A Study of the Snow-melt Runoff of Rivers

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Takao Sakai*

Abstract

In this paper, the present author has developed a new method of analysing and computing snow-melt runoff using the degree-hour factor. He has found the formulae of determining degree hours only from daily maximum and minimum air temperatures.

He has applied this method for the Saru River Basin and the Ishikari River Basin in Hokkaidô. The computed runoff coincides well with the observed runoff.

1. Introduction

The snow-melt runoff in spring is an important problem not only for flood control because it often brings flood, but also for such water utilization as hydro-electric power, irrigation and other kinds of water supply.

Studies on this subject have been performed actively in U.S.A. since about 1930.[1],[2] Especially a thermo-dynamical study of snow-melt by Wilson[3] was remarkable, and Light[4] also studied similar subjects. As researches in a field, we must notice those prosecuted at Gooseberry Creek and at Crater Lake on which interesting results were reported[5]. For predicting the total amount of runoff through the snow-melting season, the results of snow survey just before the melting season were used; for example, Clyde[6] applied this method for rivers in Uta State. But for forecasting day-to-day runoff, Linsley[7] adopted the method of degree days and applied it to rivers in California.

In Japan, Sugaya[8] performed a snow survey and an investigation of runoff in the Chûbetsu River Basin near Mt. Taisetsu in Hokkaidô in 1948, and henceforth similar researches by other investigators were carried out actively.

As known by the previous studies, kinds of heat transfer which cause snow-melt are listed in the order of importance as follows:

1) Convection from turbulent air
2) Heat by condensation of water vapour in air
3) Solar radiation
4) Heat from warm rain
5) Conduction from soil
6) Conduction from still air

Above all, important items are 1)~3), of which thermo-dynamical calculations are possible by Wilson's formulae or others. But the combination of these factors is very complicated, and the meteorological conditions of nature are widely variable;

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therefore, unless the data of observation are offered enough, application of these formulae is difficult.

The practical method which hitherto has been used is that of degree days. This method adopts air temperature as the most important and representative factor which causes snow-melt. The degree day is a quantity of daily mean air temperature which exceeds that at which snow-melt begins (usually taken as 0°C) multiplied by a day. The snow-melt or its runoff per unit degree day is called the *degree-day factor*. The observed values of degree-day factor hitherto reported by various investigators are about 0.02~0.15 in./°F·day namely 0.09~0.7 cm/°C·day.

The method using degree-day factor is effective and practical. Nevertheless, it has irrationality that even in the case when the air temperature in the daytime is so high that snow-melt actually happens, if the daily mean air temperature (which is usually taken as the mean of daily maximum and minimum values) is below 0°C, the value of degree days is calculated as zero. (cf. Fig. 1)
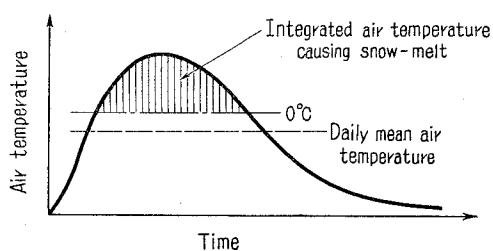

Fig. 1. Daily mean air temperature and integrated air temperature causing snow-melt.

Therefore, for more accurate treatment, the use of integrated value of air temperature above 0°C should be considered. This is why the author should propose the method using the degree-hour factor.[9],[10]

2. The Author's Method of Computation of Snow-melt Runoff

(1) Method of Computation of Integrated Air Temperature

The integrated value of air temperature above 0°C is obtained from records of thermograph which, however, is not usually obtained at a simple observatory where only daily maximum and minimum air temperatures are observed. As many actual examples show, the daily variation of air temperature is periodic but not as a sine curve; its rising period is shorter while its falling period is longer.

The author has expressed the daily variation of air temperature by the following equation.

$$T = Cte^{-\alpha t^2} \tag{1}$$

where T is an air temperature (°C) measured from the minimum air temperature, t is a time (hr) measured from the time of minimum air temperature, and C and α are constants. This equation is applied from the time of minimum air temperature of a day to ditto of the next day as one division. Taking the time length between the hours of maximum and minimum air temperature as 7 hours, constants in Eq. (1) are determined as follows.

At the time of maximum air temperature,

$dT/dt=0$, hence $\alpha=1/2t^2$, and putting $t=7$ we get $\alpha=1/98$.

From Eq. (1)

$$C=\frac{T}{t}e^{\alpha t^2}=\frac{T}{t}e^{\frac{1}{2}}$$

Let the temperature difference between daily maximum T_2 and minimum T_1 be ΔT,

when $t=7$, $T=\Delta T$.

$$\therefore \quad C=\frac{\Delta T}{7}e^{\frac{1}{2}}=\frac{\Delta T}{7}\times 1.649=0.2355\,\Delta T$$

Therefore Eq. (1) becomes

$$T=0.2355\,\Delta Tte^{-t^2/98} \tag{2}$$

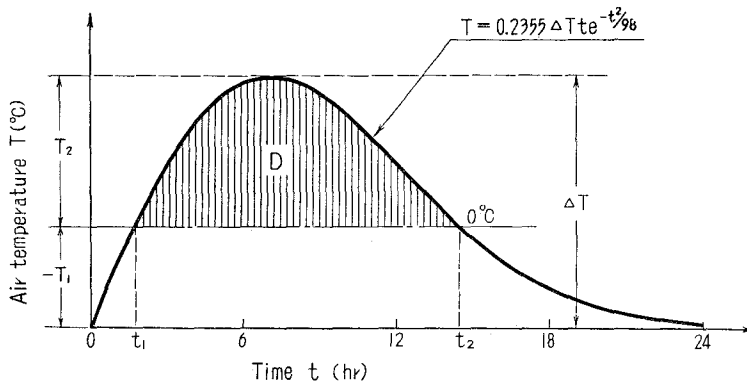The daily variation curve of T by Eq. (2) is as shown in Fig. 2.

Fig. 2. Daily variation curve of air temperature and the integrated air temperature.

For the happening of snow-melt, $T_2>0°C$, while T_1 may be $\lessgtr 0°C$.
1) $T_1<0°C$: —

In this case, the hours t_1 and t_2 when $T=0°C$, are determined by the ratio $T_2/\Delta T$, and integrating T from t_1 to t_2, the integrated value of air temperature is obtained.

Now denote

$$\left.\begin{array}{l} m=T_2/\Delta T \\ C=n\Delta T \end{array}\right\} \tag{3}$$

then at t_1 and t_2,

$$T = \Delta T - T_2$$
$$= (1-m)\Delta T = n\Delta T t e^{-\alpha t^2}$$
$$\therefore \quad t e^{-\alpha t^2} - \frac{1-m}{n} = 0 \tag{4}$$

Substituting numerical values into constants in Eq. (4), and rewriting it to the form convenient for calculation,

$$\frac{t}{10^{0.00443t^2}} - \frac{1-m}{0.2355} = 0 \tag{4'}$$

Solving the above equation, t_1 and t_2 may be determined.

Then the integrated air temperature above 0°C i.e. the degree hours D in Fig. 2 may be written

$$D = \int_{t_1}^{t_2} T dt - (\Delta T - T_2)(t_2 - t_1)$$

$$= \left\{ \frac{n}{2\alpha} \left(e^{-\alpha t_1^2} - e^{-\alpha t_2^2} \right) - (1-m)(t_2 - t_1) \right\} \Delta T$$

$$= \left\{ 11.54 \left(10^{-0.00443t_1^2} - 10^{-0.00443t_2^2} \right) - (1-m)(t_2 - t_1) \right\} \Delta T$$

$$= \xi \Delta T \tag{5}$$

In the above equation, ξ is eventually a function of m only, and is calculated as shown in Table 1.

