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On Three-dimensional Stress Distribution due to Displacement of a Cylindrical Inclusion

Kenichi G. MATSUOKA and Sumio G. NOMACHI*

Abstract

An interaction between the finite elastic body and the cylindrical inclusion is handled by solving a three-dimensional stress problem written in the cylindrical coordinate system.

The problem is analyzed by means of finite Fourier-Hankel transforms, on the assumption that the elastic body is very thick cylinder and the solid core as the inclusion keeps its sectional area unchanged duping the interaction.

The numerical calculations were carried on for the cases with the various ratio between outer and inner radii, as well as the different ratio between elastic moduli of the outer body and the inclusion

1. General expression of displacement

Three-dimensional stress problems were solve by means of finite Fourier-Hankel transforms[1),2)], and as an application of it, the correct solution concerning the bending of the thick hollow cylinder, has been obtained by the outhors with the expressions of the displacements[3)], and replacements of sine for cosine and cosine for sine, into these expressions yield another set of displacements.

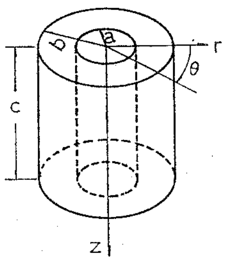

Fig. 1.

Thus obtained displacements will be taken for the problem now considered. The origin of coordinate is placed as shown in Fig. 1, in which a and b denote the inner and outer radii, and c denotes the height of the cylinder. Let u, v and w be the displacement components in the r, θ and z directions. The boundary conditions satisfying that the shearing stress vanishes and w is zero for $z=0$ and c, give the displacement vector as follows:

$$u = \frac{1}{\pi} \sum_{\nu=0}^{\infty} c_\nu (A_{\nu z r} + B_{\nu z r}) \cos \nu\theta \qquad (1)$$

$$v = \frac{1}{\pi} \sum_{\nu=1}^{\infty} (A_{\nu z r} - B_{\nu z r}) \sin \nu\theta \qquad (2)$$

$$w = \sum_{k=1}^{2} \sum_{\nu=1}^{\infty} \frac{2}{\pi c} \sum_{n=1}^{\infty} c_\nu \left[G_\nu^{(k)}(Nr)\, D_{\nu nk} + \frac{\mu+\lambda}{2\mu+\lambda} F_\nu^{(k)}(Nr) \left\{ \frac{1}{2\mu} \beta_{\nu nk} + A_{\nu nk} - B_{\nu nk} + D_{\nu nk} \right\} \right] \frac{1}{N} \sin Nz \cos \nu\theta \qquad (3)$$

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$$A_{\nu z r}=\sum_{k=1}^{2}\Bigg[\frac{a^{2\nu}b^{2\nu}}{b^{2\nu}-a^{2\nu}}\cdot\frac{r^{-(\nu+1)}}{3\mu+\lambda}\cdot\frac{1}{c}(-1)^{k-1}a_k^{2-\nu}\Big\{\frac{1}{2}\alpha_{\nu 0k}+\frac{1}{2}\beta_{\nu 0k}$$
$$+\frac{2\nu\mu-(\mu+\lambda)}{\nu+1}A_{\nu 0k}-\frac{\mu+\lambda}{\nu-1}B_{\nu 0k}\Big\}+f_{\nu p}^{(k)}(r)\Big\{\frac{1}{2\mu}\alpha_{\nu 0k}$$
$$+\frac{1}{2(2\mu+\lambda)}\beta_{\nu 0k}+\frac{3\mu+\lambda}{2\mu+\lambda}A_{\nu 0k}+\frac{\mu+\lambda}{2\mu+\lambda}B_{\nu 0k}\Big\}\frac{1}{c}$$
$$+\frac{2}{c}\sum_{n=1}^{\infty}\frac{1}{N}\Big(\frac{1}{2\mu}\chi_{\nu p}^{(k)}(Nr)\{\alpha_{\nu nk}+\beta_{\nu nk}+4\mu A_{\nu nk}+\mu D_{\nu nk}\}$$
$$-\frac{\mu+\lambda}{2(2\mu+\lambda)}\omega_{\nu p}^{(k)}(Nr)\Big\{\frac{1}{2\mu}\beta_{\nu nk}+A_{\nu nk}-B_{\nu nk}+D_{\nu nk}\Big\}\Big)\cos Nz\Bigg] \tag{4}$$

