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Pedagogical listening: understanding how teachers listen to student struggle during mathematical sense-making discussions

Andrea R. English [a], Kersti Tyson [b], Allison Hintz[c], Diana Murdoch[a,d] and Julie Anderson[c]

[a]Moray House School of Education and Sport, University of Edinburgh, Edinburgh, UK; [b]Director of Research and Evaluation, Los Alamos National Laboratory Foundation, Española, NM, USA; [c]School of Educational Studies, University of Washington, Bothell, USA; [d]European Agency for Special Needs and Inclusive Education, Brussels, Belgium

ABSTRACT

Research suggests that students' verbalisation of how they are struggling to understand something new is important for learning mathematics with conceptual understanding. However, less is known about how teachers listen while students 'think aloud' through struggle. In this study, we sought to answer the following research question: What types of listening do teachers enact when students are verbalising struggle during mathematical sense-making discussions? We detail how we created and applied a Framework for Pedagogical Listening, which extends previous theoretical and empirical research on teacher listening to identify and differentiate between five types of teacher listening: empathic, supportive, educative, self-reflective and generative. Our study involved nine teachers and their students in the US and Scotland, contexts which are focused on reform-efforts towards inquiry-oriented mathematics instruction that engages students in sense-making discussions. Our findings suggest that the five types of teacher listening in our framework are present when students verbalise struggle during sense-making discussions, and that our Pedagogical Listening Framework is a useful tool for identifying and documenting the complex ways teachers listen when students verbalise struggle. We present three vignettes of classroom interaction during mathematical discussion that illustrate the five pedagogical listening types.

1. Introduction

When students 'think aloud' about their uncertainty and confusion, it is a form of verbalising struggle. Research in mathematics education increasingly has found that teacher support of students' verbalisation of struggle during mathematical discussion fosters the learning of mathematics with conceptual understanding (e.g. Barlow et al., 2018; Hiebert & Grouws, 2007; Warshauer, 2015; Warshauer et al., 2021). This explicit

attention to supporting students' verbalisation of struggle, though relatively new, is part of long-standing research in mathematics education that has taught us that children learn mathematics with understanding, and identify as mathematicians, when they engage in collaborative sense-making discussions (e.g. Carpenter et al., 1999; Lampert, 1990). Scholars—drawing largely on philosopher John Dewey—have defined struggle in learning as the effortful attempt to grasp something new (e.g. English, 2009; Hiebert & Grouws, 2007). When students struggle *productively*, they can be considered to be in a fruitful state of uncertainty where new ideas can emerge and be explored (English, 2013). While important research has been done that identifies *struggle* as a significant form of students' verbalised thinking during mathematical discussion, far less is known about how teachers listen to student struggle as part of how they understand and support students' sense-making in relation to the learning and constructing of mathematics.

Although listening may be thought of as an 'invisible practice', the different ways teachers listen have been shown to be 'observable' within dialogic interactions in classrooms. While listening has long been mentioned as an important part of a teacher's role when children are sharing and discussing mathematical thinking (e.g. Carpenter et al., 1999; Wood et al., 2006), a growing body of research on teacher listening points to the fact that it is not merely *that* a teacher listens, but rather *how* a teacher listens, and that what they are listening *to* and *for* is consequential for student learning (Davis, 1996, 1997; Rinaldi, 2021; Schultz, 2003). Studies in mathematics education, largely building on Davis' (1996) framework for teacher listening, have empirically identified distinctions between three types of teacher listening: 'evaluative', 'interpretive' and 'hermeneutic' (e.g. Crespo, 2000; Davis, 1996, 1997; English & Doerr, 2004; Hintz & Tyson, 2015; Johnson & Larsen, 2012). These studies have indicated that when teachers enact a particular type of listening, known as 'evaluative listening', their focus tends to be on students being able to produce a correct answer, thus foregrounding accuracy and procedural memorisation over conceptual understanding and sense-making. Evaluative listening can significantly hinder students' conceptual understanding, because with its emphasis on accuracy (and often speed), it short-circuits reflective inquiry into sense-making and stymies any uncertainties in sense-making that may be within and underneath students' strategies and solutions. Additionally, the aforementioned studies have shown that when teachers enact other types of listening, students are offered the possibility of deeply exploring a mathematical issue through their own, and their peers', sense-making, rather than of merely reproducing a surface solution. Specifically, teachers' 'interpretive listening' focuses on understanding students' processes of thinking about the concepts being learned, and 'hermeneutic listening' actively builds on the students' verbalised thinking to further collaborative thinking and problem solving (e.g. Davis, 1996). These studies of listening have identified important distinctions between types of teacher listening; however, they do not specifically address the types of listening teachers enact when students are verbalising struggle.

