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FORECASTING OUTPUT GROWTH TAIL RISK USING QUANTILE REGRESSION FRAMEWORK

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Abstract

This thesis examines which financial indicator is the most accurate to model and predict the tail risk of output growth in the euro area. The CISS is more informative than other indicators that only focus on specific segment of the financial market. To capture the tail distribution information, the thesis implements quantile regression, capturing determined quantile of the output growth distribution. The forecast produced with the quantile regression for the 10^{th} and the 5^{th} quantile outperformed the standard OLS model in terms of forecasting evaluation metrics in predicting the 2008 output growth downfall, concluding that the quantile specification, combined with the CISS as financial indicator, improves the modeling and forecasting accuracy of tail risk output growth in the euro area.

Keywords: Quantile Regression; Growth at Risk; Macro-financial linkages; Time Series

1 Introduction

The first purpose of the thesis is to examine and define which measure of financial conditions is the most informative about the tail risks to output growth in the euro area, which will be defined in the thesis as the bottom 10^{th} quantile. Further analysis will also be provided for the bottom 5^{th} quantile, which can give a better understanding of the euro area output growth risk, especially if we apply this analysis to the 2008 financial crisis, where the magnitude of the shock can be considered as an extreme tail event. The second purpose, but also main to the thesis, is to find, and then demonstrate if it is present, the advantage of using a quantile regression framework while trying to model and forecast output growth, and in particular output growth declines such as recessions. The thesis will apply quantile regression framework to the 2008 financial crisis to predict the output growth downfall using each bottom quantile, especially the 5^{th} and the 10^{th}, as they should be representative of the output growth tail risk.

2 Literature

Considering the recent literature, Adrian et al (2019) found that financial conditions play a critical role for forecasting the distribution of future output growth in the US economy. During the recent years of financial turmoil, policy makers have been drawn towards the relationship between financial variables and output growth. This thesis contributes to the recent literature on tail risk to output growth, literature composed by papers such as Adrian et al (2019); Brownlees and Souza (2019); Reichlin et al (2019); Chaveleishvili and Manganelli (2019). This thesis differs from those papers by focusing the analysis on the euro area financial conditions, trying to find a relation between various financial variables and output growth tail risks. The paper of Adrian et al (2019), applied a similar analysis for the US economy, in this thesis, the analysis will be performed for the European economy, defining, and choosing the financial indicator which is the best to model output growth tail risks. Another part of the literature that should be considered is the working paper of Figueres and Jarocinski (2020), while this elaborate differs by not only modeling output growth tail risk (that is done in the first part of the working paper, but also forecasting output growth using different quantiles, evaluating those forecasts for each quantile and then performing an application for 2008 financial crisis with the regression quantile framework and the financial indicator that better explain the output growth tail risks for euro area. Similar forecasting analysis has been done in the working paper of Chaveleishvili and Manganelli (2019). This thesis differs from the working paper by performing a variables selection analysis and then using a univariate time series framework for the quantile regression model in the 2008 forecasting application. Performing univariate quantile regression analysis saves several degrees of freedom, criticality due to the availability of the data for the financial indicator CISS, which has also been used by Chaveleishvili and Manganelli (2020). Another difference from the literature is that the thesis also presents a quantile forecasting application using Gradient Boosting Regression, which is a state-of-the-art machine learning algorithm for classification and regression problems that can be applied to the quantile framework and context.

3 Quantile Regression Analysis

In this section, as first step, the optimal financial indicator will be detected using the quantile loss function called Tic Loss, which is the implicit loss function when the object of interest is a forecast of a particular quantile of a distribution, according to Giacomini and Komunjer (2005). Later in the section, different quantile regression model will be estimated with the optimal financial indicator as independent variable and output growth as dependent variable. Each model will be estimated using different quantile levels of the output growth distribution, and then the coefficients will be analyzed for each one of them, showing their dynamics through the dependent variable distribution.

3.1 Variable Selection

The Analysis starts by performing variable selection considering different financial indicators for the euro area. Sovereign spread, defined as 10-years European bond minus the 10-years German bond, reflects the riskiness of the euro area sovereign debt relative to the German debt which is less risky. The TED spread of the euro area, defined as the 3-month Euribor minus the 3-month German bond, is a financial measure that reflects the credit risk for interbank loans. The Term Spread of the euro area, defined as the 10-year German bond minus the 3-month German bond. The reason to focus on the Germany term spread is because the German bonds have virtually no risk of default, while other countries in Europe could have the term spread distorted by this risk-reward compensation at different horizons. Finally, it is considered a financial measure that aggregates financial conditions: The Composite Indicator of Systemic Stress (CISS) by Hollò et al. (2012). The CISS is defined as a non-linear aggregation of 15 raw financial stress measures, divided into 5 market-based categories: Financial intermediaries' sector, money market, bond market, bond market and foreign exchange markets, equity markets. This indicator picks up the episodes when multiple financial stress measures are simultaneously high and exhibiting high time-varying correlation. The core purpose of this indicator is to detect systemic stress episodes, which is perfect for the analysis on tail risk output growth. To perform variable selection, it is necessary to have a selection criteria. In this case the selection criteria will be a loss function to minimize. Giacomini and Komunjer (2005) argue that the Tic Loss is the implicit loss function whenever the object of interest is a forecast of a particular quantile of a distribution. The Tic Loss function works as stated by the expression:

$$L(y_{ip}, y_p) = max[q(y_i - y_{ip}), (q - 1)(y_i - y_{ip}) \quad (1)$$

Where y_{ip} is the predicted value of y, y_i is the actual-observed value and q is the quantile. For the analysis and the variable selection process, the data are quarterly, starting from the second quarter of 2000 and ending, for the training sample, in the third quarter of 2007, with a total of 30 observations. The test sample starts in the last quarter of 2007 and ends in the second quarter of 2009, for a total of 7 observations in the test sample. The variables considered are: CISS, Sovereign spread, TED spread, Term spread. All of them are taken considering from the first to the fourth lag and evaluated at 10th quantile, while the output growth is considered at time t such as y_t. A general linear regression model would be as in the equation below:

$$y_t \; = \; \beta_0 + \; \beta_1 x_{t-i} + \epsilon \quad (2)$$

Taking a similar structure to the linear regression, the general quantile regression model equation for the τ^{th} quantile is:

$$Q(y_t) \; = \; \beta_0(\tau) + \; \beta_1(\tau) x_{t-i} + \epsilon \quad (3)$$

In this specific analysis the estimation of the univariate quantile regression model is performed at 10th quantile level for each variable. Then the financial indicators will be evaluated against the test sample using the Tic Loss function (1) as selection criteria (Table 1).