Table 1.

m	t_1	t_2	ξ
.1	4.8	9.4	.29
.2	4.0	10.6	.86
.3	3.3	11.5	1.60
.4	2.8	12.5	2.49
.5	2.2	13.5	3.55
.6	1.8	14.5	4.74
.7	1.3	15.6	6.08
.8	.9	17.1	7.62
.9	.4	19.3	9.37
1.0	0	24.0	11.50

2) $T_1 = 0°C$: —

In this case, $m=1$. Denote D in this case as D_0, then from Table 1,

$$D_0 = 11.50 \Delta T \tag{6}$$

3) $T_1 > 0°C : -$

In this case, $m > 1$.

$$D = D_0 + 24 T_1$$
$$= \{11.50 + 24(m-1)\} \varDelta T$$
$$= 24(m - 0.521) \varDelta T = \xi \varDelta T \qquad (7)$$

As seen above, ξ is a linear function of m, namely varies as a straight line. Summarizing the above, $m - \xi$ relation is shown in Table 2 and Fig. 3.

Table 2. $m - \hat{\xi}$ Relation

m	.00	.02	.04	0.4	.08
.0	.0	.0	.1	.1	.2
.1	.3	.4	.5	.6	.7
.2	.9	1.0	1.1	1.3	1.5
.3	1.6	1.8	1.9	2.1	2.3
.4	2.5	2.7	2.9	3.1	3.3
.5	3.6	3.8	4.0	4.2	4.5
.6	4.7	5.0	5.3	5.5	5.8
.7	6.1	6.4	6.6	6.9	7.3
.8	7.6	7.9	8.3	8.6	9.0
.9	9.4	9.8	10.2	10.6	11.0
1.0	11.5	12.0	12.5	13.0	13.4
1.1	13.9	14.4	14.9	15.3	15.8
1.2	16.3	16.8	17.3	17.7	18.2
1.3	18.7	19.2	19.7	20.1	20.6
1.4	21.1	21.6	22.0	22.5	23.0
1.5	23.5	23.9	24.4	24.9	25.3
1.6	25.9	26.3	26.8	27.3	27.8
1.7	28.3	28.7	29.2	29.7	30.2
1.8	30.7	31.1	31.6	32.1	32.6
1.9	33.1	33.5	34.0	34.5	35.0

As described above, if only the maximum and minimum air temperatures are given in any day, from Table 2 the value of ξ is found by which degree hours D (°C hr) may be easily computed.

Equations related are summarized as follows.

$$\left. \begin{array}{l} \varDelta T = T_2 - T_1 \\ m = T_2 / \varDelta T \\ D = \xi \varDelta T \end{array} \right\} \qquad (8)$$

The above-mentioned theory has been com-
posed, assuming that the minimum air tem-
perature of a day is nearly equal to ditto of
the next day. Actually, however, there is a
little difference between these two, hence some
error exists between the computed and the
actual values of D. To remove such an error,
the minimum air temperature used for calcu-
lation should rather be taken as the mean
value of the minimum air temperature of a
day T_{1a} and that of the next day T_{1b}, namely

$$T_1 = \frac{1}{2}(T_{1a} + T_{1b}) \qquad (9)$$

Numerical example : —

$$T_{1a} = -4°C \qquad T_{1b} = -2°C$$
$$T_2 = 8°C$$

Fig. 3. $m-\xi$ curve.

From the above data, D is required.

$$T_1 = \frac{1}{2}(-4-2) = -3°C$$

$$\varDelta T = T_2 - T_1 = 8 - (-3) = 11°C$$
$$m = T_2/\varDelta T = 8/11 = 0.727$$

From Table 2, $\xi = 6.5$

$$D = \xi \varDelta T = 6.5 \times 11 = 71.5°C \text{ hr}$$

The calculated value of D by the above method nearly coincides with the
actual value ; its error is less than about $\pm 8\%$ while the former method of degree
days may contain an error about $\pm 30\%$, especially larger error when the mean
daily air temperature is near 0°C.

(2) **Integrated Temperature-Area Causing Snow-Melt**

In a river basin, snow-melt happens in the zone between the *snow line* which
is the lower limit of snow cover area and the *freezing line* which is defined as
the line where the daily maximum air temperature is 0°C.

Fig. 4 shows the *area-elevation curve* for a river basin, on which the snow
line elevation h_1 and the freezing line elevation h_2 being indicated, the percentage of
area for each of them p_1 and p_2 is easily found graphically. Then the snow-melting
area A is expressed by

$$A = (p_1 - p_2)\, A_0/100 \tag{10}$$

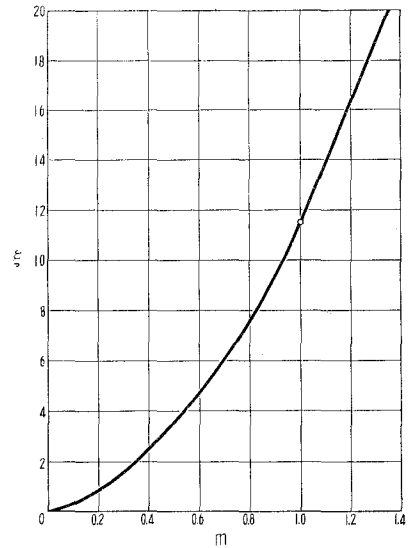where A_0 is the total basin area.

The snow line, in a snow-melting season, moves up and down as the season advances and is not constant. Moreover, the snow-line does not always coincide with a contour line; it is generally lower on northward or forestcovered slopes and is higher on southward or bare slopes. Nevertheless, it is possible to determine the mean elevation of snow-line of the whole basin and to know its movement following the progress of season in broad view.

Next, the freezing line also moves every day and is not constant. But its elevation is determined by the air temperature at an observatory if the lapse rate of air temperature is known. The lapse rate may be generally assumed as $0.6°C/100$ m in the snow-melting season. Then the freezing line elevation h_2 is

Fig. 4. Relation between the area and elevation of a snow-melting zone.

$$h_2 = h_0 + \frac{100}{0.6} T_2'' \qquad (11)$$

where h_0 is the elevation of an observatory and T_2'' is the daily maximum air temperature at the observatory.

The area-elevation curve for a river basin is important for the calculation of snow-melting area; this curve may be drawn by planimetering along contour lines, but that method requires an enormous and troublesome work. As a simpler and also precise and practical method, the author wishes to recommend the method using the elevations of grid intersections over a topographic map.[11]

The air temperature of a melting area is represented by the air temperature at the median elevation, i.e. 50% elevation h_m of the area. This h_m is the elevation for p_m, the middle point of p_1 and p_2, and may be found graphically from the area-elevation curve. The temperature difference T' between those at the observatory and at h_m is

$$T' = \frac{0.6}{100} (h_m - h_0) \qquad (12)$$

Using the relations above stated, only from daily maximum and minimum air temperatures at the observatory, dittos at h_m are determined; then by Eq. (8) D is computed, and multiplying D by A, we get the integrated temperature-area DA which is a numerical expression of a main element that causes the snow-melt runoff in a river basin.
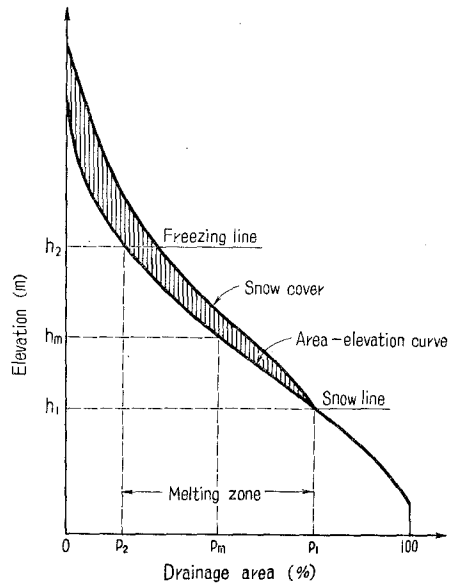
But when we do such computation extending over many days, it is more convenient to use kD instead of DA in which

$$k = A/A_0 \qquad (13)$$

Since k is a dimensionless number of ratio, kD is expressed in degree hours.

