate sufficient conditions (5. 4) and (5. 5). The inequality (5. 4) is a sufficient condition in order that the controllable and observable subsystem (A_P, B_P, K) is stable. Therefore, this condition can be easily realized by means of appropreate techniques in the finite-dimensinonal linear control theory. The second condition (5. 5) appears in connection with the control of residual mode $e_Q(k)$ by finite-dimensional controller. Recalling that $\|A_Q\|$ has been set very small (nearly equal zero) and

$$\sum_{k=0}^{\infty} \|A_Q\|^k \fallingdotseq 1$$

it is not so difficut to set the observer gain matrix K and the regulator gain matrix F to satisfy the condition (5. 5). Thus, the system Eqs. (5. 1), (5. 2) and (5. 3) works as a distribution regulator.

6. Concluding Remarks

A low-spillover reduced order model was proposed for parabolic distributed parameter system, and a practical trade-off measure was given between the order of the approximate model and the sampling interval. The reduction of observation spillover made the constrains on the number of sensors and on sensor positions in the control system almost free. As the result, it became possible for system designenrs to realize a state feeaback distribution observer-regulator without difficulty by using basic techniques in the finite-dinensional multi-input multi-output linear control theory. For the proposed distribution regulator, sufficient conditions were given in order to guarantee the output regulation of the control system.

This low-spillover reduced order model would be available for the control design of servomechanisms for distributed parameter systems of the parabolic type.

Acknowledgement

The Authrors are very grateful to Professor Shigeru Nomura and Professor Hiroshi Tazawa at Depatment of Electronics Engineering in Muroran Institute of Technology for their encouragements.

APPENDIX: Derivation of Infinite-dimensional State Equations

Let us introduce functions defined by $W_n(t, x) = \exp(\lambda_n t)\phi_n(x)$, (n = 1, 2, ...), It is easily shown that these functions satisfy the equations:

$$-\frac{\partial W_n(t,x)}{\partial t} = A W_n(t,x) \qquad \text{in D,} \tag{A. 1}$$

$$\alpha(\xi W_n(t,\xi) + (1-\alpha(\xi)) \quad \nabla\nu\, W_n(t,\xi) = 0 \quad \text{on S.} \tag{A. 2}$$

Then

$$\frac{d}{dt}\int_D T(t,x) W_n(t, x) dx = \int_D \frac{\partial T}{\partial t} W_n dx + \int_D T \frac{\partial W_n}{\partial t} dx$$

$$= \int_D \{\Delta T + cT + \sum_{i=1}^{M'} f_i(x)\psi_i(t)\} W_n dx - \int_D T\{\Delta W_n + c W_n\} dx$$

After simple manipulations and using well known Green's formula[13], we have

$$= \int_D \sum_{i=1}^{M'} f_i(x)\psi_i(t) W_n dx + \int_S \{W_n \nabla\nu\, T - T\nabla\nu\, W_n\} dS$$

Thus, we obtain an useful equation for the system interpretation

$$\int_D \frac{\partial T}{\partial t} W_n dx = -\int_D T\frac{\partial W_n}{\partial t} dx + \int_S \{W_n \nabla\nu\, T - T\nabla\nu\, W_n\} dS + \sum_{i=1}^{M'} \{\int_D f_i(x) W_n dx\}\psi_i \tag{A. 3}$$

By the way, following relations hold everywhere on the surface, and at any time:

$$\alpha T+(1-\alpha)\nabla\nu T=\sum_{i=M'+1}^{M} g_i\psi_i$$

$$\alpha W_n+(1+\alpha)\nabla\nu W_n=0$$

Then,

$$\frac{1}{\begin{vmatrix} T & \nabla\nu T \\ W_n & \nabla\nu W_n \end{vmatrix}}=\frac{\alpha}{\nabla\nu W_n\sum_{i=M'+1}^{M} g_i\psi_i}=\frac{1-\alpha}{-W_n\sum_{i=M'+1}^{M} g_i\psi_i}$$

$$=\frac{1}{(\nabla\nu W_n-W_n)\sum_{i=M'+1}^{M} g_i\psi_i}$$

Thus,

$$T\nabla\nu W_n-W_n\nabla\nu T=(\nabla\nu T=(\nabla\nu W_n-W_n)\sum_{i=M'+1}^{M} g_i\psi_i \tag{A. 4}$$

After expanding both to system distributions such as

$$T(t,x)=\sum_{i=1}^{\infty} e_i(t)\psi_i(x),\quad f_i(x)=\sum_{k=1}^{\infty} f'_{ki}\psi(x) \tag{A. 5}$$

substute them into. Eq. (A. 3) together with Eq. (A. 4). Now, we execute the integrations, and obtain infinite-dimensional state variable equations (2. 4)and (2. 5) for the distributed parameter system.

References

1) M. J. BALAS, Modal Control of Certain Flexible Dynamic Systems, SIAM J. Control and Optimization, Vol. 16, pp. 450-462, 1978

2) M. J. BALAS, Feedback Control of Linear Diffusion Processes, INT. J. Control, Vol. 29, No. 3, pp. 523-533, 1979

3) T. KOBAYASHI, Discrete-time Observers and Parameter Determination for Distributed Parameter Systems with Discrete-time Input-output Data, SIAM J. Control and Optimization, Vol. 21, No. 3, pp. 331-351, 1983

4) T. KOBAYASHI, Finite-dimensional Servomechanism Design for Parabolic Distributed-parameter Systems, INT. J. Control, Vol. 37, No. 5, pp. 975-993, 1983

5) T. KOBAYASHI, Discrete-time Servomechanism Design of Parabolic Distributed-parameter Systems, INT. J. Control, Vol. 41, No. 4, pp. 845-864, 1985

6) N. FUJII and M. HIPAI, A Finite-dimensional Asympotic Observer for a Class ofDistributed Paramer Systems, INT. J. Control, Vol. 32, No. 6, 951-962, 1980

7) For example, J. L. LIONS and E. MAGENES, Non-Homogeneous Boudary Value Problems and Applications, Vol. 2, Chapter 4, Springer-Verlag Berlin Heidlberg New York, 1972

8) R. IINO and M. TSUTSUMI, Introduction to Partial Differential Equations, Science-sya Co. Ltd, Science Library: Mathematics for Science and Engeneering, Vol. 9, pp. 14-16, 1978 (in Japanese)

9) S. ITO, Fundamental Solutions of Parabolic Differential Equations and Boundary Value Problems, Japan J. Math., Vol. 27, pp. 55-102, 1957

10) H. KOGO and T. MITA, Introduction to System Control Theory, Jikkyo-Syuppan Co. Ltd., pp. 58-59, 1981 (in Japanese)

11) T. KOBAYASHI, Discrete-time Observability for Distributed Parameter Systems, INT. J. Control, Vol. 31, No. 1, pp. 191-193, 1980

12) Y. SAKAWA and T. MATSUSHITA, Stabilization of Distributed-parameter Systems of Parabolic Type and Construction of Observer, Transactions of the Society of Instrument and Control Engineers, Vol. 11, No. 2, pp. 168-174, 1975 (in Japanese)

13) W. RUDIN, Principles of Mathematical Analysis, Kyoritsu-Syuppan Co. Ltd., pp. 242-243 1979 (Translated into Japanese by M. KONDO and N. YANAGIHARA)