Graph theory as model to understand American bullfrog invasion in Uruguay

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Abstract. *In this work it is presented a graph theory approach for the characterization of the data-set of ponds invaded by the aquatic amphibian Lithobates catesbeianus in the period 2007-2022 in Aceguá (Uruguay). The topological characterization of the network of ponds with a mobility threshold distance of* 700 *m is presented. The vulnerability analysis of ponds is carried out using centrality metrics, community analysis and evaluating the connection probabilities between nodes, the goal is to classify nodes to prioritize the invasion control.*

1. Introduction

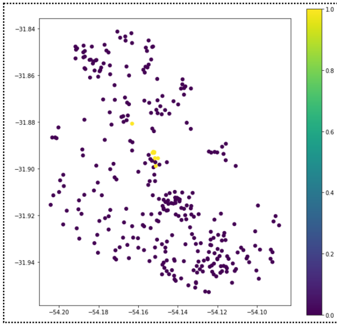
Lithobates catesbeianus, known as the American bullfrog, populations have been established at different countries such as western North America, Europe, Asia, South America, and the Caribbean [Barbosa et al. 2017]. Its ecological attributes, large body size, broad diet, frequently high population densities, and capacity to invade natural environments, facilitates its potential to impact on different taxa through predation, competition, and habitat modification. Since 2005 the bullfrog invasion has been reported in Aceguá (Cerro Largo Department, Uruguay) where there was a farm in operation from the 90s to 2000. The bullfrog was established there and began to expand in 2012, thenceforth it became an environmental problem given that accelerated the disappearance of native species due to its propagation capabilities. Here we present an approach to the problem of bullfrog invasion from the graph theory and complex networks with the goal of establishing the vulnerability of nodes invasion and establishing control mechanisms such as their removal from the network [Barabási and Pósfai 2016].

2. Graph theory

Definition 1 *A graph is a mathematical structure* $G = \{V, E\}$ *where V and E are sets, together with a map*

$$\gamma_G : E \to \{\{u, v\} : u, v \in V\}$$

The set V is called the set of vertices; to the set E set of sides or edges, and to the application γ_G *application of incidence.* $|V|$ *and* $|E|$ *represent the number of vertices and edges of G. If* $\{u, v\} \in V$ *and* $e = \{u, v\} \in E$*, we say that the edge e affects u and v. The* ***degree*** *of a vertex* $v \in V$*, denoted by* $deg(v)$*, is the number of edges incident on v.*



Node	Lon	Lat
0	-54.15004085,	-31.89564738
1	-54.15194188,	-31.89685761
2	-54.15115334,	-31.89896649
3	-54.15276887,	-31.8933096
4	-54.16310479,	-31.8807915
5	-54.15156147,	-31.89560485
6	-54.15199224,	-31.89332922
7	-54.15186839,	-31.89299244
8	-54.15240874,	-31.89288049
9	-54.154538,	-31.893438

Figure 1. Node-coordinate relationship (2007 invasion puddles)

Definition 2 *Given the sequence of degrees of a graph, the mean degree is defined as*

$$< k >= \frac{1}{N} \sum_i k_i$$

where N is the total number of vertices in the graph.

A random network is one in which the degree of each node is a random variable $k \in [0, N]$. Since all the nodes in a random graph are statistically equivalent, they all have the same binomial distribution, so the probability for any randomly chosen node of the network to have a certain degree k_i is given by $P(k_i) = P(k_i = k)$.

Definition 3 *A distribution of degrees of a network is a probability distribution*

$$p(k) = \frac{|\{i|deg(i) = k\}|}{n}$$

that is, the probability that a node has degree k.

An important property of the random networks is that the degree distribution function is that they have a maximum at the mean value of the degree and decay exponentially, which implies that almost all nodes in the network have the same number of connections [Barabási and Pósfai 2016].

Definition 4 *The average length of shortest paths for all possible pairs of nodes in the network is given by*

$$l = \frac{\sum_{i,j} d(i \to j)}{n(n-1)}$$

where n is the number of nodes.

Definition 5 *The clustering coefficient measures how densely connected the nodes are with each other in a localized area in a network*

$$C_i = \frac{|\{j, k|d(i, j) = d(i, k) = d(j, k) = 1\}|}{deg(i)(deg(i) - 1)/2}$$

The denominator is the total number of possible node pairs within the neighborhood of node i, while the numerator is the number of node pairs actually connected to each other.

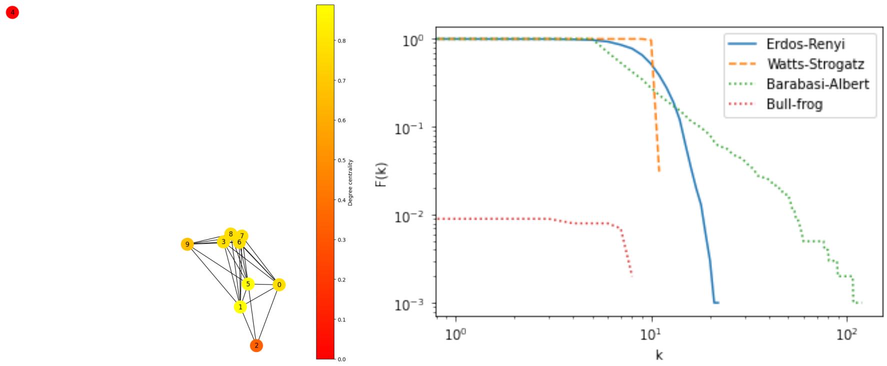


Figure 2. Degree centrality and probability distribution in 2007 invasion

3. Case study and data

Starting in 2007 and every year in spring, nocturnal sampling of the invaded and non-invaded peripheral ponds is carried out. In each pond, a survey transect of the perimeter of the ponds is made and the bullfrog specimens observed and vocalizing are recorded. With this, an invasion presence/absence matrix is collected for each system in each year, forming a data set of 341 monitored ponds with a total of 38 invaded ponds in 2022.