(3) Analysis of Runoff and the Ratio of Snow-melt Runoff

As well known, runoff consists of direct runoff and base flow runoff; the former being surface runoff which varies more suddenly by day-to-day causes and the latter being groundwater runoff which varies quite slowly. In a snow-melting season, each of them consists of snow-melt runoff and rainfall runoff.

Their relation may be written

$$Q_s = Q - (Q_r + Q_0) \qquad (14)$$

where Q is the whole discharge, Q_s is the discharge due to direct snow-melt runoff, Q_r is that due to rainfall and Q_0 is the base flow discharge.

Runoff at some period is written

$$V_s = \int Q_s dt = \int Q dt - \int (Q_r + Q_0) dt \qquad (15)$$

If the discharge is expressed by daily mean discharge, since one day is $24 \times 60 \times 60 = 0.0864 \times 10^6$ seconds, discharge and runoff are exchanged each other by the following Eq. (16).

$$\left. \begin{array}{l} V = \int Q dt = 0.0864 \times 10^6 \sum Q \\[2mm] \sum Q = V/0.0864 \times 10^6 \end{array} \right\} \qquad (16)$$

where V is the runoff (m³) at some period and $\sum Q$ is the sum of daily mean discharges (m³/sec) at the period.

Actual examples show that the day-to-day variations of kD and discharge Q correspond well each other with some lag of time, but the variation of kD is generally more sharp while that of Q is milder. This is because the snow-melt runoff (rainfall runoff also) caused by kD of a day does not end in a day but is distributed over several days and is averaged, and moreover, because the base flow exists.

Then, taking some suitable number of days continuous, and computing V_s by Eqs. (15) and (16) and meanwhile computing $\sum kD$ or $\sum DA$ in the same number of days continuous but preceding by the lag of time formerly mentioned, the ratio of snow-melt runoff i.e. the degree-hour factor f_s or f'_s may be written

$$\left. \begin{array}{l} f_s = V_s/\sum kD \\[2mm] f'_s = V'_s/\sum DA \end{array} \right\} \qquad (17)$$

where f_s is in m³/°C·hr and f'_s is in 10^{-3} mm/°C·hr if A is in km².

Remembering $k = A/A_0$, f_s is exchanged to f'_s by

$$f'_s = f_s / A_0 \qquad (18)$$

As already mentioned, when we calculate over many days, it is more convenient to use kD instead of DA, that is to say, to use f_s instead of f'_s.

However, for comparing different basins, it is better to use f'_s.

Using f_s or f'_s, the snow-melt runoff V_s may be computed by

$$\left. \begin{array}{l} V_s = f_s \; kD \\ V_s = f'_s \; DA \end{array} \right\} \qquad (19)$$

(4) Seasonal Variation of the Ratio of Snow-melt Runoff

The result of Linsley's study of rivers in California shows that the value of degree-day factor is small in early spring but increases as the season advances to summer. The result of Work's research at Crater Lake shows that the melt value per degree-day also increases as the season advances, and this is concerned with the quantity of snow-melt itself and does not contain the effect of lag of runoff.

These data show that the ratio of snow-melt runoff increases as the season advances with which coincides the result of analysis in the Saru River Basin by the author.

In computing f_s, the technique of determining base flow discharge is concerned with, and since the base flow itself varies, there exists the complexity of the problem. However, the data at Crater Lake show that the quantity of snow-melt itself varies seasonally, being unrelated to runoff.

As the factors affecting seasonal variation of f_s, the seasonal variation of solar radiation and albedo of snow and the ripening of snow, etc. may be considered.

1) *Effect of solar radiation*

In the northern semi-sphere of the earth, as the season advances from spring to summer, the altitude of the sun becomes higher and the heat of solar radiation increases. The ratio of intensity of solar radiation J for a horizontal plane to the solar constant J_0 is expressed by

$$J/J_0 = \sin\theta \, p^{1/\sin\theta} \qquad (20)$$

where θ is the elevational angle of the sun, p is the transmission coefficient of atmosphere which is 0.8~0.6 or nearly 0.7 as the mean value. If we may represent θ by that at noon, for example, at 43° North Latitude (near Sapporo, Hokkaidô), taking $p = 0.7$, J/J_0 in the middle of each month from March to June may be calculated by Eq. (20) as follows.

Table 3. Seasonal Variation of J/J_0 (43° N.L.)

Month	March	April	May	June
θ	45° 30′	57° 00′	66° 00′	70° 20′
J/J_0	.432	.547	.617	.645

However, since the counter radiation from the surface of snow is supposed nearly equal to the solar radiation in March, assuming that both are equal, the net values of heat of radiation gained by snow are as follows.

Table 4. Seasonal Variation of $[J/J_0]_{net}$

Month	March	April	May	June
$[J/J_0]$ net	0	.115	.185	.213
Ratio to April	0	1.00	1.61	1.85

Namely, the value of $[J/J_0]_{net}$ becomes nearly twice in June, as compared with that in April.

2) *Other effects*

It is supposed that as the season advances the ripening of snow makes the value of f_s increase, but it is difficult to express it numerically.

The albedo of newly fallen snow being about 0.85 and that of old snow about 0.4~0.7, the ratios of absorption of solar radiation may be approximately compared as follows.

Sort of snow	Albedo	Ratio of abs. of sol. rad.
Newly fallen snow	0.85	0.15
Old snow	0.50	0.50

Namely, the ratio of absorption of solar radiation of old snow may be more than three times as much as that of newly fallen snow.

Summarizing the above, though the seasonal variation of f_s is consid ered to be somewhat different according to regions, and though it is difficult to determine its value generally, the results by Linsley and by Work show that the ratios of values of f_s in April, May and June are approximately $1:2\sim3:4\sim6$. According to the considerations already described in this article, these ratios may be said to be almost affirmable. Therefore, in a region at the medium latitude of the northern semisphere, the ratios of values of f_s at each middle day of April, May and June may be assumed as $1:2.5:5.0$.

(5) **Calculation of Rainfall Runoff**

The elevational distribution of precipitation in a snow-melting season should be studied, as shown in Fig. 5, being divided into the following three zones.

1) Rainfall in the snowless zone below the snow-line or the unfreezable line

2) Rainfall and snowfall in the snow-melting zone

3) Snowfall above the freezing line

In the above, the unfreezable line means the line of elevation where the daily minimum air temperature is 0°C, and that does not always coincide with the snow-line but may be regarded as approximately coincident.

Also in regard to the boundary line between 2) and 3), i.e. the coincidence of the upper limit of rainfall zone with the freezing line, the same may be said. In the zone 2), both of rainfall and snowfall may occur, and the percentage
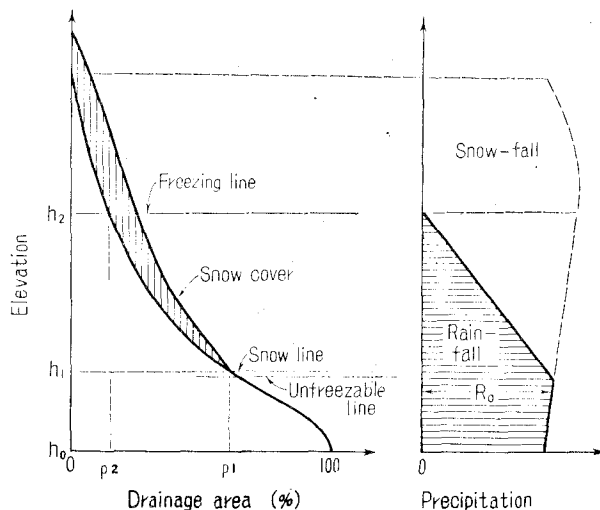

Fig. 5. Elevational distribution of rainfall in a
snow-melting season.

of rainfall will increase as the elevation decreases. The boundary line of distribution between the amount of rainfall and that of snowfall may be a curve, but may be regarded approximately as a straight line.