Table 1: **Tic Loss Quantile Report for different financial indicators at different time lags.** Tic Loss value for each variable at different time lags where the univariate quantile regression model is estimated at 10th quantile.

Quantile	Tic Loss	Variable
10th	0.00155	CISS t-2
10th	0.002	Sovereign spread t-1
10th	0.002	CISS t-3
10th	0.00239	TED spread euro area t-3
10th	0.00242	TED spread euro area t-2
10th	0.00296	CISS t-4
10th	0.00341	Term spread euro area t-4
10th	0.00343	Term spread euro area t-1
10th	0.00346	Term spread euro area t-2
10th	0.00351	Term spread euro area t-3
10th	0.00357	CISS t-1
10th	0.00794	Sovereign spread t-4
10th	0.00828	TED spread euro area t-4
10th	0.00877	TED spread euro area t-1
10th	0.00897	Sovereign spread t-2
10th	0.00906	Sovereign spread t-3

As shown in Table 1, $CISS_{t-2}$, where t is time in quarters, is the variable which minimizes the Tic Loss Function given the 10th quantile estimation of the model. Thus, the CISS best fits the 10th quantile of output growth distribution, followed by the Sovereign spread and TED spread. This is the same result found in the working paper of Figueres and Marek Jarocinski (2020). Focusing only on the CISS financial indicator, Table 1 provides the Tic Loss for all the last year quarters, in which is shown that CISS second lag minimizes the Tic Loss. The decision of using the Tic Loss function as objective function in the analysis is based on the knowledge that the it is the implicit loss function whenever the object of interest is a forecast of a particular quantile of a distribution as analyzed in the paper of Giacomini and Komunjer (2005).

Considering this foundation, the thesis will continue the analysis by using the CISS as the main explanatory variable for the quantile regression models and the forecasting analysis.

3.2 Coefficients Analysis

As shown in Exhibit 1, different quantile regression models have been built considering as dependent variable the output growth and as independent variable $CISS_{t-2}$. The models are estimated given different quantiles: 5^{th}, 10^{th}, 50^{th} and 90^{th}, considering our data sample from second quarter of 2000 to last quarter 2021. It is also estimated the OLS model, to show the different coefficients of each model. Considering the relation between output growth and CISS, we are expecting negative coefficients due to the economic reasons that an increase in the CISS will lead to a decrease of output growth. This economic thought is reflected in Exhibit 1 where all the coefficients are negative. The purple top line is the quantile regression model estimated at the 90^{th} quantile. This model has a negative coefficient of -0.0060, which is statistically significant at 10% significance level. The coefficient of $CISS_{t-2}$ at 10^{th} quantile is -0.0288, with a p-value at a significant level less than 1% (Table 3). As shown in Exhibit 1, the model estimated using OLS has a smaller in absolute term, but still negative, coefficient of -0.018 than the one of the models estimated at the 10^{th} quantile (-0.0288), highlighting that the 10^{th} quantile model captures a tail distribution dynamic that the baseline OLS model is not able to capture. The OLS model and the quantile regression model estimated at 10^{th} quantile also differs in the intercept as shown in Exhibit 1. Considering the same exhibit, it is also possible to see how the 5^{th} quantile is capturing tail distribution behavior that is neglected by the other quantile levels and the OLS model, highlighting that there are some tail risk events in the output growth distribution that can be modelled using the 5^{th} quantile model. In this model the coefficient is -0.065 (statistically significant at less than 1% as shown in Table 4). Difference in the OLS model and the 50^{th} quantile regression model is due to the underlying distribution, which is not a normal distribution, so the two models give different results. Considering now Exhibit 2, it is possible to analyze the coefficient evolution and dynamics at different quantile levels. The slope of the CISS coefficient changes at each quantile, highlighting different behaviors of the model for different sections of the output growth distribution, showing that the CISS can capture the nonlinear relationship between financial conditions and future output growth in the euro area. Considering the dynamics of CISS coefficients between different quantile levels, it is possible to conclude that the CISS has a significantly asymmetric effect on the lower tail of the output growth distribution.

Exhibit 1: **Quantile Regression coefficient analysis.** Quantile regression models estimated at different quantiles level: 5th, 10th, 50th and 90th. OLS regression model as red line.

