Analysis of Commutation Curve of d-c Machine with Tandem Brush

Analysis of Commutation Curve of *d-c* Machine with Tandem Brush

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Abstract

This paper treats of the solution of the commutation equation, *i. e.* differential epuation concerning the current in a coil short-circuited by the brush, in the case of the *Tandem Brvsh.* By this solution, commutation curves and the values of the voltage drop between brush and commutator segment are obtained for some representative conditions of commutation in *d-c* machine to examine the commutating performance of the Tandem Brush.

1. Introduction

One of the most inportant limiting factors on satisfactory operation of a *d-c* machine is the capability of transfering the necessary armature current through the brush contact at the commutator without sparking and local heating of the brushes and commutator.

The various investigations on the brush, brush holder and the contact phenomena between brush and commutator, etc., therefore, have been reported by many researchers. Nowadays quite a few *d-c* machines have the *Tandem Brush* (*i. e.* bisected brush). It has been known that this Tandem Brush gives us rather satisfactory commutation.[1), 2)]

This paper treats of the solution of the differential equation concerning the current in a coil short-circuited by the Tandem Brush supposing the same width of a brush and a segment in the case of a constant contact resistance. By numerical calculation, the short-circuit current curves and the voltage drop curves between a commutator segment and a brush have been also presented so that the following points may be clarified:

(1) The performance of the Tandem Brush with $m=1$ are inferier to that of the ordinary brush as regards the transfer of short-circuit current.

(2) In the case of the Tandem Brush with $m<1$, rather satisfactory commutation may be expected as compared with the cases of both the ordinary brush and the Tandem Brush with $m=1$.

2. Commutation Equation in the Case of Tandem Brush

The differential equation concerning the current in a coil short-circuited by the brush can be expressed by the following equation (1) by considering the contact resistance between commutator segment and a brush as the Ohmic resistance, so far as the coil resistance is neglected and the brush width is

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equal to the width of the commutator segment and the mica thickness between the segments is assumed to be zero:

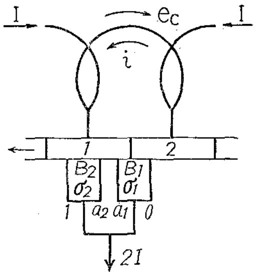

Fig. 1. The armature coil short circuited by a Tandem Brush. Component brush B_1 and B_2 have the contact surface conductivities σ_1 and σ_2, respectively.

$$L\frac{di}{dt}+v_1-v_2+e_c=0 \tag{1}$$

where

L; inductance of the short-circuited coil
i ; short circuit current in the commutated coil
v_1; voltage drop between the commutator segment No. 1 and the brush
v_2; voltage drop between the commutator segment No. 2 and the brush
e_c; commutating emf.
t ; time

In this paper the following symbols are to be used:

I ; conductor current before and after commutation
R_b; contact resistance at the surface all over the brush
T; commutating period
σ_1; surface conductivity of a leading brush B1
σ_2; surface conductivity of a trailing brush B2
m; ratio of σ_2 to σ_1 ($m=\sigma_2/\sigma_1$)

3. Commutation Curve in Case of $m=1$

(1) The solution of the commutation equation (1)

The commutation equation in case of the ordinary brush has been solved exactly by J. K. Hayashi and M. Naito.[3]

They have developed a clever method of solving the differential equation. In case of the Tandem Brush, the commutation equation can be solved by using their method as follows:

By introducing the new variables $x=t/T$, $y=i/I$, $c=e_c/2IR_b$ and $1/b=r=(2LI)/(2IR_bT)$, Eq. (1) can be divided into the following three cases, according to the contact features between the segments and a brush;

1) Case of $o\leq x\leq a_1$

In this case one can obtain the following relations;

$$v_1=(I+i)\frac{WR_b}{W-t/T}=(I+i)\frac{R_b}{1-x'} \tag{2}$$

$$v_2=(I-i)\frac{WR_b}{t/T}=(I-i)\frac{R_b}{x'} \tag{3}$$

where $x'=x/W$, $W=1+a_1-a_2$

Substitution of Eqs. (2) and (3) into Eq. (1) gives

$$\frac{dy}{dx} + b'y\left(\frac{1}{1-x'}+\frac{1}{x'}\right) = b'\left(\frac{1}{x'}-\frac{1}{1-x'}\right) - 2b'c \tag{4}$$

where $b' = Wb$.

Eq. (4) is similar to the equation obtained in the case of the ordinary brush. Therefore, the form of the solution of Eq. (4) is the same as in the case of an ordinary brush, namely,

$$y_1 = 1 - 2x' + 2(1-b'c)\left(\frac{1-x'}{x'}\right)^{b'}\int_0^{x'}\left(\frac{x'}{1-x'}\right)^{b'}dx' \tag{5}$$

Using the relation $x' = \frac{x}{W}$, one can rewrite the above equation as follows:

$$y_1 = 1 - 2\frac{x}{W} + \frac{2}{W}(1-b'c)\left(\frac{W-x}{x}\right)^{b'}\int_0^{x}\left(\frac{x}{W-x}\right)^{b'}dx \tag{6}$$

2) Case of $a_1 \leq x \leq a_2$

In the case of $a_1 \leq x \leq a_2$, one has the following relations:

$$v_1 = (I+i)\frac{WR_b}{1-a_1} \tag{7}$$

$$v_2 = (I-i)\frac{WR_b}{a_2} \tag{8}$$

Substitution of Eqs. (7) and (8) into Eq. (1) gives

$$\frac{dy}{dx} + by\left(\frac{W}{1-a_2}+\frac{W}{a_1}\right) = b\left(\frac{W}{a_1}-\frac{W}{1-a_2}\right) - 2bc \tag{9}$$

Upon putting $\alpha = W/(1-a_2) + W/a_1$ and $\beta = W/a_1 - W/(1-a_2)$, one has the following equation:

$$\frac{dy}{dx} + \alpha b y = \beta b - 2bc \tag{10}$$

The solution of Eq. (10) is

$$y = e^{-\alpha bx}\left[C_1 + \left(\frac{\beta-2c}{\alpha}\right)e^{\alpha bx}\right] \tag{11}$$

where C_1 is an integration constant. By introducing the initial condition of $y_{\text{II}} = y_{a_1} = y_{\text{I}}\mid_{x=a_1}$ at $x = a_1$, the integration constant C_1 will be decided as follows:

$$C_1 = y_{a_1}e^{\alpha b a_1} - \left(\frac{\beta-2c}{\alpha}\right)e^{\alpha b a_1} \tag{12}$$

Substitution of Eq. (12) into Eq. (11) gives

$$y_{\text{II}} = \left\{y_{a_1} - \frac{\beta-2c}{\alpha}\right\}e^{\alpha b(a_1-x)} - \left(\frac{\beta-2c}{\alpha}\right) \tag{13}$$

where α and β are the constants which can be determined by the dimensions of the component brushes B_1 and B_2.

3) Case of $a_2 \leq x < 1$

In the case of $a_2 \leq x < 1$ one has the following relations:

$$v_1 = (I+i)\frac{WR_b}{1-t/T} = (I+i)\frac{R_b}{1-x''} \tag{14}$$

$$v_2 = (I-i)\frac{WR_b}{W-(1-t/T)} = (I-i)\frac{R_b}{x''} \tag{15}$$

Substitution of Eq. (14) and (15) into Eq. (1) gives

$$\frac{dy}{dx''}+b'y\left(\frac{1}{1-x''}+\frac{1}{x''}\right)=b'\left(\frac{1}{x''}-\frac{1}{1-x''}\right)-2b'c \tag{16}$$

where $x''=\frac{1}{W}(W-1+x)$, $b'=Wb$.

Eq. (16) is similar to the form of Eq. (4), therefore the solution of Eq. (16) is similar to that of an ordinary brush, namely,

$$y=1-2x''+\left(\frac{1-x''}{x''}\right)^{b'}\left[C_2+2(1-b'c)\int\left(\frac{x''}{1-x''}\right)^{b'}dx''\right] \tag{17}$$

where C_2 is an integration constant.