The whole amount of precipitation, as well-known, generally increases as the elevation increases, but its maximum value exists a little below the top of a high mountain; hence its distribution curve will be approximately as shown in Fig. 5.

According to the above consideration, when the observatory exists at a low elevation, it is clear that in order to get the value of rainfall suitable for the whole basin, we find it necessary to correct observed values. In early spring, when the freezing line exists at a lower elevation, the necessity of such correction is great.

The snowfall in the zone 3) above the freezing line remains as snow covers and may be regarded not to run off immediately.

1) *Rainfall runoff in the snowless zone below the snow line.*

This may be treated similarly as ordinary rainfall runoff, but since near the snow line the ground is very wet, the ratio of runoff is supposed to be large. The drainage area A_1(km^2) below the snow line is

$$A_1 = \left(1 - \frac{p_1}{100}\right)A_0 \qquad (21)$$

Hence the amount of rainfall V_r (m^3) is

$$V_r = 10^{-3} f_r R\left(1 - \frac{p_1}{100}\right)A_0 \tag{22}$$

where R (mm) is rainfall and f_r is a coefficient of runoff.

2) *Rainfall runoff in the snow-melting zone above the snow line.*

Rainfall in this zone may be reserved in the snow cover as water for a long while, therefore the lag of runoff may occur with the same tendency as the snow-melt runoff.

The area of snow-melting zone being given by Eq. (10), the amount of rainfall runoff is

$$V_r = 10^{-3} f_r R(p_1 - p_2)A_0/100 \tag{23}$$

3) *Rainfall runoff when the snowless and the snowy zones are summarized.*

In this case, the area of the rainfall zone A_r is

$$A_r = \left(1 - \frac{p_2}{100}\right)A_0 = \alpha_1 A_0 \tag{24}$$

where α_1 is a coefficient which shows the ratio of the rainfall area.

Next, in Fig. 5, the rainfall below the snow line being assumed approximately uniformly distributed as R_0, the mean value of rainfall R_m is computed as follows.

When $p_2 > 0$ i.e. $h_2 < h_{max}$,

$$R_m = \left\{(h_1 - h_0)R_0 + \frac{1}{2}(h_2 - h_1)R_0\right\}\frac{1}{h_2 - h_0}$$

$$= \frac{h_2 + h_1 - 2h_0}{2(h_2 - h_0)}R_0 = \alpha_2 R_0 \tag{25}$$

where α_2 is a coefficient which shows the mean value of rainfall.

When $h_2 < h_{max}$ and $h_1 = h_0$, $\alpha_2 = 1/2$.

When $h_2 > h_{max}$, α_2 is larger than $1/2$ and approaches to 1.

Then the amount of rainfall runoff is

$$V_r = 10^{-3}\alpha_1\alpha_2 f_r R A_0 \tag{26}$$

The rainfall runoff considered above in various cases never ends in a day but continues for several days. In order to determine its distribution, we may well apply the unit graph method. Such a unit graph, however, from the point of view of its purpose, does not always have to be precise but may be approximate such as composed by triangles. For example, re-
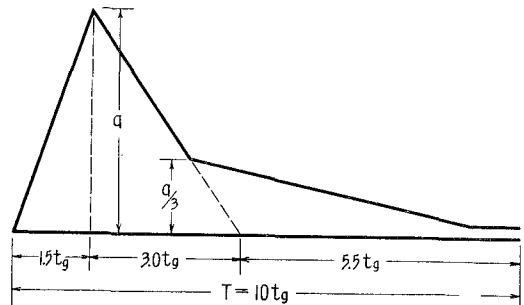

Fig. 6. Unit graph.

ferring to the distribution graph by Nakayasu[12], the unit graph as shown in Fig. 6 may be used.

Such a unit graph may be easily drawn only if the basin lag t_g is presumed. Then, from the composite hydrograph, the day-to-day percentage of runoff distribution may be determined.

(6) Calculation of Base Flow

The base flow in the snow-melting season may be said to have the following properties in broad view.

1) The base flow at the beginning of a melting season is nearly equal to the low water discharge in winter.

2) The base flow increases as the melting season advances, and holds larger value during the period when snow melts hard.

3) At the end of the melting season, the base flow decreases gradually but is still larger than the value at the beginning.

In order to determine the value of base flow, the author proposes the following method.

At first, referring to the low water discharge in winter, we should assume a constant base flow Q_0 through the whole melting season, and find f_s in every ten days by Eq. (17). Then the value of f_s will increase as the season advances, and the rate of its increase will generally exceed that already considered in Article (4). (Call it "*the standard ratio*") Because the snow-melt runoff containing base flow occurs generally a while later than the phenomenon of snow-melt itself, and since the lag accumulates gradually, the apparent value of f_s increases greatly as the season advances. Therefore keeping the seasonal variation of the value of f_s at the standard ratio, increase and correct the value of base flow at every period of ten days. The method of calculation is as follows.

Now, let f_{s1} be the value of f_s at any period when the base flow is assumed as Q_0 and let f_{s2} be the corrected value. Denote

$$\Delta f_s = f_{s1} - f_{s2}$$

To increase Q_0 by ΔQ_0 for correcting f_s is to decrease the direct snow-melt discharge by ΔQ_s which is equal to ΔQ_0; hence from Eqs. (16) and (17), the following relation is brought about.

$$\sum \Delta Q_0 = \Delta f_s \sum kD/0.0864 \times 10^6 \qquad (27)$$

Namely, the base flow at this period should be increased by $\sum \Delta Q_0$. Then the mean value of base flow which is to be added becomes

$$[\Delta Q_0]_m = \sum \Delta Q_0/n \qquad (28)$$

where n is the number of days during the period.

(7) Distribution and Composition of Runoff

The distribution of snow-melt runoff is considered to be fundamentally similar to that of rainfall runoff. However, the snow-melt runoff has a tendency of delay

as compared with the ordinary rainfall runoff; hence its value of t_q is expected to be larger. That value may be found, in an actual river basin, by the difference of the time of peak flow and the time of antecedent maximum air temperature in a day when the effect of rainfall is negligible.

In applying a unit graph to the snow-melt runoff, the distribution of D is determined by the value of m, which generally increases as the season advances, but in actual examples, during the period of maximum snow-melt, $m \doteqdot 0.8 \sim 0.9$.

Now, assuming $m = 0.8$ representatively, the percentage of D in each 4 hrs is found as shown in Fig. 7.

Fig. 7. The integrated air temperature (%) in each 4 hours.

From the hydrograph composed by the unit graph and using % of D in each 4 hrs in Fig. 7, the day-to-day distribution of runoff may be determined. However, even if the unit graph itself is used, the results will not be so different.

Summarizing the above, the discharge Q at any day is written

$$Q = Q_s + Q_r + Q_0 \tag{29}$$

(9) Forecasting Snow-melt Flood

Forecasting the snow-melt flood which is necessary for flood control is possible by applying the above-mentioned method of solution.

In a river basin, by analysis of runoff in the snow-melting season for a year or two the value of f_s proper for the basin and the tendency of base flow may be found. Therefore, if only the meteorological conditions are forecasted, daily mean discharge may be done as well.

