Journal of Engineering Research

Volume 7
Issue 5 *This is a Special Issue from the Applied Innovative Research in Engineering Grand Challenges (AIRGEC) Conference, (AIRGEC 2023), Faculty of Engineering, Horus University, New Damietta, Egypt, 25-26 October 2023*

Article 14

2023

Numerical Simulation of Parallel RLC Model Using Different Fractional Derivative Operators

ahmed nazih, Hamdy Elminir, W. E. Raslan, I. L. El-Kalla

Journal of Engineering Research (ERJ)
Vol. 7 – No. 5 2023
©Tanta University, Faculty of Engineering
ISSN: 2356-9441 https://erjeng.journals.ekb.eg/ e ISSN: 2735-4873

Numerical Simulation of Parallel RLC Model Using Different Fractional Derivative Operators

Ahmed Nazih[1], Hamdy K. Elminir[2], W. E. Raslan[3], and I. L. El-Kalla[3]

[1] Communication Engineering Department, Mansoura Higher Institute of Engineering and Technology, Mansoura, Egypt– email: ahmednazih635@gmail.com.
[2] Electrical Engineering Department, Faculty of Engineering, Kafr El -Shikh University, Egypt– email: hamdy_elminir@eng.kfs.edu.eg.
[3]Mathematics and Engineering Physics Department, Faculty of Engineering, Mansoura University, Mansoura, Egypt– emails: w_raslan@yahoo.com, al_kalla@mans.edu.eg.

***Abstract*- In the current study, the theory of fractional calculus is applied to the electric parallel RLC circuit. The aim of this article is to alter the concept of a parallel RLC circuit by applying various fractional derivative operators. A fractional RLC circuit was investigated via Caputo, Caputo-Fabrizio, and Atangana-Baleanu derivatives. The Laplace transform technique was applied to resolve the system of governing differential equations. The results for the various orders are compared to each other. When the fractional order derivative tends to be one, the system's performance is found to be very slow due to a decrease in damping capacity.**
Keywords- Fractional calculus, RLC circuit, Caputo fractional derivative, Caputo-Fabrizio fractional derivative, Atangana-Baleanu fractional derivative.

I. Introduction

Many research fields use fractional calculus, including automatic control, medical applications, civil engineering, time series, and long memory effect modeling [1-8]. The most commonly used definitions of fractional calculus are the Riemann-Liouville and Caputo derivatives with fractional orders, but they have a singularity [9]. Caputo-Fabrizo overcame this problem by devising a new fractional derivative based on the exponential function with no singularity. [10, 11]. The mathematical properties of Fabrizo were developed by Atangana [12]. Atangana-Baleanu introduced another definition of Caputo-Fabrizio based on the Mittag-Leffler function into the fractional derivative, which many researchers have used in their articles [13, 14].
A resistance, capacitor, and inductor (RCL) circuits are found in almost every electrical device. The responses of a parallel RCL circuit will be analyzed using various fractional derivative operators.
In this paper, we will compare the performance of fractional RLC circuits based on Caputo, Caputo-Fabrizo, and Atangana-Baleanu fractional derivatives. It is organised as follows: basic information is explained in the next section. In section 3, we present the mathematical model that represents the various currents in a fractional parallel RLC circuit. Section 4 discusses case studies. Finally, section 5 summarizes this paper's contribution.

II. Basic Information

This section contains various fractional calculus definitions and properties.
2.1 Riemann-Liouville integral operator
The Riemann-Liouville integral operator of order $\alpha \geq 0$ of the function is defined by

$$J_a^\alpha v(t) = \int_a^t \frac{(t-s)^{\alpha-1}\, v(s)}{\Gamma(\alpha)}\, ds \qquad (1)$$

Where $\Gamma(\alpha)$ is the gamma function. As special case, when $a = 0$, we can write $J_0^\alpha = J^\alpha$

2.2 Caputo fractional derivative
The Caputo fractional derivative of order α of the function $v(t)$ is defined by [21]

$$D_a^\alpha v(t) = J_a^{m-\alpha} \frac{d^m}{dt^m} v(t) = \int_0^t \frac{(t-s)^{m-\alpha-1}\, v^m(s)}{\Gamma(m-\alpha)}\, ds \qquad (2)$$

