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Home care routing and scheduling problem with teams' synchronization



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ABSTRACT

The demand for home care (HC) services has steadily been growing for two main types of services: healthcare and social care. If, for the former, caregivers' skills are of utter importance, in the latter caregivers are not distinguishable in terms of skills. This work focuses social care and models caregivers' synchronization as a means of improving human resources management. Moreover, in social care services, several visits need to be performed in the same day since patients are frequently alone and need assistance throughout the day. Depending on the patient's autonomy, some tasks have to be performed by two caregivers (e.g. assist bedridden patients). Therefore, adequate decision support tools are crucial for assisting managers (often social workers) when designing operational plans and to efficiently assign caregivers to tasks. This paper advances the literature by 1) considering teams of one caregiver that can synchronize to perform tasks requiring two caregivers (instead of having teams of two caregivers). 2) simultaneously modelling daily continuity of care and teams' synchronization, and 3) associating dynamic time windows to teams' synchronizations introducing scheduling flexibility while minimize service and travel times. These concepts are embedded into a daily routing and scheduling MIP model, deciding on the number of caregivers and on the number and type of teams to serve all patient tasks. The main HC features of the problem, synchronization and continuity of care, are evaluated by comparing the proposed planning with the current situation of a home social care service provider in Portugal. The results show that synchronization is the feature that most increases efficiency with respect to the current situation. It evidences a surplus in working time capacity by proposing plans where all requests can be served with a smaller number of caregivers. Consequently, new patients from long waiting lists can now be served by the "available" caregivers.

1. Introduction

Demographic and social trends are increasing the demand of home care (HC). Technological and pharmaceutical innovations have allowed people with chronic illnesses to live longer. However, this extended life is not always autonomous, often requiring some level of living assistance. In addition, social factors also contribute for a higher demand of HC. Changing family structures and latter retirement are decreasing the provision of informal care [1]. This places pressure on the social sector to increase the supply of HC services, a less expensive alternative to institutionalization and that fosters independence [2]. "Ageing in place" is a community-based care model allowing elderlies to "remain living in the community, with some level of independence, rather than in residential care" [3]. Such support frequently requires tailored living assistance solutions to extend their autonomy for as long as possible and represents a transition from the paradigm of residential care [4]. Several

policies substantiated by years of research support the deinstitutionalization of care and the promotion of community-based care [5]. Motivating this transition is the prioritization of both users' quality of life and the sustainability of care systems [5,6].

In a survey synthesizing stakeholder views on the future of health and healthcare in England, the prominent theme was the changing models of health and social care [7]. One strategy is the development of transitional care programs, interventions designed to reduce hospital readmissions [8]. Lower hospital readmissions and a reduction in 180 days mortality are the outcomes of a transitional care program involving a social worker-led assessment and personalized care planning, with the coordination of the followed home-based post-discharge care [9]. Furthermore, early discharge and home recovery is less costly than a complete recovery at the hospital [10]. A day in a hospital ward represents the consumption of highly qualified resources, such as specialized doctors, nurses, and therapists, in addition to other essential

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services. Therefore, there has been a growing interest in home care and team-based care in primary care. A recent study compares three settings of primary care delivery, namely, office-based, home-care based and mixed. The main conclusion is that even though office-based delivery reduced cost the most, home care was more often indicated as the best setting across the experiments conducted in the study, beyond reducing the number of patients per physician [11].

From the patient's perspective it is relevant to allocate the same caregiver to a patient, referred, traditionally, as continuity of care [12]. However, continuity of care in a social context has a slight difference to how it is generally perceived in home health care problems. Home health care incudes the periodic healthcare delivery and improvement recording, care is prescribed for a pre-established number of weeks [13]. Visiting frequencies vary but usually not more than once per day. In social care, caregivers monitor the evolution of the health status of a patient, spotting changes in health status throughout a week and, for more dependent patients, a day. They aim at acting timely and preventing health deterioration [14].

A Portuguese organization providing several social services, including home social care services, is the motivation for the work presented. With daily planning dynamism, a higher degree of flexibility should be allowed when building the caregiver service schedules. A manually designed planning leads the manager to assume simplifications, reducing complexity. Firstly, the patients are classified as bedridden or semi-dependent and independent routes visit either patients of one group or of the other. We refer to this kind of planning as independent planning since it is done by patient's typology. Bedridden patients usually require more physically demanding care, and their visits are performed by teams of two caregivers, hereafter named double teams, while semi-dependent patients are visited by single teams. This is a consequence of two organizational preferences: 1) allocating a team of two caregivers to bedridden patients and 2) it facilitates the planning of continuity of care of such frail patients. Sharing bedridden workload between two caregivers reduces the risk of musculoskeletal lesions and consequently lowers absenteeism. Many bedridden patients need several visits per day and reducing the combinations of visiting teams decreases the options when manually analyzing continuity of care. A second simplification is the fixed number of teams of each type. Our proposal is to solve the problem in an integrated manner, where both patient types are considered simultaneously. This allows the synchronization of two single teams to serve tasks requiring two caregivers (called integrated planning). In other words, it considers the possibility of having two single teams synchronizing and then splitting again to perform other tasks. These features bring flexibility into the plan design, rendering daily continuity of care easier to assure and could potentially reduce the caregivers needed to perform the same set of tasks.

This work proposes a new mixed integer linear program (MILP) model to optimize the daily routing and scheduling plan of HC social services. In social services, the caregivers' skill level is homogeneous. The model routes and schedules tasks requiring a fixed number of caregivers, to be performed in a specific location and within a predefined time window (TW). The model decides which team, or teams, should perform each task and when. When synchronizing two teams, the arrival of the second team must fall within a time offset. Continuity of care is modelled assuring that all tasks requested by a patient are ful-filled by, at most, two teams of any type. No type of preference or work overtime is considered. The objective is to reduce operational service and travel times, as well as freeing caregiver capacity so that the same number of caregivers can perform more tasks, serving a larger number of patients.

The novelty of this work is three-fold. First, the modelling of caregivers' teams composed of one or two (skill-homogeneous) caregivers allows the model to select the best team scheme (number of teams of each type) while adjusting the staffing levels required to serve the demand and improving the efficiency of home social care operations. Second, the simultaneous modelling of continuity of care and teams' synchronization may enable the reduction of caregivers exclusively dedicated to home care services since synchronization facilitates daily continuity of care planning. Moreover, since caregivers are skillhomogeneous and social organizations provide several other types of services (child daycare, adult daycare, meal preparation, among others), the caregivers with no assigned tasks in the homecare service, can be reassigned to other services. Lastly, the modelling of dynamic time windows associated with team's synchronization introduces further flexibility in human resources allocation within the context of home social care services. Bringing these aspects into a single model formulation one can now assess the optimal number of each type of teams and the optimal number of caregivers required to fulfill all requests. Although based on a case study, the proposed model is generic and can be applied to many other HC cases. This work also presents numerical experiments that compare 1) the solvers CPLEX and Gurobi and, 2) Independent Planning to Integrated Planning.

Fig. 1 shows, side by side, the current practice (independent planning) and the one proposed in this work (integrated planning). Currently, bedridden (BR) patients are exclusively served by double teams while semi-dependent (SD) patients are exclusively served by single teams (left illustration) and there is no synchronization of teams since it would be far too complex for managers to plan it manually. By planning the service in an integrated manner with the introduction of synchronization of single teams (right illustration in Fig. 1), one aims to improve human resources management by, if possible, making team schemes available that comprise less caregivers. Fig. 1 exemplifies the benefit of such integrated planning, and how synchronization may create more efficient service plans by making one or more caregivers available to attend new requests. In Fig. 1, left illustration, bedridden patients (red circles, requiring two caregivers) are only served by double teams (red arrows, teams with two caregivers), and semi-dependent patients (blue triangles, whose tasks require only one caregiver) are served exclusively by single teams (blue arrows, teams with one caregiver). In the depicted (left) case, there is the need of one team of each type and the routing plans for each patient type are designed independently. Therefore, 3 caregivers are needed to serve 5 patients (one single and one double team). By allowing bedridden patients to be served through single team synchronization and by jointly planning the bedridden and semi-dependent patient routes (right side illustration), it is possible to improve the human resource usage, as the team scheme is no longer fixed a priori. With the integrated planning (right side illustration), the same 5 patients can now be served by just two caregivers that work in single teams but synchronize the arrival times when visiting bedridden patients.

The remaining of the paper is structured as follows. Section 2 reviews the existing literature and deepens the novel contributions of the present work. The real-world case study serving as motivation for this work is presented in Section 3. The MILP model is detailed in Section 4. In Section 5, the results regarding both adapted literature instances and the case-study instance are displayed and discussed. Finally, Section 6 is dedicated towards the main conclusions and the directions for future work.

2. Literature review

A great deal of attention has been placed on HC problems as the improvement in management decisions may generate relevant benefits, namely decrease in both hospital admissions and hospitalization length, also reducing demand for long-term residential care facilities by promoting ageing in place [15]. The management decisions occur at the strategic or long-term, tactical or mid-term and operational or short term levels [16]. The decisions made at higher hierarchical levels restrain decisions at the levels below. The two main decisions at the operational level deal with the assignment of workers to patients and the routing and scheduling of visits, which may be done simultaneously. This work focuses the routing and scheduling decisions in HC, a problem

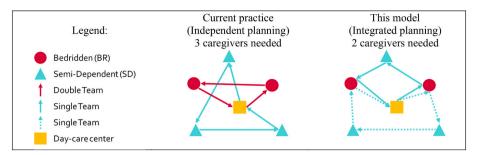


Fig. 1. Current practice (independent planning) versus suggested modelling approach (integrated planning).

known as home health care routing and scheduling problem (HHCRSP).

The HHCRSP is usually solved by extending pre-existing standard routing and scheduling problems with constraints specific to the context of HC. Most of these constraints may be classified as either temporal, assignment or geographic. Temporal constraints model time relationships and frequency of visits over time, i.e. selecting visiting patterns [17] and the TW for service provision starting time or synchronization of workers in one visit [18]. Assignment concerns are related with intrinsic attributes of caregivers and patients, without considering time nor place/movement characteristics, such as caregiver qualification [19] or preferences manifested by either patients or caregivers [20]. Finally geographic constraints handle the aspects related to the location of care workers, for example in districts [21], or the type of network between locations, such as transportation mode selection [22]. The features of HC routing and scheduling originating these specific constraints are continuity of care, temporal dependences between services and caregiver-patient assignment [23]. This work explores the first two features, as staff is homogeneous, and no preferences are considered. The literature accessed is summarized in Table 1 and emphasized below.

In general, continuity of care, also called patient-nurse loyalty [24], concerns the organizational preference of allocating the minimal number of caregivers to a patient [21,25], and if the caregiver set is skill-homogeneous, the patient should be allocated to a single caregiver. However, the working time of an organization is frequently larger than a caregivers' working shift, allowing patients to request visits separated by over a shift length. Thus, at least two caregivers must be assigned to

such patients [26]. This feature may also result in increased overtime and HC providers may allocate several caregivers to avoid incurring in that additional expense, in particular because, in the social context, there is a higher percentage of patients requiring several tasks and engaging different services [27]. Patients may request up to three [25] and even four [26] visits per day. Most published works tackling continuity of care consider each patient to be visited at most once a day and up to five times per week [20], ignoring high daily frequencies of visit. Organizations may adopt a policy of either full, partial or no continuity of care, being this feature prominently considered in long-term planning horizons [17,20,25,28,29]. Continuity of care can also be tackled through districting/clustering decomposition approaches [27], or assured by maintaining patient-caregiver assignments between two consecutive planning horizons [20,26]. Rather than an organizational policy, continuity of care has also been considered as an attribute of patient's care needs [12], or modelled inherently through preferences [27]. Many works consider a visiting pattern based on the weekly frequency of services with the objective of assuring patients to be visited by the same caregiver over the whole planning horizon. Visiting patterns have also been used as a decomposition strategy to solve multi-period problems [30].

Solution methods employed mainly include heuristics and matheuristics as the addition of the constraints associated to skillrequirement matching, work regulations and caregiver/patient preferences often make these problems intractable [21]. In Bowers et al. [28] a Clarke-Wrights (CW) savings algorithm is modified to minimize

Table 1

Characteristics of HC problems, objectives considered, and solution approached adopted. Characteristics include Time Windows (TW), Work Regulations (WR), Continuity of Care (CC), Temporal Dependencies (TeD), Overtime (OvT), Workload Balance (WB), Caregiver's Skills (Skill), Preferences (PR), Heterogeneous teams/ vehicles (HT). These characteristics may also be included as objectives. Additional objectives are related to time: Total Time (TotT), Traveling Time (TT), Service Time (ST), Waiting Time (WT); distance: Traveling Distance (TD); Number of Tasks (#T) and Number of Caregivers (#CG). Finally, the solution approach may be a combination of Exact methods (E), Matheuristic (MaH) and Heuristic (H).