But it is rather the value of peak discharge and its time of happening than the daily mean discharge that is necessary for flood forecasting. Now, let the daily mean discharge be denoted by Q, and the maximum discharge by Q_{max}, and write

$$Q_{max} = \eta Q \tag{30}$$

The value of η should be previously found in the analysis of runoff. Of course η is not constant, and will vary somewhat owing to the value of Q and others, but its approximate tendency must be found. Then using it, Q_{max} will be obtained.

Next, the time of happening of maximum discharge, in regard to the snow-melt runoff only, will be generally at midnight in the case of a small basin. In the case of a large basin, using what was obtained in each tributary, the method
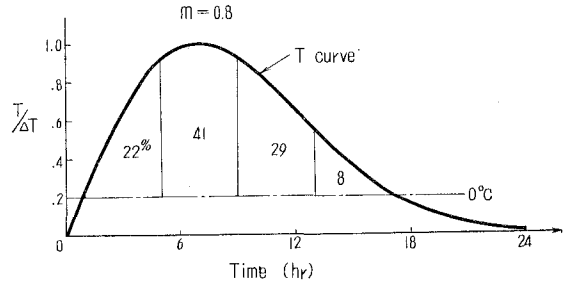
of flood routing may be applied.

3. Actual Examples of Analysis and Computation

(1) Snow-melt Runoff in the Saru River Basin

The Saru River Basin above Piratori is as shown in Fig. 8. At its riverhead lie the steep Hidaka Mountains which may be called the backbone of Hokkaidô.
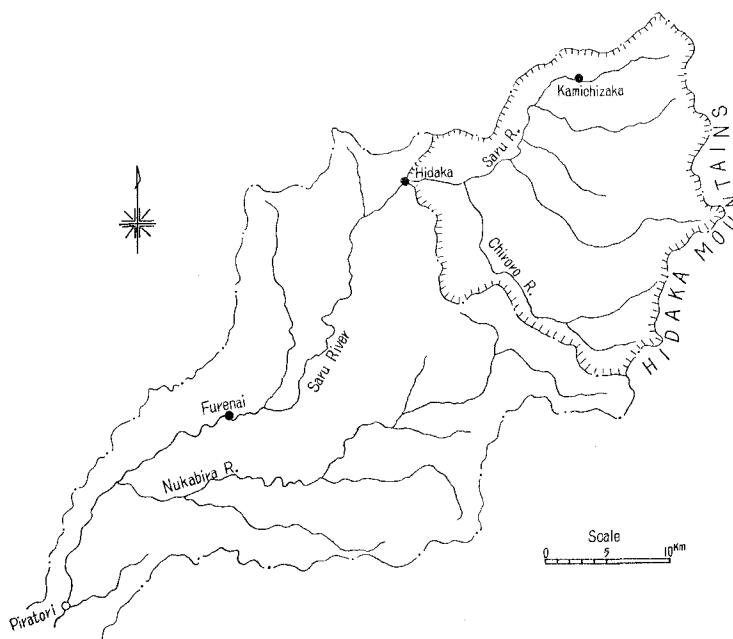

Fig. 8. The Saru River Basin.

The drainage area above the Hidaka observatory is 420 km², and its area-elevation curve is as shown in Fig. 9.

On the computation of snow-melt runoff in this river basin in the spring of 1957, the author has formerly published,[9][10] but adding the correction by the later study, wishes to describe here again. The stage-discharge curve at the Hidaka observatory is as shown in Fig. 10, using which and from the auto-record of water stage the daily mean discharge may be determined.

The actual auto-record of water stage shows that there is diurnal fluctuation by day and night and that peak flow exists near the midnight, therefore in oder to calculate runoff taking the midnight as a center, the daily mean discharge was calculated dividing each day at noon. The day-to-day variation of the snow line elevation in the basin is linear as shown in Fig. 11, and the observed values near Muroran for reference are plotted almost on the same straight line.

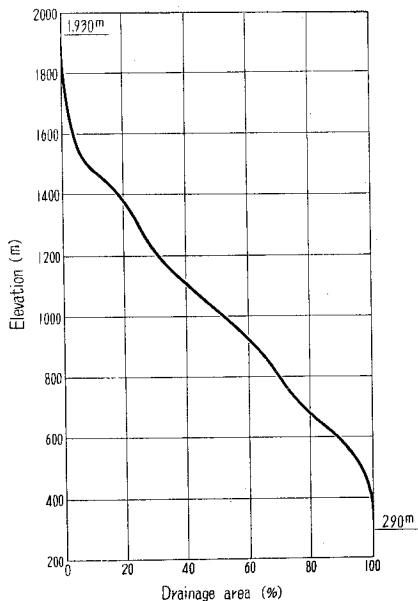Since the maximum elevation of the basin is 1930 m, the snow-melt in the

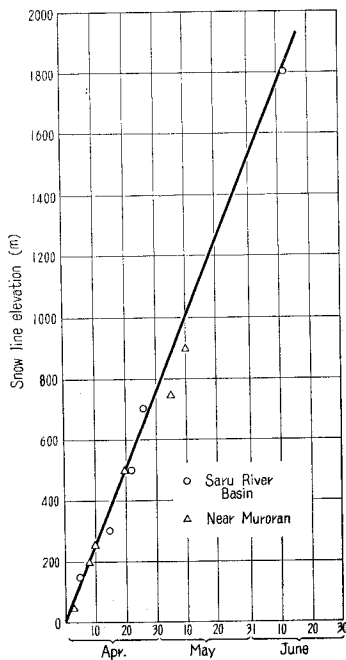**Fig. 9.** Area-elevation curve for the Saru River Basin above Hidaka.

Fig. 11. Day-to-day variation of the snow line elevation in the Saru River Basin.
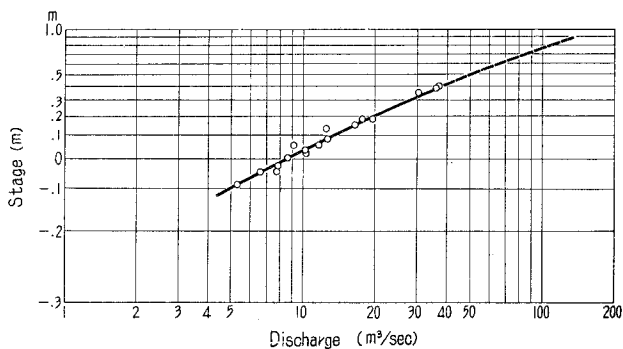

Fig. 10. Stage-discharge curve for the Saru River at Hidaka.

basin is supposed to be finished at about June 17. The author performed snow surveying in this basin at 4~5 th of April of the same year. The total runoff during the period from Apr. 5 to June 17 amounted to

$$\int Qdt = 2669 \, \text{m}^3/\text{sec} \cdot \text{day} = 230.6 \times 10^6 \, \text{m}^3$$

The result of the snow survey was formerly published by the author.[9][13] Generally, when the elevational distribution of water equivalent of snow cover is linear, the total water equivalent of snow cover V_s may be expressed by

$$V_s = C\bar{H}_w A \tag{31}$$

where $\bar{H}_w$ is the water equivalent of snow cover at the median elevation of a basin, A is the total basin area and C is a coefficient of correction due to the effect of forests.

Taking $C = 0.9$ considering the condition of forests in the basin, the total water equivalent of snow cover as of Apr. 5 becomes

$$V_s = 0.9 \times 0.663 \times 420 \times 10^6$$
$$= 250.7 \times 10^6 \, \text{m}^3$$

In the next place, the amount of rainfall was taken as the mean value of that of three observatories, i.e. Hidaka (elevation 290 m), Kamichizaka (600 m) and Furenai (87 m). Since the total rainfall during this period is 167 mm, the total volume of rainfall V_r is

$$V_r = 0.167 \times 420 \times 10^6 = 70.2 \times 10^6 \, \text{m}^3$$

Hence, the mean ratio of runoff during the period becomes

$$f = \int Q dt / (V_s + V_r)$$
$$= 230.6/(250.7 + 70.2) \doteqdot 0.72$$

The freezing line elevation h_2 is determined by taking $h_0 = 290$ m in Eq. (11).