By introducing initial condition of $y_{\text{III}}=y_{\text{II}}\,|_{x=a_2}=y_{a_2}$ at $x=a_2$, one can obtain the integration constant C_2 as follows:

$$C_2=\left(\frac{W-1+a_2}{1-a_2}\right)^{b'}\left[y_{a_2}-\frac{W-1+a_2}{W}\right]-\frac{2}{W}(1-b'c)\int\left(\frac{W-1+x}{1-x}\right)^{b'}dx\Bigg|_{x=a_2} \tag{18}$$

By substituting Eq. (18) into Eq. (17) one can obtain the solution $y=y_{\text{III}}$ in the case of $a_2\leq x<1$:

$$y_{\text{III}}=1-\frac{2}{W}(W-1+x)+\left(\frac{1-x}{W-1+x}\right)^{b'}\left[\left(\frac{W-1+a_2}{1-a_2}\right)^{b'}\left\{y_{a_2}-\frac{W-1+a_2}{W}\right\}\right.$$
$$\left.-\frac{2}{W}(1-b'c)\int_{a_2}^{x}\left(\frac{W-1+x}{1-x}\right)^{b'}dx\right] \tag{19}$$

Thus, the equations which represent the commutation curve of the Tandem Brush are as follows:

1) Case of $0\leq x\leq a_1$

$$y_{\text{I}}=1-\frac{2}{W}x+\frac{2}{W}(1-b'c)\left(\frac{W-x}{x}\right)^{b'}\int_0^x\left(\frac{x}{W-x}\right)^{b'}dx$$

2) Case of $a_1\leq x\leq a_2$

$$y_{\text{II}}=\left\{y_{a_1}-\frac{\beta-2c}{\alpha}\right\}e^{\alpha b(a_1-x)}+\frac{\beta-2c}{\alpha}$$

3) Case of $a_2\leq x<1$

$$y_{\text{III}}=1-\frac{2}{W}(W-1+x)+\left(\frac{1-x}{W-1+x}\right)^{b'}\left[\left(\frac{W-1+a_2}{1-a_2}\right)^{b'}\right.$$
$$\left.\left\{y_{a_2}-\frac{W-1+a_2}{W}\right\}+\frac{2}{W}(1-b'c)\int_{a_2}^{x}\left(\frac{W-1+x}{1-x}\right)^{b'}dx\right]$$

Figs. (2)~(7) show the commutation curves of a Tandem Brush with $a_1=0.4$, $a_2=0.6$ and $W=0.8$ in the case of $r=1/2$, 1, 2, 5, 10 and 20, respectively. In Fig. 3, dotted lines are the commutation curves of a ordinary brush at $r=1$, $W=0.8$. Fig. 8 shows the relation between the commutation curve and the value of W.

(2) Evaluation of Commution in Case of the Tandem Brush with $m=1$

When the value of the voltage drop between the segment No. 1 and the brush is about 3 ~ 3.5 volts, the *d-c* machine may have spark between the commutator segment and the brush.[4] This value of the voltage drop which is closely connected with satisfactory commutation can be obtained as follows:

$$
\left.\begin{aligned}
v_1 &= IR_bW(1+y_{\mathrm{I}})/(W-x), && (o\le x\le a_1)\\
&= IR_bW(1+y_{\mathrm{II}})/(1-a_2), && (a_1\le x\le a_2)\\
&= IR_bW(1+y_{\mathrm{III}})/(1-x), && (a_2\le x<1)
\end{aligned}\right\} \tag{20}
$$

Especially, the value of v_1 at the end of commutation is important and it becomes

$$
\lim_{x\to 1} v_1 = -IR_bW \lim_{x\to 1} \frac{d\,y_{\mathrm{III}}}{d\,x} \tag{21}
$$

Therefore, the quality of commutation can be estimated by the value of $\lim_{x\to 1} dy_{\mathrm{III}}/dx$. Table 1 shows the values of $\lim_{x\to 1} dy_{\mathrm{III}}/dx$ and $\lim_{x\to 1} v_{1R}$

Table 1.

ralues of r/Wand c		$\lim_{x\to 1} y'_{\mathrm{III}}$	$\lim_{x\to 1} v_{1R}$	values of L and RbT
$\frac{r}{W}\ge 1$	$r/_W > C$	$-\infty$	$+\infty$	$\frac{L}{W}\ge R_bT$
	$r/_W = C$	$-2/_W$	$+1$	
	$r/_W < C$	$+\infty$	$-\infty$	
$\frac{r}{W}<1$	$\frac{r}{W} \gtrless C$	$\frac{2}{W}\frac{c-1}{1-r/_W}$	$\frac{1-c}{1-r/_W}$	$\frac{L}{W}<R_bT$

From Table 1 one can understand the following relations:

1) In the case of $r/W=c$,

$\lim_{x\to 1} y'$ and $\lim_{x\to 1} v_{1R}$ have the finite values for all cases of $r/W \lesseqgtr c$.

2) In the case of $r/W<1$,

$\lim_{x\to 1} y'$ and $\lim_{x\to 1} v_{1R}$ are $\frac{2}{W}\frac{c-1}{1-r/_W}$ *i.e.* finite for any value of c.

Therefore, In the above two cases, one can expect satisfactory commutation.

3) In the case of $r/W\ge 1$ and $r/W \gtrless c$,

$\lim_{x\to 1} y'$ and $\lim_{x\to 1} v_{1R}$ are infinite. In these cases, commutation will be hard.

4) Whether the values of $\lim_{x\to 1} y'$ and $\lim_{x\to 1} v_{1R}$ are finite or not is decided by the valus of $r/_W$.

The value of v_{1R} used in Table 1, upon putting

$$
v_{1R} = \frac{v_1}{2\,IR_b} \tag{22}
$$

is expressed as follows:

$$
\left.\begin{aligned}
v_{1R} &= \frac{W}{2}\,\frac{1+y_{\mathrm{I}}}{W-x} && (o\le x\le a_1)\\
&= \frac{W}{2}\,\frac{1+y_{\mathrm{II}}}{1-a_2} && (a_1\le x\le a_2)\\
&= \frac{W}{2}\,\frac{1+y_{\mathrm{III}}}{1-x} && (a_2\le x<1)
\end{aligned}\right\} \tag{23}
$$

The values of v_{1R} in the cases of $r=1/2$, 1, 2, 5, 10 and 20 are shown in Figs. 9 ~ 14, respectively. In the same way, one can estimate the value of v_{2R} as follows:

$$
\left.\begin{aligned}
v_{2R} = \frac{v_2}{2\,IR_b} &= \frac{W}{2}\,\frac{1-y_{\mathrm{I}}}{x} && (o\le x\le a_1)\\
&= \frac{W}{2}\,\frac{1-y_{\mathrm{II}}}{a_1} && (a_1\le x \le a_2)\\
&= \frac{W}{2}\,\frac{1-y_{\mathrm{III}}}{W-1+x} && (a_2\le x<1)
\end{aligned}\right\} \tag{24}
$$

Figs. 15 ~ 20 show the values of v_{1R} in the case of r = 1/2, 1, 2, 5, 10 and 20, respectively.

In the cases of r = 2, 5, 10 and 20 the values of v_{1R} reveal the maximum at the leading edge of the trailing brush B_2 in Figs. 17 ~ 20. However, this muximum value is not so large except when the large value of c is given. On the other hand, the value of v_{1R} has a comparatively large value as compared with the value of v_{2R} even in the case of $r/W<1$. Therefore, one can understand that the brush sparking will appear easier at the trailing edge of component brush B_2 than at any other positions over the surface. This tendency is likely one of the ordinary brush. Now, upon comparing the commutation curve of Tandem Brush with that of an ordinary brush, the former has the following tendency:

1) In the former period of commutation, commutation is carried out earlier than in the case of an ordinary brush and in the latter, it is down later than in the case of an ordinary brush.

2) The greater the gap between the component brushes B_1 and B_2 is, the larger the above tendency becomes.