Authors	Year	TW	WR	CC	TeD	OvT	WB	Skill	PR	HT	Objectives	Solution Approach
Eveborn et al.	2006	Hard	х	Single	х			х	х		TC;TD;PR	MaH;H
Bredström & Rönnqvist	2008	Hard			x		x		х		TT;PR;WB	MaH
Dohn et al.	2009	Hard	х		х		x	х			#T	E
Rasmussen et al.	2011	Hard	х		х			х			TC;PR;#T	E
Bachouch et al.	2011		х		х			х			TD	E
Nickel et al.	2012	Hard	х	Multi							TD;OvT;CC;#T	MaH;H
Mutingi & Mbohwa	2013	Soft	х		х						TT;TD;ST;WB	Н
Cappanera & Scutellà	2014		х	Multi			х	х			WB	E;H
Duque et al.	2014	Soft	х	Multi				х	х		TC;PR	MaH;H
Liu et al.	2014	Hard	х		х						WB	Н
Mankowska et al.	2014	Hard			х			х			TC;WB	Н
Bowers et al.	2015		х	Multi			х		х		TT;PR	Н
Redjem & Macron	2015	Hard	х		х						TT;WT	Н
Fikar & Hirsch	2015	Hard	х		х				х		WT	MaH;H
Hewitt et al.	2016	Hard		Multi			х				TotT;#CG	Н
Wirnitzer et al.	2016	Hard	х	Multi				х			#CG	E
Cappanera et al.	2018		х	Multi							WB	MaH
Gomes & Ramos	2019	Hard	х	Multi							TT;WB	MaH;H
Malagodi et al.	2021	Hard	х	Multi		х	x		х		TT;OvT;PR;CC	E;H
Lin et al.	2021	Hard	х		х	х	х	х	х		PR;#T	E
This work		Hard	x	Single	х					x	TT;ST;#CG	E

traveling distance, while incorporating preferences regarding patient-caregiver allocations. The same algorithm is applied in Hewitt et al. [29] but imbedded into a Consistent Vehicle Routing Problem Record-to-Record algorithm, which improves the solution from the CW using three different local search methods. This latter work compares the solutions for a weekly rolling horizon with the integrated planning for the whole three months period. In Maya Duque et al. [20], a two-stage strategy based on a hierarchy of objectives is applied. Firstly, the service level objective is optimized and afterwards travel distance is minimized. Service level accounts for the benefit of a patient-caregiver assignment in a visiting scheme, which dependents on both patient's and caregiver's preferences concerning the time slot. Continuity of care, modelled as a hard constraint, is accounted for during the enumeration of feasible visiting schemes. The service level is maximized by solving a set partitioning model formulated as an integer linear problem (ILP). To improve travel distance, a randomized local search algorithm is used. In turn, the objective function in Nickel et al. [24] is a weighted sum, presenting a term penalizing the numbers of times a patient is treated by a caregiver other than the reference one. The authors describe a model for the home health care problem and proceed to suggest a two-stage solution approach composed of a constraint-programming method for the timely generation of a feasible solution, followed by the application of an adaptative large neighborhood search to improve the previously obtained solution. Exact methods have been the main determinant of solution design in Carello & Lanzarone [12], Cappanera & Scutellà [17], and Wirnitzer et al. [25]. Cappanera & Scutellà [17] optimize workload balance, proposing an ILP to assign patients and caregivers to care patterns, while simultaneously performing care plan scheduling, operator assignment and routing decisions. A robust assignment model is introduced in Carello & Lanzarone [12] aiming at minimizing overtime and a penalty derived from care discontinuities. Continuity of care is also enforced by maintaining patient-caregiver assignment between planning periods in a rolling horizon scheme. The authors account for uncertainty of patient demand by applying a cardinality-constraint approach. The work developed in Wirnitzer et al. [25] defines five different measures of continuity of care, which are tested in a MIP model minimizing the number of caregivers allocated to a patient.

The most common type of temporal dependences in HC problems is the provision of simultaneous services and temporal precedence. In the case of social HC, simultaneous services may be the personal hygiene of highly dependent patients, whereas an example of temporal precedence happens in laundry, when one caregiver might put the clothes in the washing machine and another will hang them to dry [31]. In home health care, simultaneous services seems to be the most popular temporal dependences [32]. The recurrent argument for introducing synchronization constraints, also called coordination constraints [33], is the level of physical labor required in a visit, referred to as task arduousness [34]. Caregiver routes are designed accounting for the necessity to have more than one person providing care, assuring that both caregivers arrive and start providing care at the same time [18,19,35-37]. Synchronization has also been modelled when sharing resources. For example, allowing the routes of caregivers to be performed both walking and by car, with drivers synchronizing their routes with the end of selected visits by caregivers and transport them to other relatively distant areas where they will perform other walking tour [22]. Another form of synchronization concerns different types of nurses' professional skill which must be present at the same visit [18], named double services. These double services may be synchronized or associated with a temporal precedence, in particular separation by a minimum and up to a maximum time interval. An exhaustive exploration of temporal dependencies in HC is presented in Rasmussen et al. [31], defining five types, namely, synchronization, overlap, minimum difference, maximum difference and minimum plus maximum difference. The authors explain how to model each of the previous temporal dependencies by introducing general precedence constraints. However, the most relevant type of temporal precedence remains synchronization. For a

comprehensive classification of synchronization types in routing and scheduling refer to Drexl [38].

The solution methods applied in works emphasizing temporal dependences rely mainly on heuristics, as the interdependences between routes greatly increase problem complexity [33]. Liu et al. [39] implement various schemes of a metaheuristic, composed of both tabu search and different types of local search methods, to minimize the maximal route length, a workload balance objective. Precedence is relevant due to the routing sequence of material pick-up, biological sample collection and delivery to a lab within a strict time window. A MILP for the HHCRSP with interdependent services is formulated in Mankowska et al. [18], which is solved through a new heuristic designed to tackle large problems and compared to exact solutions obtained with CPLEX. The matrix representation of a solution is the innovative aspect, more suited to handle temporal dependences in articulation with search methods such as variable large neighborhood search. Redjem & Marcon [33] propose a method of two steps, the first in which the model is solved by relaxing some constraints, leading to a multiple traveling salesman problem where each routing object is independent from each other. Then, in the second step, the relaxed constraints, namely precedence and coordination constraints, are integrated to build the final solution. The strategies resemble greedy algorithms. An evolutionary algorithm coupled with fuzzy sets theory is applied in Mutingi & Mbohwa [40] considering multiple objectives, including workload balance, cluster efficiency and minimization of TW violations. Matheuristic solution methods were applied to derive routing plans in Fikar & Hirsch [22], Bredström et al. [36] and Eveborn et al. [37]. In Eveborn et al. [37], a minimum matching problem is formulated as an ILP, which is then integrated into a repeated matching algorithm. The objective is to minimize a weighted sum function composed of several terms including travel times and costs, preferences, inconvenient working hours, among others. Bredström et al. [36] propose an optimization based-heuristic, similar to the local branching heuristic. The objective function is a weighted sum of preferences, travel times and workload balance. Fikar & Hirsch [22] consider a problem resembling a dial-a-ride problem, in which caregivers move either walking or by car, being transported by drivers, requiring vehicle routes to be synchronized with the end of some visits. A two-stage solution approach is proposed. The first stage consists of generating feasible walking-tours and selecting a promising feasible subset through a set partitioning model. Those routes serve as input to the second stage, when an initial solution is produced, including both caregivers and vehicle routes. This complex stage combines a parallel savings heuristic, an optimization model, a biased-randomized savings heuristic and a tabu search algorithm. Finally, works by Dohn et al. [19], Rasmussen et al. [31], Bachouch et al. [35] develop exact optimization solution methods. Bachouch et al. [35] present a MILP minimizing total distance traveled, and compare two commercial solvers. Cuts associated to time windows are implemented as technical improvements to reduce processing time. A branch-and-price approach is suggested in both Dohn et al. [19] and Rasmussen et al. [31]. In the former, a HHCRSP is modelled as a manpower allocation problem with time windows, job-teaming constraints and with a limit on the number of teams (MAPTWTC). The authors argue that this modelling approach is relevant in contexts where tasks require cooperation between teams of workers, which may present different skill sets. The MAPTWTC resembles a VRPTW, with teams as the routing object, instead of vehicles, moving from one task to the other. The teams deliver part of their time at each task node, diverging from the VRPTW since their routes are interconnected. The solution strategy relaxes synchronization constraints and employs a Dantzig-Wolfe decomposition to solve the resulting problem. Branching rules enforce solution integrality and synchronization of tasks. In Rasmussen et al. [31] the authors apply a Dantzig-Wolfe decomposition to the problem and model it as a set partitioning problem with side constraints, solving it using dynamic column generation within a branch-and-price framework.

Recent works include innovations such as the disagregation of

visiting tasks, to attribute priorities to subtasks and introduce flexibility in task duration so that more patients can be scheduled [41] and the distiction between service time and ardousness of a task [34]. A correlation between the profit for performing a task and its ardouness is assumed. The ideia of task/activity priorization also seems a concern in Malagodi et al. [27] and Mosquera et al. [41]. A trend for the solution of exact models is the disagregation of problems into clusters, solving the formulated problem in separate sets of patients clustered according to patient attributes [26,27].

To the best of our knowledge no work considers teams' synchronization as a means of improving human resources management. The relevance of the proposed model lies in the interaction between the proportion of services requiring two caregivers and the number of vehicles available, which is an additional limit to the number of teams/ routes. The model introduces flexibility in planning, a highlighted direction for future development in Di Mascolo et al. [42]; as more flexible models tackle uncertainties inherent to the problem, such as the proportion of service per type and the number of services per patient when continuity of care is to be enforced.

Continuity of care is usually considered on multi-period planning settings, since most published works assume patients to request at most one vist per day. In social care services, several patients resquest two or more services per day and therefore it should be assured that the same caregiver is allocated throughout the day, for health status monitoring.

A new aspect of the porposed work is the modelling of dynamic time windows, as defined in Drexl [38], which has not yet been explored in the HC literature. Since most temporal dependences in HC are related to synchronization issues, the introduction of a time window after the arrival of the first team, creating a time interval for the arrival of the second team will increase flexibility in scheduling. Additionally, most works considering daily planning do not account for the possibility of decreasing the number of caregivers. In fact, models are designed assuring that all caregivers perform a route each day. However, in our context, having a caregiver at the day-care center could help perform other tasks such as serving as a backup for unforseen or urgent requests, to inform future planning or allow vehicle maintenance if a smaller number of routes are designed.

In short, this work main innovative aspect is the proposal of a new model which allows the selection of the number of teams and their composition (either one or two caregivers), while simulatenouly syncronizing visiting teams when needed. The number of teams is bound by the number of available vehicles. It further includes two characteristics not frequently found in HHC single-day planning horizon models simultaneously: continuity of care and synchronization. Together with these features, the proposed MILP handles different task types and staffing levels.

3. Case-study

This work was motivated by the case-study of a non-profit organization delivering social care services, named APOIO. This organization provides HC services (APOIO's caregivers provide services at the patient's home) and a day-care center (patients go to APOIO facilities to spend the day). The home services fall under two main areas. The first includes activities of daily living, such as medication assistance, personal hygiene (bathing, diaper changing, dressing) and instrumental activities of the daily living (as laundry, home cleaning). The second area is meal delivery, where the meals cooked at the day-care center must be distributed by the caregivers to patients' homes. For the daycare center, additional services may be provided on request, such as patient transportation.

This organization answers about 40 daily requests to provide HC services to 25 patients and delivers 100 daily meals to patients. The maximum number of tasks by the same patient is four. Patients are classified in one of two types according to their level of autonomy: semi-dependent and bedridden patients. This separation defines the number

of caregivers required to answer a visit request. Semi-dependent patients require one caregiver while bedridden patients require two caregivers. The geographical placement of the patients and their types are represented in Fig. 2.

The patient typologies allow a decomposition of the problem in the manual design of routes by the operational manager. The manager designs three routes to visit bedridden patients, each performed by a team of two caregivers (a double team), and three routes to visit semidependent patients, each performed by a team of one caregiver (a single team). This team scheme uses all the nine available caregivers and six cars. The staff is considered homogenous since they are not characterized by any skill influencing their suitability to perform a particular task. The decomposition makes it easier to manually try to secure daily continuity of care.

Due to the working period of the organization, from 8 a.m. to 8 p.m., together with the maximum working time of a caregiver, 8 h, it becomes impossible to secure daily continuity of care to all patients. Ideally, the same team of caregivers would be responsible for answering all daily requests from a patient but, since it is not possible due to the maximum working time, a maximum of two teams should be assigned to fulfill all requests of each patient. During shifts, there must be at least a break for lunch, when caregivers return to the day-care center and lunch is provided by the organization. The wide range of social services leads to caregivers being assigned to tasks beyond the scope of HC services. One such task is lunch distribution to patients at their homes, performed daily at mid-day and requiring three caregivers. Notice that many of these clients only receive these meals. No other service is provided to them. In this context, rather than considering visits as the basic elements of the routing and scheduling problem, we consider tasks.

An option to improve the efficiency of operations is solving the problem in an integrated manner, allowing double teams to serve SD patients. Additionally, single team synchronization in tasks requiring a

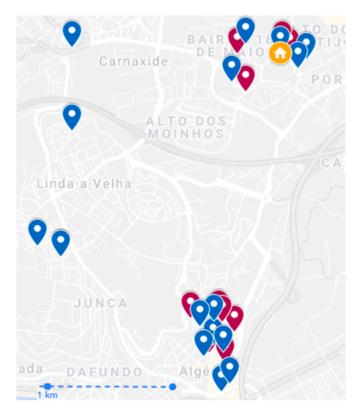


Fig. 2. Geographical placement of patients. Pink – bedridden; Blue – Semidependent; Orange – Day-care center. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

shared visit should be considered. Synchronization is overlooked due to the increased complexity of manually matching two caregivers to pay a shared visit within the same period. Nevertheless, the existence of double teams also presents advantages. A distinction is the certainty of a synchronous arrival of the two caregivers at the residencies and a second advantage is the usage of a single car to perform a route, in opposition to synchronized visits which require two cars.