$$B_{\nu z r}=\sum_{k=1}^{2}\Bigg[\frac{1}{b^{2\nu}-a^{2\nu}}\cdot\frac{r^{\nu-1}}{3\mu+\lambda}\,\frac{1}{c}(-1)^{k-1}a_k^{\nu+2}\Big\{\frac{1}{2}\alpha_{\nu 0k}-\frac{1}{2}\beta_{\nu 0k}+\frac{\mu+\lambda}{\nu+1}A_{\nu 0k}$$
$$+\frac{2\nu\mu+(\mu+\lambda)}{\nu-1}B_{\nu 0k}\Big\}-f_{\nu s}^{(k)}(r)\cdot\frac{1}{c}\Big\{\frac{1}{2\mu}\alpha_{\nu 0k}-\frac{1}{2(2\mu+\lambda)}\beta_{\nu 0k}$$
$$+\frac{\mu+\lambda}{2\mu+\lambda}A_{\nu 0k}+\frac{3\mu+\lambda}{2\mu+\lambda}B_{\nu 0k}\Big\}$$
$$+\frac{2}{c}\sum_{n=1}^{\infty}\frac{1}{N}\Big(-\frac{1}{2\mu}\chi_{\nu s}^{(k)}(Nr)\{\alpha_{\nu nk}-\beta_{\nu nk}+4\mu B_{\nu nk}-\mu D_{\nu nk}\}$$
$$-\frac{\mu+\lambda}{2(2\mu+\lambda)}\omega_{\nu s}^{(k)}(Nr)\Big\{\frac{1}{2\mu}\beta_{\nu nk}+A_{\nu nk}-B_{\nu nk}+D_{\nu nk}\Big\}\Big)\cos Nz\Bigg] \tag{5}$$

where

$$N=\frac{n\pi}{c},\quad (n=1,2,\cdots);\ \nu=0,1,2,\cdots,\quad a_1=b,\quad a_2=a,\quad a_0=a,$$

μ, λ=Lamé's constants, $c_i=\frac{1}{2}$ for $i=0$ and $c_i=1$, for $i\neq 0$,

$$\alpha_{\nu nk}=\boldsymbol{S}_\nu\boldsymbol{C}_n[\tau_{r\theta}]_{r=a_k},\quad \beta_{\nu nk}=\boldsymbol{C}_\nu\boldsymbol{C}_n[\sigma_r]_{r=a_k}$$
$$A_{\nu nk}=\frac{1}{2}(\nu+1)\boldsymbol{C}_n\Big[\boldsymbol{C}_\nu[u]+\boldsymbol{S}_\nu[v]\Big]_{r=a_k},\quad B_{\nu nk}=\frac{1}{2}(\nu-1)\boldsymbol{C}_n\Big[\boldsymbol{C}_\nu[u]-\boldsymbol{S}_\nu[v]\Big]_{r=a_k}$$
$$D_{\nu nk}=N\boldsymbol{C}_\nu\boldsymbol{S}_n[w]_{r=a_k},\quad k=1,2$$
$$\boldsymbol{C}_n[f]=\int_0^c f(x)\cos\frac{n\pi}{c}x\mathrm{d}x,\quad \boldsymbol{S}_n[f]=\int_0^c f(x)\sin\frac{n\pi}{c}x\mathrm{d}x$$

and the functions:

$$G_\nu^{(k)}(Nr)=\frac{R_{\nu\cdot\nu}^{(k)}(Nr)}{R_{\nu\cdot\nu}^{(k)}(Na_k)},\quad \chi_{\nu p}^{(k)}(Nr)=\frac{R_{\nu+1\cdot\nu}^{(k)}(Nr)}{R_{\nu\cdot\nu}^{(k)}(Na_k)},\quad \chi_{\nu s}^{(k)}(Nr)=\frac{R_{\nu-1\cdot\nu}^{(k)}(Nr)}{R_{\nu\cdot\nu}^{(k)}(Na_k)},$$

