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Isosceles Triangular Plate Under Uniform Bending Moment

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Abstract

Making use of the complex function often leads us to the solution of explicit form for the various problems in the field of mathematical physics. It is same with the bending problem of the thin plate, and many investigators found the solutions of explicit form for the peculiar case of the thin plate, which could otherwise hardly be obtained. In this paper, a new method of solving the bending of simply supported isosceles triangular plate under uniform bending moment by means of the complex function, is presented. The solution of explicit form is given to the right isosceles triangular plate under uniform bending moment.

1. Fundamental Equation

The bending of plate is governed by the equations as follows;

$$\Delta w = -M/D, \quad (1)$$

$$\Delta M = -q, \quad (2)$$

where w: deflection of the plate,

$$M = (M_x + M_y)/(1+\nu),$$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right),$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^1 w}{\partial x^2}\right),$$

$$D = Eh^3/12(1-\nu^2),$$

h: thickness of the plate,

ν: Poison's ratio.

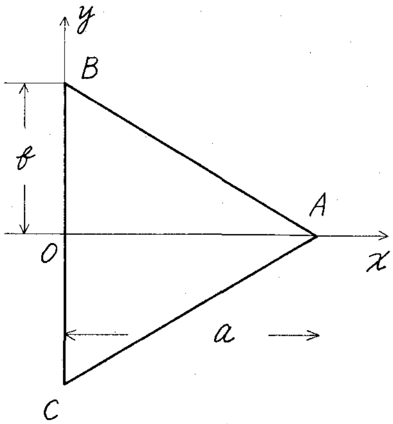

Fig. 1.

The symbolic notation Δ denotes the harmonic differentiation:

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$$

in which

$$\begin{matrix} z \\ \bar{z} \end{matrix} = x \pm iy,$$

and the equation (1) becomes

$$4\frac{\partial^2 w}{\partial z \partial \bar{z}} = -\frac{M}{D}, \quad (3)$$

which yields for the constant M

$$w=-\frac{M}{4D}z\bar{z}+\sum_{n}A_{n}z^{n}+\sum_{n}\bar{A}_{n}\bar{z}^{n}\,. \tag{4}$$

2. Boundary Conditions

A_n, $\bar{A}_n$ in the equation (4), are the integration constants which should be determined so as to satisfy the boundary conditions. Choosing the isosceles triangle as shown in Fig. 1, and supposing all sides are simply supported and bent by the moment M, we may write that the deflection vanishes at the edge $x=0$, as

$$\text{for } z+\bar{z}=0 \quad w=0\,, \tag{5}$$

and the symmetry of w with respect to the axis $y=0$, as

$$\text{for } z-\bar{z}=0 \qquad \frac{\partial w}{\partial z}-\frac{\partial w}{\partial \bar{z}}=0\,. \tag{6}$$

Thus, we have

$$A_n+\bar{A}_n(-1)^n=0\,, \qquad (n\neq 2) \tag{7}$$

$$A_2+\bar{A}_2=-M/(4D)\,, \tag{8}$$

$$A_n-\bar{A}_n=0\,, \tag{9}$$

from which n has to be an odd integer and A_n is always real.

The remainning boundary is identified by

$$my+x-a=0, \tag{10}$$

that is

$$\zeta+\bar{\zeta}=0$$

where

$$\zeta=z(1-mi)-a\,, \qquad m=a/b\,. \tag{11}$$

Thus, the simply supported condition at the above edge is expressed by

$$\text{for } \zeta=-\bar{\zeta}\,, \qquad w=0\,.$$

According to the symmetry with respect to the x axis, w is also cancelled along the line AC.

The substitution of the equation (11) into the equation (4), yields

$$w=-\frac{M}{4D}\frac{(\zeta+a)(\bar{\zeta}+a)}{(1+m^2)}+\sum_{n}A'_n\Big\{(\zeta+a)^n(1+mi)^n+(\bar{\zeta}+a)^n(1-mi)^n\Big\}\,,$$

where

$$A'_n=A_n/(1+m^2)^n\,, \qquad n=1,3,5,\cdots$$

Putting $\bar{\zeta}=-\zeta$ into the above, we have from the equation (10)

$$\sum_n A'_n\left\{(a+\zeta)^n(1+mi)^n+(a-\zeta)^n(1-mi)^n\right\}$$
$$+A_2\left\{\frac{2(a^2-\zeta^2)}{(1+m^2)}+\frac{(\zeta+a)^2(1+mi)^2}{(1+m^2)}+\frac{(\zeta-a)^2(1-mi)^2}{(1+m^2)^2}\right\}=0\,, \qquad (12)$$

which follows that

$$A'_n=0\,, \quad \text{for } n>3\,,$$

as well as

$$A'_3\left[(a^3+3a\zeta^2)(1-3m^2)-i(3a^2\zeta+\zeta^3)(3m-m^3)\right]$$
$$+\frac{A_2}{(1+m^2)^2}\left[(a^2+\zeta^2)(1-m^2)-4a\zeta mi+(1+m^2)(a^2-\zeta^2)\right]+A'_1(a-\zeta mi)=0\,.$$