Then by the author's method, from the day-to-day maximum and minimum air temperatures at the Hidaka observatory, the day-to-day kD was computed. These day-to-day kD, rainfall and discharge are shown in Fig. 12.

In analyzing the runoff, the base flow was at first assumed as $Q_0 = 8.0 \text{m}^3/\text{sec}$ which is equal to the low water discharge just before the snow-melting season and which was assumed as constant during the whole period. The rainfall runoff was calculated at every period of ten days using Eq. (26) where f_r being assumed as 0.75.

The value of f_s at every period of ten days was calculated as shown in Table 5.

In Table 5, the values of $\sum kD$ in June are small and the runoff should be rather regarded as recession, hence the values of f_s (* marked) are not reliable.

Table 5 shows that the values of f_s in April are nearly constant and equal to 0.012 on an average. Hence, taking it as a basis, and applying the standard ratio of $1 : 2.5 : 5$ for Apr. : May : June, the seasonal variation of f_s as suitable is drawn as a curve shown in Fig. 13.

From Fig. 13, the values of f_s at every period of ten days are shown in Table 6.

Using the above values of f_s, the values of base flow were corrected by Eqs. (27) and (28) as shown in Table 7.

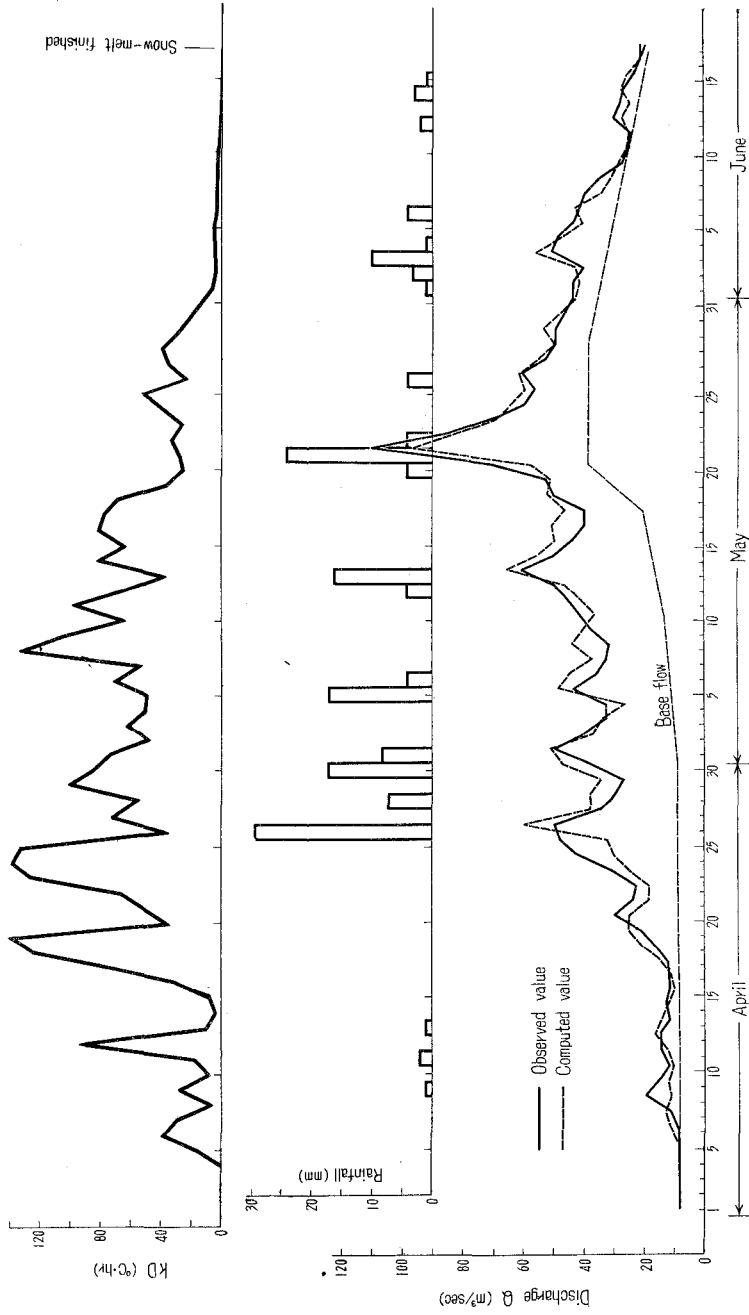Next, for determining the distribution of runoff, the lag t_g for snow-melt runoff

Fig. 12. Day-to-day kD, rainfall and discharge in the Saru River Basin.

Table 5. Calculation of f_s

Date	No. of days	$\sum Q$ (m³/s)	$\sum Q_0$ (″)	$\dfrac{.0864}{(10^6\text{m}^3)}(\sum Q - \sum Q_0)$	V_r (″)	V_s (″)	$\sum kD$ (°C hr)	f_s (10⁶m³/°C hr)
4. 5～10	6	73	48	2.2	0.1	2.1	123.4	.0170
″ 11～20	10	150	80	6.1	0.3	5.8	517.7	.0112
″ 21～30	10	337	80	22.2	11.7	10.5	847.7	.0124
5. 1～10	10	371	80	25.1	7.3	17.8	697.7	.0255
″ 11～20	10	496	80	35.9	6.8	29.1	620.7	.0469
″ 21～31	11	677	88	50.9	10.1	40.8	330.3	.1235
6. 1～10	10	391	80	26.9	6.0	20.9	26.5	.789*
″ 11～17	7	174	56	10.2	1.9	8.3	2.7	3.074*
Totals	74	2669	592	179.5	44.2	135.3	3166.7	.0427
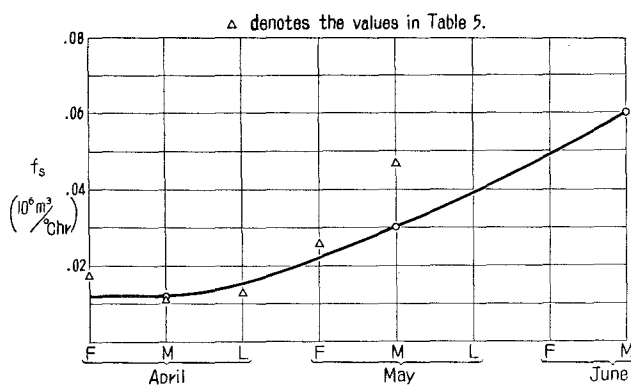
Fig. 13. Seasonal variation of f_s.

Table 6. Values of f_s (10⁶m³/°C·hr)

Ten days ＼ Month	April	May	June
First	.012	.022	.049
Middle	.012	.030	.060
Last	.015	.039	

Table 7. Values of mean Q_0 (m³/sec)

Ten days ＼ Month	April	May	June
First	8	11	31
Middle	8	20	21
Last	8	37	

was presumed as 10 hrs, and an approximate unit graph was drawn from which runoff percentage was determined as follows:

Order of day	1	2	3
Runoff %	50	30	20

The distribution of rainfall runoff may be considered not so different from that of snow-melt runoff in the case when the snow-melting zone and the snowless zone are treated together, but generally rainfall does not continue every day as snow-melt; hence the number of days of its distribution should be taken more. Here, the runoff distribution was assumed as follows:

Order of day	1	2	3	4
Runoff %	50	25	15	10

As described above, the day-to-day discharges Q_s, Q_r and Q_o were computed respectively and summed up together as Q, which is shown in Fig. 12.

Comparing both observed and computed discharges, it may be said to have fair accuracy.