APOIO serves 10 bedridden patients, representing 40% of HC patients which account for 55% of the requests, with on average 2.2 daily visits per patient. For semi-dependent patients there is only one patient requiring over a visit. The relatively high number of services on a single day increases the relevance of assuring continuity of care, particularly because for many bedridden patients, caregivers are their only human contact and are responsible for monitoring their health status. The simultaneous consideration of continuity of care generates a trade-off. Depending on the percentage of services requiring two caregivers, greater efficiency of the routing plan might require different numbers of teams of each type. Looking at the extremes, a set of exclusively semidependent patients would be satisfied with just single teams and a set of exclusively bedridden patients would more efficiently be served by double teams. Fig. 3 shows an illustrative example of how the proportion of service types may impact the solution. The allocation of tasks depends also on network characteristics formed by the residencies. If there is a semi-dependent patient requiring a visit near a bedridden patient, with compatible time windows and the service time required is not too long, it might be more efficient to allocate a double team than a single team coming from further away. Patients placing several tasks increase the complexity of enforcing continuity of care through the day. Synchronization might facilitate securing that at most a very small number of different teams visits each patient as depicted in Fig. 4. The illustration emphasizes how patients requiring less visits in a day may be served by the synchronization of two single teams.

The problem described retains some differences from what is found in the literature. Analyzing the features associated to HC routing proposed in Cissé et al. [23], a main difference concerns continuity of care. While most works address this issue over a medium to long term planning horizon, in this problem the focus is placed on daily continuity of care. Another distinction comes from the relation between service typology and the skill level of caregivers. In social care services, caregivers are homogeneous regarding skill level and service typology is reduced to classifying visits as requiring either one or two caregivers. Visits requiring two caregivers may not only be due to physical labor, but also to the aggregation of several services into one visit. These characteristics induce a potential routing/scheduling object which is the team, composed of one or two caregivers, in opposition to the caregiver as the routing object.

We expect that instead of having three double teams and three single teams, as it currently happens, a larger number of single teams, and consequently a smaller number of double teams, will increase the service operations efficiency.

4. Modelling approach

The modelling approach is based on the single depot vehicle routing

within which services must start. It assumes the vehicle must remain in that location during service provision. In our problem, the demand is set in nodes and defined as a task. It comprises both the duration of service and the number of caregivers required. A task may be served by a larger number of caregivers, but not lower. The routing object are teams of one or two caregivers, traveling by car. Once a team leaves the day-care center its composition is kept throughout the entire route length or working shift. Two teams of one caregivers are allowed to synchronize and service requests requiring two caregivers. Not all caregivers are required to leave the day-care center and the model solution proposes the number of teams is bounded by the cars available. Tasks are grouped into sets by patient to assure that at most two teams can be assigned to that patient, our definition of continuity of care.

problem with time windows (VRPTW). This problem is an extension of

the capacitated VRP where the time windows set the time interval

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The most valuable resource is the caregivers' working time. Therefore, the objective function (OF) minimizes the total working time. Since a team of two caregivers can be assigned to perform a task requiring only one caregiver, this OF aims at reducing the number of tasks served by teams of two caregivers when unnecessary. Notice, however, that cases may exist where it might be better (in terms of total working time) to assign a team of two caregivers to tasks needing only one caregiver. In addition, and as an incentive to minimize the number of caregivers, lunch breaks are accounted for in the OF.

4.1. Model formulation

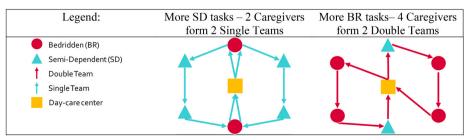
Let G = (N, A) be a directed graph, where $N = \{1, ..., n\}$ is the node set and $A = \{(i,j) : i \neq j, i \in N \setminus \{n\}, j \in N \setminus \{1\}\}$ is the arc set. Node subsets include the subset of the departing day-care center, $N_D = \{1\}$, the subset of the arrival day-care center, $N_A = \{n\}$, a task subset, N_C , and a lunch break subset, N_{LB} . The task subset, N_C , is further divided into three subsets: N_B for tasks requiring the visit of two caregivers, N_S for those requiring only one caregiver, and the lunch distribution subset, N_{LD} . In short, $N_C = N_B \cup N_S \cup N_{LD}$ and $N = N_D \cup N_B \cup N_S \cup N_{LD} \cup N_{LB} \cup N_A$. For every patient requiring more than one visit, a node subset is defined containing all the tasks related with the patient, $R_i = \{j \in N_C \land j \ge i : \text{task } j \text{ needs to be performed on patient } i\}$. Notice, that *i* is the identifier of the first task to be performed at that patient and any task in N_C belongs to one and only one patient. A final set $S = \bigcup R_i, |R_i| > 1$, created having as elements the subsets R_i . Each task node, $N_C = N_B \cup N_S \cup N_{LD}$, is characterized by a time window, $[a_i, b_i]$, and

node, $N_C = N_B \cup N_S \cup N_{LD}$, is characterized by a time window, $[a_i, b_i]$, and a service duration per caregiver, W_i . The day-care center nodes are associated to a TW representing the working period of the organization. Parameter T_{ij} is the traveling time to cross arc (i, j).

Given the existence of time windows, service duration and traveling time, all arcs are pre-processed to determine the *valid* arcs, those leading to feasible solutions. Validity is determined by calculating a measure of schedule feasibility with parameter $L_{ij} = a_i + T_{ij} + W_i - b_j$. If L_{ij} is less than or equal to zero, the arc is deemed invalid since it means that the arriving time at task *j* coming from task *i* can only occur outside *j*'s time window. Then, set *A* can be replaced by its subset A_V containing only

Fig. 3. Proportion of services & team schemes. Resources available: 2 Cars & 4 Caregivers.

It is assumed a TW overlap of the visits on the same horizontal level. Two cars limit the possible team scheme to two double teams, two single teams or one double team and one single team. In a situation with a greater proportion of SD patients the flexibility introduced in routing and scheduling by single team synchronization is more likely to reduce the number of caregivers.



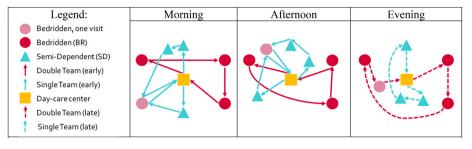


Fig. 4. Facilitating continuity of care through synchronization.

Serving BR patients requiring fewer daily services with two single teams eases the allocation of double teams to BR patients requiring CC, potentially avoiding TW overlaps and working time bottlenecks. For example, the early double team route could be unfeasible if the one visit BR patients, in the respective time slots, had to also be included, due to working time regulations or because of TW incompatibilities. This way, CC is assured by having the 3 BR patients requiring CC served by just two teams. BR patients served by synchronization are also served

by at most two single teams. In general, synchronization facilitates designing better plans regarding the caregivers required, violations to CC rules or overtime costs.

valid arcs: $A_V = \{(i, j) \in A : L_{ij} > 0\}$. Following the approach proposed by Gomes & Ramos [26]; set A_V is partitioned into: A_D with arcs departing from the day-care center, A_A with all arcs arriving to the day-care center and A_C all arcs between tasks. Finally, the team set is $V = \{1, ..., v\}$ which is partitioned into V_S , single teams, and V_D , double teams. The maximum number of teams is bounded by the number of available vehicles, Q.

Several decision variables are modelled. The binary variable x_{ijk} states if an arc (i, j) is either crossed by team k ($x_{ijk} = 1$) or not ($x_{ijk} = 0$). The non-negative variable t_{ik} accounts for the arrival time at node i by team k. For nodes served by the synchronization of two teams, an auxiliary variable τ_i is used to assess schedule feasibility, concerning the tasks served after synchronization. Notice that, when synchronizing, service provision can only begin when both caregivers are present. A τ_i value for a feasible schedule falls within $\max_{k,l \in V_S} \{t_{ik}, t_{il}\}$ and $\min_{j:(i,j) \in A_V} \{b_j - d_{k-1}\}$.

 $d_{ij} - W_i$ }. Finally, variables d_{ik} and y_{R_ik} are introduced to model continuity of care. Both are binary variables with $d_{ik} = 1$ indicating that task i is performed by team k (0 otherwise) and $y_{R_ik} = 1$ stating that patient i is visited by team k to perform at least one task ($y_{R_ik} = 1$), and 0 otherwise. A summary of the model nomenclature can be found on Appendice A.

$$\sum_{k \in V} \sum_{i,j:(i,j) \in A_V} (T_{ij} + W_i) Cg_k x_{ijk}$$

$$\tag{1}$$

$$\sum_{k \in V_j: (i,j) \in A_V} \sum_{Cg_k x_{ijk} \ge CgR_i, \forall i \in N_C$$
(2)

$$\sum_{j:(i,j)\in A_D} x_{ijk} \le 1, \forall k \in V, i \in N_D$$
(3)

$$\sum_{j:(i,j)\in A_D} x_{ijk} - \sum_{i:(j,i)\in A_A} x_{ijk} = 0, \forall k \in V$$
(4)

$$\sum_{i:(i,j)\in A_V} x_{ijk} - \sum_{i:(j,i)\in A_V} x_{jik} = 0, \forall k \in V, \forall j \in N_C$$
(5)

$$t_{ik} - t_{il} \le G_i + H\left(2 - \sum_{j: (i,j) \in A_V} x_{ijk} - \sum_{j: (i,j) \in A_V} x_{ijl}\right), i \in N_B, \forall k, l \in V_S : k \neq l$$
(6)

$$\tau_i \ge t_{ik}, \forall k \in V_S, i \in N_B \tag{7}$$

$$\tau_i + (T_{ij} + W_i) x_{ijk} \le t_{jk} + b_i (1 - x_{ijk}), \forall (i \in N_B \bigwedge k \in V_S), \forall (i, j) \in A_V$$
(8)

$$t_{ik} + (T_{ij} + W_i) x_{ijk} \le t_{jk} + b_i (1 - x_{ijk}), \forall \sim (i \in N_B \bigwedge k \in V_S), \forall (i, j) \in A_V$$
(9)

$$a_i \sum_{j: (i,j) \in A_V} x_{ijk} \le t_{ik}, \forall k \in V, \forall i \in N_C$$
(10)

$$t_{ik} \le b_i \sum_{j:(i,j) \in A_V} x_{ijk}, \forall k \in V, \forall i \in N_C$$
(11)

$$t_{ik} - t_{jk} \le H, \forall k \in V, i \in N_D, j \in N_A$$
(12)

$$a_i \le t_{ik}, \forall k \in V, i \in N_D \cup N_A \tag{13}$$

$$t_{ik} \le b_i, \forall k \in V, i \in N_D \cup N_A \tag{14}$$

$$b_p \sum_{j:(i,j)\in A_D} x_{ijk} \ge t_{pk}, \forall k \in V, p \in N_A$$
(15)

$$\sum_{j:(i,j)\in A_V} x_{ijk} = d_{ik}, \forall k \in V, i \in (N_B \cup N_S)$$
(16)

$$\sum_{i \in R_j} d_{ik} \le |R_j| y_{R_jk}, \forall k \in V, R_j \subseteq S$$
(17)

$$\sum_{k \in V} y_{R_{jk}} \le CgT, \forall R_j \subseteq S$$
(18)

$$\sum_{j:(i,p)\in A_V} x_{ipk} = \sum_{j:(i,j)\in A_D} x_{ijk}, \forall k \in V, p \in N_{LB}$$
(19)

$$\sum_{k \in V} \sum_{j: (i,j) \in A_D} x_{ijk} \le Q, \forall i \in N_D$$
(20)

$$\sum_{k \in V} \sum_{j: (i,j) \in A_D} Cg_k x_{ijk} \le CgA, \forall i \in N_D$$
(21)

$$x_{ijk} \in \{0, 1\}, \forall k \in V, (i, j) \in A$$
 (22)

$$t_{ik} \ge 0, \forall k \in V, i \in N \tag{23}$$

$$\tau_i \ge 0, \forall i \in N \tag{24}$$

(25)

 $d_{ik} \in \{0, 1\}, \forall i \in N, k \in V$

$$y_{rk} \in \{0,1\}, \forall r \in S, k \in V$$

$$(26)$$

Equation (1) is the objective function, minimizing total working time (the sum of traveling and service times) and penalizing the addition of caregivers to a solution. Inequalities (2)–(5) are routing constraints and (6)–(15) are scheduling constraints. Constraints (16)–(19) model continuity of care, while constraints (19) assure lunch breaks. Constraints (22)–(26) are domains for variables. In detail, constraint (2), assures that an acceptable number of caregivers is assigned to the task. Note that the task for lunch distribution is also included in the set N_c . Constraint (3) states that all teams may depart from the day-care center at most once, while equation (4) ensures that if a team has left the day-care center, it must return. The last routing constraint, equation (5),

assures flow continuity, stating that if a team arrives at one node it must leave the node. Regarding the scheduling constraints, constraint (6) addresses the feature that allows tasks needing two caregivers to be served by two single teams, which, ideally, should arrive at the node simultaneously. This is modelled by assuring that the maximum difference between the arrival times of two single teams to such a task *i* is at most G_i time units. If G_i is zero, the teams must arrive simultaneously. Constraint (7) assures that the value of the τ_i is greater than or equal to the maximal value of t_{ik} . Constraint (8) states the earliest time at which a team can start task *j*, after having performed task *i* through synchronization. Variable τ_i has a lower bound provided by (7) and an upper bound equal to $\min_{j:(i,j)\in A_V} \{b_j - d_{ij} - W_i\}$, which is the hypothetical maximal

arrival time of the second team that would still allow the ensuing tasks to be served. Together, constraints (6)-(8) model the dynamic time window associated to the synchronization of single teams. For a representation of the modelling approach for the dynamic time window see Fig. 5. Constraint (9) is the traditional scheduling constraint when hard TWs apply (all remaining). Constraints (10) and (11) assure arrival times to be within the tasks predefined time windows ($t_{ik} \in [a_i, b_i]$), i.e., patients will be served within the requested time window. Constraint (12)concerns the maximum route length allowed, while constraints (13) and (14) model the earliest departure and latest arrival times to the day-care center (assuring the organization working schedule). Constraint (15) guarantees that, when a team is not needed it should not leave the daycare center. Therefore, its arrival time to the day-care center should be set to zero. Equation (16) sets variable d_{ik} value according to whether (or not) team k performs task i. Constraints (17) and (18) model continuity of care. Constraint (17) sets the value of variable $y_{R,k}$ if a patient *i* is visited (or not visited) by team k and equation (18) assures that a patient is visited by at most CgT teams. Equation (19) is the lunch break constraint, assuring that every team that leaves the day-care center must have a lunch break. Constraint (20) makes sure the maximum number of teams is not exceeded, bounded by the number of available vehicles Q. Constraint (21) assures that the team scheme is possible for the number of available caregivers (CgA).