from which we find that

$$A_1=-A_2a/4\,, \qquad A_3=-A_2/(32a)\,, \qquad m^2=3\,, \qquad A_2=-M/(8D)\,,$$
$$b=a/\sqrt{3}$$

and

$$w=A_2(z^2+\bar{z}^2+2z\bar{z})-\frac{A_2}{32a}(z^3+\bar{z}^3)-\frac{A_2a}{4}(z+\bar{z})\,, \qquad (13)$$

which is the solution of the equi-lateral triangular plate and identical with what S. Woinowsky-Krieger found out.

3. Linear Transform

Let

$$z=x+icy\,, \qquad (14)$$

and

$$4\frac{\partial^2 w}{\partial z\partial\bar{z}}=\left(\frac{\partial^2 w}{\partial x^2}+\frac{1}{c^2}\frac{\partial^2 w}{\partial y^2}\right), \qquad (15)$$
$$\zeta+\bar{\zeta}=x+cmy-a\,.$$

The replacement of z by $x+icy$ into the equation (13), yields the deflection of another isosceles triangular plate under another distribution of M which will be given in the following discussion.

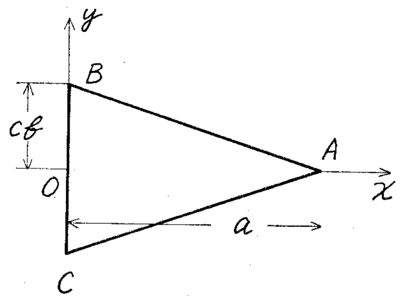

Fig. 2.

Since

$$4\frac{\partial^2 w}{\partial z\partial\bar{z}}=-M/D\,,$$

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=-M/D+\left(1-\frac{1}{c^2}\right)\frac{\partial^2 w}{\partial y^2}\,,$$

into which putting

$$-\frac{\partial^2 w}{c^2 \partial y^2}=\frac{\partial^2 w}{\partial z^2}+\frac{\partial^2 w}{\partial \bar{z}^2}-2\frac{\partial^2 w}{\partial z \partial \bar{z}},$$

we have

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=-\frac{1+c^2}{2}\frac{M}{D}-\frac{Mx(1-c^2)}{16aD}. \tag{16}$$

the right side of which is the distribution of M corresponding to the transformation mentioned above.

4. One Mohr Linear Distribution of M

Let $-\frac{M}{4D}=p+2q(z+\bar{z})$, then

$$w=pz\bar{z}+qz\bar{z}(z+\bar{z})+p(z^2+\bar{z}^2)\frac{1}{2}+A(z^3+\bar{z}^3)+A_1(z+\bar{z}) \tag{17}$$

which satisfys the conditons (5) and (6), and the remainning condition

$$\zeta+\bar{\zeta}=0, \quad w=0,$$

yields

$$A=-\frac{1+m_2}{8a}p, \quad A_1=-\frac{a}{2}p$$

$$q=\frac{m^2-3}{8a}p.$$

So that the deflection w takes the follwing form:

$$w=\frac{1}{2}p(z+\bar{z})^2-\frac{ap}{2}(z+\bar{z})-\frac{p}{8a}m^2(z-\bar{z})^2(z+\bar{z})-\frac{p}{8a}(z+\bar{z})^3. \tag{18}$$

and the corresponding M is expressed by

$$M=-4Dp\left\{1+\frac{1}{2}(m^2-3)\cdot\frac{x}{a}\right\}. \tag{19}$$

Cancelling the term of x/a between the right side of (16) and (19), we have

$$2Dp(m^2-3)-M(1-c^2)/(16)=0, \tag{20}$$

in which m is an index of the triangle and must be given as to equate the both triangular shape, namely; $m^2=3/c^2$.

5. Solution for Isosceles Triangular Plate

Writing the right side of the equation (14), we have

$$2(x+icy)=z(1+c)+\bar{z}(1-c),$$

which is substituted for z into (13), the expression of the deflection of the transformed triangular plate may be obtaind. Thus, we finally come to the deflection by the supperposition of the both results with the relation (20), as follows:

$$w=-\frac{Mc^2(1-c^2)}{192D}\left\{(z+\bar{z})^2-a(z+\bar{z})-\frac{3}{4ac^2}(z-\bar{z})^2(z+\bar{z})-\frac{1}{4a}(z+\bar{z})^3\right\}$$
$$+\frac{M}{8D}\left[\frac{x^3}{4a}-\frac{3x}{2a}\left\{z(1+c)+\bar{z}(1-c)\right\}\left\{\bar{z}(1+c)+z(1-c)\right\}+\frac{ax}{2}-4x^2\right]. \tag{21}$$

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