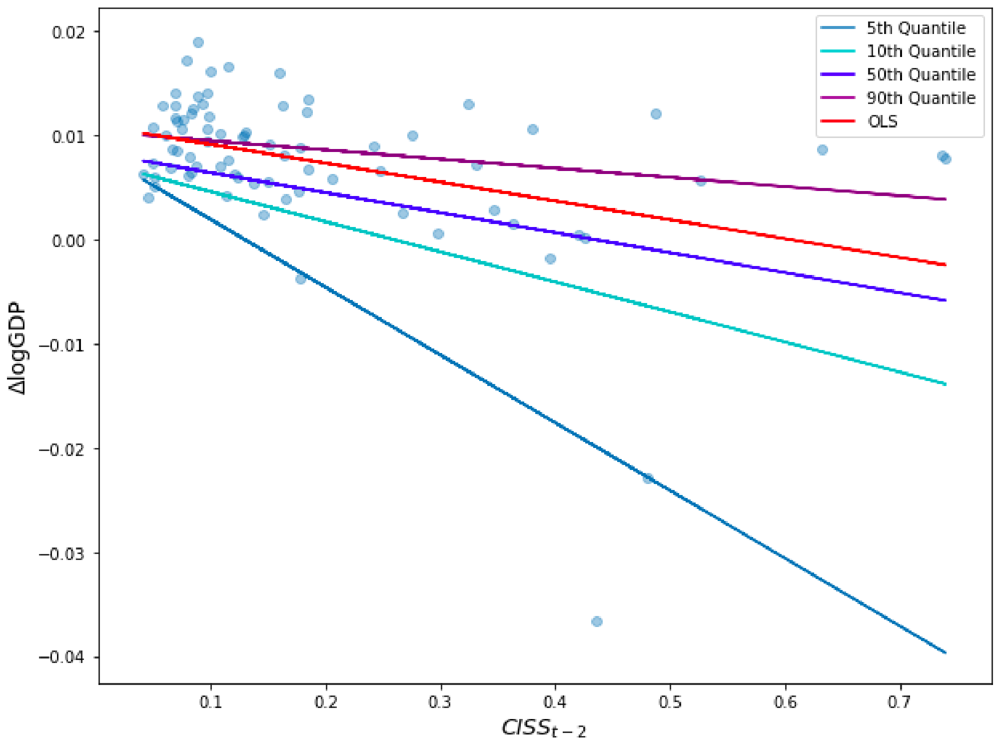


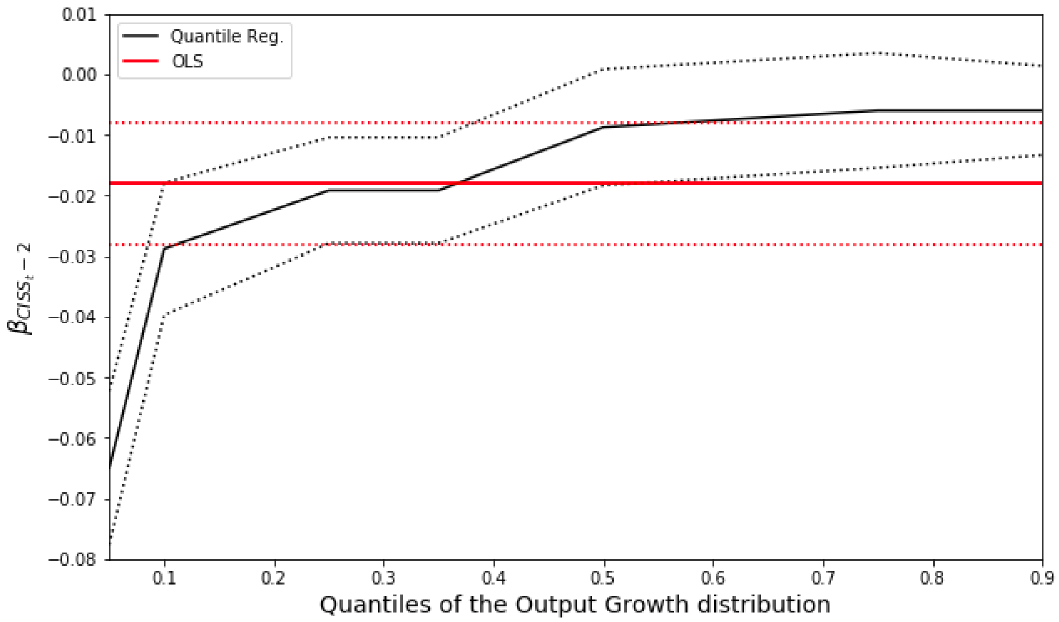
Table 3: **Quantile Regression results.** Model estimated at 10th quantile for $CISS_{t-2}$ as independent variable and output growth as dependent variable.

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	0.0075	0.001	5.335	0.000	0.005	0.010
CISS_t_2	-0.0288	0.005	-5.269	0.000	-0.040	-0.018

Table 4: **Quantile Regression results.** Model estimated at 5th quantile for $CISS_{t-2}$ as independent variable and output growth as dependent variable.

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	0.0084	0.002	5.212	0.000	0.005	0.012
CISS_t_2	-0.0650	0.006	-10.207	0.000	-0.078	-0.052

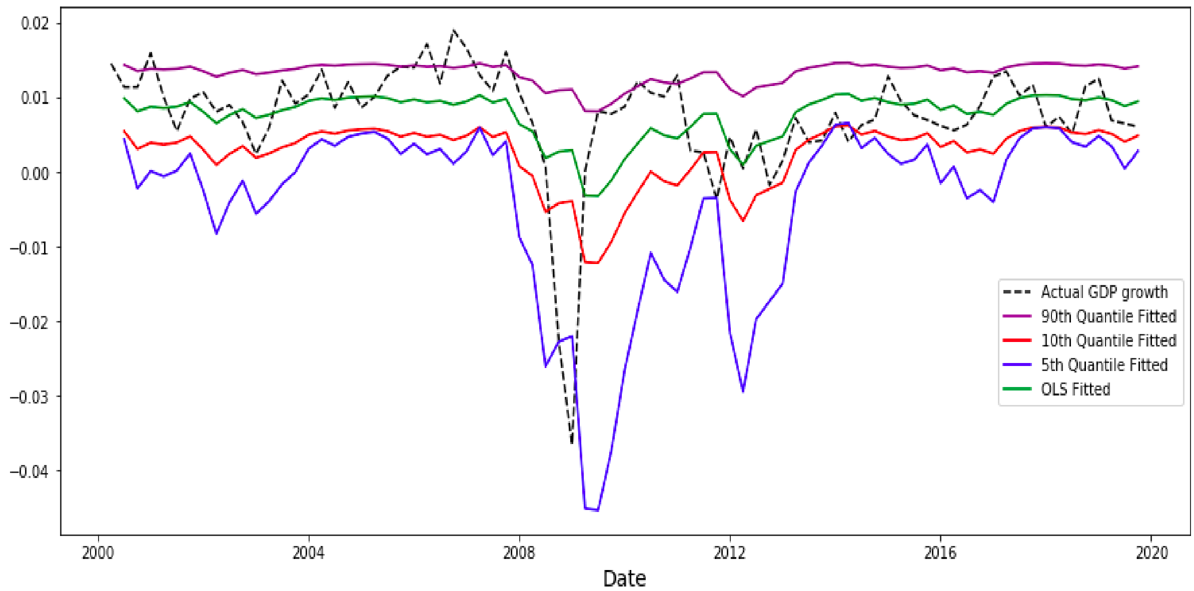
Exhibit 2: **Quantile Regression coefficient dynamics.** Evolution of the $\beta_{CISS_{t-2}}$ coefficient given different quantile levels. Confidence intervals as dotted lines.



3.3 Fitted and Actual values

Examining Exhibit 3 it is possible to analyze how different quantile models, estimated using the model (3), are fitting differently the output growth actual values observed from second quarter 2000 to last quarter 2019. The purple line is the model in equation (3) estimated considering the 90^{th} quantile. This model only captures and fits the upper part of the output growth distribution, in this particular case, the OLS model (green line) is more sensitive to capture the financial crisis of 2008 and its output growth fall. The 10^{th} quantile model is able to capture more the output growth downfall of 2008 then the OLS baseline model, but the massive improvement in terms of fitting the 2008 financial crisis is reached by modeling using the 5^{th} bottom quantile, while in Chaveleishvili, Manganelli (2019) and Figueres, Jarocinski (2020) only the bottom 10^{th} quantile is used to represent the crisis scenario, which could lead to lose explanatory power of the output growth distribution.

Exhibit 3: **Fitted Values and Actual Values**. Estimated values of output growth using different quantile levels and $CISS_{t-2}$ as independent variable. Actual output growth as black dotted line.



