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ASYMTOTIC STABILITY OF INDUCTION MOTOR* VIA SECOND METHOD OF LIAPUNOV

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Abstract

A stability analysis of an ideal three phase squirrel cage induction motor is performed by applying the second method of Liapunov to the nonlinear equations which describe the dynamic behavior of the ideal induction motor.

In general, previous stability analyses of the induction motor have been accomplished by linearizing about a steady state operating point. This has been the most feasible approach because of the complexity of the nonlinear equations. If the methods of Liapunov are applied to the nonlinear equations usually there is great computational difficulty. This paper describes a unique transformation resulting in a simplified system of equations to which the second method of Liapunov is applicable. In the determination of the stability region, a simple method is presented using the specific nature of the nonlinearities; i.e. terms involving the product of the two state variables. The asymptotic stability region which is obtained here is a region of stability in the large. This region is much larger than the local region of stability resulting from linearization about the steady state operating point.

Introduction

The squirrel cage induction motor is of great practical interest because of its low cost and high reliability. But in the past it had the disadvantage that its speed was not easily adjustable. With the advent of the silion controlled rectifier, triac and related members of the thyristor family, it has become feasible to design variable frequency inverter fed induction motor drive systems. It is well known that the variable frequency induction motor itself becomes unstable at certain operating conditions even when supplied from an ideal three phase ac power source 2,3. Since the torque produced by the motor is proportional to the product of the winding current and air gap flux, the motor is represented by nonlinear differential equations. Previous stability analyses of the induction motor have been accomplished by linearizing about a steady state operating point, but no attempt has been made to analyze directly the nonlinear system equations. In this paper Liapunov functions are used to find a simple method to predict the stability region for an ideal three phase squirrel cage type induction motor. First a new state representation for an ideal three phase induction motor is devised, for which the second method of Liapunov is applicable. Then in the determination of asymptotic stability, with the aid of the theory of matrices and vectors and polar coordinates, a simple method is presented using the specific nature of the nonlinearities;

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i.e., terms involving the product of two state variables. The asymptotic stability region which is derived here yields conditions for stability in the large, not in the small. Thus, it provides us with much information on the transient stability of such machines subject to disturbances which can often occur during transient operation of thyristor controlled variable frequency induction motor drives.

System Equations

In order to devise analytical methods for studying the stability of an induction motor, a suitable mathematical model of this machine was developed. The general procedure was as follows : first, certan assumptions were made which were very nearly correct for practical machines ; then, with the help of the d-q transformation of variables, the basic equations were developed[1]. This yields,

$$\begin{bmatrix} V_m \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_1 + \frac{p}{w_b} X_{11} & -\frac{w}{w_b} X_{11} & \frac{pX_{12}}{w_b} & -\frac{wX_{12}}{w_b} \\ \frac{w}{w_b} X_{11} & r_1 + \frac{pX_{11}}{w_b} & \frac{w}{w_b} X_{12} & p\frac{X_{12}}{w_b} \\ p\frac{X_{12}}{w_b} & -\frac{sw}{w_b} X_{12} & r_2 + p\frac{X_{22}}{w_b} & -sw\frac{X_{22}}{w_b} \\ s\frac{w}{w_b} X_{12} & p\frac{X_{12}}{w_b} & \frac{sw}{w_b} X_{22} & r_2 + \frac{pX_{22}}{w_b} \end{bmatrix} \begin{bmatrix} i_{1d} \\ i_{1q} \\ i_{1d} \\ i_{2q} \end{bmatrix} \tag{1}$$

and

$$\frac{2H}{w_b} w_2 B \frac{w_2}{w_b} + T_L = X_{12}(i_{1q} i_{2d} - i_{1d} i_{2q}) \tag{2}$$

