



23 European Conference on Fracture – ECF23

# Accuracy of models of confined concrete in rectangular columns using different proposals for the prediction of failure of the FRP

Mariana Jesus<sup>a,b,\*</sup>, Paulo Silva Lobo<sup>a,c</sup>

<sup>a</sup>*CERIS, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal*

<sup>b</sup>*DEC, NOVA School of Science and Technology, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal*

<sup>c</sup>*Departamento de Engenharia Civil e Geologia, Faculdade de Ciências Exatas e da Engenharia, Universidade da Madeira, Campus Universitário da Penteada, 9020-105 Funchal, Portugal*

## Abstract

Confinement with externally applied fiber reinforced polymers (FRP), such as carbon, glass and aramid-based composites, results in notorious improvement of ductility and strength. Several constitutive models, regarding stress–strain relationship, have been proposed. However, few models exist for square and rectangular columns confined with FRP when compared with the number of models for circular concrete columns, and even fewer models satisfactorily predict the failure strain of FRP. In this paper, the accuracy of existing models for the prediction of the failure strain of the FRP is evaluated. Comparison of analytical results with experimental test results of concrete columns reported in the literature is presented, focusing different parameters such as strength, maximum strain and strain energy density.

© 2022 The Authors. Published by Elsevier B.V.

This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0>)

Peer-review under responsibility of the scientific committee of the 23 European Conference on Fracture – ECF23

## Keywords:

Failure Strain; FRP sheets; Confinement.

## 1. Introduction

Confinement with external application of FRP improves ductility and strength of concrete elements. Several models have been proposed over the years. The first studies regarding confinement of columns, with proposed equations for the prediction of maximum strength and axial strain, and the stress-strain behaviour of columns confined with FRP, were presented by [Richard and Abbott \(1975\)](#) and [Mander et al. \(1988\)](#). For subsequent studies, two categories of models were established: theoretical models built around physical concepts and, design-oriented models built around experimental evidence and calibration of parameters.

\* Corresponding author.

*E-mail address:* [mc.jesus@campus.fct.unl.pt](mailto:mc.jesus@campus.fct.unl.pt)

The design-oriented model by [Faustino et al. \(2014\)](#) for carbon fiber reinforced polymers (CFRP) and by [Jesus et al. \(2018\)](#) for glass fiber reinforced polymers (GFRP) are based on the constitutive model for circular columns based on the stress-strain relationship by [Richard and Abbott \(1975\)](#) and the peak strength proposed by [Mander et al. \(1988\)](#) modified by the reduction coefficient proposed by [Mirmiran et al. \(1998\)](#), which relates the corner radius of the column with the side of the column, allowing to transform a square/rectangular cross-section into an equivalent circular cross-section. For each models, the authors calibrated the parameters based on experimental tests from literature. Also, the model by [Lam and Teng \(2003\)](#) and the model by [Wei and Wu \(2012\)](#) are based on stress-strain response from [Mander et al. \(1988\)](#). The design-oriented model by [Manfredi and Realfonzo \(2001\)](#) was based on the theoretical model by [Spoelstra and Monti \(1999\)](#) which consider the influence of the geometry of the column. The authors proposed a reduction coefficient related to the geometry of the cross-section and a coefficient of effectiveness for the lateral stress based on studies by [Rochette and Labossiere \(2000\)](#).

The accuracy of different proposals for the prediction of FRP failure regarding circular columns were presented by [Silva Lobo and Jesus \(2022\)](#). In the present work, the accuracy of design-oriented models with square and rectangular cross-section confined with CFRP, GFRP and aramid fiber reinforced polymers (AFRP) were assessed with different proposals for the failure strain of the FRP.

## 2. Reported test results

The experimental tests for columns with square cross-section confined with CFRP chosen for comparison with the mentioned numerical models are from [Lam and Teng \(2003\)](#) (S2R15 and S4R15) and [Paula \(2003\)](#) (QR2C2 and QR2C3). For columns with rectangular cross-section confined with CFRP, the experimental results considered are from [Lam and Teng \(2003\)](#) (R4R15 and R4R25), [Rocca \(2007\)](#) (B2) and [Zeng et al. \(2018\)](#) (R2Lr45). For square columns confined with GFRP, the experimental results considered are from [Rousakis and Karabinis \(2012\)](#) (BS1G6), [Rousakis et al. \(2007\)](#) (AgL6M) and [Tastani et al. \(2006\)](#) (FSG2 and FSG4). Regarding AFRP, the experimental tests considered are those by [Silva Lobo et al. \(2018\)](#) (AS) and [Rochette and Labossiere \(2000\)](#) (S25-A3, S25-A6 and S25-A9). It should be noted that due to the lack of experimental results in the literature, no columns with rectangular cross-section confined with GFRP or AFRP were analyzed.