(3) The value of c which gives the linear commutation

The value of c which gives the linear commutation is obtained by the equations (4), (9) and (16) as follows:

$$\left.\begin{aligned} C_{\mathrm{I}} &= r - W\frac{1-W}{W-x} && (o \leq x \leq a_1) \\ C_{\mathrm{II}} &= r - \frac{W}{1-a_2} + \left(\frac{W}{1-a_2}+\frac{W}{a_2}\right)x && (a_1 \leq x \leq a_2) \\ C_{\mathrm{III}} &= r - W + \frac{Wx}{a_1-a_2+x} && (a_2 \leq x < 1) \end{aligned}\right\} \tag{25}$$

By commutation curves shown in Figs. 2 ~ 7 and Eq. (25), one can know that it is necessary to give the commutating emf., in order to get the linear commutation, as follows:

1) In the former period of commutation, to give smaller value of the commutating emf. than the value in the case of an ordinary brush.

2) In the latter period of commutation, to give larger value of the commutation emf. than the value in the case of an ordinary brush.

4. Commutation Curve in Case of $m<1$

(1) The solution of the commutation equation (1)

The differential equation concerning the current in a coil short-circuited by the Tandem Brush, where surface conductivities of the leading and trailing brushes are σ_1 and σ_2, respectively, can also be expressed in the same way as the equation (1).

$$L\frac{di}{dt} + v_1 - v_2 + e_c = 0 \tag{26}$$

By using the same variables introduced in the former Section 3, Eq. (26) can be divided into the following three cases according to the contact features between the segments and a brush.

1) Case of $o \geq x \geq a_1$

In this case one can obtain the following relations:

$$v_1 = (I+i)\frac{1}{(1-a_2)\sigma_2+(a_1-t/T)\sigma_1} = (I+i)\frac{R_b}{1-x'} \tag{27}$$

$$v_2 = (I-i)\frac{1}{(t/T)\sigma_1} = (I-i)\frac{R_b}{x'} \tag{28}$$

where $x' = x/W'$, $W' = a_1 + m(1-a_2)$ and $m = \sigma_2/\sigma_1$.

Substitution of Eqs. (27) and (28) into Eq. (26) gives

$$\frac{dy}{dx'} + b_1 y\left(\frac{1}{1-x'}+\frac{1}{x'}\right) = b_1\left(\frac{1}{x'}-\frac{1}{1-x'}\right) - 2b_1 c \tag{29}$$

where $b_1 = W'b$

Eq. (29) is similar to Eq. (4), therefore one can get a similar solution to Eq. (5) as follow:

$$y_{\mathrm{I}} = 1 - 2x' + 2(1-b_1 c)\left(\frac{1-x'}{x'}\right)^{b_1}\int_o^{x'}\left(\frac{x'}{1-x'}\right)^{b_1} dx' \tag{30}$$

Using the relation $x' = x/W'$, one can rewrite Eq. (30) as follow:

$$y_{\mathrm{I}} = 1 - 2\frac{x}{W'} + \frac{2}{W'}(1-b_1 c)\left(\frac{W'-x}{x}\right)^{b_1}\int_o^{x}\left(\frac{x}{W'-x}\right)^{b_1} dx \tag{31}$$

2) Case of $a_1 \leq x \leq a_2$

$$v_1 = (I+i)\frac{1}{(1-a_2)\sigma_2} = (I+i)\frac{W'R_b}{(1-a_2)m} \tag{32}$$

$$v_2 = (I-i)\frac{1}{a_1\sigma_1} = (I-i)\frac{W'R_b}{a_1} \tag{33}$$

Substitution of Eqs. (32) and (33) into Eq. (26) gives

$$\frac{dy}{dx} + by\left\{\frac{W'}{m(1-a_2)}+\frac{W'}{a_1}\right\} = b\left\{\frac{W'}{a_1}-\frac{W'}{m(1-a_2)}\right\} - 2bc. \tag{34}$$

Upon putting

$$\alpha' = \frac{W'}{m(1-a_2)} + \frac{W'}{a_1}, \qquad \beta' = \frac{W'}{a_1} - \frac{W'}{m(1-a_2)}$$

Eq. (34) becomes as follw:

$$\frac{dy}{dx} + \alpha' by = b(\beta' - 2c) \tag{35}$$

The solution of Eq. (35) can be obtained as follows:

$$y_{\mathrm{II}} = \left\{y_{a1} - \frac{\beta'-2c}{\alpha'}\right\} e^{\alpha' b(a_1-x)} + \frac{\beta'-2c}{\alpha'} \tag{36}$$

The above solution has been derived by deciding the integration constant at the initial condition: $y_{\mathrm{II}} = y_{\mathrm{I}}|_{x=a_1} = y_{a1}$, at $x = a_1$.

(3) Case of $a_2 \leq x < 1$

$$v_1 = (I+i)\frac{1}{(1-t/T)\sigma_2} = (I+i)\frac{W''R_b}{1-x} = (I+i)\frac{R_b}{1-x''} \tag{37}$$

$$v_2 = (I - i)\frac{1}{a_1\sigma_1 + (x - a_2)\sigma_2} = (I - i)\frac{W'' R_b}{(a_1/m) - a_2 + x} = (I - i)\frac{R_b}{x''} \tag{38}$$

where

$$x'' = \frac{(a_1/m) - a_2 + x}{(a_1/m) - a_2 + 1} = \frac{W'' - 1 + x}{W''}$$

$$W'' = (a_1/m) - a_2 + 1 = \frac{1}{m} W'$$

Substitution of Eqs. (37) and (38) into Eq. (26) gives

$$\frac{dy}{dx''} + b_2 y \left(\frac{1}{1 - x''} + \frac{1}{x''} \right) = b_2 \left(\frac{1}{x''} - \frac{1}{1 - x''} \right) - 2 b_2 c \tag{39}$$

where $b_2 = W''b$

Eq. (39) can be solved by the same way as the solution of Eq. (29). Thus, one can obtain the solution of Eq. (39) as follows:

$$y_{\text{III}} = 1 - 2x'' + \left(\frac{1 - x''}{x''} \right)^{b_2} \left[C_3 + 2(1 - b_2 c) \int \left(\frac{x''}{1 - x''} \right)^{b_2} dx'' \right] \tag{40}$$

By introducing the initial condition of $y_{\text{III}} = y_{\text{II}} \mid_{x = a_2} = y_{a_2}$ at $x = a_2$, the integration constant C_3 can be decided as follows:

$$C_3 = \left\{ \frac{a_1}{m(1 - a_2)} \right\}^{b_2} \left\{ y_{a_2} + \frac{a_1 - m(1 - a_2)}{a_1 + m(1 - a_2)} \right\} - \frac{2}{W''}(1 - b_2 c) \int \left\{ \frac{(a_1/m) - a_2 + x}{1 - x} \right\}^{b_2} dx \Bigg|_{x = a_2} \tag{41}$$

By substituting Eq. (41) into Eq. (40), one can obtatn the solution of Eq. (39) as follow:

$$y_{\text{III}} = 1 - \frac{2}{W''}\left(\frac{a_1}{m} + x - a_2\right) + \left(\frac{1 - x}{a_1/m + x - a_2} \right)^{b_2} \left[\left(\frac{a_1/m}{1 - a_2} \right)^{b_2} \left\{ y_{a_2} + \frac{a_1/m - 1 + a_2}{W''} \right\} + \frac{2}{W''}(1 - b_2 c) \int_{a_2}^{x} \left\{ \frac{a_1/m + x - a_2}{1 - x} \right\}^{b_2} dx \right] \tag{42}$$

Thus, the equations which represent the commutation curve of a Tandem Brush with $m<1$ are as follows:

1) Case of $o \leq x \leq a_1$

$$y_{\text{I}} = 1 - 2\frac{x}{W'} + \frac{2}{W'}(1 - b_1 c) \left(\frac{W' - x}{x} \right)^{b_1} \int_0^x \left(\frac{x}{W' - x} \right)^{b_1} dx$$