4 Forecasting Analysis

In this section, two different forecasting analysis will be performed using the financial indicator CISS and the quantile regression framework. The first analysis will be applied to the 2008 financial crisis, and the second one to the covid-19 shock in Europe. The analysis will focus on how the financial indicator CISS is a critical variable to model financial crisis, while it is not as explanatory if used to model and forecast non-financial shocks as the covid-19 was. The quantile framework will allow us to focus on the part of the distribution representing the output growth downfall. Attention will be provided for the bottom 10^{th} quantile and the bottom 5^{th} quantile while performing the quantile regression analysis and forecasting.

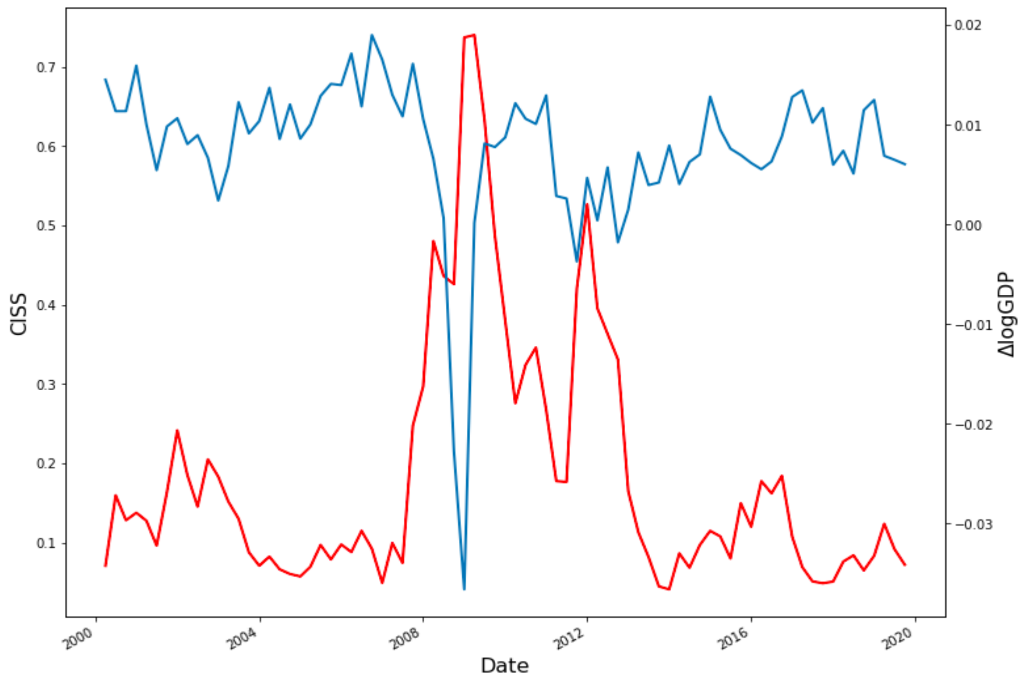
4.1 The 2008 Financial Crisis

Following the logic covered in the variable selection chapter, the forecasting section considers the general regression quantile model as defined in equation (3). It will be estimated using different quantile thresholds with the $CISS_{t-2}$ as independent variable which is the most precise financial indicator in terms of the Tic Loss function. As covered earlier in the thesis, the CISS is the indicator that picks up the episodes when multiple financial stress measures are simultaneously high and exhibiting high time-varying correlation. The core purpose of this indicator is to detect systemic stress episodes, which is perfect for the analysis on tail risk output growth. That being said, it is possible to examine the high inverse correlation between output growth and the CISS, especially during crises that can be explained by the financial sector of the economy as the 2008 financial crisis was. The CISS in Europe started to increase sharply already in the last quarter of 2007, as the financial crisis started in the US economy, going from 0.0745 in the third quarter of 2007 to 0.2477 in the last quarter of 2007. This highlights that the financial indicators were already detecting financial stability changes in the European economy. After few quarters the CISS doubled, reaching 0.4807 in the second quarter of 2008. After continued increases, the CISS reached the peak in the second quarter of 2009, with a value of 0.7401. Considering output growth, it stayed stable around 1% for the 2007, starting to approach zero only in the early 2008, while the CISS was already increasing rapidly. In the first quarter of 2009 the output growth reached the deepest peak during the financial crisis, touching the minimum value of -3.66%. The CISS during the same period reached 0.7370, which is the second maximum historical value. The absolute peak of CISS will be reached only the next quarter, at a value of 0.7401 (Table 5). Exhibit 4 provides the historical values of output growth and the CISS, highlighting their high negative correlation especially during the events of the financial crisis. The CISS stayed at high level in the years after the 2008 crisis as the financial turmoil persisted, in fact, during 2012 Europe was hit by the sovereign debt crisis.

Table 5: **Output growth and CISS values during and after the 2008 financial crisis.**

Date	Output Growth	CISS	CISS percentage change
2007-04-01	0.01300	0.0996	nan%
2007-07-01	0.01084	0.0745	-25.20%
2007-10-01	0.01612	0.2477	232.48%
2008-01-01	0.01056	0.2971	19.94%
2008-04-01	0.00658	0.4804	61.70%
2008-07-01	0.00063	0.4362	-9.20%
2008-10-01	-0.02280	0.4260	-2.34%
2009-01-01	-0.03661	0.7370	73.00%
2009-04-01	0.00015	0.7401	0.42%
2009-07-01	0.00813	0.6323	-14.57%
2009-10-01	0.00776	0.4870	-22.98%
2010-01-01	0.00873	0.3799	-21.99%
2010-04-01	0.01216	0.2758	-27.40%
2010-07-01	0.01061	0.3241	17.51%
2010-10-01	0.01008	0.3461	6.79%
2011-01-01	0.01295	0.2674	-22.74%
2011-04-01	0.00285	0.1777	-33.55%

Exhibit 4: **Output growth and CISS historical observations.** Quarterly data of the financial indicator CISS (red line) and output growth (blue line).