wherer

$V_m \triangleq$ stator applied voltage transformed to d-q coordinates

$r_1 \triangleq$ stator resistance

$r_2 \triangleq$ rotor resistance

$X_{11} \triangleq$ stator leakage reactance plus magnetizing reactance at w_b

$X_{22} \triangleq$ rotor leakage reactance plus magnetizing reactance at w_b

$X_{12} \triangleq$ magnetizing reactance at w_b

$p \triangleq \frac{d}{dt}$

$s \triangleq$ slip

$w \triangleq$ electrical angular velocity of applied stator voltage

$w_b \triangleq$ the rated maximum stator frequency

$w_2 \triangleq$ rotor angular velocity

$H \triangleq$ inertia constant

$B \triangleq$ viscous friction coefficient

$T_l \triangleq$ fixed load torque

$i_{1d} \triangleq$ instantaneous stator direct-axis current

$i_{1q} \triangleq$ instantaneous stator quadrature-axis current

$i_{2d} \triangleq$ instantaneous rotor direct-axis current

$i_{2q} \triangleq$ instantaneous rotor quadrature-axis current

For simplicity a special set of state variables is defined using the following linear transformation.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} \triangleq \begin{bmatrix} X_{11} & 0 & X_{12} & 0 & 0 \\ 0 & X_{11} & 0 & X_{12} & 0 \\ X_{12} & 0 & X_{22} & 0 & 0 \\ 0 & X_{12} & 0 & X_{22} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{1d} \\ i_{1q} \\ i_{2d} \\ i_{2q} \\ \frac{w_2}{w_b} \end{bmatrix} \quad (3)$$

Next the steady state operating point ($y_{10}, y_{20}, y_{30}, y_{40}, y_{50}$) is transferred to the origin by defining new state variables.

$$z_1 \triangleq y_1 - y_{10}$$
$$z_2 \triangleq y_2 - y_{20}$$
$$z_3 \triangleq y_3 - y_{30}$$
$$z_4 \triangleq y_4 - y_{40}$$
$$z_5 \triangleq y_5 - y_{50}$$

Then, (1) and (2) become

$$\dot{\vec{z}} = A\vec{z} + \vec{g}(\vec{z}) \quad (4)$$

where

$$\dot{\vec{z}} \triangleq [\dot{z}_1 \ \dot{z}_2 \ \dot{z}_3 \ \dot{z}_4 \ \dot{z}_5]^T \quad (5)$$

$$\vec{z} \triangleq [z_1 \ z_2 \ z_3 \ z_4 \ z_5]^T \quad (6)$$

$$A \triangleq w_b \begin{bmatrix} r_1B_r & \frac{w}{w_b} & -r_1B_m & 0 & 0 \\ -\frac{w}{w_b} & r_1B_r & 0 & -r_1B_m & 0 \\ -r_2B_m & 0 & r_2B_s & \left(\frac{w}{w_b} - y_{50}\right) & -y_{40} \\ 0 & -r_2B_m & -\left(\frac{w}{w_b} - y_{50}\right) & r_2B_s & y_{30} \\ MB_my_{40} & -MB_my_{30} & -MB_my_{20} & MB_my_{10} & -MB \end{bmatrix} \quad (7)$$

$$\vec{g}(\vec{z}) \triangleq w_b[0 \quad 0 \quad -z_4z_5 \quad z_3z_5 \quad MB_m(z_1z_4 - z_2z_3)]^T \quad (8)$$

$$M \triangleq \frac{1}{2Hw_b}$$

$$B_s \triangleq \frac{X_{11}}{X_{12}^2 - X_{11}X_{22}}$$

$$B_r \triangleq \frac{X_{22}}{X_{12}^2 - X_{11}X_{22}}$$

$$B_m \triangleq \frac{X_{12}}{X_{12}^2 - X_{11}X_{22}}$$

The resulting system equation (4) is a nonlinear fifth order autonomous differential equation.

Stability Analysis

On the basis of Liapunov's second method, the stability of systems with nonlinear terms involving the product of two state variables is investigated.
Consider (4)

$$\dot{\vec{z}} = A\vec{z} + \vec{g}(\vec{z})$$
$$\dot{\vec{z}} = A\vec{O} + \vec{g}(\vec{O}) = \vec{O}$$

Next choose a positive definite diagonal matrix Q where

$$\dot{V}(\vec{z}) = -\vec{z}^{\,T} Q\vec{z} \tag{9}$$

and then solve the matrix equation

$$A^T R + RA = -Q \tag{10}$$

for the matrix R , where also

$$V(\vec{z}) = \vec{z}^{\,T} R\vec{z} \tag{11}$$

If R is positive definite then the linearized system is asymptotically stable. This V-function also may be used to determine the instability of the system. If R is not a positive definite matrix nor a positive semi-definite matrix, the origin of the system is unstable. When the system is unstable near the origin, it is of no practical value and thus it is not necessary to investigate stability elsewhere.