4.2. Symmetry breaking constraints

The symmetry breaking constraint (28) reduces the search space. As teams of the same type are indistinguishable, there is a symmetry inherent to some solutions. Equation (27) attributes a meaning to the variables $x_{iik} : i \in N_D$, which were previously meaningless since arcs departing and arriving to the same node are not allowed. It acquires the meaning of activity, assuming the value 1 if the team leaves the day-care center and zero otherwise. Equations (27) and (28) state that, for each set of teams, a team can only be active if the previous team is already active.

$$x_{iik} = \sum_{j:(i,j) \in A_D} x_{ijk}, \forall i \in N_D, k \in V$$
(27)

$$x_{iik} \ge x_{ii(k+1)}, \forall i \in N_D, \forall k \in V_S : k < |V_S| \lor k \in V_D : k < |V_D|$$

$$(28)$$

4.3. Modelling different operational management scenarios

The general formulation may model a variety of operational management policies by selecting which constraints to activate. In comparison to the current situation, four different operating policies are identified. The first encompasses all described features and represents the option to assure CC and allow synchronization (wSyn wCC). The second and third are the disjunct consideration of each of the former problem features. A policy considering synchronization and disregarding CC (wSyn woCC), and its reverse, the policy of not allowing synchronization but enforcing continuity of care (woSyn wCC). Finally, a last policy disregards both features (woSyn woCC). In all policies mentioned the reduction of the number of caregivers employed to satisfy care demand is implicitly introduced in the model by accounting for the lunch break time as working time. A second attribute basal to the model but distinct from the current operational policies is the permission for double teams to serve SD patients. The last operational policy allows a wider understanding of both the impacts of solving the integrated problem. The identified operating policies define the scenarios analyzed in the case study section. The selective (de)activation of constraints to form the models reflecting these policies is presented in Table 2.

Fig. 5. Variables and parameters modelling the dynamic time-window.

In the situation presented, two single teams (team k and team l) synchronize to serve bedridden patient i. Then team k proceeds to visit a semi-dependent patient p while team l visits a semi-dependent patient u. The offset between the arrival of the two synchronizing teams is established in constraint (6) and is defined with the arrival of the first team (k). This offset allows the definition of the dynamic time window (dTW) to facilitate synchronization scheduling. However, service provision can only start after the arrival of the second team (1). Hence, the auxiliary variable τ_i is defined. Its bounds are set by constraint (7) where the lower bound is established with the arrival time of the second team (t_{il}) and by constraint (8), assuring that both teams can perform the respective ensuing visits, i.e. $au_i \leq \min_{j:(i,j)\in A_V} \{b_j - d_{ij} W_i$. In the provided example, τ_i range is $[t_{il}, b_u -$



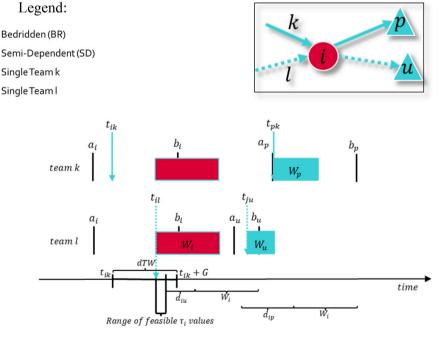


Table 2

Constraints in scenarios analyzed.

Planning Type	Abbreviation	Feature description	Constraints
Integrated Planning (Int)	wSyn_wCC wSyn_woCC woSyn_wCC woSyn_woCC	With Synchronization, With Continuity of care With Synchronization, Without Continuity of care Without Synchronization, With Continuity of care Without Synchronization, Without Continuity of care	All All, except (16)–(19) and (25), (26) All, except (6)–(8) and (24). (9) is applied to all nodes. Combine changes of wSyn_woCC and woSyn_wCC
Independent Planning (Ind)	Current	Single teams serve SD patients, double teams serve BR patients, exclusively.	Equal to woSyn_woCC but (2) becomes an equality

5. Results and discussion

This section comprises two parts, the first containing numerical experiments and the second, presents the results regarding the case-study that motivated this work. The numerical tests compare different strategies for plan design, namely addressing the problem in an integrated way (with the possibility of synchronization) versus solving the problem in an independent way. The strategies are tested by studying the plan outcomes for a varying proportion of bedridden patients.

5.1. Numerical experiments

The proportion of task types is a relevant aspect to explore in this context [36]. Therefore, further analysis was conducted to improve the knowledge of the impact that double services produce on the solution. A series of tests were performed on part of the instances provided by Mankowska et al. [18], which were adapted to fit the social care context studied. The problem instances vary in size and, for each size, there are ten different instances. This section adapts and analyses tests performed on the instances consisting of 10 and 25 visiting locations. The models were implemented using the commercial modelling software GAMS 34.2 and solved with the CPLEX 20.1 and Gurobi 9.1.1 solvers on a computer with a Intel(R) Core(TM) i9-10850K CPU @ 3.60 GHz 3.60 GHz and with a RAM of 128 GB. The time limit imposed was 3 h. We should note that, for the home care context (in particularly for our partner organization and similar organizations), this time limit is acceptable as the requests are similar from day to day, and from week to week, so the planning step can be lengthened by a few hours.

5.1.1. Gurobi vs CPLEX

The results concerning the comparison of Gurobi and CPLEX are presented only for the size-25 instances, as size-10 are easily solved by both solvers. In general, Gurobi performed better than CPLEX. Regarding solution quality, Gurobi finds better solutions in 23 out of 90 instances. Also, solutions found by Gurobi are at least as good as those found by CPLEX in 86 out of 90 instances (see Fig. 6 "Best Know Objective, #BKO" column). CPLEX reached a better solution than Gurobi in only 4 instances. However, the objective function values of those solutions are only 0.4% better than the Gurobi ones. Gurobi proves optimality for 57 out of 90 instances (63%) while CPLEX solves only 32 instances to optimality (36%). Concerning instances that were not solved to optimality, Gurobi only leaves 9 with a gap above 4% (10% of the instances) while CPLEX is not able to close the gap below 4% in 33 instances (37%). Furthermore, CPLEX was unable to produce a feasible solution for one of the instances.

Concerning the instances for which optimality has not proven by either solver (33 out of 90 instances), Gurobi always provides better gap values than CPLEX, either by finding better solutions (18 out of 33) or by providing better lower bounds (14 out of 33) and even by finding a solution when CPLEX finds none (1 out of 33).

In terms of computational time tests, Gurobi reaches the time limit of 3 h in 33 (37%) out of the 90 instances, while CPLEX reaches it for 60 (67%). For instances with proven optimality within the allotted time by both solvers, 30 out of 90, Gurobi always needs less computational time, (on average 61% less than CPLEX).

Comparing Gurobi objective function values after 0.5 h and 3 h of computational time, Gurobi is able to prove optimality for 39 instances (43%) after half an hour of computational time. Moreover, for other 16 instances (18%) half an hour is enough for Gurobi to find the solution that will be proven to be optimal after 3 h of computational time. Within the first 30 min, in 13 other instances, the BKO is found. This means that for a total of 68 instances (75%) the OF will not be improved with the additional 2.5 h of computational time. For the remaining 22 instances, the improvement of the OF value is on average 0.35% and the maximum variation is 2.56%. This clearly demonstrates that even after short computational times the solutions found by Gurobi are of very high

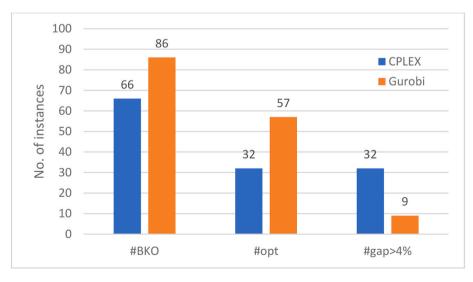


Fig. 6. Performance comparison between CPLEX and Gurobi solvers. #BKO – No. of instances reaching the best-known objective; #opt – No. of instances reaching optimality; #gap>4% - No. of instances with gap larger than 4%, after 3 h of computational time.

quality. Results for this analysis are available at Appendice G.

Given the above, Gurobi was chosen as the solver, producing all the results presented hereafter.

5.1.2. Independent vs Integrated planning

The set of tests consists of varying the percentage of bedridden (BR %) tasks and assessing how it affects the number of caregivers providing service and the objective function value, comparing independent and integrated planning outcomes. The comparisons between independent and integrated planning focused mainly on 1) the OF behavior as a function of the BR% and 2) the number of instances for which it was possible to reduce the number of caregivers needed by solving the integrated planning problem, for each BR%. Appendice B clarifies the adaptations made to the original instances. The OF value can be interpreted as a cost, measured as the sum of traveling and service times. In general terms, the cost in minutes for the integrated planning is always lower than that of the independent planning, and in the worst case it would be equal. For both instance sizes, as the BR% increases the difference between costs of the integrated and independent problem solutions decreases (see Fig. 7 for size-10 and Fig. 8 for size-25 instances). This results from a reduction of the pool of feasible solutions as BR% increases. When the proportion of BR patients is higher there are less opportunities to synchronize thus reducing the ability of the model to introduce flexibility and yielding solutions closer to the independent planning.

Nevertheless, even for high BR% the model can identify solutions reducing the number of caregivers required (see Fig. 9 for size-10 and Fig. 10 for size-25) by transferring SD request to double teams. For size-10 instances it is always possible to reduce one caregiver in at least one of the instances. In general, the greater the proportion of BR patients the less likely is the integrated strategy to reduce the number of caregivers. The only odd result was that for 40 BR%, which would be better adjusted to the overall trend if another instance had yielded a solution with one less caregiver. A deeper analysis revealed the possibility to solve that instance with one less caregiver, but with a higher cost (420 min with 3 caregivers vs 429 min with 2 caregivers) and thus the model selected the solution with 3 caregivers. In these tests the OF is to minimize the sum of traveling and service times as there is no lunch break in the instances.

Results for size-25 instances (see Fig. 10) are not so straightforward. Variations in the number of caregivers, between the independent and integrated plannings, range from reducing two caregivers to increasing one caregiver. The largest reduction occurs for 8 BR%, with 80% of the solutions having at least one less caregiver and a reduction of two caregivers for one instance. Then, the number of integrated solutions reducing the caregivers decreases as BR% increases until 40 BR% in which there is no solution reducing the number of caregivers. In contrast to the size-10 instances, there are solutions increasing the number of caregivers between 32 BR% and 52 BR%. This happens because, when solving the problem independently, there is always at least one double team to which longer routes are allocated as BR% increases. For size-25

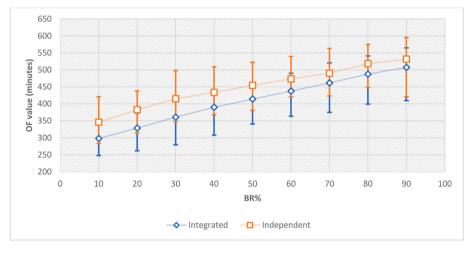


Fig. 7. Average objective function values. These values are obtained from each BR% batch tests of the size-10 instances, with maximal and minimal values displayed.

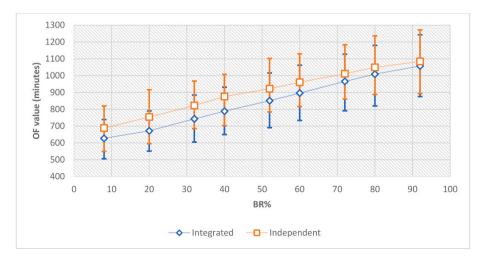


Fig. 8. Average objective function values. These values are obtained from each BR% batch tests of the size-25 instances, with maximal and minimal values displayed.

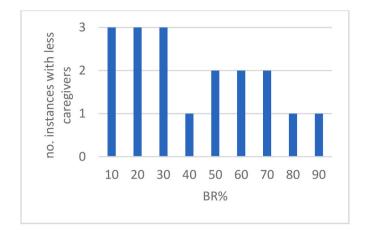


Fig. 9. Number of size-10 instances with one less caregiver in the integrated planning, of each BR% batch.

instances there are five available caregivers and having two double teams would leave only one caregiver to serve SD demand. Therefore, the independent solutions have one large double route and satisfy the SD demand with two or three caregivers. Integrated solutions tend to use between 4 and 5 caregivers where BR patients are served through synchronization of single teams and hence the larger average number of caregivers. Also, some single routes in the integrated solutions visited just one or two patients, which is undesirable in the real world. An unexpected result is the reduction of the used caregivers for four instances of the 92 BR%, due to the shifting of SD demand to double teams, suppressing the single team. A more practical conclusion is that the number of team schemes with potential to improve the solution would be impossible to assess in manual planning.

To summarize, in general, as the proportion of BR patients increases so does the number of caregivers, and the difference in OF values of the two plans decreases. Nevertheless, in the integrated planning the cost is always lower and the proportion of problems with a smaller number of caregivers increases. Results suggest that different solutions could be obtain if the number of caregivers had been penalized in the objective function. For instance, in the integrated problem, some solutions propose routes with just one or two tasks while others, if solved with one less caregiver, would imply a negligible cost increase. Finally, the combinatorial behind possible team schemes makes it difficult for manual planners to select the best composition of teams without decision support tools. A table with a summary of the tests results regarding teams' schemes and computational performance, respectively, is available in Appendice C and Appendice D for size-10 instances and Appendice E and Appendice F for size-25 instances.