A recent study in mathematics education has indicated an important link between teacher listening and student verbalisation of struggle by noting that when teachers listen carefully to 'aspects of student struggle' they can make "appropriate responses to build on students' ideas and thinking" (Warshauer, 2015, p. 380). Beyond the field of mathematics education, within the growing field of philosophies of listening in education, several important conceptual distinctions in the types of listening teachers enact during

discussion have been made (e.g. Haroutunian-Gordon, 2010; Haroutunian-Gordon & Laverty, 2011; Waks, 2015). Moreover, within this field, research indicates that there is a significant—but understudied—connection between *how* teachers listen to students' struggles and students' ability to reflectively work through struggle so that their struggles may become productive (English, 2009, 2013).

In this article, we describe our exploratory study focused on the following research question:

- What types of listening do teachers enact when students are verbalising struggle during mathematical sense-making discussions?

We describe how, building on the above lines of research, we first developed and then applied a Pedagogical Listening Framework aimed at offering greater specification of the types of listening teachers enact when focused on seeking out, hearing, understanding and responding to students verbalising struggle as part of students' work to make sense of mathematics.

Our study purposefully focused within the US and Scottish contexts since, both contexts are engaged in major reforms centred on moving mathematics instruction towards inquiry and discussion, and away from direct instruction. These reforms call on teachers to facilitate student 'communication and explanation of thinking' (Curriculum for Excellence Scotland CFES, 2012, p. 2), to 'elicit' and respond to 'student thinking' (National Council of Teachers of Mathematics NCTM, 2014, p. 10), and to 'maximize mathematical discourse and student contributions' (National Council of Supervisors of Mathematics NCSM and TODOS: Mathematics for ALL, 2016, p. 5), as part of fostering just and equitable opportunities for all children to learn mathematics. This nuanced work requires teachers to listen to students' sense-making about mathematics. More specifically, in the US context, NCTM makes explicit that effective teaching entails 'support' for students' 'productive struggle' (National Council of Teachers of Mathematics (NCTM), 2014, p. 10, 48–49). Relatedly, in the Scottish reform context, teachers are called on to enhance student conceptual understanding by 'us[ing] student misconceptions and wrong answers', implying student struggle (Curriculum for Excellence Scotland CFES, 2012, p. 2). Additionally, the Scotland *Mathematics Excellence Group's* (2011) report on instructional changes needed to make said reforms effective, recommends that teachers change *how* they listen from an 'evaluative' form of listening—that assesses the correctness of student answers—to 'interpretive listening' focused on understanding student thinking (p. 10). Our study aimed to learn more about how teachers listen as part of the work of taking up these reform practices.

2. Conceptual framework

According to theories of democratic, dialogic, anti-oppressive education (Dewey, 1916/2008, 1933/2008; Freire, 1970/2000; van Manen, 1991), learning involves inquiry, discussion, reflection and collaboration. Teaching is viewed as a reflective practice that involves dialogue in the form of asking questions and listening in ways that foster learner reflection and diverse forms of participation. Deweyan and Freirean philosophies of education underscore the significance of the learner's struggle that begins when the

learner encounters a limit to his or her established knowledge or ability and becomes uncertain. This encounter lands the learner in an 'indeterminate' (Dewey) or 'limit' (Freire) situation. Such situations can serve to incite the learner's reflective analysis of why they are uncertain and how they might find their way within, through, and out of uncertainty; this process is a process of learning that can lead to transformed understanding (Dewey, 1933/2008; Freire, 1970/2000). On this basis, we view struggle as constitutive of transformative learning (English, 2013; Murdoch et al., 2021).