Using the V-funcution of (11) we can now determine $\dot{V}(\vec{z})$ for the nonlinear systems as follows :

$$\dot{V}(\vec{z}) = -\vec{z}^{\,T} Q\vec{z} + 2\vec{z}^{\,T} R\vec{g}(\vec{z}) \tag{12}$$

Consider the last terms of (12)

$$2\vec{z}^{\,T} R\vec{g}(\vec{z}) = 2w_b\vec{z}^{\,T} R \begin{bmatrix} 0 \\ 0 \\ -z_4z_5 \\ z_3z_5 \\ MB_m(z_1z_4 - z_2z_3) \end{bmatrix}$$

$$= -2X_1z_4z_5 + 2X_2z_3z_5 + 2X_3z_1z_4 - 2X_3z_2z_3 \tag{13}$$

where

$$x_1/w_b \triangleq r_{13}z_1 + r_{23}z_2 + r_{33}z_3 + r_{43}z_4 + r_{53}z_5$$
$$x_2/w_b \triangleq r_{14}z_1 + r_{24}z_2 + r_{34}z_3 + r_{44}z_4 + r_{54}z_5$$
$$x_3/w_b \triangleq MB_m(r_{15}z_1 + r_{25}z_2 + r_{35}z_3 + r_{45}z_4 + r_{55}z_5) \tag{14}$$

and r_{ij} are the elements of the symmetric matrix R.
Therefore the total derivative of the Liapunov function (12) may be expressed in the following general form :

$$\dot{V}(\vec{z}) = -\vec{z}^{\,T}Q\vec{z} + 2\sum_{i,j,k=1}^{n} C_{ijk}X_i z_j z_k \tag{15}$$

where the C_{ijk}'s are constants.

Let $\triangle_1, \triangle_2 \cdots, \triangle_5$ be the positive diagonal elements of the matrix Q.
Then a simple $\dot{V}(\vec{z})$ can be derived due to the nature of the nonlinearities (terms involving the product of two state variables).

$$\dot{V}(\vec{z}) = -\vec{z}^{\,T}\begin{bmatrix} \triangle_1 & 0 & 0 & -X_3 & 0 \\ 0 & \triangle_2 & X_3 & 0 & 0 \\ 0 & X_3 & \triangle_3 & 0 & -X_2 \\ -X_3 & 0 & 0 & \triangle_4 & X_1 \\ 0 & 0 & -X_2 & X_1 & \triangle_5 \end{bmatrix}\vec{z} = -\vec{z}^{\,T}Q'\vec{z} \tag{16}$$

For asymptotic stability, Q' must be positive or positive semi-definite. From Sylvester's Theorem the conditions for positive definiteness of the matrices Q' are

$$\dot{V}(\vec{z}) \triangleq \triangle_1 X_1^2 + \triangle_2 X_2^2 + \triangle_5 X_3^2 = \vec{z}^{\,T}L\vec{z} \leqslant \triangle_5\triangle_i\triangle_j \tag{17}$$

where $\triangle_i\triangle_j$ is the smaller one of $\triangle_1\triangle_4$ and $\triangle_2\triangle_3$. To determine the region of asymptotic stability, a constant K can be found such that the surface
$V(\vec{z}) = K$ lies entirely within the region where
$\dot{V}(\vec{z})$ is negative or negative semi-definite; i. e.
$\dot{V}_r(\vec{z}) \leqslant \triangle_5\triangle_i\triangle_j$. Both $V(\vec{z})$ and $\dot{V}_r(\vec{z})$ have quadratic forms. R is a positive definite matrix, L is a positive semi-definite matrix for a stable system. Both are real symmetric matrices. $V(\vec{z}) = K$ is a closed hypersurface and $\dot{V}_r(\vec{z}) = \triangle_5\triangle_i\triangle_j$ space is an open hypersurface in five dimensions. Since