5.2. Case-study results

This section characterizes the current solution, which serves as a baseline for comparison with the solutions obtained in the integrated scenarios established in Section 4.3. considering different operational management policies. The main objective of this case-study is to assess the simultaneous inclusion of synchronization and daily continuity of care features into the design of social HC routes. The focus is on how it affects the organization's capacity (measured through caregivers' working time). Currently, APOIO has 9 caregivers assigned to the HC services, working 8 h a day (480 min) adding up to 4 320 min of caregivers' working time. The route length is assumed to be the total working time caregivers, i.e., the sum of service time, traveling time, waiting time and lunch time. The value for parameter G_i is constant for all nodes since the partner organization was unable to discriminate offset values for each node. Nevertheless, the operational managers understood the impact on human resource allocation of adding the dynamic time-window and agreed to compromise on an offset value of 10 min. The managers are highly averse to having caregivers waiting for more than that time for each other as it could give rise to dissatisfaction among the caregivers.

5.2.1. Current situation

The APOIO's current routing is made of two independent route sets. One set includes three routes performed by single teams while the other has three routes performed by double teams. Data supplied by the organization didn't allow to assess how the routes were performed exactly. Therefore, we computed optimized routes following the information provided by the social worker in charge of this service. All results presented in this section are in fact better than those that were really executed.

According to the optimal solution (see Table 3), on average, the routes of single teams are shorter than those of double teams (379 min versus 465 min, on average values). Moreover, two out of three routes of two caregivers are at maximum work capacity (480 min, R.2 and R.3 of double teams' columns). Another interesting aspect is the relative allocation of time within a route when compared to home healthcare problems. In all APOIO's teams, the amount of time spent traveling between locations is significantly lower than the time spent providing service. Residences are quite close to each other, and service provision demands long periods of time to be completed. More relevant is the discrepancy of service time distribution, both within and between team types, corresponding longer service times to more arduous routes.

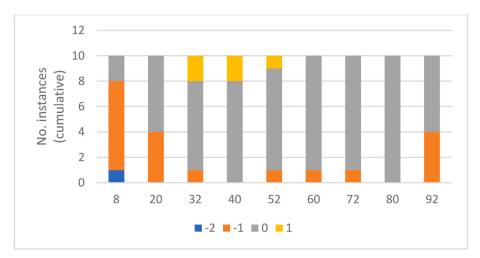


Fig. 10. Cumulative number of size-25 instances with variations in the number of caregivers between -2 and +1, for each BR% batch.

Table 3

Current solution time allocation per route.

Time allocation (minutes)	Single Tea	ams			Double Te	ams	Caregiver Capacity			
	R. 1	R. 2	R. 3	Avg.	R. 1	R. 2	R. 3	Avg.	Absolute	% of total
Length	319.7	378.5	440.1	379	433.8	480	480	465	3 925.9	91
Traveling	12.1	15.2	14.1	14	1.6	18.6	14.3	12	110.4	3
Service	157	124	214	165	230	159	280	223	1 833	42
Waiting	90.5	179.3	152	141	142.2	242.4	125.7	170	1 442.4	33
Lunch	60	60	60	60	60	60	60	60	540	13

Among single teams, the difference between the greater and lesser service times (R.2 and R.3) is 90 min (214 min versus 124 min), while for double teams (R.2 and R.3) it is 121 min (280 min versus 159 min), which is very significant. Also, on average, double team routes have 223 min of service time while single team routes have 165 min, a consequence of having more bedridden tasks and the impossibility of allocating them to two single teams.

Analyzing the caregivers' total working capacity (last two columns in Table 3), one may see that the total amount of time allocated to traveling is 3%. About 91% of the caregiver's total working time is allocated, meaning that patient admissions must be fitted into the schedule holes, usually resulting from other patients leaving the HC services, such as being admitted to a hospital, institutionalized, or even dying.

Fig. 11 shows the distribution of service time throughout a working day for each team. It highlights services occurring predominantly before 2 p.m., aligned with the nature of the most requested service types, which are personal hygiene and accompanied feeding (lunch). Consequently, there are two routes mainly in the morning, and a route mainly in the afternoon/evening for both team types.

5.2.2. Impact of synchronization and continuity of care

This section analyzes the introduction of teams' synchronization and continuity of care in the design of real-world routing and scheduling plannings. To do so, four scenarios, as defined in Section 4.3., are compared with the current situation presented above. Notice that when saying "no synchronization" one is assuming that only double teams can perform tasks needing two caregivers. The main results are presented in Table 4, showing the total working time (OF) and its disaggregation into service time (ST), traveling time (TT), lunch time (LT) the number of caregivers and teams by type. The lower bound on service time is that of

the current situation, corresponding to having all SD tasks served by single teams.

The current situation performs better than all studied scenarios in terms of travel times. This shorter travel time is achieved by using more caregivers and teams. The two scenarios without synchronization (woSyn_wCC and woSyn_woCC) present the smallest traveling times among the four integrated planning scenarios, but the highest service times. The reason is that tasks needing one caregiver can now be assigned to double teams, counting twice to the OF. Two SD tasks with late TWs are moved between team types, allowing the reduction of single teams. Indeed, it is impossible to solve the independent problem with less than three teams of each type since the latter tasks generate a bottleneck. Simultaneous single services in the morning require two teams to start working early, demanding that a third be allocated to a latter route. A few late SD tasks and their assignment to double teams may result in the reduction of single teams needed and thus less caregivers. This effect is especially relevant for the scenario with none of the features (woSyn woCC), for which there is a slight decrease in the OF. As previously emphasized, the OF is almost completely determined by lunch and service times. Reducing one caregiver meant that two tasks from SD patients with a service duration of 40 min were moved from a single to a double team. This move implies an increment of 40 min of service time to the OF (from 1 833 to 1 873 min). When double teams are allowed to serve SD patients, the service time is no longer a zero-sum component of the OF. However, reducing the number of caregivers also represents an optimization benefit, since there is one less lunch time penalty. Nevertheless, it was unexpected that the operational management policy differing the least from the current situation would already be able to serve the demand with one less caregiver.

Regarding the number of both teams and caregivers, synchronization

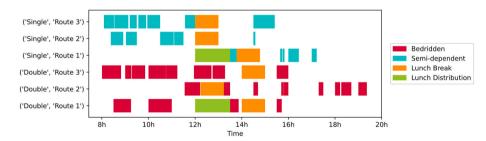


Fig. 11. Temporal representation of tasks, in routes, considered in independent planning, of the current solution.

Table 4

Characteristics of the scenarios: OF – Total Working Time; ST – Service Time; TT – Travel Time; LT-Lunch Time; #Caregivers – Number of Caregivers; #SingleT – Number of Single Teams; #DoubleT – Number of Double Teams.

	OF	ST	TT	LT	#Caregivers	#SingleT	#DoubleT
Current Situation	2 483	1 833	110	540	9	3	3
wSyn_wCC	2 397	1 846	131	420	7	3	2
wSyn_woCC	2 384	1 833	131	420	7	3	2
woSyn_wCC	2 477	1 873	124	480	8	2	3
woSyn_woCC	2 477	1 873	124	480	8	2	3

seems the feature with the greatest impact. Without synchronization, solutions always propose three double teams. This result corroborates that the organization is currently working on the limit of its double teams' capacity regarding the policy of patient-team allocation. The number of double teams only changes when allowing single team synchronization. Whenever, synchronization is allowed, all tasks can be served with less two caregivers. Consequently, the total time is smaller since there are less caregivers. It is interesting to notice that the solution wSyn_woCC that has the lowest bound on service time, increases the traveling time by just 21 min and reduces two caregivers. However, there are patients served by more than two teams.

Fig. 12 depicts a temporal representation of the solution of scenario wSyn_wCC. For a geographical representation of the routes in the current and proposed solutions see Appendice H. Two single team routes which synchronize are observed. They start their shift by visiting two semi-dependent patients each. Then, they team up and visit three bedridden patients. At the end of the morning period, one is assigned the task of lunch distribution while the other has the lunch break. In the afternoon period they team up twice to visit bedridden patients and in between, one has the lunch break while the other visits a semidependent patient. These two routes point out the possibility of other features that could potentially improve operations, such as carpooling services. This feature is outside the scope of this work. These single teams serve only three bedridden patients, two of them with two tasks and one with three tasks. A surprising result, as we expected that the synchronization would be selected in patients not requiring continuity of care and that the routes would have less tasks with synchronization.

Despite appearing to be the same according to Table 4, the solutions proposed in scenarios woSyn_woCC and woSyn_wCC are different (see Table 5). They differ considerably with respect to both route length and service time. Analyzing both solutions, one sees that a whole cyclic segment of two routes (starting on the day-care center after lunch and back) is exchanged between two double teams. Although this change has no impact on the total working time (the solutions are alternate optima), the workload distribution between the plans varies and social workers may have a preference.

6. Conclusions

Articulating home social and health services is regarded as an approach to decrease health care costs and increase health outcomes. The demographic and social trends along with the changes observed in medical care paradigm will increase the demand for home care support. This will put additional pressure on existing home care organizations to improve their efficiency to be able to provide care to a larger number of patients. In this work we proposed a new optimization model for the routing and scheduling of home social care caregivers assuring continuity of care and allowing for team synchronization when tasks demand for two caregivers. Although generic, the proposed model can easily be tailored so that features relevant for a social care organization partner are considered. Among our goals is the understanding of how these features, namely single-day continuity of care and synchronization, introduce flexibility in the routing and scheduling plan of this services provider with respect to its current situation.

The MIP model solution provides information concerning the optimal number of caregivers to answer patient requests, as well as how to organize them into teams of one and two caregivers. It also assigns visits to teams and designs the visiting sequences. These decisions are affected by two main problem features: teams' synchronization and continuity of care. The objective function minimizes service and travel times, but also includes the lunch break as an "incentive" to reduce the number of caregivers since each added caregiver penalizes the objective value in 60 min.

All studied scenarios reduced the number of caregivers needed and, consequently, the total time (the sum of all route lengths in the resulting plan) when compared to the current situation. This reduction is achieved due to the allocation of semi-dependent patients to double teams (i.e., allocation of tasks demanding one caregiver to double teams). However, when assigning two caregivers to a task needing only one, the second caregiver remains inactive while waiting for the colleague (e.g. feeding can only be done by one person). Therefore, there is a trade-off between reducing the number of caregivers and allocation of tasks needing one caregiver to double teams. We are then able to conclude that synchronization of single teams increases the available service capacity. This increased capacity enables the organization to serve new patients, reducing the waiting lists.

Although synchronization is only modelled for single teams, the model is able to propose solutions that can be viewed as a double team being "split" to visit semi-dependent patients. This is shown when modelling synchronization and continuity of care simultaneously. In the optimal solution there are two single teams that work together almost all morning, then perform different tasks around lunch time, and get back together at the end of the day to visit some bedridden patients that need more than one visit per day.

The observations made throughout this study suggest some future work directions. The first concerns the exploration of car sharing or carpooling services. In the operational policy with synchronization and continuity of care features, the solution included two single routes synchronizing at three different points in time, which suggests a potential benefit in sharing a vehicle thereby reducing traveling costs. A second concerns the existence of alternative solutions consisting of the transfer of cyclic parts of routes between analogous teams. Applying a lexicographic approach optimizing workload equity would distinguish these solutions. The demanding physical nature of these services is a strong argument supporting the implementation of multi-objective solution approaches to suggest plans with evenly distributed workloads.

Table 5

Characteristics of routes in each solution.

Solution	Length	(min.)		Service	e Time (n	iin.)
	Max	Min	Variation	Max	Min	Variation
Current Situation	480	320	-33%	280	124	-56%
wSyn_wCC	480	420	-13%	358	206	-42%
wSyn_woCC	480	420	-13%	339	193	-43%
woSyn_wCC	480	390	-19%	278	159	-43%
woSyn_woCC	480	360	-25%	315	159	-50%

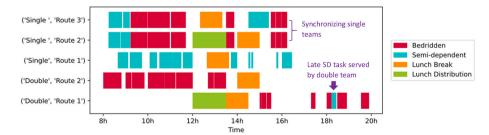


Fig. 12. Temporal representation of tasks, in routes, from scenario wSyn_wCC.

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Thirdly, the provided data shows tasks requiring two caregivers that in fact are activities that can be done in sequence by one person. There may be further advantages in discretizing task [41] and assessing the trade-off between the aggregating activities (making caregivers to work in parallel) or disaggregating activities (allowing a caregiver to perform the task in sequence).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendices.

A. Nomenclature

- *i*,*j*,*p* Nodes
- k,l Teams

Sets and subsets

- N All nodes
- N_D Departing day-care center node
- N_A Arrival day-care center node
- *N_C* All task nodes
- N_B Bedridden-type task nodes
- NsSemi-dependent-type task nodesNLDLunch distribution task
- N_{LD} Lunch distribution t N_{LB} Lunch Break task
- NLBLunch BrownAAll arcs
- A All arcs
- *A_D* Valid arcs departing from the day-care center
- *A_A* Valid arcs arriving to the day-care center
- *A_C* Valid arcs between tasks
- All valid arcs
- V All teams
- *V_S* Single teams
- *V_D* Double teams
- R_i Subset of tasks all corresponding to patient i
- S Set formed by the subsets R_i

Parameters

- *a_i* Earlier time of arrival to a node
- b_i Later time of arrival to a node
- *H* Maximum length allowed for a route
- W_i Duration of service of task *i* per caregiver
- T_{ij} Traveling time between two nodes
- *L_{ij}* Schedule feasibility measure
- G_i Temporal offset between the arrival of two single teams to a bedridden patient *i*
- *CgT* Maximum number of teams to visit any patient
- *CgA* Number of caregivers available
- CgR_i Number of caregivers required for task i
- Cg_k Number of caregivers in team k
- *Q* Number of vehicles available

Variables

- x_{ijk} Binary variable, = 1 if arc (i, j) is traversed by team k; 0 otherwise
- t_{ik} Arrival time to node *i* by team *k*
- τ_i Auxiliary variable to assess schedule feasibility after synchronizing at i
- d_{ik} Binary variable, = 1 if task *i* is performed by team *k*; 0 otherwise
- y_{R_ik} Binary variable, = 1 if team k performs at least one task of patient task set R_i ; 0 otherwise

Data availability

Data will be made available on request.