Table 1. Experimental results

Author	Specimen	Geometry			FRP Properties					Concrete Properties				
		$B$ [mm]	$2r/B$ [-]	$H/B$ [-]	type	no. layers	$t_j$ [mm]	$E_j$ [GPa]	$\varepsilon_{ju}$ [%]	$\varepsilon_{lu}$ [%]	$f_{co}$ [MPa]	$\varepsilon_{co}$ [%]	$f_{cc}$ [MPa]	$\varepsilon_{cc}$ [%]
<a href="#">Lam and Teng (2003)</a>	R4R15	150	0.20	1.50	CFRP	4	0.17	257	1.76	1.070	41.50	0.20	50.00	1.20
<a href="#">Lam and Teng (2003)</a>	R4R25	150	0.33	1.50	CFRP	4	0.17	257	1.76	0.740	41.50	0.20	56.79	1.04
<a href="#">Rocca (2007)</a>	B2	318	0.19	2.00	CFRP	5.5	0.17	291	0.93	0.470	30.50	0.20	30.58	0.30
<a href="#">Zeng et al. (2018)</a>	R2Lr45	290	0.31	1.50	CFRP	2	0.33	245.6	1.71	1.130	39.60	0.25	43.00	2.25
<a href="#">Paula (2003)</a>	QR2C2	150	0.27	1.00	CFRP	2	0.18	240	1.55	1.250	21.20	0.23	56.31	2.96
<a href="#">Paula (2003)</a>	QR2C3	150	0.27	1.00	CFRP	2	0.18	240	1.55	1.581	21.20	0.23	53.00	2.74
<a href="#">Lam and Teng (2003)</a>	S2R15	150	0.20	1.00	CFRP	2	0.17	257	1.76	0.970	33.70	0.20	50.49	0.87
<a href="#">Lam and Teng (2003)</a>	S4R15	150	0.20	1.00	CFRP	4	0.17	257	1.76	0.910	24.00	0.20	58.40	1.50
<a href="#">Rousakis and Karabinis (2012)</a>	BS1G6	200	0.30	1.00	GFRP	6	0.15	73	4.5	1.113	25.55	0.21	49.13	1.09
<a href="#">Rousakis et al. (2007)</a>	AgL6M	200	0.30	1.00	GFRP	6	0.14	65	2.8	0.676	33.04	0.17	44.29	0.60
<a href="#">Tastani et al. (2006)</a>	FSG2	200	0.25	1.00	GFRP	2	0.17	75	2.1	0.819	14.76	0.30	31.15	1.43
<a href="#">Tastani et al. (2006)</a>	FSG4	200	0.25	1.00	GFRP	4	0.17	75	2.1	0.819	14.76	0.30	32.57	1.75
<a href="#">Silva Lobo et al. (2018)</a>	AS	200	0.35	1.00	AFRP	1	0.20	120	2.5	2.870	17.80	0.51	24.81	2.65
<a href="#">Rochette and Labossiere (2000)</a>	S25-A3	152	0.33	1.00	AFRP	3	0.42	13.6	1.69	1.120	43.00	0.20	51.17	0.30
<a href="#">Rochette and Labossiere (2000)</a>	S25-A6	152	0.33	1.00	AFRP	6	0.42	13.6	1.69	1.270	43.00	0.20	51.17	0.30
<a href="#">Rochette and Labossiere (2000)</a>	S25-A9	152	0.33	1.00	AFRP	9	0.42	13.6	1.69	0.940	43.00	0.20	53.32	0.30

$B$  and  $H$  are the sides of the column,  $r$  is the corner radius to the side of the column,  $no. layers$  is a reference to the number of layers of FRP used,  $t_j$  is the design thickness of one FRP sheet,  $E_j$  is the Young's modulus of the FRP,  $\varepsilon_{ju}$  is the ultimate strain provided by the manufacturer,  $\varepsilon_{lu}$  is the observed experimental failure strain,  $f_{co}$  is the unconfined concrete strength,  $\varepsilon_{co}$  is the strain corresponding to  $f_{co}$ ,  $f_{cc}$  is the peak strength and  $\varepsilon_{cc}$  is the strain corresponding to  $f_{cc}$ .

### 3. Proposals for the prediction of failure of the FRP

The equations found in literature for the prediction of failure of the FRP are presented in Table 2. Some of the authors proposed equations of the prediction of failure of FRP regarding reduction factors observed during the experimental tests (Ilky et al. (2004); Lam and Teng (2003); Toutanji et al. (2010)). Others proposed equations regarding the influence of the geometry, as is the corner radius and the side of the column (Diego et al. (2019); Faustino et al. (2014); Lin and Teng (2020); Manfredi and Realfonzo (2001); Wang et al. (2016)). In the particular case of Lim and Ozbakkaloglu (2014), the authors related the failure strain of the FRP with the unconfined concrete strength and with the Young’s modulus of the FRP.

Table 2. Equations of  $\epsilon_{lu} / \epsilon_{ju}$  for columns with square and rectangular cross-section.

Author	$\epsilon_{lu} / \epsilon_{ju}$	Geometry	Note:
Diego et al. (2019)	$0.46 \times \left(\frac{2r}{B}\right)^{0.25} + 0.14$	(1) Square	for CFRP
Faustino et al. (2014)	$0.70 \times \left(\frac{2r}{B}\right)^{0.23}$	(2) Square	for CFRP
Ilky et al. (2004)	0.70	(3) Square / rectangular	for CFRP
Lam and Teng (2003)	0.586 0.788 0.851 0.624 0.632	(4) Square / rectangular	for CFRP for HM CFRP for AFRP for FRP
Lim and Ozbakkaloglu (2014)	$0.9 - 0.75 \times \frac{E_f}{10^6} - 2.3 \times \frac{f_{co}}{10^3}$	(5) Square / rectangular	for FRP
Lin and Teng (2020)	$0.727 \times \left(\frac{2r}{B}\right)^{0.288}$	(6) Square	for FRP
Manfredi and Realfonzo (2001)	$1.17 \times \left(\frac{r}{B}\right) + 0.10$	(7) Square	for FRP
Toutanji et al. (2010)	0.43	(8) Square / rectangular	for FRP
Wang et al. (2016)	$1 - 0.38 \times \left(\frac{B}{100}\right)^{0.41}$ 0.33	(9) Square with $B \in [100, 400]mm$ Square with $B > 400mm$	for CFRP for CFRP

### 4. Comparison of numerical and experimental results

The comparison between numerical results and experimental tests, regarding different proposals for the prediction of the failure of the FRP, focus on the analysis of different parameters such as  $f_{cc}$ ,  $\epsilon_{cc}$  and strain energy density ( $W$ ) for all three FRP. Each model was combined with the equations presented in Table 2. The error of columns with square cross-section confined with CFRP, regarding  $f_{cc}$  and  $\epsilon_{cc}$ , is presented in Table 3. The error can be obtained by:  $error (\%) = [(t_v - n_v)/t_v] \times 100$ , were  $t_v$  is the value of the specimen and  $n_v$  is the value of the numerical model.