$$\lambda_{\min}\|\vec{z}\|^2 \leqslant V(z) = K \leqslant \lambda_{\max}\|\vec{z}\|^2$$

where

$$\|\vec{z}\| \triangleq \sqrt{\vec{z}^{\,T}\vec{z}}$$

$\lambda_{\max} \triangleq$ maximum eigenvalue of R

$\lambda_{\min} \triangleq$ minimum eigenvalue of R

therefore

$$\sqrt{\frac{K}{\lambda_{\max}}} \leqslant \|\vec{z}\| \leqslant \sqrt{\frac{K}{\lambda_{\min}}} \tag{18}$$

Also since

$$\dot{V}_r(\vec{z}) = \triangle_5\triangle_i\triangle_j \quad \lambda'_{\max}\|\vec{z}\|^2$$

where $\lambda'_{max} \triangleq$ maximum eigenvalue of L
Therefore

$$\|\vec{z}\| \leq \sqrt{\frac{\triangle_5\triangle_i\triangle_j}{\lambda'_{\max}}} \tag{19}$$

From (18) and (19) a sufficient condition for the $V(\vec{z}) = K$ surface to be entirely within the

$\dot{V}_r(\vec{z}) = \triangle_5\triangle_i\triangle_j$ surface is

$$\sqrt{\frac{\triangle_5\triangle_i\triangle_j}{\lambda'_{\max}}} = \sqrt{\frac{K_1}{\lambda_{\min}}} \tag{20}$$

or

$$K_1 = \frac{\triangle_5\triangle_i\triangle_j\lambda_{\min}}{\lambda'_{\max}} \tag{21}$$

Therefore the system is asymptotically stable inside the hypersurface defined by

$$V(\vec{z}) = \vec{z}^{\,T} R \vec{z} = K_1 \tag{22}$$

The region defined by (22) may be much smaller than the largest possible region. Thus, more investigation is necessary to find a larger K. Consider polar coordinates

$$z_{1u} = \sin\theta_1 \sin\theta_2 \sin\theta_3 \cos\theta_4$$
$$z_{2u} = \sin\theta_1 \sin\theta_2 \sin\theta_3 \sin\theta_4$$
$$z_{3u} = \sin\theta_1 \sin\theta_2 \cos\theta_3$$
$$z_{4u} = \sin\theta_1 \cos\theta_2$$
$$z_{5u} = \cos\theta_1$$

where

$z_{iu} \triangleq$ component of the vector $\vec{z}_u$ on the ith axis form the origin

$\theta_i \triangleq$ angle between ith axis and (i-1)th space where

$$O \leqslant Q_i \leqslant \pi$$

$$\|\vec{z}_u\| = \sqrt{\sum_{i=1}^{5} z_{iu}^2} = 1 \tag{23}$$

Thus a unit vector in any direction can be generated by changing θ_i.

Let r be the length of the vector to the $V_r = \Delta_5\Delta_i\Delta_j$ surface from the origin ; then the component of this vector along each axis is (r)(z_{iu}). From (17), for a point on the V_r surface

$$r\vec{z}_u L r\vec{z}_u = \triangle_5\triangle_i\triangle_j$$

Therefore

$$r = \sqrt{\frac{\triangle_5\triangle_i\triangle_j}{\vec{z}_u^{\,T} L \vec{z}_u}} \tag{24}$$

when the V surface and $\dot{V}_r$ surface are tangent at a point, then the distances to the two surfaces at this point must be equal. From (11) and (24), K is then

$$K = r^2(\vec{z}_u^{\,T} R \vec{z}_u) \tag{25}$$

Next K is calulated for each generated unit vector using (25). The minimum K, designed $K_{minimum}$, among all calculated K's defines the region of asymptotic stability in the large. This region is the interior of the hypersurface defined by