$$F_\nu^{(k)}(Nr)=\frac{N}{\{R_{\nu\cdot\nu}^{(k)}(Na_k)\}^2}\Big[R_{\nu\cdot\nu}^{(k)}(Na_k)\Big\{rR_{\nu-1\cdot\nu}^{(k)}(Nr)-a_{k-1}R_{\nu\cdot\nu+1}^{(k)}(Nr)\Big\}$$
$$-R_{\nu\cdot\nu}^{(k)}(Nr)\Big\{a_kR_{\nu-1.\nu-1}^{(k)}(Na_k)-a_{k-1}R_{\nu\cdot\nu+1}^{(k)}(Na_k)\Big\}\Big],$$

$$\omega_{\nu p}^{(k)}(Mr)=\frac{N}{\{R_{\nu\cdot\nu}(Na_k)\}^2}\Big[R_{\nu\cdot\nu}^{(k)}(Na_k)\Big\{rR_{\nu\cdot\nu}^{(k)}(Nr)-a_{k-1}R_{\nu+1.\nu+1}^{(k)}(Nr)\Big\}$$
$$-R_{\nu+1\cdot\nu+1}^{(k)}(Nr)\Big\{a_kR_{\nu-1\cdot\nu-1}^{(k)}(Na_k)-a_{k-1}R_{\nu\cdot\nu+1}^{(k)}(Na_k)\Big\}\Big],$$

$$\omega_{\nu s}^{(k)}(Nr)=\frac{N}{\{R_{\nu\cdot\nu}^{(k)}(Na_k)\}^2}\Big[R_{\nu\cdot\nu}^{(k)}(Na_k)\Big\{rR_{\nu\cdot\nu}^{(k)}(Nr)-a_{k-1}R_{\nu-1\cdot\nu-1}^{(k)}(Nr)\Big\}$$
$$-R_{\nu-1\cdot\nu}^{(k)}(Nr)\Big\{a_kR_{\nu+1\cdot\nu}^{(k)}(Na_k)-a_{k-1}R_{\nu\cdot\nu-1}^{(k)}(Na_k)\Big\}\Big],$$

$$g_\nu^{(k)}(r)=\frac{a^\nu b^\nu}{b^{2\nu}-a^{2\nu}}\left\{\left(\frac{r}{a_{k-1}}\right)^\nu-\left(\frac{a_{k-1}}{r}\right)^\nu\right\},$$

$$f_{\nu p}^{(k)}(r)=\frac{a^\nu b^\nu}{b^{2\nu}-a^{2\nu}}\left\{\frac{a_{k-1}}{2(\nu+1)}\left(\frac{r}{a_{k-1}}\right)^{\nu+1}-\frac{a_{k-1}}{2}\left(\frac{a_{k-1}}{r}\right)^{\nu-1}\right\}$$
$$+\frac{1}{r^{\nu+1}}\cdot\frac{2\nu a^{2\nu}b^{3\nu}}{(b^{2\nu}-a^{2\nu})^2}\left\{\frac{a^2-b^2}{4(\nu+1)}a_{k-1}^{-\nu}+\frac{a_{k-1}^\nu}{4(\nu-1)}(a^{-2(\nu-1)}-b^{-2(\nu-1)})\right\},$$

$$f_{\nu s}^{(k)}(r)=\frac{a^\nu b^\nu}{b^{2\nu}-a^{2\nu}}\left\{\frac{1}{2}a_{k-1}\left(\frac{r}{a_{k-1}}\right)^{\nu+1}+\frac{a_{k-1}}{2(\nu-1)}\left(\frac{a_{k-1}}{r}\right)^{\nu-1}\right\}$$
$$+r^{\nu-1}\frac{2\nu a^\nu b^\nu}{(b^{2\nu}-a^{2\nu})^2}\left\{\frac{a_{k-1}^{-\nu}}{4(\nu+1)}(a^{2(\nu+1)}-b^{2(\nu+1)})+\frac{a_{k-1}^\nu}{4(\nu-1)}(a^2-b^2)\right\},$$
$$R_{ij}^{(k)}(Nr)=I_i(Nr)K_j(Na_{k-1})-(-1)^{i+j}I_j(Na_{k-1})K_i(Nr).$$

which may correspond to the bending behaviour of the beam on the elastic subgrade as well that of the pile struck into the earth.