Table 3. Error of model predictions compared to experimental results for columns with square cross-section confined with CFRP.

Specimen	Model	Equation (1)		Equation(2)		Equation (3)		Equation (4)		Equation (5)		Equation (6)		Equation (7)		Equation (8)		Equation (9)	
		$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$
QR2C2	Faustino et al. (2014)	25.54	71.28	47.71	16.28	35.92	-11.42	43.54	6.15	37.89	-6.99	48.84	19.09	60.39	6.18	52.54	28.55	45.62	11.15
	Lam and Teng (2003)	40.70	59.09	38.59	55.14	30.16	37.85	35.44	48.96	31.48	40.72	39.50	56.86	50.58	75.13	42.57	62.44	37.00	52.07
	Manfredi and Realfonzo (2001)	39.34	48.31	35.66	40.54	20.52	3.04	30.05	27.70	22.88	9.46	37.24	43.92	54.79	77.36	42.62	55.07	32.80	34.12
	Wei and Wu (2012)	49.24	44.62	48.03	41.58	43.31	30.11	46.26	37.21	44.04	31.85	48.55	42.88	54.95	59.93	50.30	47.35	47.14	39.36
QR2C3	Faustino et al. (2014)	20.89	68.98	42.82	9.56	31.92	-20.36	40.02	-1.39	34.01	-15.58	45.65	12.59	57.92	-1.35	49.57	22.81	42.23	4.01
	Lam and Teng (2003)	37.00	55.81	34.75	51.54	25.80	32.86	31.41	44.86	27.21	35.96	35.72	53.40	47.50	73.13	38.98	59.42	33.07	48.22
	Manfredi and Realfonzo (2001)	35.55	44.16	31.65	35.77	15.56	-4.74	25.68	21.90	18.06	2.19	33.32	39.42	51.97	75.55	39.04	51.46	28.61	28.83
	Wei and Wu (2012)	46.07	40.17	44.79	36.89	39.77	24.50	42.90	32.17	40.54	26.38	45.34	38.29	52.14	56.71	47.20	43.12	43.83	34.49
S2R15	Faustino et al. (2014)	33.12	-19.52	32.08	-29.05	7.14	-611.60	28.75	-56.14	14.89	-68.08	32.85	-22.04	36.73	42.02	33.61	-14.81	29.92	-47.07
	Lam and Teng (2003)	9.82	5.53	7.94	-0.91	-3.40	-44.10	2.61	-20.33	0.28	-29.29	9.31	3.81	21.89	40.79	10.73	8.53	4.39	-13.66
	Manfredi and Realfonzo (2001)	10.26	-14.81	6.54	-27.44	-17.71	-121.58	-4.54	-67.62	-9.71	-88.29	9.25	-18.25	29.87	48.34	12.02	-9.07	-0.83	-53.85
	Wei and Wu (2012)	20.06	-7.44	19.08	-11.40	13.17	-34.26	16.29	-22.35	15.08	-27.03	19.80	-8.48	26.58	20.78	20.55	-5.46	17.20	-18.85
S4R15	Faustino et al. (2014)	38.43	-59.51	35.34	-71.87	14.41	-148.15	25.97	-107.40	8.33	-102.46	26.37	-42.01	54.98	14.84	39.91	-53.43	29.16	-95.64
	Lam and Teng (2003)	18.34	-12.20	15.10	-22.68	-4.52	-93.23	5.90	-54.32	-0.17	-76.61	17.47	-15.00	39.25	45.58	19.93	-7.19	8.96	-43.50
	Manfredi and Realfonzo (2001)	21.54	-11.55	17.24	-25.58	-7.96	-129.11	5.04	-71.00	-2.71	-104.40	20.44	-14.89	47.80	59.25	23.67	-4.87	8.97	-55.63
	Wei and Wu (2012)	37.46	-27.02	35.85	-34.38	26.24	-69.97	31.32	-51.46	28.35	-62.37	37.02	-29.84	48.04	15.78	38.25	-25.04	32.81	-45.90

The stress-strain curves for square columns confined with CFRP were analysed for the models with the lower error values of  $f_{cc}$  and  $\epsilon_{cc}$  (see Fig. 4). The comparison of the  $W$  and the  $\epsilon_{lu}$  of the FRP of the models with the  $W$  and  $\epsilon_{lu}$  of the experimental test, respectively, for the smallest value of  $f_{cc}$  and  $\epsilon_{cc}$ , are presented in Table 4.

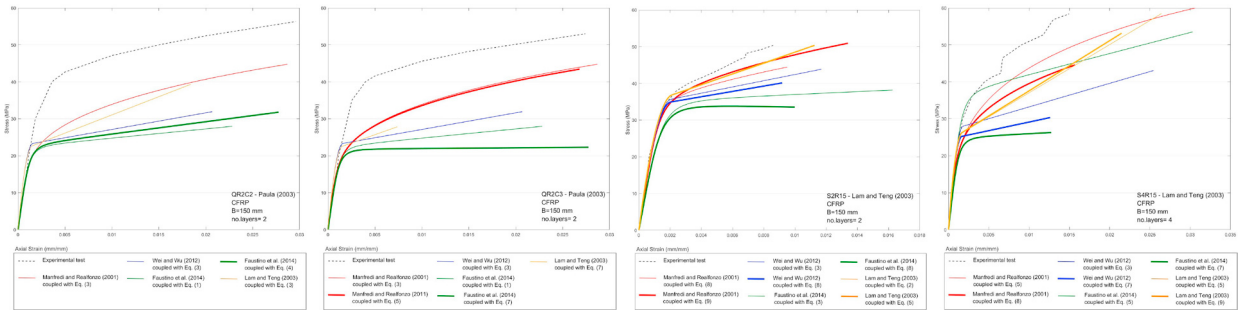


Fig. 1. Comparison of numerical stress-strain curves with experimental data for columns with square cross-section confined with CFRP.