When $a = 0$ we write ${}_0^\alpha D = D^\alpha$
The Laplace transform of (2) is defined by

$$L[D^\alpha v(s)] = s^\alpha v(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1}\, v^k(0) \qquad (3)$$

When $n = [R(\alpha)] + 1$. From this expression we have two special cases

$$L[D^\alpha v(t)] = s^\alpha v(s) - s^{\alpha-1}\, v(0)\ ,\ 0 < \alpha \leq 1 \qquad (4)$$

$$L[D^\alpha v(t)] = s^\alpha v(s) - s^{\alpha-1}\, v(0) - s^{\alpha-2} v'(0)\ ,\ \ 1 < \alpha \leq 2 \quad (5)$$

2.3 Caputo - Fabrizio fractional operator
The Caputo- Fabrizio fractional derivative is defined by [10, 11]

$${}_0^{cf}D_t^\alpha\, v(t) = \frac{M(\alpha)}{1-\alpha} \int_0^t v'(s)\, e^{-\alpha\frac{t-x}{1-\alpha}}\, dx \qquad (6)$$

Where $M(\alpha)$ is a normalization function such that $M(0) = M(1) = 1$
The Laplace transform of (6) is defined by

$$L[\,{}_0^{cf}D_t^\alpha\, v(t)] = \frac{sv(s) - v(0)}{s + \alpha(1-s)} \qquad (7)$$

2.4 Atangana – Baleanu fractional operator
The Atangana – Baleanu fractional derivative is defined by [15-20]

$${}_a^{ABC}D_t^\alpha\, v(t) = \frac{M(\alpha)}{1-\alpha} \int_a^t v'(s)\, E_\alpha\left(-\alpha\, \frac{(t-s)^\alpha}{1-\alpha}\right) ds \qquad (8)$$

The Laplace transform of (8) is defined by

$$L[\,{}_a^{ABC}D_t^\alpha\, v(t)] = \frac{M(\alpha)}{1-\alpha}\, \frac{s^\alpha v(s) - s^{\alpha-1} v(0)}{s^\alpha + \dfrac{\alpha}{1-\alpha}} \qquad (9)$$

doi: 10.21608/erjeng.2023.235486.1239

Journal of Engineering Research (ERJ)
Vol. 7 – No. 5 2023
©Tanta University, Faculty of Engineering
ISSN: 2356-9441 https://erjeng.journals.ekb.eg/ e ISSN: 2735-4873

III. Mathematical Model

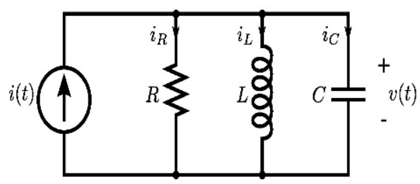

Figure 1. Parallel RLC circuit.

Consider the circuit shown in figure 1, which includes a resistor, capacitor and inductor connected in parallel, with $i_L(0) = i_o$ representing the initial inductor current, $v_c(0) = v_o$ indicating the initial capacitor, and $i_s(t)$ showing the current source [27].
Where i_R is the resistor current, i_L is the inductor current, i_c is the capacitor current and $v(t)$ is the voltage source of the circuit in figure 1.
The governing mathematical equation of the circuit is

$$CD_t^{\beta} v(t) + \frac{1}{L} j^{\alpha} V(t) + \frac{1}{R} V(t) + i_0 = i_s(t) \quad (10)$$

Since α and β are the voltage and current parameters of a fractional order inductor and capacitor, respectively [26]. From equation 10, using the Caputo fractional operator in the Laplace domain, the voltage of the capacitor may be represented as

$$v_{C1}(s) = \frac{s^{\alpha+\beta-1} v_c(o) + \frac{1}{C} s^{\alpha} I_s(s) - \frac{1}{C} I_0 s^{\alpha-1}}{s^{\alpha+\beta} + \frac{1}{RC} s^{\alpha} + \frac{1}{LC}} \quad (11)$$

In the Laplace domain, the voltage of the capacitor using the Caputo-Fabrizo fractional operator can be expressed as

$$v_{C2}(s) = \frac{s + \beta(1-s)\left(I_s - \frac{I_0}{s}\right) + C v_c(0)}{Cs + \left(s + \beta(1-s)\right)\left(\frac{1}{L} s^{-\alpha} + \frac{1}{R}\right)} \quad (12)$$