2) Case of $a_1 \leq x \leq a_2$

$$y_{\text{II}} = \left\{ y_{a_1} - \frac{\beta' - 2c}{\alpha'} \right\} e^{-\alpha' b(x - a_1)} + \frac{\beta' - 2c}{\alpha'}$$

3) Case of $a_2 \leq x < 1$

$$y_{\text{III}} = 1 - \frac{2}{W''}\left(\frac{a_1}{m} + x - a_2\right) \left(\frac{1 - x}{a_1/m + x - a_2} \right)^{b_2} \left[\left(\frac{a_1/m}{1 - a_2} \right)^{b_2} \left\{ y_{a_2} + \frac{a_1/m - 1 + a_2}{W''} \right\} + \frac{2}{W''}(1 - b_2 c) \int_{a_2}^{x} \left\{ \frac{a_1/m + x - a_2}{1 - x} \right\}^{b_2} dx \right]$$

Figs. 21 ~ 26 show the commutation curves in the cases of $r = 1/2$, 1, 2, 5, 10 and 20 respectively, where the commutating emf. is constant and the dimensions of the buush are $a_1 = 0.4$, $a_2 = 0.6$, $W = 0.8$ and $m = 1/4$. In Fig. 22, commutation curves of the ordinary brush and the Tandem Brush with $m = 1$ are entered to compare with the performance of three brushes.

(2) Evaluation of commutation in case the Tandem Brush with $m < 1$

Upon putting
$$v_{1R} = v_1 / 2IR_b\,,$$
$$v_{2R} = v_2 / 2IR_b\,,$$
one can obtain the values of v_{1R} and v_{2R} as follows:

$$\left.\begin{aligned} v_{1R} &= \frac{W'}{2}\,\frac{1+y_{\mathrm{I}}}{W'-x} && (o \le x \le a_1) \\ &= \frac{W'}{2}\,\frac{1+y_{\mathrm{II}}}{1-a_2} && (a_1 \le x \le a_2) \\ &= \frac{W''}{2}\,\frac{1+y_{\mathrm{III}}}{1-x} && (a_2 \le x < 1) \end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned} v_{2R} &= \frac{W'}{2}\,\frac{1-y_{\mathrm{I}}}{x} && (o \le x \le a_1) \\ &= \frac{W'}{2}\,\frac{1-y_{\mathrm{II}}}{a_1} && (a_1 \le x \le a_2) \\ &= \frac{W''}{2}\,\frac{1-y_{\mathrm{III}}}{W''-1+x} && (a_2 \le x < 1) \end{aligned}\right\} \tag{44}$$

From above equations of y and v_{1R}, one obtains the values of $\lim\limits_{x\to 1} y'$ and $\lim\limits_{x\to 1} v_{1R}$ as the following Table 2:

Table 2.

values of r/W'' and c		$\lim\limits_{x\to 1} y'_{\mathrm{III}}$	$\lim\limits_{x\to 1} v_{1R}$	values of L/W'' and R_bT
$\frac{r}{W''} > 1$	$\frac{r}{W''} > C$	$-\infty$	$+\infty$	$\frac{L}{W''} > R_bT$
	$\frac{r}{W''} = C$	$-\frac{2}{W''}$	$+1$	
	$\frac{r}{W''} < C$	$+\infty$	$-\infty$	
$\frac{r}{W''} = 1$	$\frac{r}{W''} > C$	$-\infty$	$+\infty$	$\frac{L}{W''} = R_bT$
	$\frac{r}{W''} = C$	$-\frac{2+K}{W''}$	$1+\frac{K}{2}$	
	$\frac{r}{W''} < C$	$+\infty$	$-\infty$	
$\frac{r}{W''} < 1$	$\frac{r}{W''} \gtreqless C$	$\frac{2}{W''}\,\frac{c-1}{1-r/W''}$	$\frac{1-c}{1-r/W''}$	$\frac{L}{W''} < R_bT$

In Table 2, K is the first term in Eq. (41) as
$$K = \left\{\frac{a_1}{m\,(1-a_2)}\right\}^{b_2} \left\{ y_{a2} + \frac{a_1 - m(1-a_2)}{a_1 + m(1-a_2)} \right\}$$

In case of the surface resistance of the trailing brush being larger than that of the leading brush, the effective width of the brush as a whole reduces to $W' = a_1 + m\,(1-a_2)$ when the width of the trailing brush is conventionally converted into the leading one of the brush. This fact is equivalent to the reduction of the commutation period in the range, $o \le x \le a_1$.

On the other hand, the conventional conversion of the leading brush into the trailing one results in the fact that the width of the whole brush is enlarged as $W' = (a_1/m) + (1 - a_2)$ in the range, $a_2 \leq x < 1$. This manner of understanding is equivalent to the concept of assuming an increase of the commutation period.

In another range $a_1 \leq x \leq a_2$, while the segment mica lies between the component brushes, the contact resistances remain constant. In this period the commutation curve has the exponential form according to Eq. (36). Thus, the use of Tandem Brush which has the condition $m<1$ enables us to obtain a satisfactory commutation by making the value of $\lim_{x\to 1} dy/dx$ and $\lim_{x\to 1} v_{IR}$ smaller than that of an ordinary brush even when $r \geq 1$.

From Table 2, one can also express the performance of the Tandem Brush with $m<1$ as follows:

Under the two conditions of

(a) $\frac{r}{W''} \geq 1$ and $\frac{R}{W''} = C$

(b) $\frac{r}{W''} > 1$ and $\frac{r}{W''} \gtreqless C$

the value of $\lim_{x\to 1} v_{IR}$ is finite. Therefore, we should consider the realization of the above two conditions. It must be difficult in practice, however to realize the conditian (a) but the condition (b) may be realized by making a brush with the value of W'' as Eq. (45):

$$W'' = \left[\left(\frac{\sigma_1}{\sigma_2} - 1\right) a_1 + \{a_1 + (1 - a_2)\}\right] \tag{45}$$

(3) The values of c which gives the linear commutation

The value of c which gives the linear commutation is obtained from the equations (29), (34) and (39) as follows:

$$\left.\begin{array}{ll} C_{\mathrm{I}} = r - W' \ (1 - W') / (W' - x) & (o \leq x \leq a_1) \\ C_{\mathrm{II}} = r - W'/m\ (1 - a_2) + W'\{1/m(1 - a_2) + 1/a_1\}\ x & (a_1 \leq x \leq a_2) \\ C_{\mathrm{III}} = r - W'' \ (1 - W'') / (W'' - x) & (a_2 \leq x < 1) \end{array}\right\} \tag{46}$$

5 Summary

The present writer has solved the differential equation concerning the current in a coil short circuited by a Tandem Brush in order to obtain the following results by regarding the contact resistance between the commutator and a brush as an Ohmic resistance, so far as the brush width is equal to the width of the commutator segment and the mica thickness is assumed to be zero:

(1) In the case of a Tandem Brush a linear commution can be reallized, so far as the commutating emf. is given by the equation (25) or (46) to the respective cases.

(2) When the commtating emf. is constant and the Tandem Brush has the

value of $m=1$, the value of $\lim_{x\to 1} di/dt$ is finite in the case of $L/W < R_bT$ and then a satisfactory commutation can be realized. On the contrary, the value of $\lim_{x\to 1} di/dt$ is infinite in the case of $L/W \geq R_bT$ except for a case of $r/W=c$.

(3) In case of $m=1$, the value of W is smaller than 1. Therefore, the performance of commutation in case of the Tandem Brush is worse than in that of the ordinary brush as regards the transfer of the current.

(4) In the case of a Tandem Brush with different surface conductivities of two component brushes, the value of $\lim_{x\to 1} di/dt$ is finite under the condition of $L/W < R_bT$. So, a satisfactory commutation can be easily realized.

(5) The value of $\lim_{x\to 1} di/dt$ is infinit in the case of $L/W'' \geq R_bT$ except for a case of $r/W''=c$. However, the larger the $W''=(a_1/m)+1-a_2$ is, the easier the commutation becoms.