Table 4. Error of the strain energy density and failure strain prediction for square specimens confined with CFRP.

Model	QR2C2			QR2C3			S2R15			S4R15		
	Equation	$W$	$\epsilon_{lu}$	Equation	$W$	$\epsilon_{lu}$	Equation	$W$	$\epsilon_{lu}$	Equation	$W$	$\epsilon_{lu}$
Faustino et al. (2014)	(1)	60.16	41.65	(1)	54.77	53.86	(3)	-63.11	-27.01	(5)	-101.54	-26.11
	(4)	47.19	27.46	(7)	51.54	74.90	(8)	11.00	21.98	(7)	53.60	58.03
Lam and Teng (2003)	(3)	60.79	13.20	(3)	55.48	31.37	(2)	4.20	12.29	(5)	-64.41	-26.11
							(5)	-31.17	-14.26	(9)	-24.44	-6.62
Manfredi and Realfonzo (2001)	(3)	27.42	13.20	(3)	17.60	31.37	(8)	-1.55	21.98	(5)	-116.62	-26.11
				(5)	25.22	34.19	(9)	-61.33	-0.02	(8)	16.85	16.84
Wei and Wu (2012)	(3)	60.34	13.20	(3)	54.97	31.37	(3)	-25.77	-27.01	(3)	-33.24	-35.38
							(8)	8.14	21.98	(7)	49.31	58.03

Regarding the specimens QR2C2 and QR2C3, it is noted that  $\epsilon_{cc}$  is obtained with less error for the model by Manfredi and Realfonzo (2001) coupled with equation (3) and the model by Faustino et al. (2014) coupled with equation (7), respectively. In the case of  $f_{cc}$  and  $W$ , no model is sufficiently accurate in the prediction of the experimental values. Regarding the value of  $\epsilon_{lu}$  it is noted that equation (3) is the most accurate with an error of 13.2 % and 31.37% for specimens QR2C2 and QR2C3, respectively.

For specimens S2R15 and S4R15, the model by Lam and Teng (2003) coupled with Equation (5) presents the smallest error for the prediction of  $f_{cc}$ , while the model by Lam and Teng (2003) coupled with (2) and the model by Manfredi and Realfonzo (2001) coupled with (8) presents the smallest error for the prediction of  $\epsilon_{cc}$ , respectively. Regarding  $W$ , the model by Manfredi and Realfonzo (2001) coupled with equation (8) present the smallest error for specimen S2R15 and no model is representative of  $W$  for specimen S4R15. When analysing  $\epsilon_{lu}$ , for both specimens, equation (9) is the most accurate, presenting an error of 0.02 % for specimen S2R15 and 6.62% for specimen S4R15.

The errors between numerical models and experimental tests, regarding  $f_{cc}$  and  $\epsilon_{cc}$ , for columns with rectangular cross-section confined with CFRP are presented in Table 5.

The stress-strain curves for rectangular columns confined with CFRP were analysed for the models with the lower error values of  $f_{cc}$  and  $\epsilon_{cc}$  (see Fig. 4). The comparison of the  $W$  and the  $\epsilon_{lu}$  of the FRP of the models with the  $W$  and  $\epsilon_{lu}$  of the experimental test, respectively, for the smallest value of  $f_{cc}$  and  $\epsilon_{cc}$ , are presented in Table 6.

In stress-axial strain response of specimen R4R15, the model by Wei and Wu (2012) coupled with Equation (5), is a good predictor for the full response of experimental test, with the smallest error for  $f_{cc}$ ,  $\epsilon_{cc}$  and  $W$ , with equation (5) presenting an error of 0.29% in the prediction of  $\epsilon_{lu}$ . For specimen R4R25, in a generalized perspective, the model by Wei and Wu (2012) coupled with equation (8) presents the smallest error for the stress-strain response, also, equation (8) is the most accurate for the prediction of  $\epsilon_{lu}$  with an error of 2.27%.

Regarding specimens B2 and R2Lr45, no model is able to predict the curve behaviour of the specimen. In accordance, the error values of  $f_{cc}$ ,  $\epsilon_{cc}$  and  $W$  are high.

Table 5. Error of model predictions compared to experimental results for columns with rectangular cross-section confined with CFRP.