4.2 Forecasting the 2008 Financial Crisis

To forecast output growth during the 2008 financial crisis, the thesis will start the analysis by splitting the data into training and test sample. The independent variable is the $CISS_{t-2}$, for this reason the training sample is composed of the CISS values starting from the second quarter of 2000 and ending in the third quarter of 2007, for a total of 30 observations. The test sample is composed of observed output growth values, the data that the model is built to forecast. It starts in the last quarter of 2007 and ends in the second quarter of 2009, for a total of 7 observations, to capture all the output growth downfall of the financial crisis. Then, the quantile regression models are estimated according to equation (3) for the 5th, 10th, 25th, 75th and 90th quantile of output growth, using the independent variable $CISS_{t-2}$. After training the models for each quantile, they are used to predict output growth. The results of the forecasts are compared with the test sample using the MSE, RMSE and the MAE as evaluation metrics. The MAE is included for sake of completeness, and it can be easily neglected during this forecasting evaluation analysis as the MSE and its root transformation account and penalize for large errors more, which is in line with the logic of forecasting analysis during a considerable output growth downfall as the 2008 financial crisis was. The OLS model is also included in the analysis to compare its forecasts with the quantile regression ones, to establish a real advantage of using a quantile regression framework to predict output growth during a crisis. The 10^{th} output growth forecasts produced by the quantile regression model are more accurate than the values forecasted by the other quantile models (excluding the 5^{th} one) and by the OLS model as shown in Exhibit 5, where the forecasts of each quantile regression model are compared to the test sample of output growth. This intuition is provided by Exhibit 5 in a graphic representation by analyzing the forecasts line and concluding that those are closer to the output growth negative peak, which corresponds to the first quarter of 2009. Table 6 provides forecasting evaluation metrics for all the models estimated. The 10^{th} quantile regression model has a MSE of 0.000169 (RMSE of 0.013), the smallest compared to the other quantile models (excluding the 5^{th} quantile) and most importantly, it is significantly smaller than the MSE of the Ordinary Least Square model (0.000288), concluding that the quantile regression, if considering the right quantile, could improve the forecasting analysis of output growth during crisis. The 10^{th} quantile was the one considered representative of an economic crisis and the one upon which the analysis is developed in the paper of Chaveleishvili and Manganelli (2019) and in the paper of Figueres and Jarocinski (2020). In this thesis the analysis is pushed forward by estimating a quantile regression model considering the 5^{th} quantile. Following the analysis for the 10^{th} quantile, the 5^{th} should be able to capture the crisis even more, representing the bottom 5% of the output growth distribution such as tail events. The forecast produced by the 5^{th} quantile regression model is more accurate in terms of MSE (or its square transformation the RMSE) than the forecast of the 10th quantile model as it is shown in Table 6, dropping from 0.000169 to 0.000145. Considering Exhibit 5, the 5th quantile forecast is capturing the output growth downfall peak more than all the other models, suggesting that even if the forecasting evaluation metrics are close to the 10th quantile ones, the 5th quantile forecast should be considered capable of capturing more extreme negative tail events. Table 7 shows the single forecasted values for each quantile model and for each quarter. The 5th and the 10th quantile models are the only ones that capture the downfall of output growth, forecasting significant negative values while other models forecast remain positive or close to zero for the entire test sample period. The 5th quantile forecast is the closest one to the output growth negative peak of 2009 first quarter, with a forecasted value of -2.04% against the actual value of -3.66%, while the 10th quantile forecast is -0.78%, not even close to the negative peak or the 5th quantile forecast. The 5th quantile forecast referred to the last quarter of 2008 is -2.39%, against the actual value of -2.28%, while the 10th quantile forecast is -0.98%. In this last example, the 5th quantile model is way more accurate to capture the output growth downfall. In Conclusion, forecasting output growth with a regression quantile framework, using the 10th and the 5th quantile, leads to significant improvements in the forecasts that would have been obtained using the OLS regression model without the quantile approach. As already analyzed, forecasting using the 10th quantile instead of the OLS, reduces the MSE from the OLS value of 0.000288, to 0.000169, value of the 10th quantile model (Table 6). Moreover, further improvements in the forecasts can be reached by considering also for the 5th quantile, which was not examined in other papers as Chaveleishvili and Manganelli (2019), Figueres and Jarocinski (2020). That being said, the MSE of the 5th quantile model is halved compared to the OLS MSE, representing significant improvements in the forecast accuracy of output growth during the crisis. Lastly, as shown in Table 7, the 5th and the 10th quantile models capture the output growth negative peak during the last quarter of 2008 and first quarter of 2009, while the OLS model continued to forecast positive values during this entire period of the financial crisis.

Exhibit 5: **Quantile regression models forecasts and test sample.** The quantile regression models are trained using different quantile levels and the $CISS_{t-2}$ as independent variable.

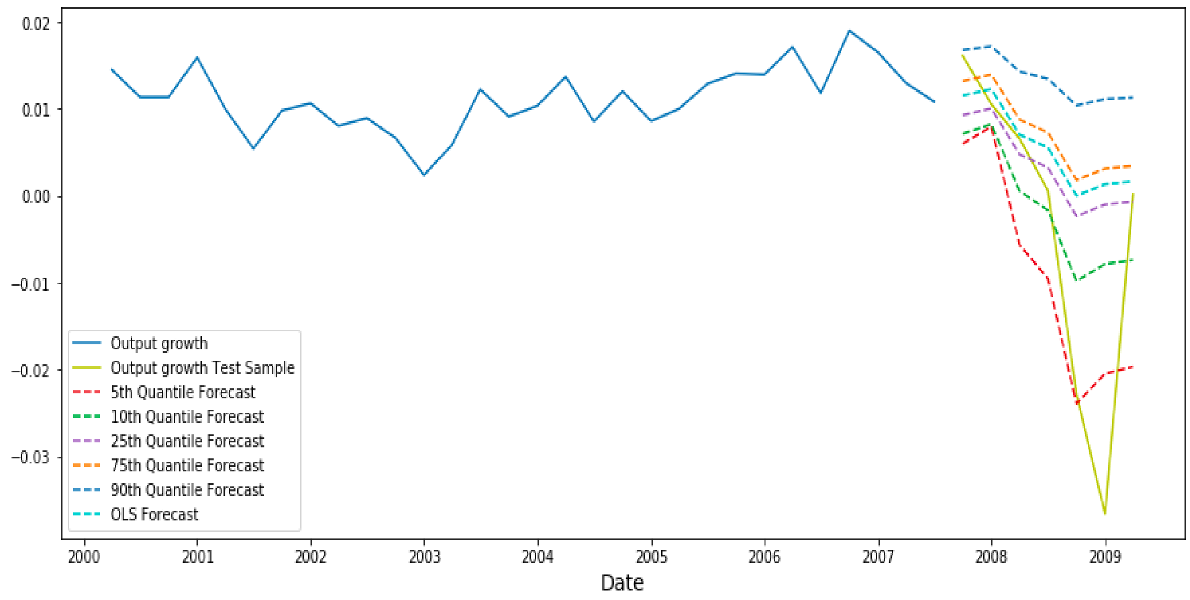


Table 6: **Forecasting Evaluation Metrics.** Mean absolute error, mean squared error and root mean squared error are calculated for each quantile model forecast.