$$V(\vec{z}) = \vec{z}^{\,T} R \vec{z} = K_{\text{minimum}} \tag{26}$$

The approximate tangent points of two surfaces $V(\vec{z})$ and $V_r(\vec{z})$ are obtained by equating the distances to the surfaces. Exact tangent points in the vicinity of these approximate points may be determined in the following way. If the two surfaces $V(\vec{z})$, and $\dot{V}_r(\vec{z})$ are tangent, then they have the same tangent plane and the same vector normal to this plane. A vector normal to this plane at a point $\vec{z}_p$ is the gradient of $V(\vec{z})$ and $\dot{V}_r(\vec{z})$. Therefore

$$\vec{\nabla} V(\vec{z}) \Big|_{\vec{z}_p} = q \quad \dot{V}_r(\vec{z}) \Big|_{\vec{z}_p} \tag{27}$$

where q is a scalar, and

$$\dot{V}_r(\vec{z}) \Big|_{\vec{z}_p} = \triangle_5 \triangle_i \triangle_j \tag{28}$$

The solution of (27)and (28) gives the exact tangent point $\vec{z}_p$.

Examples

The theoretical results are verified numerically using the digital computer. Two different machines are taken as examples. One is stable and the other is unstable. The per unit machine parameters are as follows:

	Machine I (stable)	Machine II (unstable)
r_1	0.036	0.025
r_2	0.0425	0.008
X_{11}	2.853	4.1
X_{22}	2.784	4.1
X_{12}	2.74	4.
V_m	1.025	120/377
w_b	377.	377.
w	377.	120.
H	0.5	0.1
B	0.02	0.
T_l	1.0	0.

Computational results are given in the following sections.

Machine I

Steady State Operating Point:

$$\begin{bmatrix} y_{10} \\ y_{20} \\ y_{30} \\ y_{40} \\ y_{50} \end{bmatrix} = \begin{bmatrix} 1.8737E-02 \\ -9.8525E-01 \\ -1.4893E-01 \\ -9.2096E-01 \\ 9.4978E-01 \end{bmatrix}$$

Linearized System:

$$A=(377)\begin{bmatrix} -0.2303E+0 & 0.1000E+01 & 0.2267E+00 & 0.0 & 0.0 \\ -0.1000E+01 & -0.2303E+00 & 0.0 & 0.2267E+00 & 0.0 \\ 0.2676E+00 & 0.0 & -0.2786E+00 & 0.5022E-01 & 0.9210E+00 \\ 0.0 & 0.2676E+00 & -0.5022E-01 & -0.2786E+00 & -0.1489E+00 \\ 0.1597E-01 & -0.2365E-02 & -0.1704E-01 & -0.3073E+03 & -0.5305E-04 \end{bmatrix}$$

Matrix R for Q = w_b I in (10) :

$$R=\begin{bmatrix} 0.2794E+01 & 0.1055E-01 & 0.5821E+00 & -0.1008E+01 & -0.1056E+00 \\ 0.1055E-01 & 0.2329E+01 & 0.9978E+00 & 0.5502E+00 & 0.2640E+01 \\ 0.5821E+00 & 0.9978E+00 & 0.2298E+01 & -0.1906E-01 & -0.4271E+00 \\ -0.1008E+01 & 0.5502E+00 & -0.1906E-01 & 0.2238E+01 & 0.6703E+00 \\ -0.1056E+00 & 0.2640E+01 & -0.4271E+00 & 0.6703E+00 & 0.1280E+03 \end{bmatrix}$$

Determinants of Principal Minors of R :

0.2794E + 01 0.7905E + 01 0.1444E + 02 0.2550E + 02 0.3175E + 04

This shows R is positive definite.