Specimen	Model	Equation (3)		Equation (4)		Equation (5)		Equation (8)	
		$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$
R4R15	Lam and Teng (2003)	-15.36	-76.72	-10.04	-42.90	-11.29	-50.57	-2.87	-1.92
	Wei and Wu (2012)	-1.23	-5.60	1.59	4.01	0.93	1.71	5.47	17.83
R4R25	Lam and Teng (2003)	-4.98	-124.30	0.27	-80.57	-0.96	-90.47	7.33	-27.70
	Wei and Wu (2012)	3.74	-27.76	7.34	-15.91	6.49	-18.78	12.26	1.02
B2	Lam and Teng (2003)	-9.75	-261.90	-8.11	-205.87	-8.48	-218.20	-5.89	-137.50
	Wei and Wu (2012)	-6.44	-229.60	-5.40	-202.67	-5.64	-209.07	-3.98	-164.37
R2Lr45	Lam and Teng (2003)	-11.97	31.93	-8.70	43.08	-9.84	39.31	-4.31	56.56
	Wei and Wu (2012)	-6.65	47.48	-4.39	51.64	-5.17	50.19	-1.30	57.59

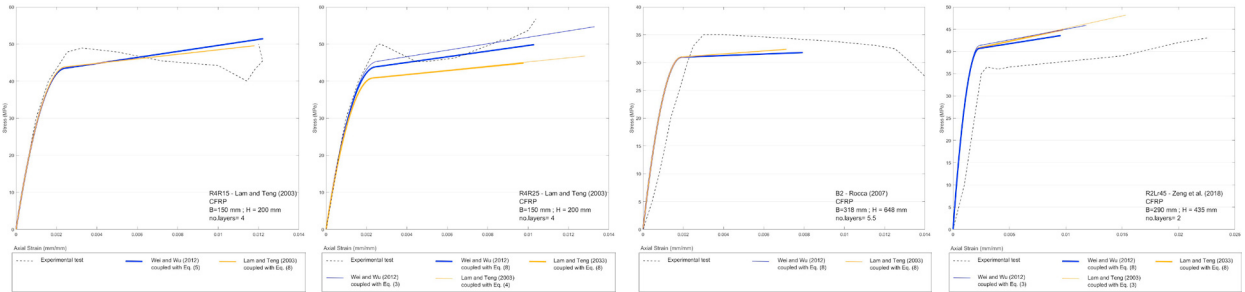


Fig. 2. Comparison of numerical stress-strain curves with experimental data for columns with rectangular cross-section confined with CFRP.

Table 6. Error of the strain energy density and failure strain prediction for rectangular specimens confined with CFRP.

Model	R4R15			R4R25			B2			R2Lr45		
	Equation	W	$\epsilon_{lu}$	Equation	W	$\epsilon_{lu}$	Equation	W	$\epsilon_{lu}$	Equation	W	$\epsilon_{lu}$
Lam and Teng (2003)	(8)	-5.09	29.27	(4)	-93.27	-39.14	(8)	52.28	14.91	(3)	21.35	-5.93
				(8)	-28.82	-2.27				(8)	53.14	34.93
Wei and Wu (2012)	(5)	0.29	-0.64	(3)	-33.52	-66.49	(8)	46.72	14.91	(3)	41.50	-5.93
				(8)	4.57	-2.27				(8)	55.02	34.93

The errors between numerical models and experimental tests, regarding  $f_{cc}$  and  $\epsilon_{cc}$ , for columns with square cross-section confined with GFRP are presented in Table 7.

Table 7. Error of model predictions compared to experimental results for columns with square cross-section confined with GFRP.

Specimen	Model	Equation (4)		Equation (5)		Equation (6)		Equation (7)		Equation (8)	
		$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$	$f_{cc}$	$\epsilon_{cc}$
BS1G6	Jesus et al. (2018)	-80.09	-432.84	-101.49	-568.99	-65.35	-340.46	-32.06	-139.63	-53.99	-270.09
	Lam and Teng (2003)	-10.43	-186.52	-25.64	-287.32	-0.13	-124.53	22.21	-10.91	7.73	-81.06
	Manfredi and Realfonzo (2001)	-6.54	-444.95	-25.74	-704.59	7.52	-292.66	37.86	-46.79	18.44	-192.66
	Wei and Wu (2012)	12.60	-30.93	4.01	-51.15	18.50	-16.47	31.59	18.07	23.06	-4.84
AgL6M	Jesus et al. (2018)	-81.81	-563.70	-94.02	-721.85	-72.45	-448.74	-49.15	-198.82	-64.88	-360.84
	Lam and Teng (2003)	-8.10	-83.11	-16.22	-132.32	-2.20	-50.50	10.61	9.33	2.30	-27.63
	Manfredi and Realfonzo (2001)	-10.78	-187.39	-28.52	-305.04	2.03	-113.45	26.40	4.20	11.26	-66.39
	Wei and Wu (2012)	3.97	-44.97	-0.88	-61.80	7.54	-32.13	15.47	-1.39	10.30	-21.87
FSG2	Jesus et al. (2018)	-5.56	-117.81	-13.41	-182.16	0.74	-70.94	14.50	12.22	3.61	-51.15
	Lam and Teng (2003)	37.10	14.50	32.49	-7.82	40.50	29.17	46.49	50.54	41.93	34.86
	Manfredi and Realfonzo (2001)	37.14	-36.83	25.42	-107.70	45.21	5.27	56.16	59.30	48.39	20.71
	Wei and Wu (2012)	43.11	-30.76	40.47	-51.05	45.08	-14.90	48.65	16.43	45.92	-7.86
FSG4	Jesus et al. (2018)	-56.97	-53.43	-76.22	-98.57	-42.33	-20.57	-13.08	38.00	-35.88	-6.63
	Lam and Teng (2003)	25.00	-9.26	16.17	-45.65	31.49	14.57	42.98	49.45	34.24	23.83
	Manfredi and Realfonzo (2001)	21.11	-45.71	4.86	-125.14	33.04	1.71	51.45	61.14	37.92	18.86
	Wei and Wu (2012)	37.24	-58.66	32.40	-86.40	40.84	-36.95	47.40	5.97	42.39	-27.31

Table 8. Error of the strain energy density and failure strain prediction for square specimens confined with GFRP.