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B. Instances adaptation

Table B1

General characteristics of the instances used in relation to the original instances

Instance Characteristics	Observation
Service Duration	Common to all nodes and different across instances.
Time Windows	Mainly placed in first half of the working day period. $a_i - b_i = 120$ minutes
Route length	A limit of H was added.
	H = 480 minutes
Time travels	Reduced to a third to better represent the studied context.

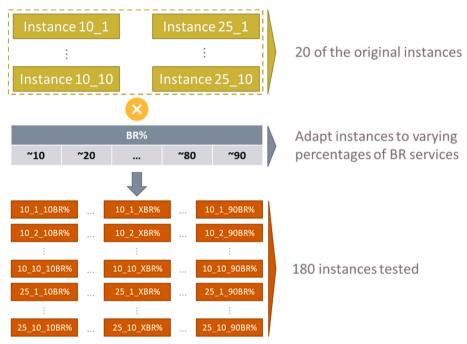


Fig. B1. Scheme of the adaptation of the instances

The variation of BR tasks was simulated by considering all tasks homogeneous regarding skills required. To generate the different sets of BR tasks the decreasing numerical order of the nodes' identifier was used. For example, for the instances of 10 nodes, 20 BR% meant that the set of SD tasks contained the nodes {2, ...,9} and the set of BR contained {10,11}, while nodes 1 and 12 represent the day-care center.

C. Team scheme results of tests with size-10 instances

BR%	id	Independ	dent		Integrate	ed CPLEX		Integrate	ed Gurobi		var_#Cgv
		#S	#D	#Cgv	#S	#D	#Cgv	#S	#D	#Cgv	
10	1	1	1	3	2	0	2	2	0	2	1
	2	1	1	3	2	0	2	2	0	2	1
	3	1	1	3	3	0	3	3	0	3	0
	4	1	1	3	3	0	3	3	0	3	0
	5	1	1	3	2	0	2	2	0	2	1
	6	1	1	3	3	0	3	3	0	3	0
	7	1	1	3	3	0	3	3	0	3	0
	8	1	1	3	3	0	3	3	0	3	0
	9	1	1	3	2	0	2	2	0	2	1
	10	1	1	3	2	0	2	2	0	2	1
<u> </u>		-	-							0.5	0.5
Avg.		1	1	3	2,5	0	2,5	2,5	0	2,5	0,5
20	1	1	1	3	3	0	3	3	0	3	0
	2	1	1	3	3	0	3	3	0	3	0
	3	1	1	3	3	0	3	3	0	3	0
	4	1	1	3	3	0	3	3	0	3	0
	5	1	1	3	2	0	2	2	0	2	1
	6	1	1	3	3	0	3	3	0	3	0
	7	1	1	3	3	0	3	3	0	3	0
	8	1	1	3	3	0	3	3	0	3	0
	9	1	1	3	2	0	2	2	0	2	1
	10	1	1	3	2	0	2	2	0	2	1
								—			

3R%	id	Independ	lent		Integrated CPLEX			Integrated Gurobi			var_#Cg
		#S	#D	#Cgv	#S	#D	#Cgv	#S	#D	#Cgv	
Avg.		1	1	3	2,7	0	2,7	2,7	0	2,7	0,3
Avg. 30	1	1	1	3	3	0	3	$\frac{2,7}{3}$	0	3	0
	2	1	1	3	2	0	2	2	0	2	1
	3	1	1	3	3	0	3	3	0	3	0
	4	1	1	3	1	1	3	1	1	3	0
	5	1 1	1 1	3 3	3 3	0 0	3 3	3 3	0 0	3 3	0 0
	6 7	1	1	3	3	0	3	3	0	3	0
	8	1	1	3	3	0	3	3	0	3	0
	9	1	1	3	2	0	2	2	0	2	1
	10	1	1	3	2	0	2	2	0	2	1
Avg.		1	1	3	2,5	0,1	2,7	2,5 3	0,1	2,7	0,3
10	1	1	1	3	3	0	3	3	0	3	0
10	2	1	1	3	3	0	3	3	0 0	3	0
	3	1	1	3	3	0	3	3	0	3	0
	4	1	1	3	1	1	3	1	1	3	0
	5	1	1	3	3	0	3	3	0	3	0
	6	1 1	1	3	3	0	3	3	0	3	0
	7 8	1	1 1	3 3	1 3	1 0	3 3	1 3	1 0	3 3	0 0
	9	1	1	3	3	0	3	3	0	3	0
	10	1	1	3	2	0	2	2	0	2	1
wa	—	1	1	3		0.2	2,9	2.5	0.2	2,9	0,1
Avg. 50			1 1		$\frac{2,5}{3}$	0,2		$\frac{2,5}{3}$	$\frac{0,2}{0}$		
50	1	1 1		3		0	3			3	0
	2 3	1	1 1	3 3	3 3	0 0	3 3	3 3	0 0	3 3	0 0
	4	1	1	3	1	1	3	1	1	3	0
	5	1	1	3	3	0	3	3	0	3	0
	6	1	1	3	3	0	3	3	0	3	0
	7	1	1	3	1	1	3	1	1	3	0
	8	1	1	3	3	0	3	3	0	3	0
	9 10	1 1	1 1	3 3	2	0 0	2 2	2 2	0 0	2	1 1
	10	1 1	<u> </u>	3	$\frac{\frac{2}{2,4}}{2}$	<u> </u>		<u> </u>	<u> </u>	2	
Avg. 50	_	1	$\frac{1}{1}$	3	2,4	$\frac{0,2}{0}$	2,8	$\frac{\frac{2}{2,4}}{2}$	$\frac{0,2}{0}$	2,8 2	0,2
0	1	1		3	2	0	2	2			1
	2	1	1	3	3	0	3	3	0	3	0
	3	1	1	3	3	0	3	3	0	3	0
	4 5	1 1	1 1	3 3	1 3	1 0	3 3	1 3	1 0	3 3	0 0
	6	1	1	3	3	0	3	3	0	3	0
	7	1	1	3	1	1	3	1	1	3	0
	8	1	1	3	3	0	3	3	0	3	0
	9	1	1	3	3	0	3	3	0	3	0
	10	1	1	3	2	0	2	2	0	2	1
vg.		1	$\frac{1}{1}$	3 3	2,4	$\frac{0,2}{0}$	2,8 2	$\frac{2,4}{2}$	$\frac{0,2}{0}$	2,8	0,2
vg. 0	1	$\frac{1}{1}$	1	3	$\frac{2,4}{2}$	0	2	2	0	2	1
	2	1	1	3	3	0	3	3	0	3	0
	3	1	1	3	3 3	0	3	3 3 1	0	3	0 0
	4	1	1	3	1	1	3	1	1	3	0
	5	1	1	3	3	0	3	3 3	0	3	0
	6	1 1	1 1	3	3 1	0 1	3	3 1	0 1	3	0 0
	7 8 9 10	1	1	3	3	0	3		0	3	0
	9	1	1	3	3 3	0	3	3	0	3	0
	10		1	3	2		2	2	0	2	1
vo		1	1	3	24	$\frac{\frac{0}{0,2}}{0}$	2.8	24	0.2	2.8	0,2
vg. 0		-	<u> </u>	5	<u>2,7</u>	0,2	2,0	2,4	0,2	2,0	0,2
U	1 2 3	1 1 1 1	1 1 1 1	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	$\frac{2}{2,4}$ 3 3 3	0 0	3 3 3 2 2,8 3 3	3 2 2,4 3 3 3 1	$ \begin{array}{c} 0\\ 0,2\\ 0\\ 0\\ 0 \end{array} $	2,8 2 3 3 3 3 3 3 3 3 2 2,8 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	0 0
	∠ 3	1	1	э 3	<u>э</u>	0	3	<i>з</i>	0	<i>з</i>	0
	4	1	1	3	3 1	0	3	1	1	3	0
	5	1	1	3	3	0	3	3	0	3	0
	5 6	1	1	3	3	0	3	3	0	3	0
		1	1	3		1	3	2	1	4	0
	7 8 9	1	1	3	1 3 2 2,5	0	3 3 2 2,9	3 2 3 3 2 2,6	0	$\frac{3}{2}$	0
	9 10	1 1	1	3	3	0	3	3	0 0 0,2	3	0
.vg.	10	1 1	1 1	3	۷	0 0,2	<u>∠</u>	2	0	<u> </u>	1 0,1
		1	1	0				0.6	0.0	0	0.1

BR%	id	Independ	dent		Integrate	Integrated CPLEX			ed Gurobi		var_#Cgv
		#S	#D	#Cgv	#S	#D	#Cgv	#S	#D	#Cgv	
90	1	1	1	3	3	0	3	3	0	3	0
	2	1	1	3	3	0	3	3	0	3	0
	3	1	1	3	3	0	3	3	0	3	0
	4	1	1	3	1	1	3	1	1	3	0
	5	1	1	3	3	0	3	3	0	3	0
	6	1	1	3	3	0	3	3	0	3	0
	7	1	1	3	0	1	2	0	1	2	1
	8	1	1	3	3	0	3	3	0	3	0
	9	1	1	3	3	0	3	3	0	3	0
	10	1	1	3	2	0	2	2	0	2	1
Avg.		1	1	3	2,4	0,2	2,8	2,4	0,2	2,8	0,2

D. Computational results of tests with size-10 instances

BR%	id	Objective Fun	ction (min.)			Computatio	nal Time (s)	Computational Time (s)			
		Ind.	Int_c	Int_g	Var (%)	Ind.	Int_c	Int_g			
0	1	365,4	312,6	312,6	-14,4	0	0	0			
	2	283,7	256,8	256,8	-9,5	0	0	0			
	3	345,1	324,7	324,7	-5,9	0	0	0			
	4	385,5	314,8	314,8	-18,3	0	0	1			
	5	337,4	247,8	247,8	-26,6	0	0	0			
	6	324,1	322,7	322,7	-0,4	0	0	0			
	7	347,4	289,2	289,2	-16,8	0	0	0			
	8	339,1	275,1	275,1	-18,9	0	0	0			
	9	420,6	337,5	337,5	-19,8	0	0	1			
	10	311,5	298,7	298,7	-4,1	0	0	0			
Avg.	_	346,0	298,0	298,0	-13,5	0	0	0,2			
20	1	382,2	343,7	343,7	-10,1	0	0	0			
	2	314,6	278,1	278,1	-11,6	0	0	0			
	3	425,9	372,7	372,7	-12,5	0	1	1			
	4	414,3	341,5	341,5	-17,6	0	1	1			
	5	353,6	261,8	261,8	-26	0	0	0			
	6	403,1	361,4	361,4	-10,3	0	0	0			
	7	339,9	315,3	315,3	-7,2	0	0	0			
	8	370,6	297,3	297,3	-19,8	0	0	0			
	9	437,7	366,1	366,1	-16,4	0	0	0			
	10	385	347,7	347,7	-9,7	0	0	0			
Avg.		382,7	328,6	328,6	-14,1	0	0,2	0,2			
30	1	403,1	365,5	365,5	-9,3	0	0	0			
	2	348	309	309	-11,2	0	0	0			
	3	498	404,2	404,2	-18,8	0	2	1			
	4	438,2	416,2	416,2	-5	0	17	27			
	5	362,1	279,4	279,4	-22,8	0	0	1			
	6	424,2	381,8	381,8	$^{-10}$	0	0	0			
	7	424,1	350,6	350,6	-17,3	0	0	0			
	8	393	334,3	334,3	-14,9	0	0	0			
	9	446,1	398,7	398,7	-10,6	0	0	1			
	10	412	368,4	368,4	-10,6	0	0	0			
Avg. 40	_	414,9	360,8	360,8	-13,1	0	1,9	3			
40	1	440,2	392,9	392,9	-10,7	0	0	0			
	2	367,8	325,9	325,9	-11,4	0	0	1			
	3	508,7	432	432	-15,1	0	1	2			
	4	455,2	433,3	433,3	-4,8	0	15	37			
	5	379,1	307,8	307,8	-18,8	0	1	0			
	6	435,1	412,4	412,4	-5,2	0	0	0			
	7	439,9	439	439	-0,2	0	0	0			
	8	404,3	349,4	349,4	-13,6	0	0	1			
	9	467,9	420,1	420,1	-10,2	0	1	1			
	10	439,5	387,1	387,1	-11,9	0	0	0			
Avg.	_	433,8	390,0	390,0	-10,2	0	1,8	4,2			
50	1	458,5	423	423	-7,7	0	0	1			
	2	380,9	346,5	346,5	-9	0	1	1			
	3	522,4	458,7	458,7	-12,2	0	4	3			
	4	480,3	450,8	450,8	-6,1	0	40	25			
	5	430,7	340,7	340,7	-20,9	0	1	1			
	6	467	441,5	441,5	-5,5	0	0	0			