Quantile	MAE	MSE	RMSE
5th	0.0103	0.000145	0.012
10th	0.00984	0.000169	0.013
25th	0.00981	0.000249	0.0158
35th	0.00952	0.000237	0.0154
50th	0.00939	0.000225	0.015
75th	0.0118	0.000324	0.018
90th	0.0171	0.00054	0.0232
OLS	0.0106	0.000288	0.017

Table 7: **Models forecasts and output growth actual values during 2008 financial crisis.**

	OLS Forecast	5th Quantile Forecast	10th Quantile Forecast	25th Quantile Forecast	75th Quantile Forecast	90th Quantile Forecast	Actual Values
2007-10-01	0.01153	0.00601	0.00715	0.00930	0.01321	0.01680	0.01612
2008-01-01	0.01229	0.00798	0.00826	0.01007	0.01396	0.01722	0.01056
2008-04-01	0.00705	-0.00564	0.00055	0.00477	0.00878	0.01431	0.00658
2008-07-01	0.00556	-0.00952	-0.00165	0.00326	0.00730	0.01348	0.00063
2008-10-01	0.00002	-0.02393	-0.00981	-0.00234	0.00182	0.01040	-0.02280
2009-01-01	0.00135	-0.02045	-0.00784	-0.00099	0.00314	0.01114	-0.03661
2009-04-01	0.00166	-0.01965	-0.00739	-0.00068	0.00345	0.01131	0.00015

4.3 Features Engineering

Considering the forecasting analysis that has been done in the previous section, the purpose of this analysis is to improve the quantile model from equation (3) by training and testing other models using different features combinations. The CISS is considered in different lags and combined with the $CISS_{t-2}$, which minimizes the Tic Loss function. The previous quarter of output growth is also included as an independent variable in the set of model combinations accounting for memory and information in the past output growth observations. The final purpose is to obtain more accuracy in the forecasts for the 5th and 10th quantiles than in the ones obtained only using $CISS_{t-2}$ as independent variable. The results of the train and test analysis of the different models is reported in Table 8, in which all forecasting evaluation metrics are exposed for each variable combination used. The models are evaluated at the 5th and the 10th quantile, priority in the evaluation metrics is given to the mean squared error, reflecting the logic of the previous section. Table 8 shows that the combination of $CISS_{t-2}$ and $CISS_{t-3}$ leads to a better forecast accuracy than the model estimated only using $CISS_{t-2}$ as independent variable, performing better at both 5th and 10th quantile for all the metrics considered. Table 9 provides a comparison between the MSE of the 5th and the 10th quantile and the OLS MSE, concluding that the forecasting accuracy increases if applying a quantile specification to the regression framework. Equation (4) represents the mathematical specification of the new quantile regression model, while Exhibit 6 shows a graphical representation of the different quantile forecast.

$$Q(y_t) = \beta_0(\tau) + \beta_1(\tau)CISS_{t-2} + \beta_2(\tau)CISS_{t-3} + \epsilon \quad (4)$$

Table 8: **Forecasting evaluation metrics for features optimization.** Different features are considered and combined to increase the accuracy of the forecast.

Quantile	MAE	MSE	RMSE	Variable
5th	0.00896	0.000131	0.0114	CISS t-2 + CISS t-3
5th	0.0103	0.000145	0.012	CISS t-2
10th	0.00949	0.000148	0.0122	CISS t-2 + CISS t-3
10th	0.00984	0.000169	0.013	CISS t-2
5th	0.0141	0.000247	0.0157	CISS t-1 + CISS t-2
10th	0.0114	0.000248	0.0158	GDP log diff t-1 + CISS t-2
5th	0.0112	0.000259	0.0161	GDP log diff t-1 + CISS t-2
10th	0.0159	0.000322	0.018	CISS t-1 + CISS t-2

Exhibit 6: **Quantile regression models forecasts and test sample.** The forecasts are obtained using model (4) at all the different quantile levels in the chart legend. The OLS forecasts are also reported to complete the analysis.

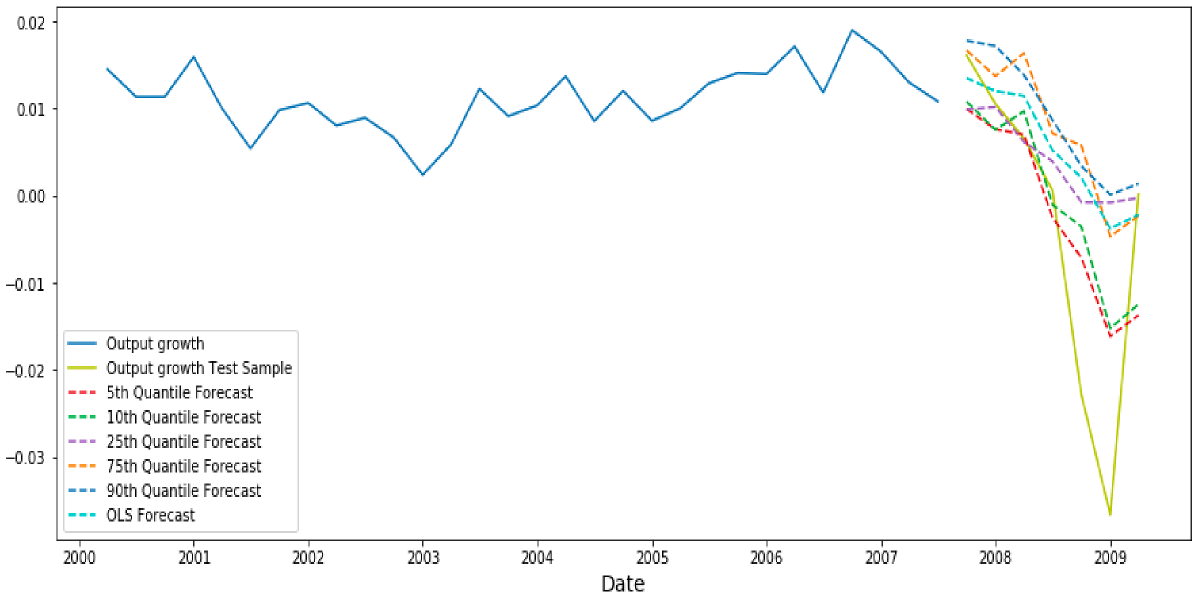


Table 9: **Forecasting evaluation metrics for the feature optimized model and the OLS.** The quantile models are estimated using equation (4), while the OLS model is estimated using the same independent variables but without the quantile specification.