Model	BS1G6			AgL6M			FSG2			FSG4		
	Equation	$W$	$\epsilon_{lu}$	Equation	$W$	$\epsilon_{lu}$	Equation	$W$	$\epsilon_{lu}$	Equation	$W$	$\epsilon_{lu}$
Jesus et al. (2018)	(7)	-189.15	-11.38	(7)	-139.64	-14.11	(6)	-39.21	-25.04	(7)	44.61	18.69
Lam and Teng (2003)	(6)	-114.14	-107.81	(6)	13.11	-112.88	(7)	36.35	36.86	(8)	-10.67	-41.98
	(7)	11.83	-11.38	(7)	29.24	-14.11	(5)	35.37	-107.64	(4)	26.46	-106.04
Manfredi and Realfonzo (2001)	(4)	-500.05	-152.29	(6)	-23.77	-112.88	(5)	-38.15	-107.64	(5)	-96.05	-167.39
	(7)	-2.88	-11.38	(7)	55.82	-14.11	(6)	50.37	-25.04	(6)	37.17	-61.02
Wei and Wu (2012)	(5)	-42.65	-217.99	(5)	7.16	-221.11	(5)	14.00	-107.64	(5)	-22.27	-167.39
	(8)	16.75	-73.85	(7)	50.07	-14.11	(8)	42.62	-10.26	(7)	25.46	18.69

The stress-strain curves for square columns confined with GFRP were analysed for the models with the lower error values of  $f_{cc}$  and  $\epsilon_{cc}$  (see Fig. 4). The comparison of the  $W$  and the  $\epsilon_{lu}$  of the FRP of the models with the  $W$  and  $\epsilon_{lu}$  of the experimental test, respectively, for the smallest value of  $f_{cc}$  and  $\epsilon_{cc}$ , are presented in Table 8.

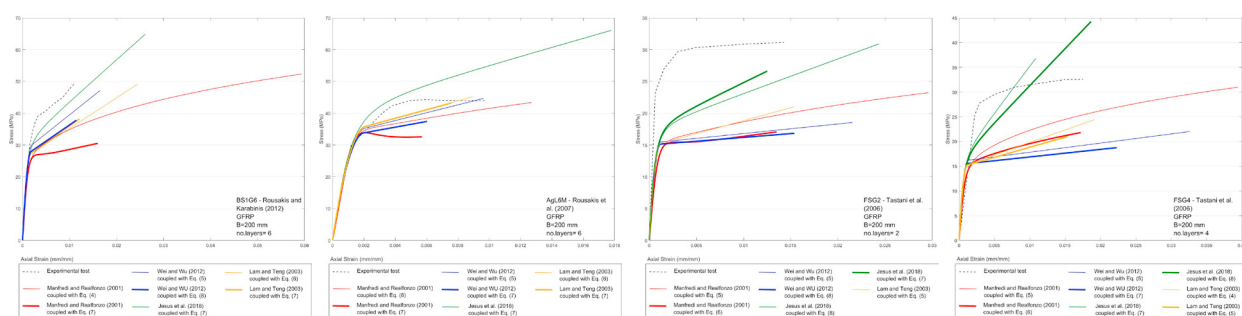


Fig. 3. Comparison of numerical stress-strain curves with experimental data for columns with square cross-section confined with GFRP.

In the case of specimen BS1G6, the model by Wei and Wu (2012) and the model by Manfredi and Realfonzo (2001), both coupled with equation (7), presents the smallest error of  $\epsilon_{cc}$  and  $W$ , respectively. Also, equation (7) presents the smallest value of error in the prediction of  $\epsilon_{lu}$  with an error of 11.38%. The model of Lam and Teng (2003) coupled with equation (6) presents the better fit for  $f_{cc}$ .

In the stress-axial response of specimen AgL6M, the model by Wei and Wu (2012) coupled with equation (5) and equation (7) are representative of the behaviour of the experimental test. The analysis of the  $W$  for specimen AgL6M shown that the model by Wei and Wu (2012) coupled with equation (5) presents the smallest value of error, in accordance to the graphic visualization, and, the prediction of  $\epsilon_{lu}$  with the smallest error is from equation (7) and its equal to 14.11%.

For specimens FSG2 and FSG4 and regarding the stress-strain behaviour of the experimental test, no model is sufficient accurate to predict the behaviour of both specimens. Regarding the prediction of  $\epsilon_{lu}$ , for specimens FSG2 and FSG4, equation (8) and (7), are the most accurate with an error of 10.26% and 18.69%, respectively.

The errors between numerical models and experimental tests, regarding  $f_{cc}$  and  $\epsilon_{cc}$ , for columns with square cross-section confined with AFRP are presented in Table 9.

The stress-strain curves for square columns confined with AFRP were analysed for the models with the lower error values of  $f_{cc}$  and  $\epsilon_{cc}$  (see Fig. 4). The comparison of the  $W$  and the  $\epsilon_{lu}$  of the FRP of the models with the  $W$  and  $\epsilon_{lu}$  of the experimental test, respectively, for the smallest value of  $f_{cc}$  and  $\epsilon_{cc}$ , are presented in Table 10.

In stress-axial strain response of specimen AS, the model by Manfredi and Realfonzo (2001) coupled with equation (6) presents the smallest error for the prediction of  $f_{cc}$ , while, the model by Wei and Wu (2012) coupled with equation (6) presents the smaller error in the prediction of  $\epsilon_{cc}$ . The model by Lam and Teng (2003) coupled with equation (4) presents the smallest error of  $W$  in comparison with the experimental test. Besides, in accordance to the graphic observation, the model by Manfredi and Realfonzo (2001) coupled with equation (6) is the closest response to the specimen stress-axial strain behaviour, however, equation (4) presents the smallest error in the prediction of  $\epsilon_{lu}$  and its equal to 25.87%. Regarding the specimens S25-A3, S25-A6 and S25-A9, no model is able to predict the full response of the experimental test. In accordance, the error values of  $W$  are high.