(2) Snow-melt Runoff in the upper Ishikari River Basin

The upper Ishikari River Basin above Inô is as shown in Fig. 14.

The drainage area is 3430 km², and its area-elevation curve is as shown in Fig. 15.
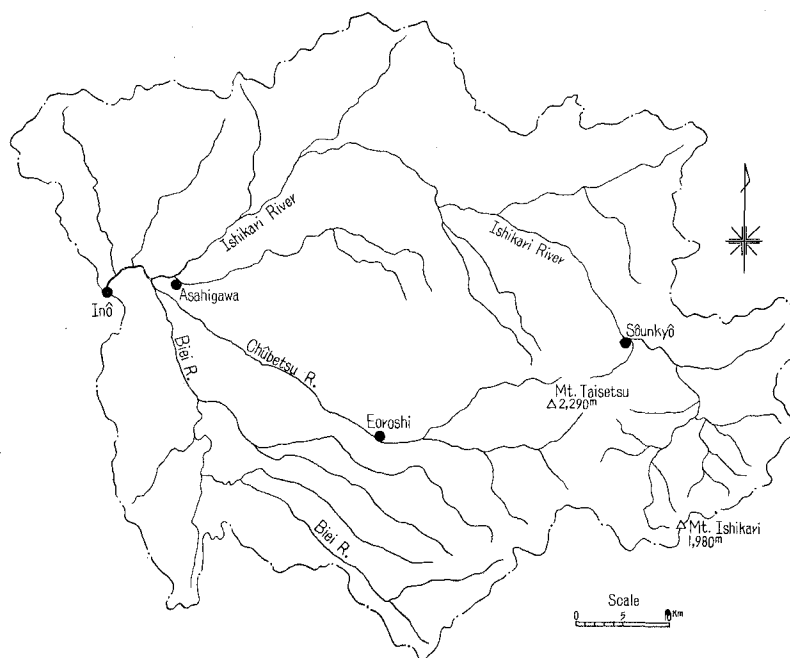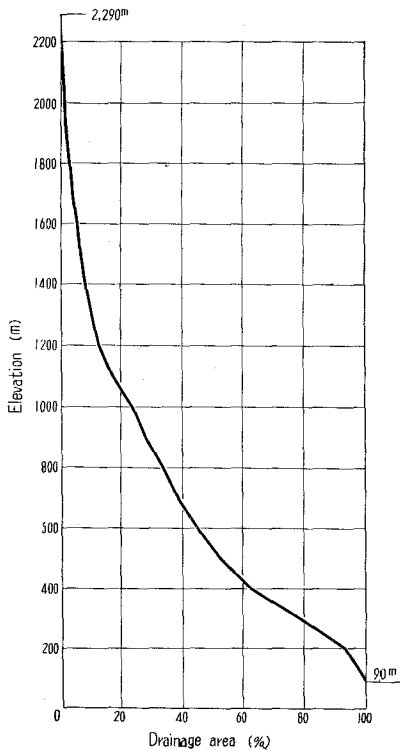

Fig. 14. The upper Ishikari River Basin.

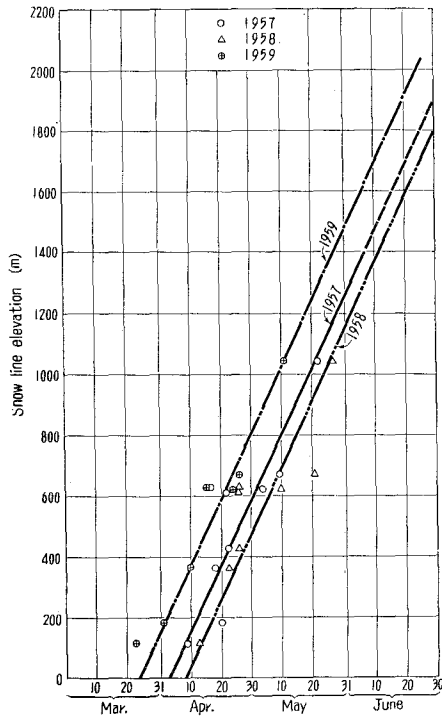**Fig. 15.** Area-elevation curve for the upper Ishikari River Basin above Inô.

Fig. 16. Day-to-day variation of the snow line elevation of the Ishikari River Basin.

The day-to-day variation of the snow line elevation in the basin is linear as shown in Fig. 16, and in spite of having some differences between the positions of lines every year, its mean velocity of rising is constant indicating 22 m/day while that of the Saru River Basin was 25 m/day.
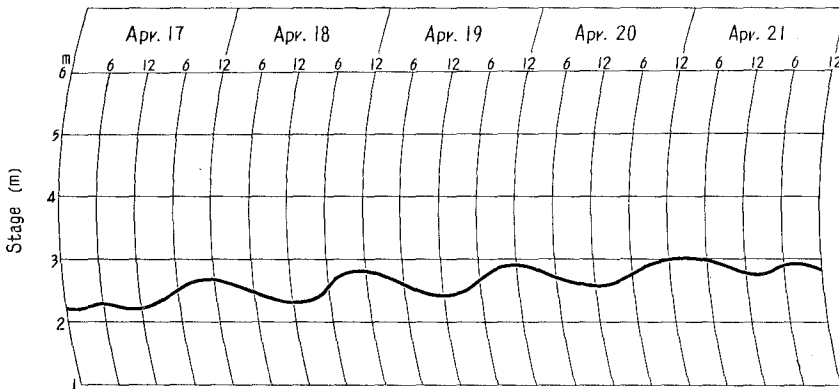
Fig. 17. A part of the auto-record of water stage of the Ishikari River at Inô.

The lapse rate of air temperature, according to the result of investigation in the basin, indicated 0.6°C/100 m which coincides with what is described in meteorology.

For calculation of kD, as a meteorological observatory, Asahigawa (elevation 113 m) was adopted.

As observatories of rainfall, Asahigawa (113 m), Eoroshi (360 m) and Sôunkyô (625 m) were selected. The mean of observed values at these three places is considered to give the suitable value as the mean of whole basin in view of plane as well as of elevation.

Fig. 18. Stage-discharge curve for the Ishikari River at Inô.

The auto-record of water stage at Inô in the spring of 1957 shows that the main snow-melting period is April, hence analysis was performed from the end of March till 10th of May.

Fluctuation of water stage by day and night is seen also and more prominent than in the case of the Saru River. Fig. 17 shows a part of it.

The stage-discharge curve at Inô is as shown in Fig. 18.

The day-to-day kD, rainfall and discharge are as shown in Fig. 19.

Fig. 19 shows that the day-to-day variation of kD and that of discharge correspond well each other with a lag of half a day.

In analyzing the runoff, the base flow was at first assumed as $Q_0 = 40 \text{ m}^3/\text{sec}$ which is equal to the low water discharge just before the snow-melting season and which was assumed as constant during the whole period. The rainfall runoff was calculated at every period of ten days using Eq. (26) where f_r being assumed as 0.75 referring to the case of the Saru River.