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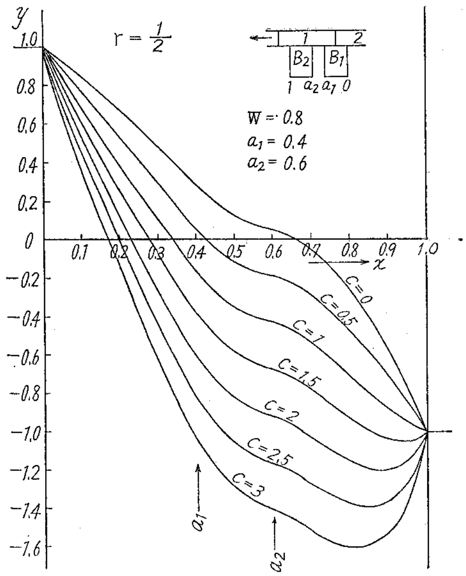
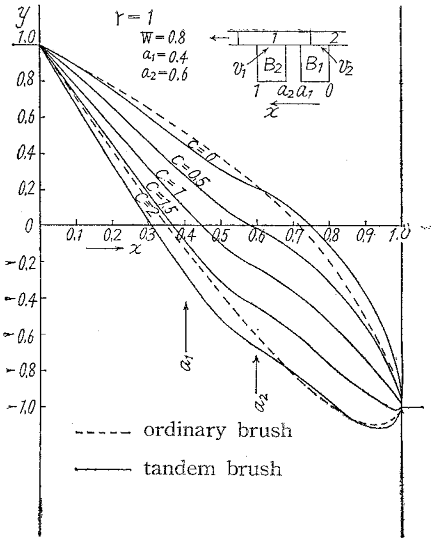


Fig. 2



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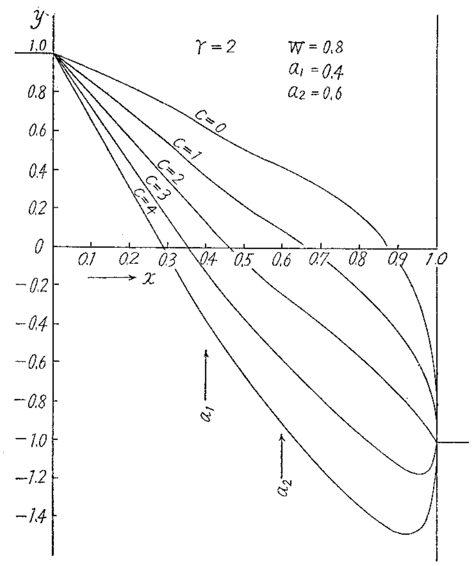

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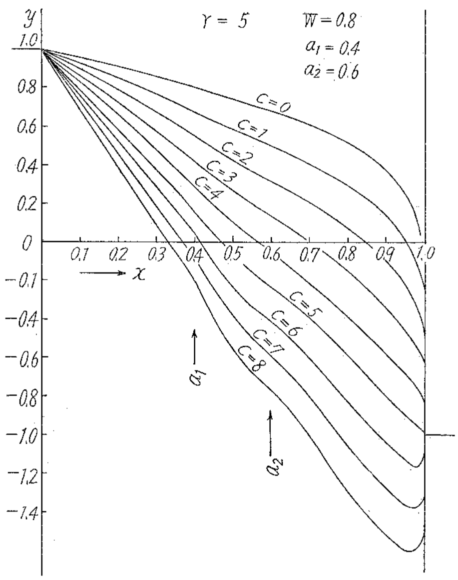

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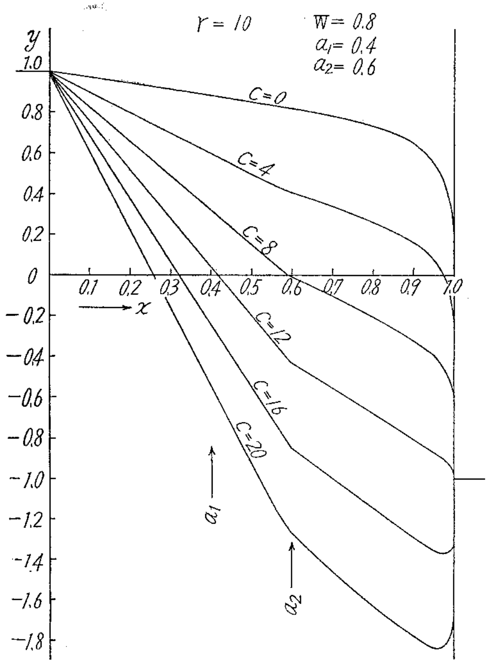

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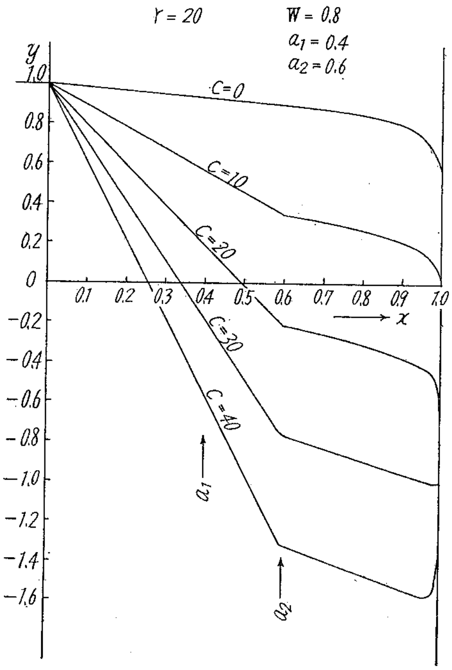


Fig .7

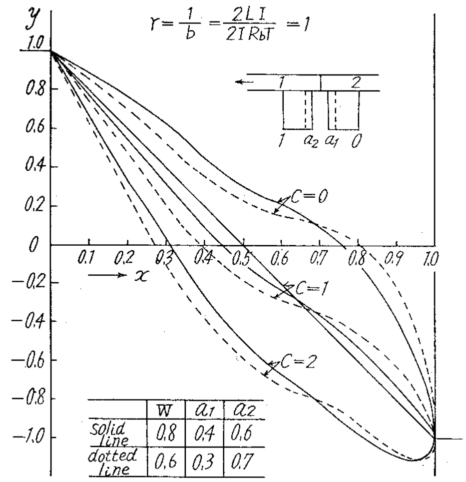
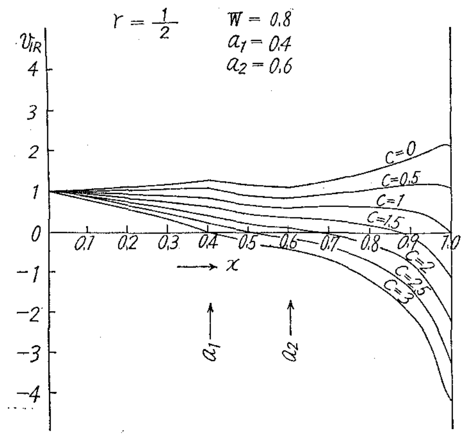


Fig. 8 Reration between the commutation curves and values of W.