Table 9. Error of model predictions compared to experimental results for columns with square cross-section confined with AFRP.

Specimen	Model	Equation (4)		Equation (5)		Equation (6)		Equation (7)		Equation (9)	
		$f_{cc}$	$\varepsilon_{cc}$	$f_{cc}$	$\varepsilon_{cc}$	$f_{cc}$	$\varepsilon_{cc}$	$f_{cc}$	$\varepsilon_{cc}$	$f_{cc}$	$\varepsilon_{cc}$
AS	Lam and Teng (2003)	-5.96	3.08	-2.67	11.72	6.63	33.88	15.96	52.08	10.93	42.83
	Manfredi and Realfonzo (2001)	-32.06	-150.57	-23.21	-111.70	0.32	-21.13	18.95	42.26	9.82	11.32
	Wei and Wu (2012)	5.34	-29.50	7.42	-22.49	13.38	-1.55	19.53	21.96	16.19	8.90
S25-A3	Lam and Teng (2003)	5.74	-88.23	6.45	-81.17	9.62	-52.40	12.45	-31.80	10.80	-43.20
	Manfredi and Realfonzo (2001)	11.86	-370.00	14.89	-323.33	25.97	-156.67	29.27	-53.33	28.59	-110.00
	Wei and Wu (2012)	8.67	-86.20	9.15	-82.53	11.31	-65.17	13.29	-47.97	12.13	-58.20
S25-A6	Lam and Teng (2003)	-4.52	-160.23	-3.05	-145.53	3.26	-88.43	8.94	-47.00	5.63	-69.83
	Wei and Wu (2012)	1.94	-134.07	2.89	-127.63	7.02	-98.63	10.84	-69.17	8.59	-86.90
	Manfredi and Realfonzo (2001)	-13.71	-503.33	-9.00	-443.33	10.36	-213.33	21.93	-66.67	16.33	-143.33
S25-A9	Lam and Teng (2003)	-10.11	-231.80	-8.03	-210.13	1.09	-124.17	9.23	-62.33	4.46	-96.57
	Manfredi and Realfonzo (2001)	-27.80	-606.67	-21.83	-530.00	2.68	-253.33	19.55	-80.00	10.64	-173.33
	Wei and Wu (2012)	-0.35	-175.83	0.96	-167.33	6.80	-127.63	12.15	-87.90	9.00	-111.77

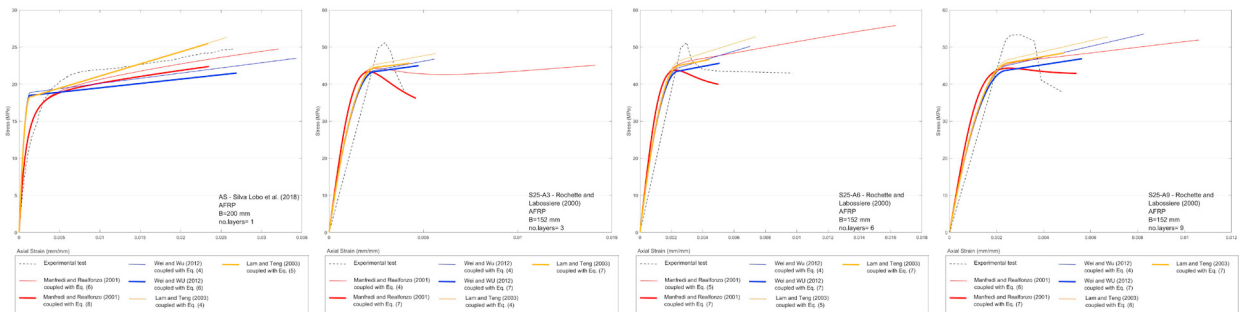


Fig. 4. Comparison of numerical stress-strain curves with experimental data for columns with square cross-section confined with AFRP.

Table 10. Error of the strain energy density and failure strain prediction for square specimens confined with AFRP.

Model	Equation	AS		S25-A3		S25-A6		S25-A9				
		W	$\varepsilon_{lu}$	W	$\varepsilon_{lu}$	W	$\varepsilon_{lu}$	W	$\varepsilon_{lu}$			
Lam and Teng (2003)	(4)	0.89	25.87	(4)	-71.48	-28.41	(5)	16.29	-6.08	(6)	-62.48	5.11
	(5)	11.55	33.01	(7)	-19.86	55.88	(7)	57.18	60.78	(7)	-6.31	47.43
Manfredi and Realfonzo (2001)	(6)	-20.60	53.20	(4)	-351.16	-28.41	(5)	-106.82	-6.08	(6)	-170.04	5.11
	(8)	18.33	62.54	(7)	-24.73	55.88	(7)	51.55	60.78	(7)	-16.83	47.43
Wei and Wu (2012)	(4)	-26.85	25.87	(4)	-64.61	-28.41	(4)	23.80	-14.14	(4)	-104.26	-53.00
	(6)	6.41	53.20	(7)	-33.23	55.88	(7)	50.25	60.78	(7)	-22.51	47.43

### 5. Conclusions

The accuracy of existing confinement models coupled with different proposals for the prediction of the failure strain of the FRP was evaluated through comparison of analytical results with experimental test results for columns with square cross-section confined with CFRP, GFRP and AFRP, and columns with rectangular cross-section confined with CFRP. Due to the lack of experimental tests reported in the literature, it was not possible to assess rectangular columns confined with both GFRP and AFRP.