2. The formulas of stress components

The stress components are related to the displacement components by the well-known Hooke's law:

$$\sigma_r=2\mu\frac{\partial u}{\partial r}+\lambda\cdot e \tag{6}$$

$$\sigma_\theta=2\mu\left(\frac{u}{r}+\frac{\partial v}{r\partial\theta}\right)+\lambda\cdot e \tag{7}$$

$$\sigma_z=2\mu\frac{\partial w}{\partial z}+\lambda\cdot e \tag{8}$$

$$e=\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial v}{r\partial\theta}+\frac{\partial w}{\partial z} \tag{9}$$

$$\tau_{r\theta}=\mu\left(\frac{\partial u}{r\partial\theta}+\frac{\partial v}{\partial r}-\frac{v}{r}\right) \tag{10}$$

$$\tau_{\theta z}=\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{r\partial\theta}\right) \tag{11}$$

$$\tau_{zr}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right) \tag{12}$$

where σ_r : the normal stress in the r direction
σ_θ : the normal stress in the θ direction
σ_z : the normal stress in the z direction
$\tau_{r\theta}$: the shearing stress around the z axis
$\tau_{\theta z}$: the shearing stress around the r axis
τ_{zr} : the shearing stress around the θ axis

The eqs. (1)~(5), through the eqs. (6)~(11), lead to the stress components as followes :

$$\sigma_r=\frac{2\mu}{\pi}\sum_{\nu=0}^{\infty}c_\nu\left(\frac{\partial A_{\nu zr}}{\partial r}+\frac{\partial B_{\nu zr}}{\partial r}\right)\cos\nu\theta+\lambda\cdot e \tag{13}$$

$$\sigma_\theta=\frac{2\mu}{\pi}\sum_{\nu=0}^{\infty}c_\nu\left(\frac{\nu+1}{r}A_{\nu zr}-\frac{\nu-1}{r}B_{\nu zr}\right)\cos\nu\theta+\lambda\cdot e \tag{14}$$

$$\begin{aligned}\sigma_z=\frac{2}{\pi}\sum_{k=1}^{2}\sum_{\nu=0}^{\infty}c_\nu\Bigg[&\left\{\frac{\lambda}{2(2\mu+\lambda)}\beta_{\nu0k}+\frac{\mu\lambda}{2\mu+\lambda}(A_{\nu0k}-B_{\nu0k})\right\}g_\nu^{(k)}(r)\cos\nu\theta\\&+\sum_{n=1}^{\infty}\Bigg(G_\nu^{(k)}(Nr)\cdot\frac{2\mu\lambda}{2\mu+\lambda}\left\{\frac{1}{2\mu}\beta_{\nu nk}+A_{\nu nk}-B_{\nu nk}+\frac{2(\mu+\lambda)}{\lambda}D_{\nu nk}\right\}\\&+\frac{2\mu(\mu+\lambda)}{2\mu+\lambda}F_\nu^{(k)}(Nr)\left\{\frac{1}{2\mu}\beta_{\nu nk}+A_{\nu nk}-B_{\nu nk}+D_{\nu nk}\right\}\Bigg)\cos Nz\Bigg]\end{aligned} \tag{15}$$

$$\begin{aligned}e=\frac{2}{\pi c}\sum_{k=1}^{2}\sum_{\nu=0}^{\infty}c_\nu\Bigg[&\left\{\frac{1}{2(2\mu+\lambda)}\beta_{\nu0k}+\frac{\mu}{2\mu+\lambda}(A_{\nu0k}-B_{\nu0k})\right\}g_\nu^{(k)}(r)\\&+\sum_{n=1}^{\infty}\frac{1}{2\mu+\lambda}\left\{\beta_{\nu nk}+2\mu(A_{\nu nk}-B_{\nu nk}+D_{\nu nk})\right\}G_\nu^{(k)}(Nr)\cos Nz\Bigg]\cos\nu\theta\end{aligned} \tag{16}$$