In the Laplace domain, the voltage of the capacitor can be represented as using the Atangana – Baleanu operator

$$v_{C3}(s) = \frac{CM(\beta) s^{\beta-1} v_c(0) + (s^{\beta} - \beta s^{\beta} + \beta)\left(I_s(s) - \frac{I_o}{s}\right)}{CM(\beta) s^{\beta} + (s^{\beta} - \beta s^{\beta} + \beta)\left(\frac{1}{L} s^{-\alpha} + \frac{1}{R}\right)} \quad (13)$$

IV Case Study

In this section we will discuss an example to describe the under damping (namely figures 2, 3, 4, 6, 7, 8 and 9), over damping (e.g., figures 6, 10 and 11) and critical damping response (figure 12). The model's parameters were chosen as $I_s(t) = 4u_o(t)$, $i_L(0) = 0$, $v_c(0) = 0$, $R = 60\ \Omega$, $L = 200\ mH$, $C = 120\ mF$.
We obtained the following time response for the RLC circuit based on different values of α and β by using FORTRAN code for finding the inverse Laplace transform for equations (11, 12 and 13).

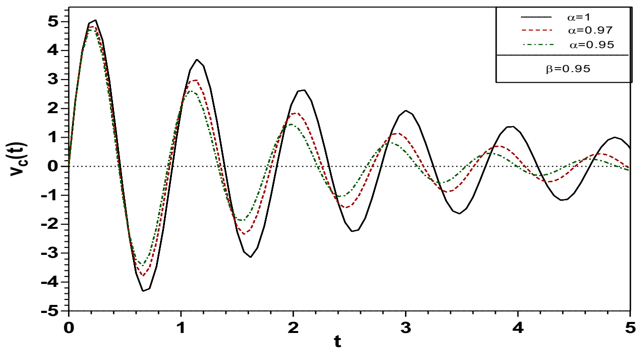

Figure 2. Numerical results for RLC circuit based on Caputo fractional operator; for $\alpha = 0.95$, 0.97 and 1 with $\beta = 0.95$.

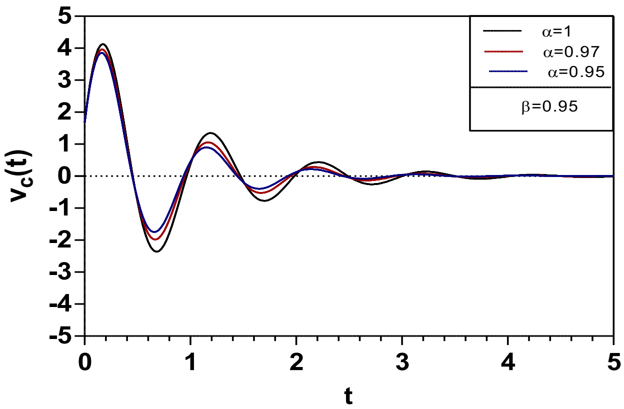

Figure 3. Numerical results for RLC circuit based on Caputo-Fabrizo fractional operator; for $\alpha = 0.95$, 0.97 and 1 with $\beta = 0.95$.

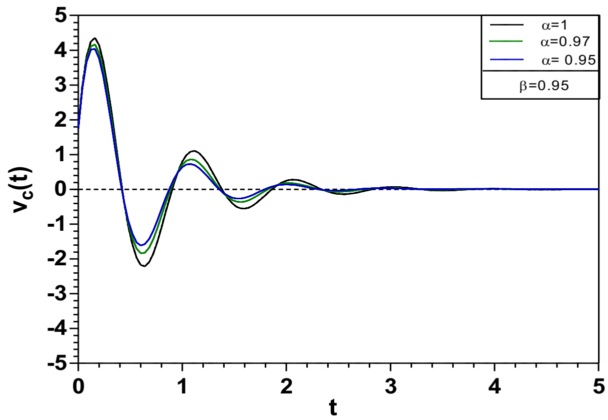

Figure 4. Numerical results for RLC circuit based on Atangana – Baleanu fractional operator; for $\alpha = 0.95$, 0.97 and 1 with $\beta = 0.95$.

doi: 10.21608/erjeng.2023.235486.1239

Journal of Engineering Research (ERJ)
Vol. 7 – No. 5 2023
©Tanta University, Faculty of Engineering
ISSN: 2356-9441 https://erjeng.journals.ekb.eg/ e ISSN: 2735-4873