The models proposed by Manfredi and Realfonzo (2001) coupled with equations (3) and (5) present the most accurate stress-axial strain response for columns with square cross-section confined with CFRP. Regarding columns with rectangular cross-section confined with CFRP, the model by Wei and Wu (2012) coupled with equations (5) and (8) delivered the most accurate stress-axial strain response. In two specimens, no model adequately predict the stress-axial strain curve.

The model by Wei and Wu (2012) coupled with equation (5) presents the most adequate response for columns with square cross-section confined with GFRP. For columns with square cross-section confined with AFRP, the model proposed by Manfredi and Realfonzo (2001) coupled with equation (6) was the most accurate in the case of specimen

AS. For the other specimens, high values of error were obtained and no model adequately predicts the stress-axial strain response of the specimens.

The use of different proposals for the prediction of the failure strain of the FRP together with existing models for confined concrete columns allowed to obtain better stress-axial strain curves, regarding the prediction of the peak strength and its corresponding strain, the strain energy density and the failure strain of the FRP. The use of equations (5), (7) and (8) allowed to improve the response of models in 60% of cases and the use of equation (4) allowed for better results in 15% of cases.

## References

- Diego, A., Arteaga, Á., & Fernández, J. (2019). Strengthening of square concrete columns with composite materials. Investigation on the FRP jacket ultimate strain. *Composites Part B: Engineering*, 162, 454-460.
- Faustino, P., Chastre, C., & Paula, R. (2014). Design model for square RC columns under compression confined with CFRP. *Composites Part B: Engineering*, 57, 187-198.
- Ilki, A., Kumbasar, N., & Koc, V. (2004). Low strength concrete members externally confined with FRP sheets. *Structural Engineering and Mechanics*, 18(2), 167-194.
- Jesus, M., Silva Lobo, P., & Faustino, P. (2018). Design models for circular and square RC columns confined with GFRP sheets under axial compression. *Composites Part B: Engineering*, 141, 60-69.
- Lam, L., & Teng, J. (2003). Design-oriented stress-strain model for FRP-confined concrete in rectangular columns. *Journal of reinforced plastics and composites*, 22(13), 1149-1186.
- Lim, J. C., & Ozbakkaloglu, T. (2014). Design model for FRP-confined normal-and high-strength concrete square and rectangular columns. *Magazine of Concrete Research*, 66(20), 1020-1035.
- Lin, G., & Teng, J. G. (2020). Advanced stress-strain model for FRP-confined concrete in square columns. *Composites Part B: Engineering*, 197, 108149.
- Mander, J., Priestley, M., & Park, R. (1988). Theoretical stress-strain model for confined concrete. *Journal of structural engineering*, 114(8), 1804-1826.
- Manfredi G, Realfonzo R. (2001). Models of concrete confined by fiber composites. In: Proc., 5th Annual Symp. on Fibre-Reinforced-Plastic Reinforced for Concrete Structures. Thomas Telford; 2001:865–74.
- Mirmiran A, Shahawy M, Samaan M, Echary HE, Mastrapa JC, Pico O. Effect of Column Parameters on FRP-confined Concrete. *Journal of Composites for Construction*, 1998; 2(4):175-184.
- Paula, R. (2003). Influência da geometria das secções no confinamento de pilares de betão armado com compósitos de CFRP.(Master Dissertation) Inst Sup Técnico/UTL, Lisboa.
- Richard, R., & Abbott, B. (1975). Versatile elastic-plastic stress-strain formula. *Journal of the Engineering Mechanics Division*, 101(4), 511-515.
- Rocca S (2007). Experimental and analytical evaluation of FRP-confined large size reinforced concrete columns.(Doctoral dissertation) University of Missouri-Rolla.
- Rochette, P., & Labossiere, P. (2000). Axial testing of rectangular column models confined with composites. *Journal of composites for construction*, 4(3), 129-136.
- Rousakis, T. C., & Karabinis, A. I. (2012). Adequately FRP confined reinforced concrete columns under axial compressive monotonic or cyclic loading. *Materials and structures*, 45(7), 957-975.
- Rousakis, T. C., Karabinis, A. I., & Kioussis, P. D. (2007). FRP-confined concrete members: Axial compression experiments and plasticity modelling. *Engineering Structures*, 29(7), 1343-1353.
- Silva Lobo, P., Faustino, P., Jesus, M., & Marreiros, R. (2018). Design model of concrete for circular columns confined with AFRP. *Composite Structures*, 200, 69-78.
- Silva Lobo, P., & Jesus, M. (2022). Accuracy of models of concrete in circular columns using different proposals for the prediction of failure of the confining FRP. *Procedia Structural Integrity*, 37, 788-795.
- Spoelstra, M., & Monti, G. (1999). FRP-confined concrete model. *Journal of composites for construction*, 3(3), 143-150.
- Tastani, S. P., Pantazopoulou, S. J., Zdoumba, D., Plakantaras, V., & Akritidis, E. (2006). Limitations of FRP jacketing in confining old-type reinforced concrete members in axial compression. *Journal of Composites for Construction*, 10(1), 13-25.
- Toutanji, H., Han, M., Gilbert, J., & Matthys, S. (2010). Behavior of large-scale rectangular columns confined with FRP composites. *Journal of Composites for Construction*, 14(1), 62-71.
- Wang, D. Y., Wang, Z. Y., Smith, S. T., & Yu, T. (2016). Size effect on axial stress-strain behavior of CFRP-confined square concrete columns. *Construction and Building Materials*, 118, 116-126.
- Wei, Y., & Wu, Y. (2012). Unified stress-strain model of concrete for FRP-confined columns. *Construction and Building Materials*, 26(1), 381-392.
- Zeng, J. J., Lin, G., Teng, J. G., & Li, L. J. (2018). Behavior of large-scale FRP-confined rectangular RC columns under axial compression. *Engineering structures*, 174, 629-645.