Eigenvalue of R :

0.3061E + 01, 0.3749E + 01, 0.1355E + 01, 0.333E + 01, 0.1281E + 01

Therefore $\lambda_{max} = 0.1281E + 03$

$\lambda_{min} = 0.1333E + 01$

Eigenvalues of L/w_b3 :

0.0, 0.0, 0.8027E + 01, 0.6413E + 01, 0.3688E + 01

Therefore $\lambda_{max} = (0.8027E + 01)\,(377)^3$

Minimum K in (25) :

$K_{minimum} = 0.4341$

Therefore, the region of asymptotic stability is the interior of the hypersurface defined by (26)

$$V(\vec{z})=\vec{z}^{\,T}R\vec{z}=0.4341$$

From (18)

$$0.0582=\sqrt{\frac{0.4341}{128}}\leqslant\|\vec{z}\|\leqslant\sqrt{\frac{0.4341}{1.333}}=0.57$$

The size of this region of asymptotic stability is much larger than the region calculated from (21) and (22)

$$V(\vec{z})=\vec{z}^{\,T}R\vec{z}=K_1=\frac{(377)(377)(377)(1.333)}{(8.027)(377)^3}=0.167$$

The solution of (27) and (28) is substituted into (11) yielding

$$K = \vec{z}_p R \vec{z}_p$$
$$= 0.4358$$

This K is very close to $K_{minimum}$

Machine II

Steady State Operating Point :

$$\begin{bmatrix} y_{10} \\ y_{20} \\ y_{30} \\ y_{40} \\ y_{50} \end{bmatrix} = \begin{bmatrix} 0.1915E-01 \\ -0.9996E+00 \\ 0.1868E-01 \\ -0.9753E+00 \\ 0.3180E+00 \end{bmatrix}$$

Linearized System :

$$A = (377) \begin{bmatrix} -0.1265E+00 & 0.3183E+00 & 0.1234E+00 & 0.0 & 0.0 \\ -0.3183E+00 & -0.1265E+00 & 0.0 & 0.1235E+00 & 0.0 \\ 0.3851E+01 & 0.0 & -0.4049E-01 & 0.0 & 0.9753E+00 \\ 0.0 & 0.3951E-01 & 0.0 & -0.4049E-01 & 0.1868E-01 \\ 0.6388E-01 & 0.1223E-02 & -0.6547E-01 & -0.1254E-02 & 0.0 \end{bmatrix}$$

Matrix R for Q = w_sJ in (10) :

$$R = \begin{bmatrix} -0.2173E+02 & -0.1457E+02 & -0.1518E+02 & 0.1108E+02 & -0.1141E+03 \\ -0.1457E+02 & -0.3464E+02 & 0.1879E+02 & -0.1935E+01 & -0.1384E+03 \\ -0.1518E+02 & 0.1819E+02 & -0.3203E+02 & 0.1975E+02 & -0.1167E+01 \\ 0.1108E+02 & -0.1934E+02 & 0.1934E+02 & 0.1795E+02 & 0.5391E+01 \\ -0.1141E+03 & -0.1384E+03 & -0.1167E+01 & 0.3414E+02 & -0.6865E+03 \end{bmatrix}$$

Determinant of Principal Minors of R :

− 0.2173E + 02 0.5403E + 03 0.6656E + 04 0.96682 + 05
0.6113E + 07

This shows R is indefinite, therefore the operating point is unstable.

Conclusion

The stability analysis described in this paper uses the nonlinear differential equations for the induction motor. These equations are simplified by a unique transformation of variables. An approach using Liapunov functions for systems with nonlinearities involving the products of pairs of state variables is then developed to determine regions of asymptotic stability. This provides information on the stability of such machines subject to disturbances from the normal steady state operating point. Such disturbances often occur during transient operation of thyristor controlled variable frequency induction motor drives. It should be noted that if R is found to be indefinite or negative definite for a positive definite Q, then the reduced linearized system is unstable. Thus, the nonlinear system is not asymptotically stable. In the determination of the stability region, the simple method developed here is also applicable to these classes of nonlinear systems with nonlinear terms involving the product of the two state variables such as fed back bilinear systems.

The author has recently proved that the solution of this induction motor system is bounded applying Yoshizawa's boundedness theorem[9] and practical stability concept which results from LaSalle and Lefschetzs' work[10]. Therefore the solution does not diverge: asymptotically stable or oscillatory. A global behavior of the solution will be found according to the definiteness of matrix R. This will be presented later.

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