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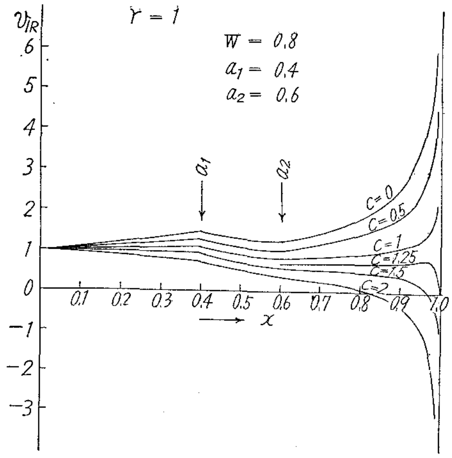

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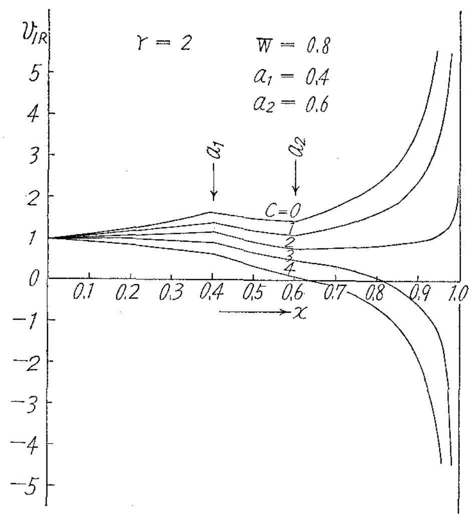
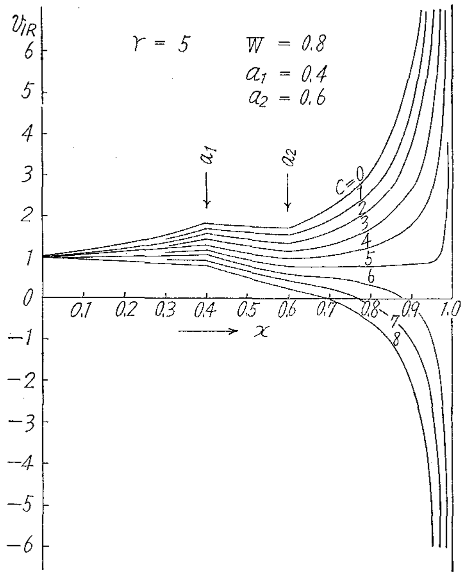


Fig. 11



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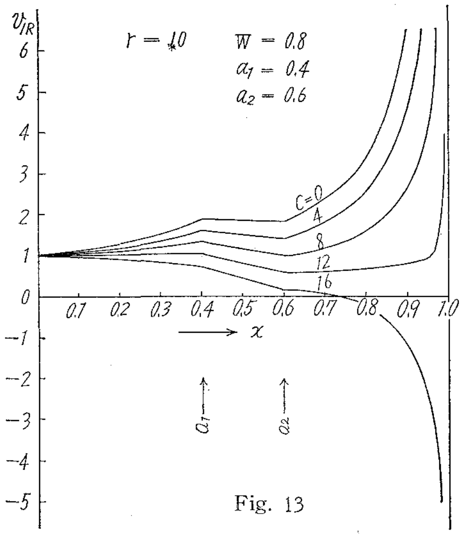
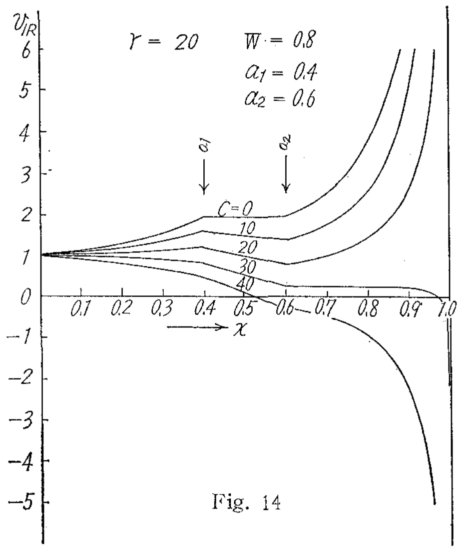


Fig. 13



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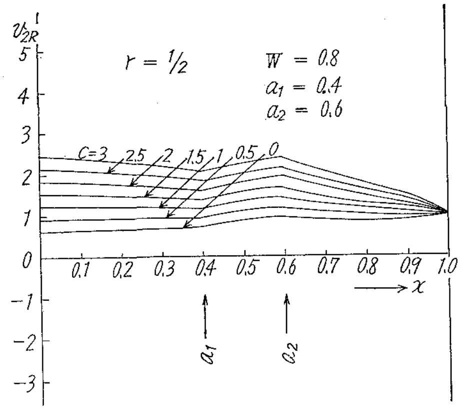

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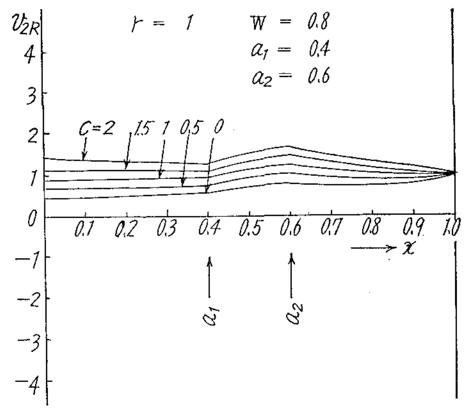

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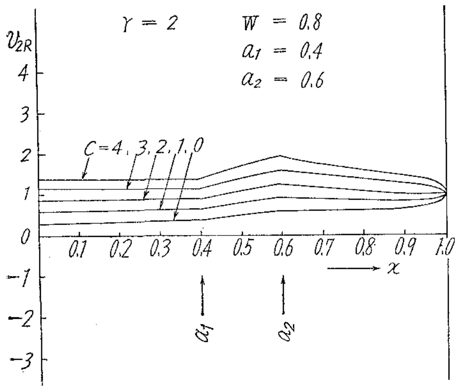
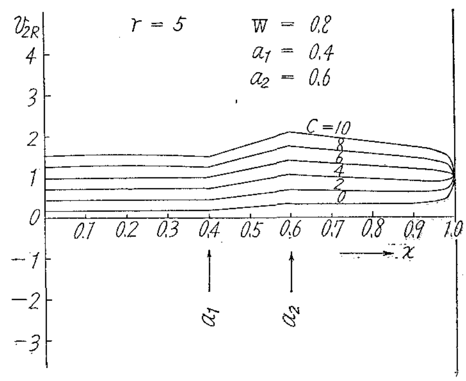


Fig 17



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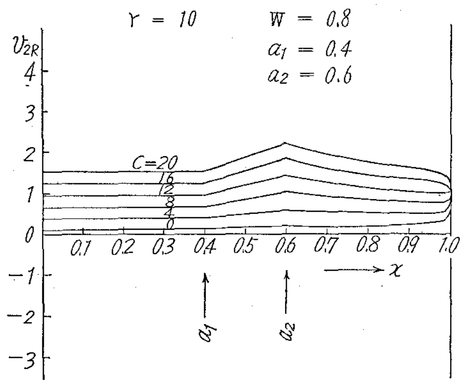

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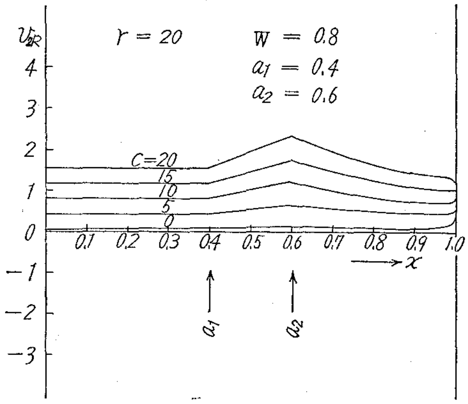

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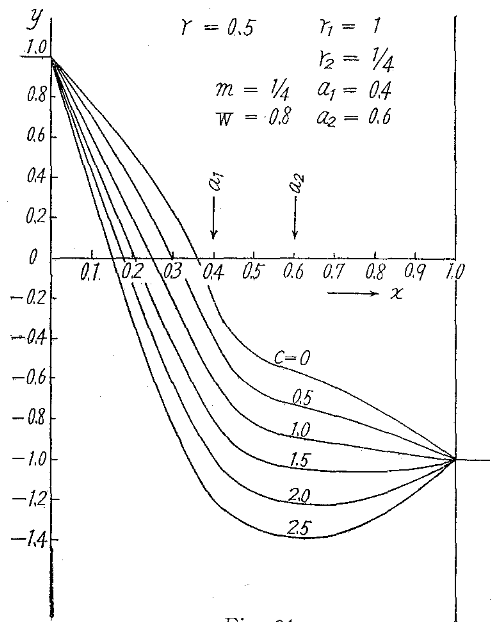