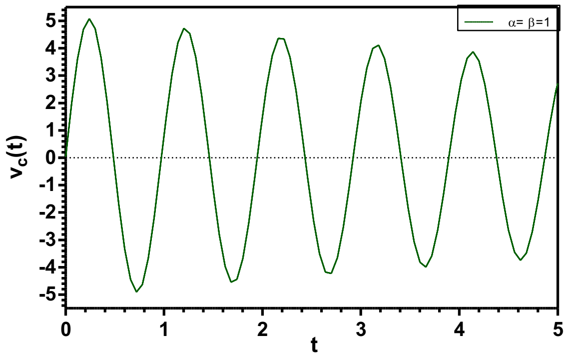

Figure 5. Numerical results for RLC circuit in the classical case; for $\alpha = \beta = 1$.

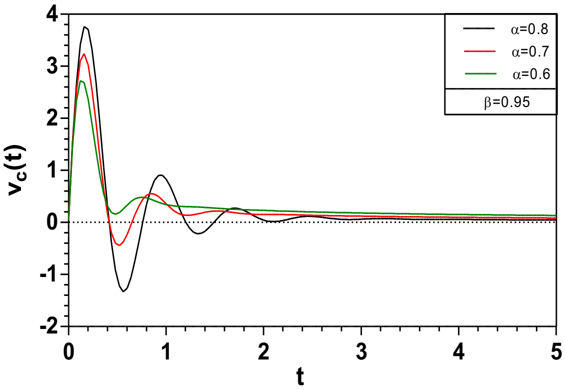

Figure 6. Numerical results for RLC circuit based on Caputo fractional operator; for $\alpha = 0.6$, 0.7 and 0.8 with $\beta = 0.95$.

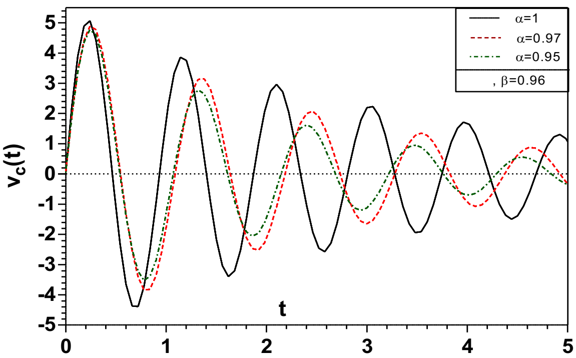

Figure7. Numerical results for RLC circuit based on Caputo fractional operator; for $\alpha = 0.95$, 0.97 and 1 with β=0.96.

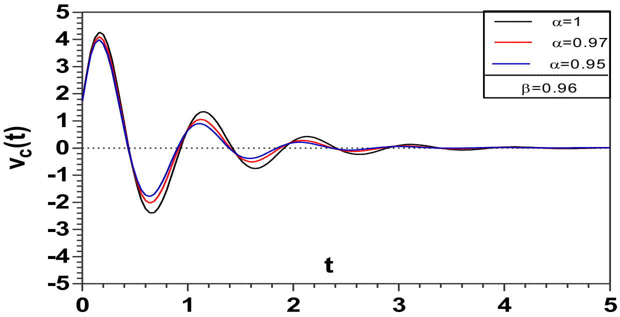

Figure8. Numerical results for RLC circuit based on Caputo-Fabrizo fractional operator; for $\alpha = 0.95$, 0.97 and 1 with β=0.96.

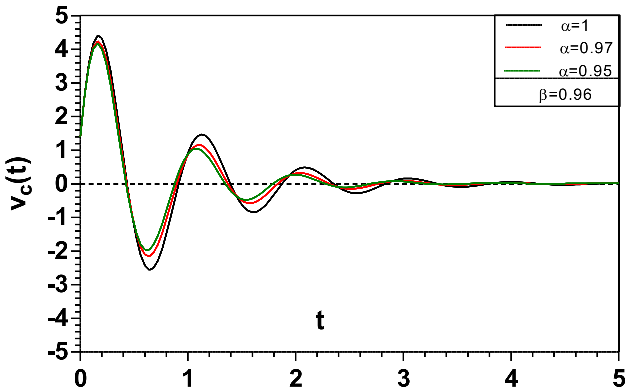

Figure 9. Numerical results for RLC circuit based on Atangana- Baleanu fractional operator; for $\alpha = 0.95$, 0.97 and 1 with $\beta = 0.95$.