Quantile	MAE	MSE	RMSE
5th	0.00896	0.000131	0.0114
10th	0.00949	0.000148	0.0122
OLS	0.0105	0.000251	0.0158

4.4 Forecasting the Covid-19 Shock

In this section the quantile forecasting analysis will be pushed forward by implementing an analysis for the Covid-19 shock. After increasing the forecasting accuracy by implementing the model in equation (4), in this section it will be used and applied to forecast the output growth shock due to the covid-19 pandemic. The model (4) is trained with the CISS training sample, starting in the second quarter of the 2000 and ending in the third quarter of 2019, for a total of 78 observations. The output growth test sample starts from the last quarter of 2019 and ends in the second quarter of 2020, for a total of 3 observations as the analysis is only considering the output growth downfall and not the positive correction in the quarters after. In Table 10 the forecasting evaluation metrics for each quantile model are reported, highlighting that the 5th and the 10th quantile models still the best ones to predict a crisis. Although the findings of those models are not as good as the ones founded for the 2008 financial crisis. During the first and second quarter of 2020 the output growth fell and became negative, while the models that were trained, regardless of the quantile, are not able to capture the output growth downfall, forecasting positive values for all the period and only values close to zero for the 5th quantile one (Table 11), showing how wide the difference between the forecasted values of output growth using model (4) and the actual output growth values during the shock are. In conclusion, the CISS fails to capture the covid-19 shock due to the nature of the indicator, which is a financial one while the shock was a supply side shock not originating in the financial sector of the economy.

Table 10: **Covid-19 forecasting evaluation metrics for each quantile model.**

Quantile	MAE	MSE	RMSE
5th	0.04887	0.00462	0.06797
10th	0.04993	0.004905	0.07004
25th	0.05079	0.005063	0.07116
35th	0.05188	0.005168	0.07189
50th	0.05384	0.005391	0.07342
75th	0.05679	0.005698	0.07549
90th	0.05849	0.005874	0.07664
OLS	0.05361	0.005365	0.07325

Table 11: **Models forecasts and output growth actual values during the covid-19 shock.**

	Actual Values	OLS Forecast	5th Quantile Forecast	10th Quantile Forecast	25th Quantile Forecast	50th Quantile Forecast	75th Quantile Forecast	90th Quantile Forecast
2019-10-01	0.0060	0.0088	1.2985e-03	0.0051	0.0062	0.0088	0.0122	0.0144
2020-01-01	-0.0274	0.0092	8.5609e-05	0.0043	0.0063	0.0098	0.0124	0.0138
2020-04-01	-0.1118	0.0096	2.5585e-03	0.0053	0.0067	0.0098	0.0126	0.0141

4.5 Gradient Boosting Regression

Gradient boosting regression is a machine learning technique, more in details it is a supervised learning algorithm. Supervised learning means modeling the data relationship between measured features (covariates) and the associated label. An algorithm of this kind analyzes the data and extrapolate a function from them, as the regression analysis does, which is in fact a supervised learning algorithm. Gradient boosting is an ensemble method where multiple weak models are created and then combine to get better performance. The gradient boosting algorithm involves training weak learners, which are low bias estimators, to predict the outcome of the target variable. Once the first step of training is finished, then another model is trained using the residual from the previous model. After the first step, the following learners are not trying to predict the target variable, but the residuals of the previous learner. This process of predicting residuals continues until no further improvements are possible. In the next section an application of the model is provided for the 2008 financial crisis, while in appendix the mathematical derivation of the algorithm is presented, following the paper of Friedman (1999).

4.5.1 Gradient Boosting Regression Application

Following the analysis that has been done in forecasting the output growth using a quantile regression framework, in this section the gradient boosting algorithm is applied to the 2008 financial crisis, to assess if this machine learning technique is able to out-perform the quantile regression forecast presented in section 4.2 and 4.3. This machine learning algorithm will also be evaluated and compared with the forecasts of the standard OLS regression. The train and test sample are the same presented in section 4.2: The training sample is composed of the financial indicator values starting from the second quarter of 2000, ending in the third quarter of 2007, for a total of 30 observations. The test sample is composed of output growth values, the data that the model is built to forecast. The test sample starts in the last quarter of 2007 and ends in the second quarter of 2009. It is composed of 7 observations. Then the machine learning model is trained using $CISS_{t-2}$ and $CISS_{t-3}$ as features (independent variables). Output growth at time t is the target variable (dependent variable). Table 12 provides the forecasting evaluation metrics for the machine learning model estimated using the quantile loss function as in equation (1). The table highlights that the Gradient Boosting Regression model is performing poorly if compared to the simpler OLS regression model, which has a MSE of 0.000251, smaller than all the MSE of the machine learning models. The MAE is pointing in the same direction of the

MSE, concluding that the non-quantile OLS model performed better in terms of forecasting evaluation metrics compared to the quantile gradient boosting regression. The gap is even more evident if considering the MSE and the MAE of the quantile regression model estimated using model (4) and compared it to the forecasting evaluation metrics of the quantile gradient boosting regression (Table 13). As reported in Table 13, the quantile regression model, with the same covariates of the quantile gradient boosting regression, is out-performing in terms of forecasting accuracy the machine learning algorithm, presenting a MSE at the 5^{th} quantile of 0.000131, while the gradient boosting at the same quantile has a MSE value of 0.000489, more than three times the quantile regression MSE value. The difference still considerable if examine the 10^{th} quantile of both algorithms. The gradient boosting has a MSE of 0.000463, while the quantile regression model has a MSE of 0.000148, more than a third compared to the machine learning forecasting metric. In conclusion, the quantile regression model outperformed the gradient boosting regression (with quantile specification) in terms of forecasting accuracy of the 2008 output downfall.

Table 12: **Quantile gradient boosting regression forecasting evaluation metrics.**

Quantile	MAE	MSE	RMSE
5th	0.01542	0.000489	0.02211
10th	0.01494	0.0004624	0.0215
25th	0.0137	0.0004306	0.02075
35th	0.0136	0.0004219	0.02054
50th	0.01415	0.0004335	0.02082
75th	0.01629	0.0005246	0.0229
90th	0.01444	0.0004192	0.02048
OLS	0.01051	0.000251	0.01584

Table 13: **Quantile regression model and gradient boosting regression compared.**

Forecasting evaluation metrics accuracy of the quantile gradient boosting regression and quantile regression model. Values ordered by MSE.

Algorithm	Quantile	MAE	MSE	RMSE
Quantile Regression	5th	0.00896	0.000131	0.0114
Quantile Regression	10th	0.00949	0.000148	0.0122
Gradient Boosting Regression	10th	0.015	0.000463	0.0215
Gradient Boosting Regression	5th	0.0153	0.000489	0.0221

5 Conclusion

In the thesis it has been found that the financial indicator CISS is the most accurate in modeling and predicting output growth downfall during financial crisis. While the quantile regression framework models each quantile of the distribution, capturing the non-linear relationship and the coefficient dynamics, the standard OLS regression model fails in capturing the left tail behavior of the output growth distribution. The economic intuition of a negative correlation between the CISS and output growth is reflected in the coefficient of the quantile regression models estimated using different quantiles of the output growth distribution, and as the quantile become smaller, the CISS coefficient become smaller in absolute terms. A CISS shock in the left tail of output growth distribution, such as the 5^{th} or the 10^{th}, has a more negative impact than shocks in other quantiles of the distribution.