The value of f_s at every period of ten days was calculated as shown in Table 8.
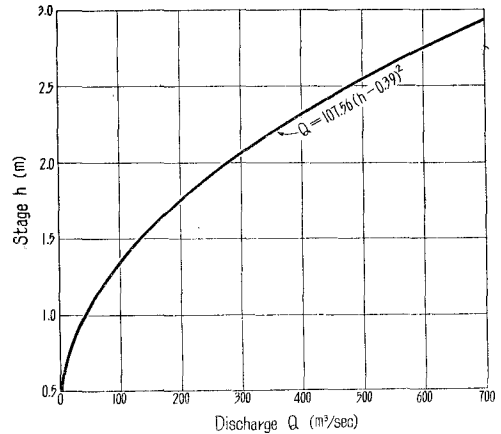
Table 8. Calculation of f_s

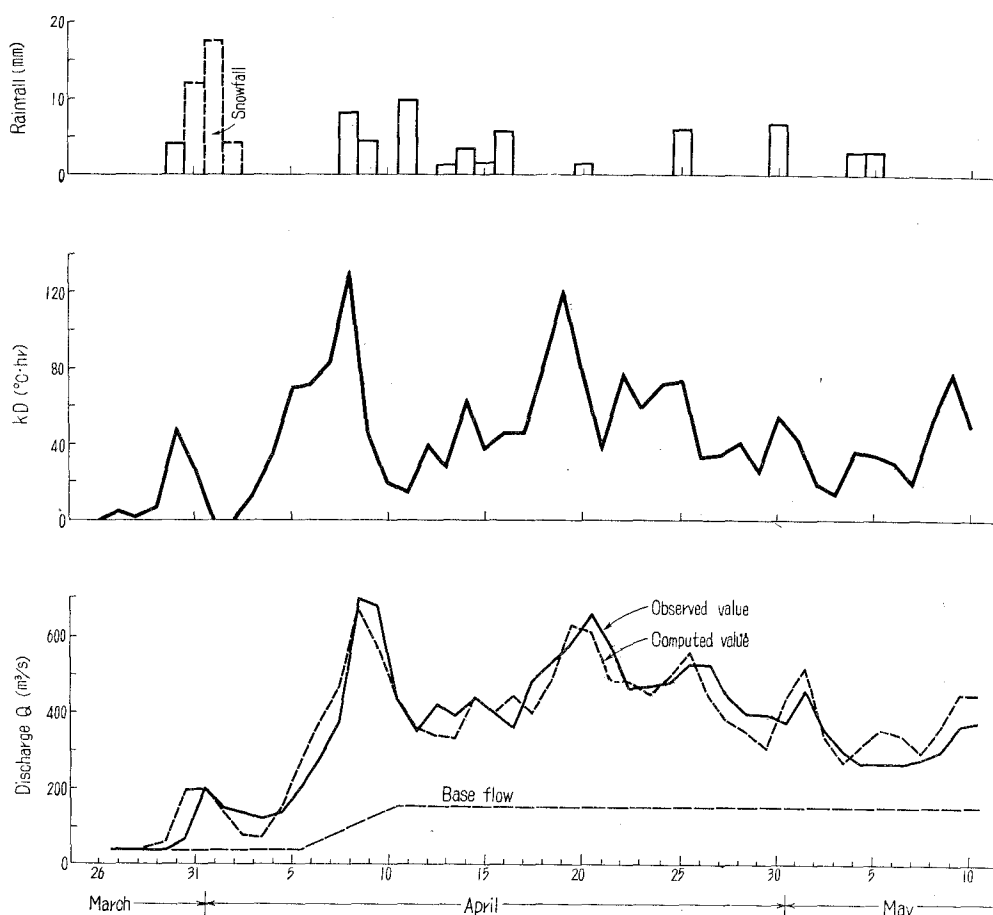| Date	No. of days	$\sum Q$ (m³/s)	$\sum Q_0$ (″)	$.0864$ $(\sum Q - \sum Q_0)$ (10^6m^3)	V_r (″)	V_s (″)	$\sum kD$ (°C hr)	f_s $(10^6 \text{m}^3/\text{°C hr})$
4. 1～10	10	3176	400	240	13	227	462.6	.491
″ 11～20	10	4553	400	359	31	328	543.7	.603
″ 21～30	10	4619	400	364	23	341	500.0	.682
5. 1～10	10	3195	400	241	12	229	371.9	.616
Totals	40	15543	1600	1204	79	1125	1878.2	.599

Fig. 19. Day-to-day kD, rainfall and discharge in the upper Ishikari River Basin.

Table 8 shows that the value of f_s does not indicate great variation during the period; hence, it is considered that it may be constant during the whole period by correcting the base flow properly. The value of f_s is smaller for the first ten days of April and is larger after the middle ten days of April having the peak at the last ten days of April, but there is no great variation from the middle ten days of April till the first ten days of May. Hence, taking the value of f_s after the middle ten days of April together and comparing it to that of the first ten days of April, the value of base flow was corrected to $150 \text{ m}^3/\text{sec}$ after the middle ten days of April.

The corrected value of f_s during the whole period is $0.43 \times 10^6 \text{m}^3/°\text{C hr}$ which is converted to $0.125 \text{ mm}/°\text{C hr}$. Furthermore, it is equivalent to $0.30 \text{cm}/°\text{C} \cdot \text{day}$ which nearly coincides with the value of $0.28 \text{ cm}/°\text{C day}$ which was found by Ohtsubo and others[14] in the runoff of the Ishikari River at Ebetsu.

The snow-melt runoff distribution was, from a unit graph taking $t_g = 12 \text{ hrs}$,

determined as follows (the same as for the case of the Saru River):

Order of day	1	2	3
Runoff %	50	30	20

In regard to the rainfall runoff, since the length l of the main river until the source is nearly equal to 120 km, the basin lag t_g is, by Nakayasu's formula[12]

$$t_g = 0.27 l^{0.7}$$
$$= 0.27 \times 120^{0.7} = 7.7 \fallingdotseq 8 \text{ hr}$$

However, according to the actual data, the difference of time between the peak of rainfall and that of flow is found to be about 12 hrs, which coincides with t_g of the snow-melt runoff. Hence, the runoff distribution was here assumed as follows (the same as for the case of the Saru River):

Order of day	1	2	3	4
Runoff %	50	25	15	10

As described above, the day-to-day discharges Q_s, Q_r and Q_0 were computed respectively and summed up together as Q which is shown in Fig. 19.

Comparing both observed and computed discharges, it may be said to have fair accuracy.

4. Conclusions

From what has been described above, the following conclusions are arrived at.

1) The snow-melt runoff of a river is important for flood control as well as for water utilization.

2) The thermo- dynamical calculation of various factors causing snow-melt is possible, but its application to the snow-melt runoff in an actual and vast river basin is difficult.

3) The method is practical which adopts air temperature as a main factor determining the amount of snow-melt, but the method of degree days formerly used contains some irrationality and shows a great error when the mean daily air temperature is near 0°C.

4) The author has proposed to use degree hours and introduced the formulae to determine degree hours only by daily maximum and minimum air temperatures and shown m-ξ relation for the convenience of calculation.

5) The values of degree hours calculated by the author's theory have satistactory accuracy as compared with values from actual auto-records.

6) In the computation of snow-melt runoff, the area-elevation curve of a basin plays an important part. To draw the curve, the method of grid intersections is found convenient.

7) The integrated temperature-area DA or kD at the median elevation of

the snow-melting zone may be regarded as representative of the quanty which causes snow-melt.

　　8)　The day-to-day variation of the snow line elevation during the melting period is about linear in broad view.

　　9)　The ratio of snow-melt runoff increases as the season advances, but at a shorter period, it may be regarded as constant.

　　10)　The snow-melt runoff has a tendency of delay as compared with ordinary rainfall runoff, and the same is said in regard to the rainfall runoff during a snow-melting period.

　　11)　On the elevational distribution of rainfall during a snow-melting period, special consideration is necessary which should be applied for the calculation of runoff.

　　12)　Runoff distribution may be determined by applying the unit graph method.

　　13)　The base flow increases as a melting season advances, and decreases gradually, after either having a wide peak or keeping a larger value. Its variation may be calculated by assuming a standard ratio of values of f_s in each month.

　　14)　If the meteorological conditions are forcasted, the prediction of snow-melt flood is possible.

　　15)　The computed values of runoff by the author's method in the Saru River Basin and the Ishikari River Basin coincide well with the observed values.

Acknowledgements

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