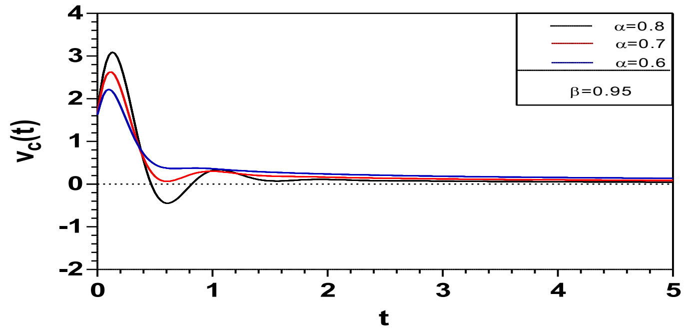

Figure10. Numerical results for RLC circuit based on Caputo-Fabrizo fractional operator; for $\alpha = 0.6$, 0.7 and 0.8 with $\beta = 0.95$.

doi: 10.21608/erjeng.2023.235486.1239

Journal of Engineering Research (ERJ)
Vol. 7 – No. 5 2023
©Tanta University, Faculty of Engineering
ISSN: 2356-9441 https://erjeng.journals.ekb.eg/ e ISSN: 2735-4873

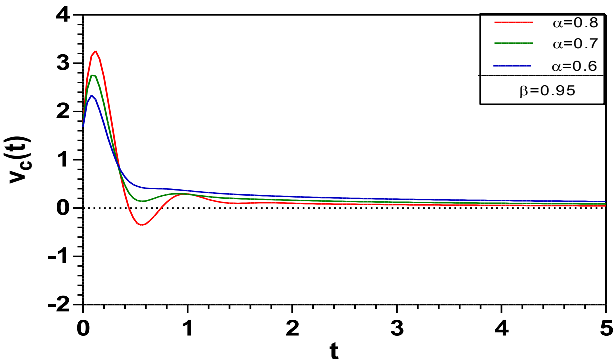

Figure11. Numerical results for RLC circuit based on Atangana- Baleanu fractional operator; for $\alpha = 0.6$, 0.7 and 0.8 with $\beta = 0.95$.

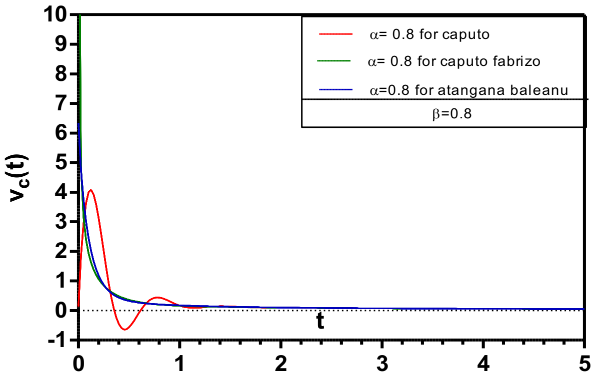

Figure.12. Numerical results for RLC circuit based on Caputo, Caputo Fabrizo and Atangana- Baleanu fractional operators; for $\alpha = \beta = 0.8$.

Conclusions

The mathematical solutions of a fractional RLC circuit were studied using Caputo, Fabrizo, and Atangana-Baleanu fractional derivatives with various values of α and β.

For all fractional operators, decreasing the fractional inductance operator (α) reduces the amplitude of the oscillation while increasing its damping capacity.

We note that from the figures, the transition between damping cases may be achieved by changing the values of fractional parameters α and β. The under damping case was obtained (from α and β which equals 0.95 to 1), over damping response (its α value between 0.6 and 0.7 and β equals 0.95) and critical damping behavior at α and β equal 0.8.

Overshoot is the divergence between the output signal and its maximum value; the greater the gap, the lower the system's performance.

As indicated in all previous figures, the responses of this system are extremely slow due to decreasing damping capacity, as illustrated in figure 5, which represents a classical case. The Caputo operator, as shown in figures (2, 6, 7 and 12), causes the system to be slow with a large overshoot. The Caputo-Fabrizo operator, as indicated in figures (3, 8, 10 and 12) and Atangana- Baleanu, as illustrated in figures (4, 9, 11 and 12), compares the response of these models with Caputo- Fabrizo and Atangana- Baleanu are very quick with a relatively modest overshoot. As a result, the caputo operator is unsuitable for usage in this model.