BR%	id	Objective Fun	ction (min.)			Computatio	nal Time (s)	
		Ind.	Int_c	Int_g	Var (%)	Ind.	Int_c	Int_g
	7	466,7	453,3	453,3	-2,9	0	0	0
	8	403,3	371,7	371,7	-7,8	0	1	1
	9	484,7	446,9	446,9	-7,8	0	2	2
	10	450,1	407,9	407,9	-9,4	0	0	0
Avg. 50	—	454,5	414,1	414,1	-8,9	0	4,9	3,4
50	1	471,7	441,5	441,5	-6,4	0	1	1
	2	420,9	377,3	377,3	-10,4	0	1	0
	3	539,3	487,4	487,4	-9,6	0	4	4
	4	490	490	490	0	0	46	37
	5	458,5	363,4	363,4	-20,7	0	3	2
	6	486,8	462	462	-5,1	0	0	0
	7	453,3	453,3	453,3	0	0	0	0
	8	442,9	408	408	-7,9	0	3	3
	9	503,6	469,1	469,1	-6,9	0	4	1
	10	467,2	424,6	424,6	-9,1	0	1	0
Avg.		473,4	437,7	437,7	-7,6	0	6,3	4,8
70	1	486,4	455,8	455,8	-6,3	0	1	1
	2	423,6	397,1	397,1	-6,3	0	1	1
	3	546,7	513,6	513,6	-6,1	0	7	3
	4	505,1	505,1	505,1	0	0	44	35
	5	438,4	374,8	374,8	-14,5	0	1	2
	6	562,9	520,7	520,7	-7,5	0	0	0
	7	482,2	482,2	482,2	0	0	0	0
	8	452,9	427,9	427,9	-5,5	0	8	6
	9	522,8	497,5	497,5	-4,8	0	3	3
	10	482,3	441,8	441,8	-8,4	0	0	1
Avg. 30	—	490,3	461,7	461,7	-5,9	0	6,5 1	5,2
30	1	503,3	472,7	472,7	-6,1	0	1	0
	2	505,9	437,3	437,3	-13,6	0	2	3
	3	573,6	541	541	-5,7	0	5	4
	4	518,2	518,2	518,2	0	0	14	15
	5	448,7	399,3	399,3	-11	0	2	1
	6	574,8	540,5	540,5	-6	0	0	0
	7	511,1	511,1	511,1	0	0	0	0
	8	488,3	457,8	457,8	-6,2	0	19	10
	9	549,1	533,4	533,4	-2,9	0	6	4
	10	510,1	464,5	464,5	-8,9	0	1	1
Avg.	_	518,3	487,6	487,6	-6,0	0	5	3,8
Avg. 90	1	514,7	496,1	496,1	-3,6	0	2	2
	2	517,1	467,3	467,3	-9,6	0	4	4
	3	595,5	563,3	563,3	-5,4	0	6	4
	4	533	533	533	0	0	10	3
	5	421,1	409,6	409,6	-2,7	0	2	3
	6	592,6	561,7	561,7	-5,2	0	0	0
	7	546,4	526,7	526,7	-3,6	0	0	0
	8	493,6	472,4	472,4	-4,3	0	27	31
	9	578,9	564,8	564,8	-2,4	0	8	5
	10	523,3	479,5	479,5	-8,4	0	0	0
Avg.	—	531,6	507,4	507,4	-4,5	0	5,9	5,2

E. Team scheme results of tests with size-25 instances

BR%	id	Indepen	dent		Integrat	ed CPLEX			Integrat	ed Gurobi		
		#S	#D	#Cgv	#S	#D	#Cgv	var_Cgv	#S	#D	#Cgv	var_Cgv
10	1	3	1	5	4	0	4	-1	4	0	4	-1
	2	2	1	4	4	0	4	0	4	0	4	0
	3 4	3 3	1 1	5 5	4 4	0 0	4 4	$^{-1}$	4 4	0 0	4 4	$^{-1}$
	4 5	3	1	5	3	0	3	$^{-1}$ -2	3	0	3	$^{-1}$ -2
	6	3	1	5	4	0	4	$^{-1}$	4	0	4	-1
	7	3	1	5	4	0	4	-1	4	0	4	$^{-1}$
	8	3	1	5	4	0	4	-1	4	0	4	-1
	9 10	2 2	1 1	4 4	4 4	0 0	4 4	0 0	3 4	0 0	3 4	$-1 \\ 0$
<u> </u>												
Avg.	—	2,7	1	4,7	3,9	0	3,9	-0,8	3,8	0	3,8	-0,9
20	1	3	1	5	4	0	4	-1	4	0	4	-1
	2 3	2 2	1 1	4 4	4 4	0 0	4 4	0 0	4 4	0 0	4 4	0 0
	4	3	1	5	5	0	5	0	5	0	5	0
	5	3	1	5	4	0	4	-1	4	0	4	-1
	6	2	1	4	3	0	3	$^{-1}$	3	0	3	$^{-1}$
	7	3	1	5	5	0	5	0	5	0	5	0
	8 9	3 2	1 1	5 4	4 4	0 0	4 4	$-1 \\ 0$	4 4	0 0	4 4	$-1 \\ 0$
	10	2	1	4	4	0	4	0	4	0	4	0
A												
Avg.	_	2,5	1	4,5	4,1	0	4,1	-0,4	4,1	0	4,1	-0,4
30	1	1	2	5	4	0	4	-1	5	0	5	0
	2 3	2 1	1 2	4 5	5 5	0 0	5 5	1 0	5 5	0 0	5 5	1 0
	4	3	1	5	5	0	5	0	5	0	5	0
	5	2	1	4	5	0	5	1	5	0	5	1
	6	2	1	4	3	1	5	1	5	0	5	1
	7	3	1	5	5	0	5	0	5	0	5	0
	8 9	1 2	2	5 4	5	0 0	5 4	0 0	5 4	0 0	5 4	0 0
	9 10	2	1 1	4	4 5	0	4 5	0	4	0	4	0
A		<u>-</u> 1,9	1,3	4,5	4,6	0,1	4,8	0,3	4,8	0	4,8	0,3
Avg. 40	-		$\frac{1,3}{2}$		5				4,8 5			
40	1 2	1 2	2	5 4	5	0 0	5 5	0 1	5	0 0	5 5	0 1
	3	1	2	5	5	0	5	0	5	0	5	0
	4	1	2	5	5	0	5	0	5	0	5	0
	5	1	2	5	5	0	5	0	5	0	5	0
	6	2	1 2	4	5	0	5	1	5	0	5	1
	7 8	1 1	2	5 5	5 5	0 0	5 5	0 0	5 5	0 0	5 5	0 0
	9	1	2	5	5	0	5	0	5	0	5	0
	10	2	1	4	4	0	4	0	4	0	4	0
Avg.	_	1,3	1,7	4,7	4,9	0	4,9	0,2	4,9	0	4,9	0,2
	1		2									
50	1 2	1 2	2	5 4	5 5	0 0	5 5	0 1	5 5	0 0	5 5	0 1
	3	1	2	5	5	0	5	0	5	0	5	0
	4	1	2	5	5	0	5	0	5	0	5	0
	5	1	2	5	5	0	5	0	5	0	5	0
	6 7	1 1	2 2	5 5	5 5	0 0	5 5	0	5 5	0 0	5 5	0 0
	8	1	2	5	5	0	5	0	5	0	5	0
	9	1	2	5	5	0	5	0	5	0	5	0
	10	1	2	5	5	0	5 5	0	4	0	4	-1
Avg.	_	1,1	1,9	4,9	5	0	5	0,1	4,9	0	4,9	0
Avg. 60	<u></u>	$\frac{\frac{1}{1,1}}{1}$	2 2 1,9 2 2 2 2 2 2 2 2	5 5 4,9 5 5	5 5 5 5 5 3	$\frac{0}{0}$	5	0	5 4 4,9 5 5	$\frac{0}{0}$	5 4 4,9 5	0
00	1 2	1	∠ 2	5	3	0	5	0	5	0	5	0
	3	1	2	5	4	0	4	-1	4	0	4	$^{-1}$
	4	1	2	5	5	0	5	0	5	0	5	0
	5	1		5	5	0	5	0	5	0	5	0
	6 7	1 1	2	5 5	5 5	0	5	0 0	5 5	0	5	0 0
	8	1	2 2	5		0 0	5 5	0	5	0 0	5 5	0
	9	1	2	5	5	0	5	0	5	0	5 5	0
	10		2	5	5	0	5	0	5	0	5	0
	10											
Avg.	10	1 1	2 2 2 2	5 5	5 5 5 4,7	0 0,1	5 5 5 4,9	-0,1	5 4,9	0 0	5 4,9	-0,1

BR%	id	Indepen	ident		Integrat	ed CPLEX			Integrat	ed Gurobi		
		#S	#D	#Cgv	#S	#D	#Cgv	var_Cgv	#S	#D	#Cgv	var_Cg
70	1	1	2	5	5	0	5	0	5	0	5	0
	2	1	2	5	1	2	5	0	3	1	5	0
	3	1	2	5	4	0	4	$^{-1}$	4	0	4	$^{-1}$
	4	1	2	5	3	1	5	0	3	1	5	0
	5	1	2	5	3	1	5	0	5	0	5	0
	6	1	2	5	5	0	5	0	5	0	5	0
	7	1	2	5	5	0	5	0	5	0	5	0
	8	1	2	5	5	0	5	0	5	0	5	0
	9	1	2	5	5	0	5	0	5	0	5	0
	10	1	2	5	5	0	5	0	3	1	5	0
Avg.		1	2	5	4,1 5	0,4	4,9	-0,1	4,3 5	0,3	4,9	-0,1
80	1	1	2	5	5	0	5	0	5	0	5	0
	2	1	2	5	_	_	_	_	3	1	5	0
	3	1	2	5	5	0	5	0	5	0	5	0
	4	1	2	5	3	1	5	0	3	1	5	0
	5	1	2	5	5	0	5	0	5	0	5	0
	6	1	2	5	5	0	5	0	5	0	5	0
	7	1	2	5	3	1	5	0	3	1	5	0
	8	1	2	5	5	0	5	0	5	0	5	0
	9	1	2	5	5	0	5	0	5	0	5	0
	10	1	2	5	2	1	4	$^{-1}$	3	1	5	0
Avg.		1	2	5	4,2	0,3	4,9	-0,1	4,2	0,4	5	0
90	1	1	2	5	3	1	5	0	3	1	5	0
	2	1	2	5	3	1	5	0	2	1	4	-1
	3	1	2	5	2	1	4	-1	2	1	4	-1
	4	1	2	5	3	1	5	0	3	1	5	0
	5	1	2	5	1	2	5	0	5	0	5	0
	6	1	2	5	3	1	5	0	5	0	5	0
	7	1	2	5	3	1	5	0	3	1	5	0
	8	1	2	5	2	1	4	-1	4	0	4	$^{-1}$
	9	1	2	5	3	1	5	0	5	0	5	0
	10	1	2	5	0	2	4	-1	2	1	4	-1
Avg.	_	1	2	5	2,3	1,2	4,7	-0,3	3,4	0,6	4,6	-0,4

F. Computational results of tests with size-25 instances

BR%	id	Objective Function	on						Computational Time (s)		
		Independent	CPLEX			Gurobi					
			OF	gap	Var(%)	OF	gap	Var(%)	Ind.	CPLEX	Gurobi
10	1	716,2	651,8	0	-9	651,8	0	-9	9	103	79
	2	549,5	521,2	0	-5,2	521,2	0	-5,2	9	4879	1908
	3	665	641,1	0	-3,6	641,1	0	-3,6	10	10	4
	4	762	707,6	0	-7,1	707,6	0	-7,1	11	95	52
	5	570,8	505,8	0	-11,4	505,8	0	-11,4	66	435	318
	6	819,7	738,7	0	-9,9	738,7	0	-9,9	61	2353	767
	7	599,7	526,8	0	-12,2	526,8	0	-12,2	6	163	105
	8	659,6	585,8	0	-11,2	585,8	0	-11,2	11	174	85
	9	814	727	5,5	-10,7	724,3	1,8	-11	311	10800	10800
	10	730	669,2	2,2	-8,3	669,2	0	-8,3	83	10800	1820
Avg.	_	688,7	627,5	0,8	-8,9	627,2	0,2	-8,9	57,7	2981,2	1593,8
20	1	820,7	681,6	0	-16,9	681,6	0	-16,9	1090	89	88
	2	595	562,6	2,8	-5,4	562,6	0	-5,4	6	10800	4801
	3	741,6	688,7	0	-7,1	688,7	0	-7,1	2	41	14
	4	819,8	759,5	0	-7,4	759,5	0	-7,4	9	972	326
	5	633,8	551,1	2,6	$^{-13}$	551,1	0	$^{-13}$	64	10800	5786
	6	848,8	790,2	2,8	-6,9	790,2	0	-6,9	49	10800	1610
	7	643,6	565,8	0	-12,1	565,8	0	-12,1	4	587	175
	8	689,7	618,8	0	-10,3	618,8	0	-10,3	7	144	72
	9	915,7	786,6	4,3	-14,1	786,6	1	-14,1	493	10800	10800
	10	833	715	0,7	-14,2	715	0	-14,2	482	10800	2816
Avg.	_	754,2	672,0	1,3	-10,7	672,0	0,1	-10,7	220,6	5583,3	2648,8
30	1	843,3	742,7	0	-11,9	742,7	0	-11,9	13	1128	331
	2	685,1	649,9	4,6	-5,1	649,9	3,7	-5,1	2	10800	10800
	3	835,4	766,1	0	-8,3	766,1	0	-8,3	7	372	100
	4	869,1	825,2	0	-5,1	825,2	0	-5,1	5	4610	498
										(continued o	n next page