$$\tau_{r\theta}=\frac{\mu}{\pi}\sum_{\nu=1}^{\infty}\left\{\frac{\partial A_{\nu zr}}{\partial r}-\frac{\partial B_{\nu zr}}{\partial r}-\frac{\nu+1}{r}A_{\nu zr}-\frac{\nu-1}{r}B_{\nu zr}\right\}\sin\nu\theta \tag{17}$$

$$\begin{aligned}\tau_{\theta z}=\frac{2}{\pi c}\sum_{k=1}^{2}\sum_{\nu=1}^{\infty}\sum_{n=1}^{\infty}\Bigg[&\left\{\chi_{\nu p}^{(k)}(Nr)-\frac{\nu}{Nr}G_\nu^{(k)}(Nr)\right\}\alpha_{\nu nk}-\frac{\nu}{Nr}G_\nu^{(k)}(Nr)\left\{\beta_{\nu nk}-2\mu D_{\nu nk}\right\}\\&+2\mu\left\{\chi_{\nu p}^{(k)}(Nr)A_{\nu nk}+\chi_{\nu s}^{(k)}(Nr)B_{\nu nk}\right\}-\frac{2\nu}{Nr}F_\nu^{(k)}(Nr)\cdot\frac{\mu(\mu+\lambda)}{2\mu+\lambda}\\&\times\left\{\frac{1}{2\mu}\beta_{\nu nk}+A_{\nu nk}-B_{\nu nk}+D_{\nu nk}\right\}\Bigg]\sin Nz\sin\nu\theta\end{aligned} \tag{18}$$

$$\begin{aligned}\tau_{zr}=\frac{2}{\pi c}\sum_{k=1}^{2}\sum_{\nu=0}^{\infty}\sum_{n=1}^{\infty}c_\nu\Bigg[&-\frac{\nu}{Nr}G_\nu^{(k)}(Nr)\left\{\alpha_{\nu nk}+\beta_{\nu nk}-4\mu B_{\nu nk}\right\}\\&-\chi_{\nu p}^{(k)}(Nr)\left\{\beta_{\nu nk}-2\mu(A_{\nu nk}+B_{\nu nk})\right\}+\frac{\mu+\lambda}{2\mu+\lambda}\left\{\omega_{\nu p}^{(k)}(Nr)+\frac{\nu}{Nr}F_\nu^{(k)}(Nr)\right\}\\&\times\left\{\beta_{\nu nk}-2\mu(A_{\nu nk}+B_{\nu nk}+D_{\nu nk})\right\}\Bigg]\cos\nu\theta\sin Nz\end{aligned} \tag{19}$$

3. Boundary conditions

As previously discribed, the periodic boundary condition is taken for $z=0$ and $z=c$, so what we need now is to find the condition on the interface between the elastic body and the inclusion, and the condition on the surface of the outer radius $r=b$.

To simplify the further discussion, the condition is assumed that u and w are zero as well as $\tau_{r\theta}$ vanishes on the outer surface:

$$u=0 \text{ for } r=b, \qquad \therefore \quad A_{\nu n1}=-B_{\nu n1} \tag{20}$$

$$w=\tau_{r\theta}=0 \text{ for } r=b, \qquad \therefore \quad C_{\nu n1}=0 \text{ and } \alpha_{\nu n1}=0 \tag{21}$$

On the interface $r=a$, the shearing stress vanishes, the radial displacement is continuous, and the radial stress occurs to hold an equilibrium state with the beam action by the inclusion, so that the boundary conditions are written as follows:

$$\tau_{r\theta}=0 \text{ for } r=a, \qquad \therefore \quad \alpha_{\nu n2}=0 \tag{22}$$

$$\tau_{zr}=0 \text{ for } r=a, \tag{23}$$

$$E_i I \frac{d^4 u_0}{dz^4}=\int_0^{2\pi} \sigma_r a \cos\theta d\theta, \tag{24}$$

which is transformed into

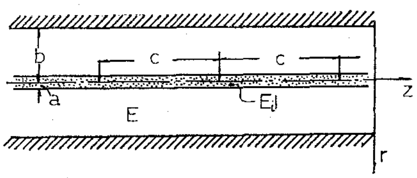

Fig. 2.

$$E_i I\left\{N^4 C_n[u_0]+N^2\left(\left[\frac{du_0}{dz}\right]_{z=0}-(-1)^n\left[\frac{du_0}{dz}\right]_{z=c}\right)\right.$$
$$\left.+\left((-1)^n\left[\frac{d^3u_0}{dz^3}\right]_{z=c}-\left[\frac{d^3u_0}{dz^3}\right]_{z=0}\right)\right\}$$
$$=a\int_0^{2\pi} C_n[\sigma_r]\cos\theta d\theta$$

where u_0: the displacement of the center of the inclusion,
E_i: the elastic modulus of the inclusion, I: the moment of inartia.