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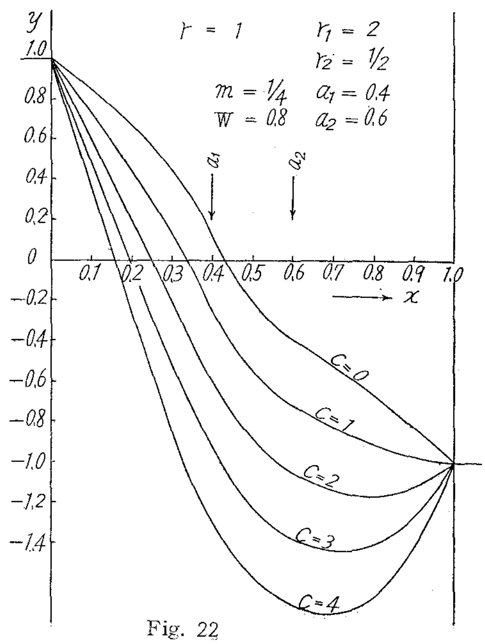

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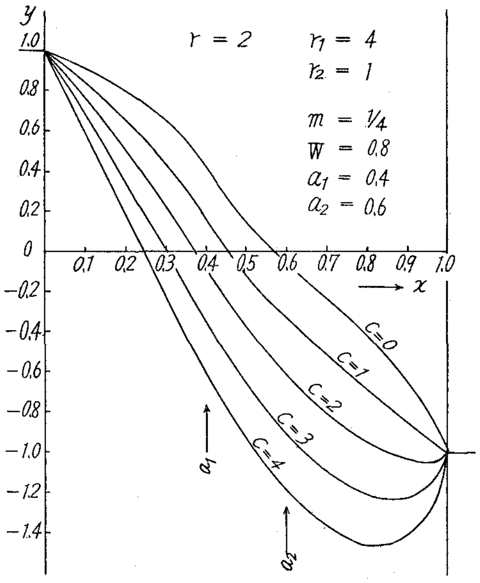

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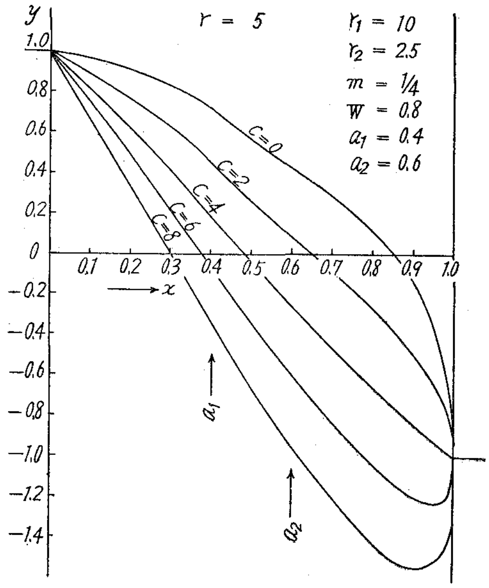

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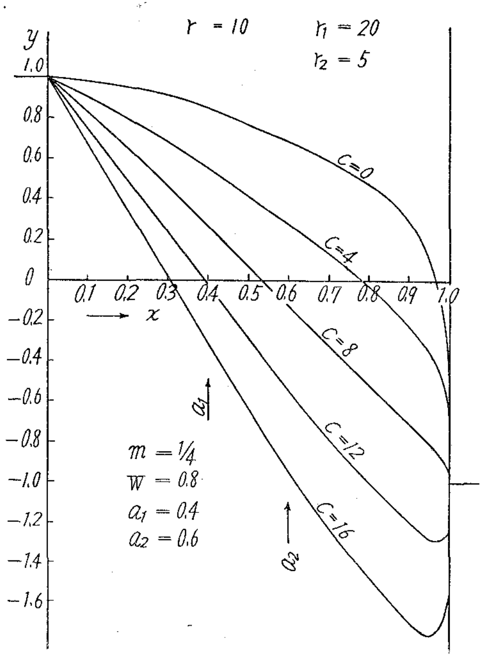

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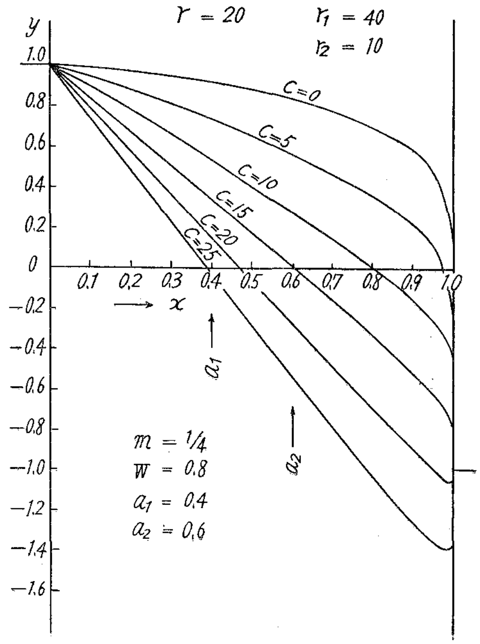

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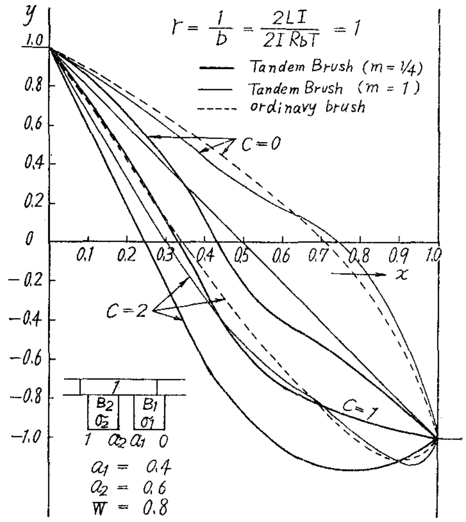


Fig. 27 Comparison of the commutation curves with Tandem Brushes m = 1/4 and m = 1 to the ordinary brush in case of r = 1