References

[1] R.D. Keyser, C. Muresan, C. Ionescu, A novel auto-tuning method for fractional order pi/pd controllers, ISA Trans 62 (2016) 268–275.

[2] C. Muresan, J .Machado, M. Ortigueira, Special issue, dynamics and control of fractional order systems. Int J Dyn Control 5 (2017) 1–3.

[3] I. Podlubny, Fractional order systems and $PI^{\lambda}D^{\mu}$ controllers, IEEE Trans Autom Control 44(1) (1999) 208–214.

[4] P. Lino, G. Maione, Design and simulation of fractional order controllers of injection in cng engines, 7th IFAC Symposium on Advances in Automotive Control 1 (1) (2013) 582 – 587.

[5] C. Ionescu, T. Machado, RD. Keyser, Modeling of the lung impedance using a fractional order ladder network with constant phase elements, IEEE Trans Biomed Circuits Syst 5(1) (2011) 83–89.

[6] C. Vastarouchas, G.Tsirimokou, T. Freeborn, C. Psychalinos, Emulation of an electrical-analogue of a fractional order human respiratory mechanical impedance model using ota topologies, AEU - Int J Electron Commun (7) (2017) 201–208.

[7] A. Beltempo, M. Zingale, O. Bursi,L. Deseri , A fractional order model for aging materials: an application to concrete, Int J Solids Struct 138 (2018)13–23.

[8] M. Du, Z. Wang, H. Hu, Measuring memory with the order of fractional derivative, Sci Rep (3) (2013) 1–3.

[9] A. Atangana, B.S.T. Alkahtani, Analysis of the Keller–Segel model with a fractional derivative without singular kernel, Entropy, 17, (2015) 4439–4453.

[10] M. Caputo, M. Fabrizio, A new definition of fractional derivative without singular kernel, Prog. Fract. Differ, Appl. 1(2) (2015) 1–13.

[11] J. Losada, J.J Nieto, Properties of a new fractional derivative without singular kernel, Prog. Fract. Differ, Appl. 1(2) (2015) 87–92.

[12] A. Atangana, On the new fractional derivative and application to nonlinear fishers reaction–diffusion equation.,Appl. Math. Comput. 273 (2016) 948–956.

[13] A. Atangana, J.F Gomez-Aguilar, Decolonisation of fractional calculus rules, breaking commutativity and associativity to capture more natural phenomena, Eur. Phys. J. plus 133(4) (2018) 166.

[14] A. Atangana, J.F. Gomez-Aguilar, Fractional derivatives with no-index law property, application to chaos and statistics, Chaos Solitons Fractals 114 (2018) 516–535.

[15] A. Atangana, D. Baleanu, New Fractional Derivatives with Nonlocal and Non-Singular Kernel, Theory and Application to Heat Transfer Model. Therm. Sci. 20 (2016) 763–769.

[16] B.S.T. Alkahtani, Analysis on non-homogeneous heat model with new trend of derivative with fractional order, Chaos Solitons Fractals 89 (2016) 566–571.

[17] B.S.T. Alkahtani, Chua's circuit model with Atangana-Baleanu derivative with fractional orderm, Chaos Solitons Fractals 89 (2016) 547–551.

[18] O.J.J. Algahtani , Comparing the Atangana–Baleanu and Caputo–Fabrizio derivative with fractional order, Allen Cahn model, Chaos Solitons Fractals 89 (2016) 552–559.

[19] A. Coronel-Escamilla,J.F. Gmóez-Aguilar, M.G. López-López, V.M .Alvarado-Martínez, G.V. Guerrero-Ramírez, Triple pendulum model involving fractional derivatives with different kernels, Chaos Solitons Fractals 91 (2016) 248–261.

[20] A. Atangana, I. Koca, Chaos in a simple nonlinear system with Atangana-Baleanu derivatives with fractional order, Chaos Solitons Fractals 89 (2016) 447–454.

[21] I .Podlubny, Fractional Differential Equations, Academic Press: New York, NY, USA, (1999).