Because the inclusion keeps its initial section during strained, the surface of the inclusion displaces by

$$u_a=u_0\cos\theta, \qquad v_a=-u_0\sin\theta, \tag{25}$$

which are identical with the radial displacement on the inner surface of the elastic body.

So that

$$u)_{r=a} = \frac{2}{\pi c} \sum_{\nu=0}^{\infty} \sum_{n=0}^{\infty} c_\nu c_n (A_{\nu n2} + B_{\nu n2}) \cos Nz \cos \nu\theta \tag{26}$$

and the cosine transformation of u is found as

$$u_a = \frac{2}{c} \sum_{n=0}^{\infty} c_n \boldsymbol{C}_n [u_0] \cos Nz \cos \theta \, . \tag{27}$$

Equating the eqs. (26) and (27), we have

$$\boldsymbol{C}_n [u_0] = A_{\nu n2} + B_{\nu n2} \tag{28}$$

and $\nu = 1$.

This means that the displacements and stresses on this case, correspond to the eqs. (1)$\sim$(5) and (13)$\sim$(19) with $\nu = 1$.

Hence, the right side of eq. (24) becomes

$$a \int_0^{2\pi} \boldsymbol{C}_n [\sigma_r] \cos \theta \, d\theta = a\beta_{\nu n2} \tag{29}$$

in which $\nu = 1$.

The inclusion also takes the periodic boundary condition in the z direction as the outer body does. The shearing stress appears for $z=0$ and $z=c$.

Denoting the resultant of the prescribed shearing stress by P, we finally get

$$E_i I N^4 \left\{ A_{1n2} + B_{1n2} \right\} - a\beta_{1n2} = \left\{ 1 + (-1)^n \right\} P \tag{30}$$

The mathematical definition requires the following equations:

$$A_{\nu zr})_{r=b} = \frac{2}{c} \sum_{n=0}^{\infty} c_\nu A_{\nu n1} \cos Nz \tag{31}$$

$$A_{\nu zr})_{r=a} = \frac{2}{c} \sum_{n=0}^{\infty} c_\nu A_{\nu n2} \cos Nz \tag{32}$$

$$B_{\nu zr})_{r=b} = \frac{2}{c} \sum_{n=0}^{\infty} c_\nu B_{\nu n1} \cos Nz \tag{33}$$

$$B_{\nu zr})_{r=a} = \frac{2}{c} \sum_{n=0}^{\infty} c_\nu B_{\nu n2} \cos Nz \tag{34}$$

in which $B_{\nu nk}$ comes with $(\nu - 1)$ and the eqs. (33) and (34) vanish in case of $\nu - 1$.

As a result, the unknown constants $\beta_{\nu n1}$, $\beta_{\nu n2}$, $A_{\nu n1}$, $A_{\nu n2}$ and $C_{\nu n2}$ are to be determined and this can be done by the boundary conditions (20), (23), (30), (31) and (32).

4. Numerical examples

The numerical calculation was carried on for many cases varying the ratio b/a, the ratio c/a, and the different ratio between the elastic moduli of the outer

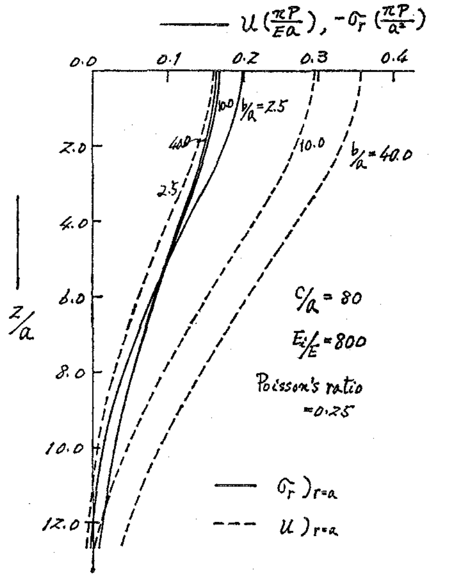

Fig. 3.