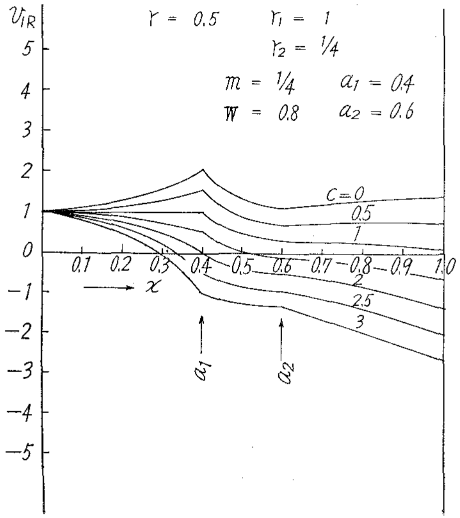


Fig. 28

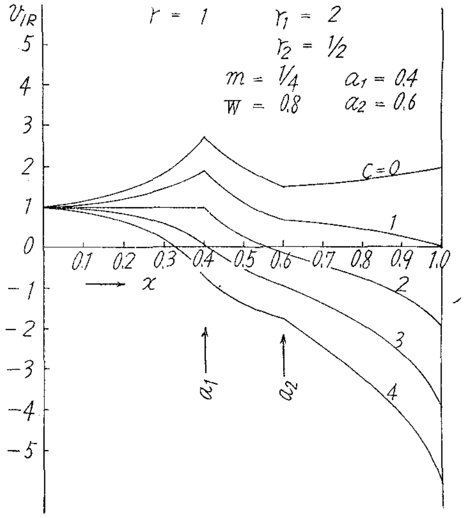


Fig. 29

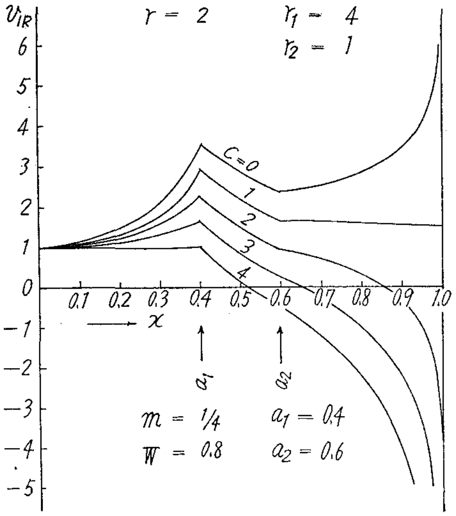


Fig. 30

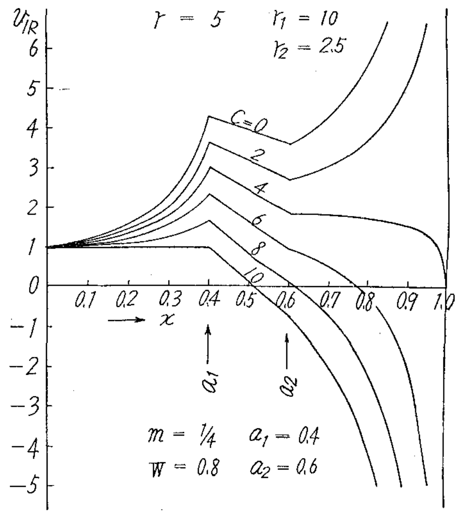


Fig. 31

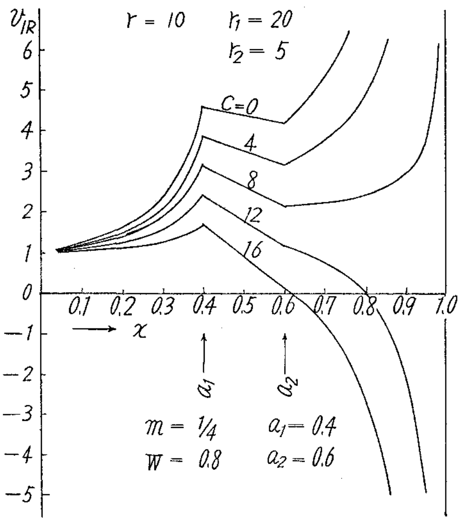
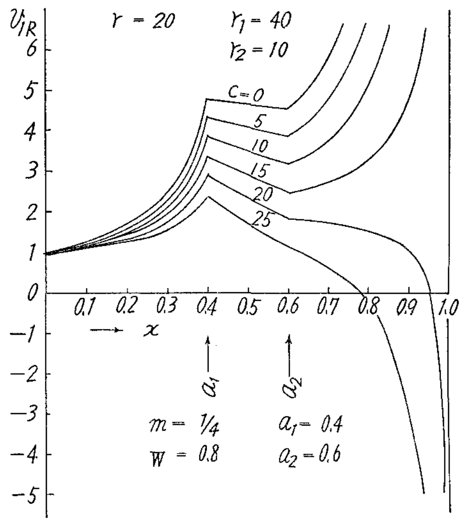


Fig. 32



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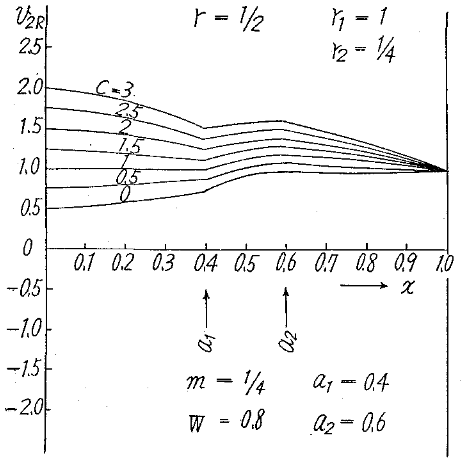

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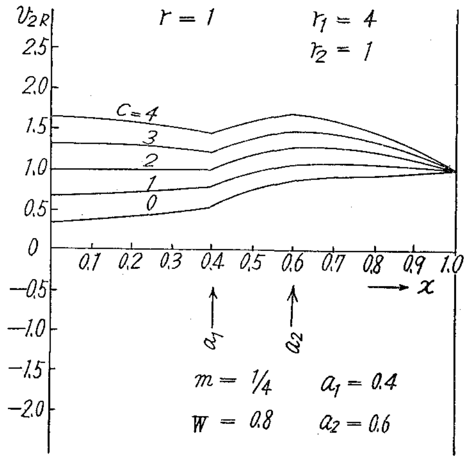

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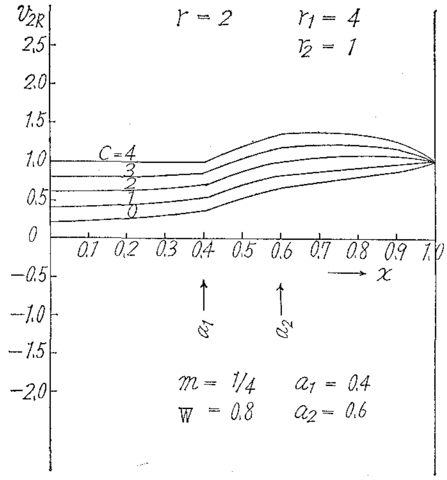

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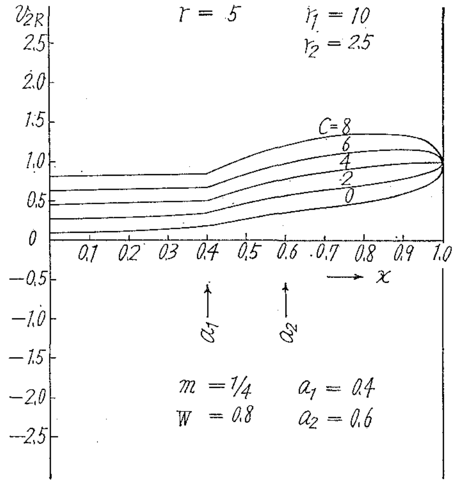

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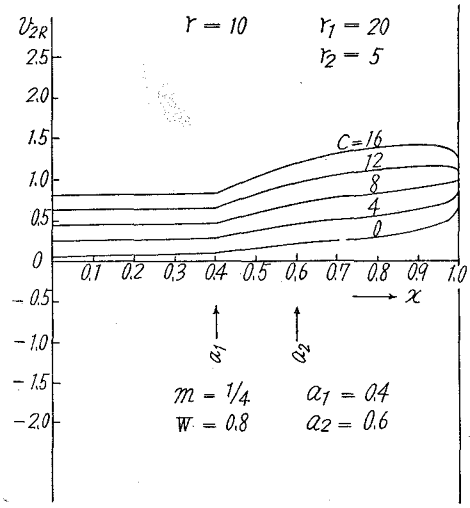
v_{2R}
2.5
2.0
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2.0
r = 10
$r_1 = 20$
$r_2 = 5$
C=16
12
8
4
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
→ x
a_1
a_2
m = 1/4
W = 0.8
$a_1 = 0.4$
$a_2 = 0.6$

Fig. 38

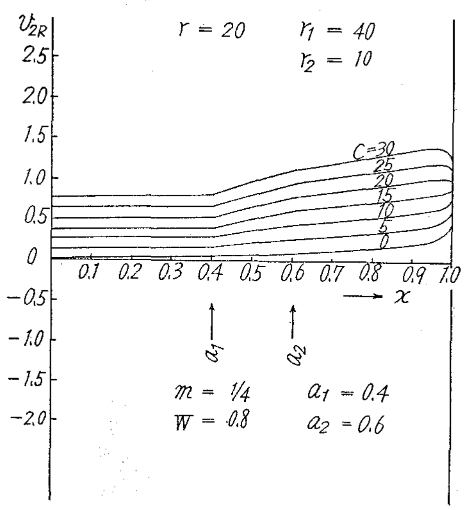
v_{2R}
2.5
2.0
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2.0
r = 20
$r_1 = 40$
$r_2 = 10$
C=30
25
20
15
10
5
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
→ x
a_1
a_2
m = 1/4
W = 0.8
$a_1 = 0.4$
$a_2 = 0.6$

Fig. 39