[22] A. M. A. El-Sayed, S. H. Behiry, W. E. Raslan, Adomian's decomposition method for solving an intermediate fractional advection dispersion equation, Computers Mathematics with Applications, 59(5) (2010) 1759-1765.

[23] A. El-sayed, I. L. El-Kalla, E. Ziada, Analytical and numerical solutions of nonlinear fractional differentail equations, Appl Numer Math, 60 (2010) 788-797.

[24] I. L. El-Kalla, Error Estimate of the series solution to a class of fractional diferential equations, Commun Nonlinear Sci Numer Simulat 16 (2011)1408-1413.

doi: 10.21608/erjeng.2023.235486.1239

Journal of Engineering Research (ERJ)
Vol. 7 – No. 5 2023
©Tanta University, Faculty of Engineering
ISSN: 2356-9441 https://erjeng.journals.ekb.eg/ e ISSN: 2735-4873

[25] K. Oldham, J. Spanier, the Fractional Calculus, Academic Press, New York (1974).

[26] I. L. El- Kalla, New results on the analytic summation of Adomian series for some classes of differential and integral equations' Applied Mathematics and Computation 217(8) (2010) 3756-3763.

[27] Steven, T. Karris.: Circuit Analysis II with MATLAB Computing and Simulink SimuPower Systems Modeling, (2010)

[28] H. Khan,, F. Jarad, T. Abdeljawad, A.A. Khan, singular ABC fractional differential equation with p-Laplacian operator. Chaos Solitons Fractals, 129, (2019)56–61.

[29] N.Sene, K. Abdelmalek, K, Analysis of the fractional diffusion equations described by Atangana-Baleanu-Caputo fractional derivative. Chaos Solitons Fractals, 127 (2019), 158–164.

[30] B. Almarri, AH. Ali., K.S Al-Ghafri, A. Almutairi, O. Bazighifan, J. Awrejcewicz, Symmetric and Non-Oscillatory Characteristics of the Neutral Differential Equations Solutions Related to p-Laplacian Operators. Symmetry,14, (2022) 566.

[31] B. Almarri, A.H. Ali, A.M. Lopes, O. Bazighifan, Nonlinear Differential Equations with Distributed Delay: Some New Oscillatory Solutions. Mathematics, 10 (2022) 995.

[32] A. Ali, FS. Alshammari, S. Islam, M.A. Khan, S. Ullah, Modeling and analysis of the dynamics of novel coronavirus (COVID-19) with Caputo fractional derivative. Results Phys., 20(103669) (2021)1-9.

[33] M. Sultana., U. Arshad, MN. Alam, O. Bazighifan, S.Askar, Awrejcewicz, J.: New Results of the Time-Space Fractional Derivatives of Kortewege-De Vries Equations via Novel Analytic Method. Symmetry, 13, (2021)2296.

[34] M. Sultana., U. Arshad, AH. Ali, O. Bazighifan, AA. Al–Moneef, K. Nonlaopon, New Efficient Computations with Symmetrical and Dynamic Analysis for Solving Higher-Order Fractional Partial Differential Equations. Symmetry, 14 (2022)1653.

[35] S. Qureshi, M.M. Chang, A.A. ShaikhA.A, Analysis of series RL and RC circuits with time–invariant source using truncated M, Atangana beta and conformable derivatives. J. Ocean. Eng. Sci, 6 (2021) 217–227.

[36] Z. Sabir, S. Saoud, M.A.Z. Raja, A.A.Hafiz Abdul Wahab, Heuristic computing technique for numerical solutions of nonlinear fourth order Emden–Fowler equation. Math. Comput. Simul, 178 (2020) 534–548.

[37] Z. Sabir, M.A.Z. Raja, A. Arbi, G.C. Altamirano, J. Cao, J, Neuro–swarms intelligent computing using Gudermannian kernel for solving a class of second order Lane–Emden singular nonlinear model. AIMS Math, 6 (2021)2468–2485.

[38] J.F. Gomez-Aguilar, A. Atangana, A, New insight in fractional differentiation: Power, exponential decay and Mittag-Leffler laws and applications. Eur. Phys. J. Plus, 132 (2017) 1–23.

[39] N. Sene, K. Abdelmalek, Analysis of the fractional diffusion equations described by Atangana-Baleanu-Caputo fractional derivative. Chaos Solitons Fractals, 127 (2019) 158–164.

doi: 10.21608/erjeng.2023.235486.1239