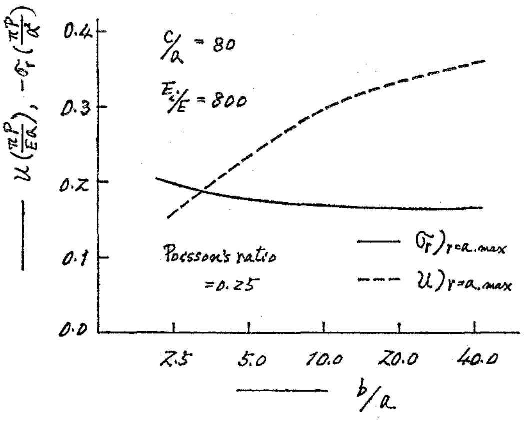

Fig. 4.

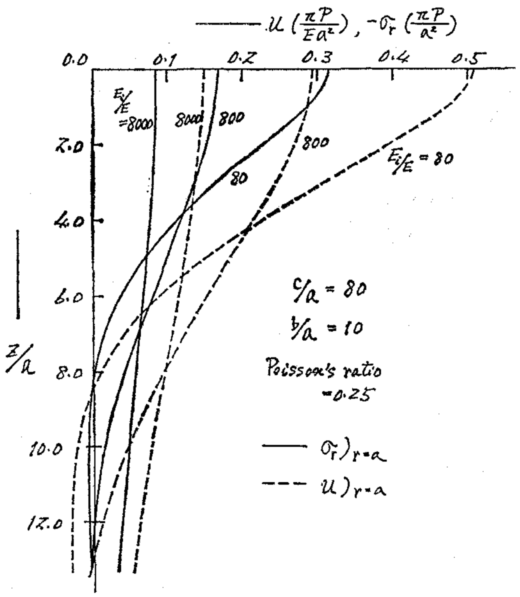

Fig. 5.

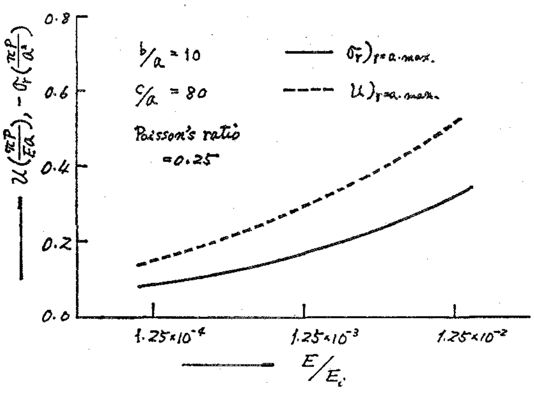

Fig. 6.

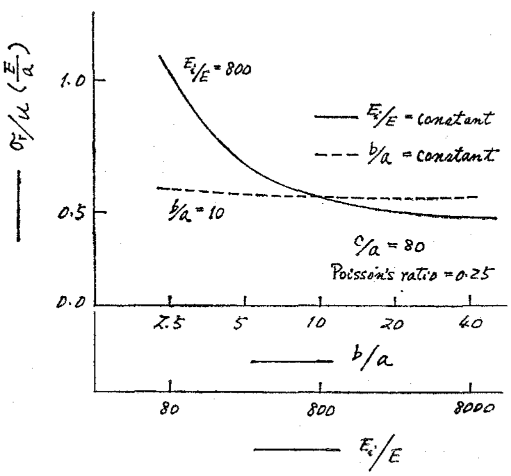

Fig. 7.