Forecasting output growth with a regression quantile framework using the 10^{th} quantile outperformed in terms of forecasting accuracy the standard OLS regression model, which fails to capture the tail behavior. Further improvements in the forecast accuracy can be reached by estimating at the 5^{th} quantile of the output growth distribution, and by so a better fit of the 2008 financial crisis is reached. Performing features selection and optimization leads to select $CISS_{t-2}$ and $CISS_{t-3}$ as independent variables for the quantile models, which increase the forecasting accuracy at the 5th and 10th quantile level. In particular, forecasting output growth during the 2008 financial crisis using the 5th quantile and $CISS_{t-2}$ and $CISS_{t-3}$ as independent variables lead to a MSE of 0.000131, a significant accuracy improvement from the OLS MSE of 0.000251. What can be moved as the main limitation of the analysis is the data availability. For the euro area the CISS was only available starting from the early 2000s, while in the US this financial indicator has more historical observations. The data are considered with quarterly time frame, and predicting the 2008 cuts the data sample, limiting the training data only to 30 observations. The loss of degrees of freedom can be considered not brutally impacting the analysis, has the last model in section 4.3 is estimated considering only two independent variables. Indeed, this reduces the sample data to few observations less than 30 to train the model with the intent to predict the 2008 financial crisis. Despite the remarkable results, the CISS financial indicator is not able of modeling and predict with accuracy the covid-19 shock. The reason is found in the fact that the CISS is a financial stress indicator, and the covid-19 was a different kind of shock. This result could be expected if considered the economic theory behind the indicator and the shock. The CISS financial indicator is not able to predict this specific output growth downfall, even if the quantile specification highlights that the forecasts of the lower left quantiles are more accurate that the OLS model. Lastly, the Gradient Boosting Regression was implemented to improve the Quantile Regression model accuracy. Unfortunately, the machine learning algorithm failed in performing better than the quantile regression model. In conclusion, it has been found that the CISS is more accurate than the other commonly used financial indicators in modeling and predicting output growth downfall during financial crisis. The 10th, but more important, the 5th quantile provides significant improvements in the forecasting accuracy than the standard OLS regression model. This suggests that without quantile specification we are losing explanatory power and prediction accuracy while modeling and forecasting financial crisis in the euro area.

Appendix

Gradient Boosting Regression Algorithm

Below the logics and methods of the gradient boosting algorithm are presented following the paper of Friedman (1999) to briefly introduce the fundamental concept of the machine learning model. The first step is to initialize the model with a constant value.

$$F_0(x) = \ argmin_y \sum_{i=1}^{n} L(y_i, \gamma) \ \ (5)$$

Where F_0 is the initial value, and L is the generic loss function that can be specified in the algorithm. In this analysis the quantile loss function will be considered later during the algorithm application, while in the mathematical specification below it is reported the squared loss function as in the Friedman (1999) paper. After initializing the first value, the residuals are calculated as follow:

$$r_{im} = \ -\left[\frac{\partial L\big(y_i, F(x_i)\big)}{\partial F(x_i)}\right]_{F(x)= \ F_{m-1}(x)} for \ i = 1, \dots, n \ \ (6)$$

The residuals r_{im} are calculated by taking the derivative of the loss function with respect of the previous prediction F_{m-1} and multiplying it by -1. The equation is resolved for the residuals r_{im}.

$$r_{im} = \frac{\partial (y_i - \ F_{m-1})^2}{\partial F_{m-1}} \ \ (7)$$

Taking the derivative from equation (7) we obtain the following result:

$$r_{im} = \ 2(y_i - F_{m-1}) \ \ (8)$$

After this step, the regression tree is trained with feature x against r and create terminal node R_{jm} for $i = 1, \dots, J_m$. Where j represents a terminal node in the decision tree, m denotes the tree index, and J the total numbers of leaves. The objective is to find γ_{jm} that minimizes the loss function on each terminal node j.

$$\gamma_{jm} = argmin_y \sum_{x_i \in R_{jm}} L((y_i, F_{m-1}(x_i) + \gamma) \; for \; j = 1, \dots, J_m \quad (9)$$

Plugin the loss function we obtain equation (10):

$$\gamma_{jm} = argmin_y \sum_{x_i \in R_{jm}} (y_i - F_{m-1}(x_i) - y)^2 \quad (10)$$

By taking the derivative of γ_{jm} , and setting it equal to zero from equation (10), the following result is obtained:

$$\gamma = \frac{1}{n_j} \sum_{x_i \in R_{jm}} r_{im} \quad (11)$$

Where n_j stands for the number of samples in the terminal node j. This means that the γ_{jm} which minimize the loss function is the average of the residuals r_{im} in the terminal node R_{jm}. In the last iteration, the prediction is updated providing equation (12). v is the learning rate ranging between 0 and 1 which controls the degree of contribution of the additional tree prediction γ to the combined prediction F_m. A bigger learning rate increases the effect of the additional tree prediction, but it also increases the chance of the model to overfit the training data.

$$F_m(x) = F_{m-1}(x) + v \sum_{j=1}^{J_m} \gamma_{jm} 1(x \in R_{jm}) \quad (12)$$

References

Figueres, Juan Manuel and Jarocinski, Marek. 2020. “Vulnerable growth in the Euro Area: Measuring the financial conditions”. *ECB,* Working Paper No. 2458.

Manganelli, S. and Chavleishvili, S. 2019. “Forecasting and stress testing with quantile vector autoregression”. *ECB,* Working Paper No. 2330.

Friedman, Jerome. 1999. “Greedy Function Approximation: A Gradient Boosting Machine”. *The Annal of Statistics,* Vol. 29, No.5, pp 1189-1232 (44 pages).

Adrian, T., N. Boyarchenko and D. Giannone. 2019. “Vulnerable growth”, *American Economic Review*: 109 (4), 1262-1289.

Giacomoni, R. and Komunjer, I. 2005. “Evaluation and Combination of Conditional Quantile Forecasts”, *Journal of Business and Economic Statistics*. Vol. 23, No. 4, pp. 416-431 (16 pages).

Koenker, R. 2005. *Quantile regression*. Econometric Society Monograph, Cambridge University Press.

Hollo, D., Kremer, and M. Lo Duca. 2012. “CISS – A compositive indicator of systemic stress in the financial system”. *ECB,* Working Paper No. 1426.