The American bullfrog case study, the set of nodes V is given by the geographic positions of the ponds invaded, that is $(Lon, Lat) \in V$, and the set of edges E is given by the connections between invaded ponds with 700 m of treshhold [Descamps and Vocht 2016]. The degree of a node is related to the number of edges attached to it. The basic intuition is that nodes with more connections are more influential and important in a network. In other words, in the case of the network associated with the bullfrog, the nodes of the highest degree correspond to the central nodes of the frog's movement.

The figure 1 shows the geographic position-node correspondence of the invaded ponds, the yellow dots correspond to the ponds invaded in 2007 when periodic monitoring of the invasion of the species began. In the bullfrog 2007 invasion graph $|V| = 10, |E| = 30$, each node degree is

$$deg(v) = \{V_0 : 7, V_1 : 8, V_2 : 3, V_3 : 7, V_4 : 0, V_5 : 8, V_6 : 7, V_7 : 7, V_8 : 7, V_9 : 6\}.$$

The set of edges is given by:

$$e = \{(0,1),(0,2),(0,3),(0,5),(0,6),(0,7),(0,8),(1,2),(1,3),(1,5),(1,6),(1,7), (1,8),(1,9),(2,5),(3,5),(3,6),(3,7),(3,8),(3,9),(5,6),(5,7), (5,8),(5,9),(6,7),(6,8),(6,9),(7,8),(7,9),(8,9).\}$$

4. Results

The figure 2 left side shows the graph and related node degree centrality for the start of detection in 2007, the assumption of propagation was generated by a farm located at coordinate $(-54.154538, -31.893438)$ corresponding to node 9 of the graph, the connections between nodes was established by the reported biological invasion threshold distance of 700 m [Descamps and Vocht 2016].

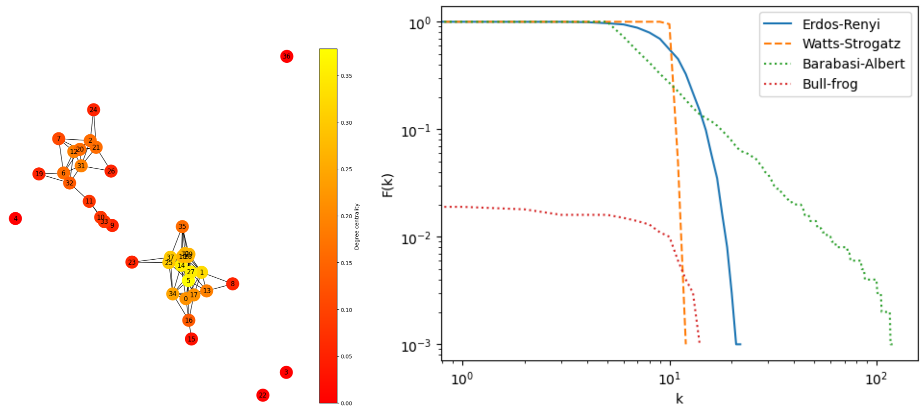


Figure 3. Degree centrality and probability distribution in 2022 invasion

The 2007 invasion graph has average degree of $< k >= 6.00$ with average short path $l = 1.04$ and clustering coefficient of $C_i = 0.81$, indicating a large number of components connected in the network, related to the movements of the frog between pairs of invasion ponds. It is noteworthy that most of the paths in the network use the center of the graph $[1, 5]$ and the peripheral nodes $[0, 3, 6, 7, 8, 9]$. The 2022 invasion graph, figure 3 left side, $|V| = 38$ and $|E| = 113$, that's means in 2022 a total of 38 invaded ponds were found, with a total of 113 possible connections between ponds. The average degree of the network is $< k >= 5.94$, $l = 1.59$ and the clustering coefficient is $C_i = 0.64$. This network analysis shows the current invasion progress fifteen years after the apparition was reported. It is possible to perceive that node number 4 of the 2007 graph was completely invaded and formed an independent community, however the component that is mostly connected continues to be the one centered on the nodes $[1, 5]$ where the invasion arose. The node probability distribution study, figure 3 and 2 right side showed "small world" behavior for the Bullfrog invasion networks, which implies high connectivity and communicability between invaded nodes and a exponential network behavior, this was possible to observe by comparing with widely studied complex networks randomness (Erdos-Renyi), small world (Watts-Strogatz) as well as scale invariant (Barabasi-Albert).

5. Conclusion

The American bullfrog invasion network in Aceguá (Cerro Largo Department, Uruguay) shows a behavior of randomness with a trend of exponential growth network. This property may have implications in that the new apparitions are equally probable in the invasion zone, so as a control, the nodes associated with new apparitions could be eliminated from the network to avoid the continued expansion.

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