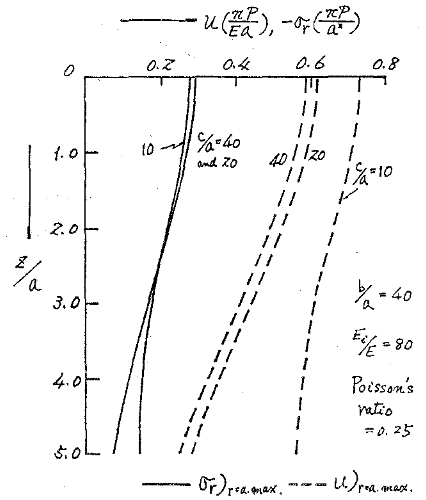

Fig. 8.

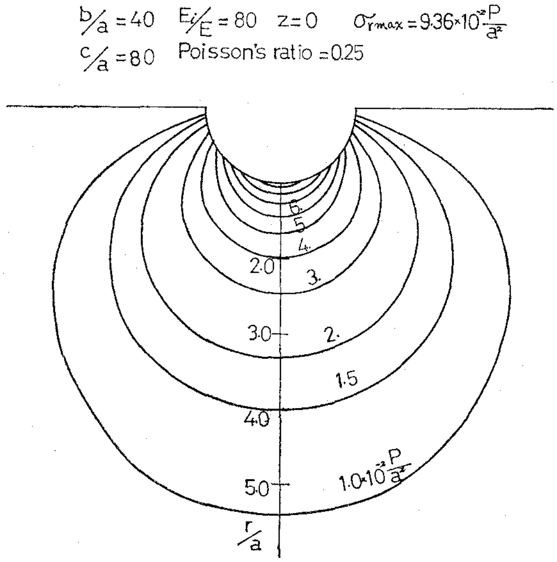

Fig. 9.

body and the inclusion. Poisson's ratio of the elastic body is taken as 0.25.

The distributions of the radial displacement u and the radial stress σ_r in the z direction for $\theta=0$, with various value of b/a are shown in Fig. 3 from which we fined that the shapes of the displacement curve are quite similar, while the magnitudes are quite different one another. The ratio σ_r and u takes approximately a constant value for each ratio b/a.

The connection of the maximum value of u and σ_r with the ratio b/a is illustrated as in Fig. 4, which shows that σ_r decreases down to a constant value as b/a increases, while the value of u increases with the increment of b/a. The ratio σ/u_r, therefore, tends to zero for $b/a \to \infty$.

The subgrade coefficient which is conventionally used in the theory of the beam on elastic subgrade, is effected not only by the elastic property of the subgrade, but also the size ratio between the subgrade and the beam.

The distributions of u and σ_r with the variation of the ratio between the elastic body and the inclusion, are drawn in Fig. 5, which shows that the inclusion has the smaller elastic modulus, the distribution has the more prompt variation in the z direction. So that the value of σ_r/u widely changes.

Letting the elastic modulus of the inclusion be constant and varying the modulus of the outer body, we can find the maximum values of u and σ_r as shown in Fig. 6.

Fig. 7 show the relation of σ_r/u with the variation of b/a and E_i/E respectively. We see that σ_r/u takes almost the constant value with the variation of E_i/E, while it gradually decreases as b/a increases. Though σ_r/u tends to zero for $b \to \infty$, the rate of decrement is very small.

Fig. 8 shows the distribution of u and σ_r in the z direction, with the variation of c/a. Fig. 9 shows the isochromatic line concerning σ_r, which promply decreases when r/a increases.

6. Closing remarks

The effect of the displacement of the cylindrical inclusion on the stress distribution in the elastic body is studied by treating the chick elastic cylinder with the co-centered cylindrical inclusion which behaves as a beam. In this manner, we write the problem in the cylindrical co-ordinate system, which can be conveniently handled by means of finite Fourier-Hankel transforms.

Carring on the various numerical calculation, we come to the conclusion:

a) the interaction between the elastic body and the inclusion, depends not only on the both elastic properties but also on the size ratio between them,

b) σ_r/u, which coincides with the subgrade coefficient, approximately takes a constant value for a larger b/a and $z/a<3.0$ in spite of any value of E_i/E.

The numerical results can not lead us to a theoretical judgement, on whether the coeventional theory of beam on elastic subgrade stands for the elastic theory or not. We however, can say the beam on elastic subgrade can practically go for the engineering use.

The calculation was carried on by FACOM 230-60 of the computer center on the campus of Hokkaido University.

(Received May. 27, 1974)

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