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BR%	id	Objective Function	on	Objective Function							
		Independent	Gurobi	Gurobi							
		I.	OF	gap	Var(%)	OF	gap	Var(%)	Ind.	CPLEX	Gurol
	5	685,2	604,9	3,8	-11,7	604,9	0	-11,7	18	10800	7189
	6	949,6	884,4	3	-6,9	884,4	1,8	-6,9	8	10800	10800
	7	692	620,1	0	-10,4	620,1	0	-10,4	5	92	94
	8	788	688,6	0	-12,6	688,6	0	-12,6	12	1272	341
	9	967,7	852,9	4,7	-11,9	852,9	0	-11,9	49	10800	10230
	10	908,8	797,9	2,9	-12,2	795,2	1	-12,5	102	10800	10800
Avg.		822,4	743,3	1,9	-9,6	743,0	0,7	-9,6	22,1	6147,4	5118,
0	1	884,9	786,8	0,7	-11,1	786,8	0	-11,1	10	10800	781
	2	702,6	675,1	4,6	-3,9	675,1	3	-3,9	3	10800	10800
	3	877,4	811,1	0	-7,6	811,1 892,4	0 0	-7,6 -8,3	1 7	493	214
	4 5	973,6 747,3	892,4 649,4	1 5,3	-8,3 -13,1	892,4 649,4	0 1,5	-8,3 -13,1	12	10800 10800	2318 1080
	6	1007,5	946,1	6	-6,1	931,4	1,3	-7,6	8	10800	1080
	7	763,4	668,8	0	-12,4	668,8	0	-12,4	8	4533	529
	8	847,9	733,1	0	-13,5	733,1	0	-13,5	8	1567	446
	9	988,7	901,6	6,6	-8,8	901,6	1,3	-8,8	10	10800	1080
	10	965,8	843,2	4,3	-12,7	832,8	0	-13,8	321	10800	10692
vg.		875,9	790,8	2,9	-9,8	788,3	0,7	-10,0	38,8	8219,3	5818,
0	1	950	870,2	0,6	-8,4	870,2	0	-8,4	7	10800	934
	2	812,4	737,8	4,6	-9,2	737,8	2,6	-9,2	16	10800	1080
	3	918,7	868,2	0	-5,5	868,2	0	-5,5	1	597	311
	4	1020,9	957,9	1,3	-6,2	957,9	0	-6,2	7	10800	1349
	5 6	784,3 1102,3	690,6 1017,2	2,4 4,8	-11,9 -7,7	690,6 1015,7	0 2,2	$^{-11,9}$ -7,9	5 55	10800 10800	1224 1080
	7	811	717,3	2,2	-11,6	717,3	0	-11,6	2	10800	2709
	8	869,7	783,4	0,4	-9,9	783,4	0	-9,9	7	10800	365
	9	1016,2	970	6,4	-4,5	970	2,1	-4,5	5	10800	1080
	10	935,5	905,2	1,3	-3,2	900,3	0	-3,8	1	10800	2018
vg.		922,1	851,8	2,4	-7,8	851,1	0,7	-7,9	10,6	9779,7	4131
0	1	938,6	901,5	1,1	-4	901,5	0	-4	2	10800	627
•	2	870,7	839,1	10,1	-3,6	789,1	3,3	-9,4	9	10800	1080
	3	975,7	914,4	0	-6,3	914,4	0	-6,3	3	840	325
	4	1088,8	1035,5	4,5	-4,9	1030,1	1,8	-5,4	11	10800	1080
	5	816,6	737,1	6,2	-9,7	733,4	3,1	-10,2	9	10800	1080
	6	1129,6	1061,9	4,5	-6	1061,9	0,4	-6	3	10800	1080
	7 8	849,2	757,2	1 0	-10,8 -9	757,2	0 0	-10,8 -9	2 7	10800	3403 380
	8 9	894,3 1046,6	813,8 1029	0 9,4	_9 _1,7	813,8 1009,3	4	-9 -3,6	2	5682 10800	380 1080
	10	999,6	953,2	3,8	-4,6	953,2	0	-3,0 -4,6	2	10800	9360
vg.	_	961,0	904,3	4,1	-6,1	896,4	1,3	-6,9	5,0	9292,2	6809
0	1	974	947,5	0	-2,7	947,5	0	-2,7	1	3897	519
	2	886	888	11,2	0,2	868,4	8,1	-2	11	10800	1080
	3 4	1056,9 1140,1	983,3 1119,3	0 6,5	-7 -1,8	983,3 1119,4	0 5,3	-7 -1,8	6 10	1178 10800	517 1080
	5	860,9	804,8	0,3 9,1	-6,5	791,1	3,3 4,4	-1,8 -8,1	54	10800	1080
	6	1183,1	1127,1	3	-4,7	1127,1	0	-4,7	3	10800	7465
	7	891,4	818	3,7	-8,2	816,2	1,6	-8,4	14	10800	1080
	8	921,6	867,1	0	-5,9	867,1	0	-5,9	1	1746	344
	9	1127,6	1096,5	9,2	-2,8	1084,6	3,6	-3,8	37	10800	1080
	10	1074,1	1071,6	5,2	-0,2	1055,3	0	-1,8	11	10800	3971
vg.		1011,6	972,3	4,8	-4,0	966,0	2,3	-4,6	14,8	8242,1	6681
0	1	1008,8	993,8	1,5	-1,5	993,4	0	-1,5	1	10800	3351
	2	933,6	-	_	-	901,8	7,6	-3,4	16	10800	1080
	3	1072,9	1027,8	0	-4,2	1027,8	0	-4,2	1	503	59
	4	1202,8	1164,6	6	-3,2	1162,7	5	-3,3	36	10800	1080
	5	888,2	819,7	6,7	-7,7	819,8	4,3	-7,7	109	10800	1080
	6 7	1236,5 914,1	1184,1 865	4,5	-4,2	1179,6 865	1,4 0	-4,6 -5,4	9	10800 10800	1080 6831
	8	914,1 923,1	805 899	3,3 0	$^{-5,4}_{-2,6}$	805 899	0	-5,4 -2,6	8 0	10800	488
	9	1184	1139,3	7	-3,8	1140,1	5	-3,7	70	10800	1080
	10	1127,2	1116,5	7,7	-0,9	1100,1	1,2	-2,4	9	10800	1080
vo		1049,1	1023,3		-3,7	1008,9		-3,9	25,9	8875,7	7552
vg.	—			4,1			2,5				
0	1	1070,9	1055,7	2,1	-1,4	1055,7	0	-1,4	9	10800	1780
	2	958,2	932,2	6,5	-2,7	920,6	4	-3,9	91	10800	1080
	3 4	1103,2 1226,2	1083,5 1198,1	0 34	$^{-1,8}$ -2,3	1083,5 1198,1	0 18	$^{-1,8}$ -2,3	5 28	222 10800	36 1080
	4 5	1226,2 892,8	862,3	3,4 8,8	-2,3 -3,4	875,2	1,8 6,8	$^{-2,3}$	28 79	10800	1080
	5	0,2,0	002,0	0,0	-3,7	070,2	0,0	-2	12	10000	1000

21

BR%	id	Objective Function	on	Computational Time (s)							
		Independent	CPLEX			Gurobi					
			OF	gap	Var(%)	OF	gap	Var(%)	Ind.	CPLEX	Gurobi
	6	1271,7	1250,5	5,5	-1,7	1243	0,9	-2,3	58	10800	10800
	7	928	895,6	2,3	-3,5	895,6	0	-3,5	6	10800	1006
	8	981,9	954,6	0,7	-2,8	954,6	0	-2,8	4	10800	640
	9	1241,9	1212,8	8	-2,3	1212	6,7	-2,4	891	10800	10800
	10	1171,4	1166,7	6,2	-0,4	1137	0	-2,9	168	10800	5397
Avg.	_	1084,6	1061,2	4,4	-2,2	1057,5	2,0	-2,5	133,9	9742,2	6285,9

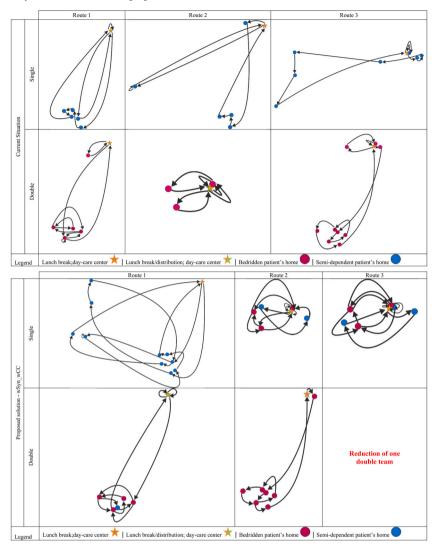
G. Results of solution quality analysis at 0.5h and 3h for size-25 instances

BR%	id	Objective Function	on (min.)		gap (%)	gap (%)		
		3 h	0.5 h	Var (%)	3 h	0.51		
10	1	651,8	651,8	0,00	0	0		
	2	521,2	521,2	0,00	0	0,8		
	3	641,1	641,1	0,00	0	0		
	4	707,6	707,6	0,00	0	0		
	5	505,8	505,8	0,00	0	0		
	6	738,7	738,7	0,00	0	0		
	7	526,8	526,8	0,00	0	0		
	8	585,8	585,8	0,00	0	0		
	9	724,3	724,8	0,07	1,8	4,2		
	10	669,2	669,2	0,00	0	0,2		
A	10							
Avg.	_	627,2	627,3	0,0	0,2	0,5		
20	1	681,6	681,6	0,00	0	0		
	2	562,6	562,6	0,00	0	2,3		
	3	688,7	688,7	0,00	0	0		
	4	759,5	759,5	0,00	0	0		
	5	551,1	551,5	0,07	0	2,8		
	6	790,2	790,2	0,00	0	0		
	7	565,8	565,8	0,00	0	0		
	8	618,8	618,8	0,00	0	0		
	9	786,6	786,6	0,00	1	2,9		
	10	715	715	0,00	0	2,9 0,7		
Avg.		672,0	672,0	0,0	0,1	0,9		
30	1	742,7	742,7	0,00	0	0		
50	2	649,9	650,7	0,12	3,7	4,7		
	3	766,1	766,1	0,00	0	,,, 0		
	4	825,2	825,2	0,00	0	0		
	5	604,9	604,9	0,00	0	2,4		
	6	884,4	887,6	0,36	1,8	3,3		
	7	620,1	620,1	0,00	0	0		
	8	688,6	688,6	0,00	0	0		
	9	852,9	852,9	0,00	0	3,6		
	10	795,2	801,7	0,82	1	3,0 2,6		
	10							
Avg.	_	743,0	744,1	0,1	0,7	1,7		
40	1	786,8	786,8	0,00	0	0		
	2	675,1	675,1	0,00	3	3,9		
	3	811,1	811,1	0,00	0	0		
	4	892,4	892,4	0,00	0	0,4		
	5	649,4	649,4	0,00	1,5	2,8		
	6	931,4	932,9	0,16	1,3	2,9		
	7	668,8	668,8	0,00	0	0		
	8	733,1	733,1	0,00	0	0		
	9	901,6	901,7	0,01	1,3	3,2		
	10	832,8	832,8	0,00	0	1		
Avg.	—	788,3	788,4	0,0	0,7	1,4		
	1							
50	1	870,2	870,2	0,00	0	0		
	2	737,8	737,8	0,00	2,6	3,3		
	3	868,2	868,2	0,00	0	0		
	4	957,9	957,9	0,00	0	0		
	5	690,6	690,6	0,00	0	0		
	6	1015,7	1015,7	0,00	2,2	3,6		
	7	717,3	717,3	0,00	0	0,5		
	8	783,4	783,4	0,00	0	0		
	9	970	981	1,13	2,1	5,9		

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BR%	id	Objective Function	gap (%)			
		3 h	0.5 h	Var (%)	3 h	0.51
	10	900,3	900,3	0,00	0	0,3
Avg.	—	851,1	852,2	0,1	0,7	1,4
60	1		901,5	0,00	0	0
60	1 2	901,5 789,1	901,5 807,9	2,38	3,3	0 6,7
	2 3	914,4	807,9 914,4	2,38	3,3 0	0,7
	4	1030,1	1030,1	0,00	1,8	2,2
	5	733,4	733,4	0,00	3,1	4
	6	1061,9	1061,9	0,00	0,4	4
	7	757,2	757,2	0,00	0,4	0,9
	8	813,8	813,8	0,00	0	0,5
	9	1009,3	1024,5	1,51	4	6,5
	10	953,2	953,2	0,00	0	1,4
	10					
Avg.		896,4	899,8	0,4	1,3	2,3
70	1	947,5	947,5	0,00	0	0
, 0	2	868,4	868,4	0,00	8,1	9
	3	983,3	983,3	0,00	0	0
	4	1119,4	1119,4	0,00	5,3	5,9
	5	791,1	791,9	0,10	4,4	6
	6	1127,1	1127,1	0,00	0	1,5
	7	816,2	821,5	0,65	1,6	3,3
	8	867,1	867,1	0,00	0	0
	9	1084,6	1094,8	0,94	3,6	6
	10	1055,3	1072,9	1,67	0	3,7
Avg.	—	966,0	969,4	0,3		3,5
	-				2,3	
80	1	993,4	993,4	0,00	0	0,8
	2	901,8	901,8	0,00	7,6	8,3
	3	1027,8	1027,8	0,00	0	0
	4	1162,7	1192,45	2,56	5	8
	5	819,8	819,8	0,00	4,3	5,2
	6	1179,6	1183,3	0,31	1,4	2,4
	7	865	865	0,00	0	1,4
	8	899	899	0,00	0	0
	9	1140,1	1140,1	0,00	5	6
	10	1100,1	1100,8	0,06	1,2	2,8
Avg.		1008,9	1012,3	0,3	2,5	3,5
90	1	1055,7	1055,7	0,00	0	0
	2	920,6	941,1	2,23	4	6,8
	3	1083,5	1083,5	0,00	0	0
	4	1198,1	1215,5	1,45	1,8	4,1
	5	875,2	882,5	0,83	6,8	9,2
	6	1243	1243,7	0,06	0,9	2,1
	7	895,6	895,6	0,00	0	0
	8	954,6	954,6	0,00	0	0
	9	1212	1217,4	0,45	6,7	7,7
	10	1137	1137	0,00	0	1,4
Avg.	—	1057,5	1062,7	0,5	2,0	3,1

H. Geographical representation of current situation and proposed solution



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