Dialogue is viewed as having potential to overturn traditional, oppressive forms of teaching that assign authority, knowledge and power to the teacher—what Freire (1970/2000) calls 'banking education' or Dewey (e.g. 1916/2008) calls 'traditional' or 'old' education. These theories highlight the need for a rich understanding of teacher listening as critical to dialogue that supports learner agency. For teachers to engage in forms of listening that support learner agency, they must explicitly shift away from the traditional banking model of *talking at* students, towards *listening with* students (English, 2016).

These ideas support our two foundational premises: First, that teachers actively listen *to* and *for* the untapped and emerging resources and ideas of students. Second, particular types of teacher listening support, while others hinder, students' knowledge, experiences and ideas from emerging within their processes of sense-making and co-construction of mathematical ideas; these processes include persevering through struggle towards new understanding.

2.1. Inquiry-oriented mathematics

Our study focuses on *inquiry-oriented* mathematics, which centres students' learning mathematics with conceptual understanding. Instead of learning how to reproduce processes modelled by the teacher in order to follow algorithms and compute a correct answer (i.e. the traditional teaching model known as the Initiate-Respond-Evaluate (IRE) pattern), students in inquiry-oriented mathematical discussions engage in individual and collaborative problem generating and solving that supports sense-making, understanding, and the collective generation of new understandings of mathematics. Within inquiry-oriented mathematics, we narrow upon mathematical *practices*, or ways of *doing* and *being* mathematical, in particular, the practices of seeking out, identifying and making sense of problems, and persevering in solving problems (National Council of Teachers of Mathematics NCTM, 2014). Doing mathematics involves an iterative process of reasoning and revising. Perseverance through this iterative process is an important facet of learning mathematics with sense-making. This democratic process necessarily includes moments of being uncertain, getting 'stuck', asking questions, reflecting, revising, experimenting and listening, while struggling to grasp and generate new understandings (Ball & Bass, 2008; Lakatos et al., 1976; Polya, 1981; Schoenfeld, 2002).

2.2. Struggle in learning

Dewey argues that struggle, described as 'wrestling with the conditions of [a] problem at first hand', is constitutive of all reflective learning processes (Dewey, 1916/2008, p. 167). He associates struggle with being in states of 'uncertainty,' 'doubt,' 'confusion,' 'felt difficulty' and the like (e.g. Dewey, 1916/2008, 1933/2008). We further locate struggle

in aspects of learning described in later philosophical and empirical studies that refer to cognitive-affective phenomena such as 'disequilibrium' (Piaget, 1960), 'puzzlement' (Passmore, 1967), 'confusion' (Brown, 1993), 'cognitive conflict' (Hoyles, 1985), aporetic experiences (Burbules, 2000), 'cognitive surprise' (Scheffler, 2010), 'difficulty' (Bjork & Bjork, 2011), and 'failure' (Kapur, 2015; Oser & Spychiger, 2005). Such phenomena can arise when a learner comes to an impasse in thinking (e.g. when a learner hears seemingly anomalous or contradictory information, or unclear ideas) signalling a *limit* to existing knowledge or ability, also referred to as a 'discontinuity in learning' (English, 2013). These impasse encounters are considered precursors to critical reflection on the nature of this limit or discontinuity, the determination of a problem, and any effortful sustained inquiry into a given problem—all of which are essential aspects of learning with understanding and becoming a critical thinker (e.g. Dewey, 1916/2008, 1933; English, 2013). Using English's (2013) conceptual framework for reflective and transformative learning, we identified discontinuities (indicated by expressed confusion, frustration or similar) as marking the beginning of a learner struggle.

We distinguish between a *productive*, *unproductive* and *destructive* struggle. Productive struggle is considered essential to mathematics learning (Hiebert & Grouws, 2007; National Council of Teachers of Mathematics NCTM, 2014). We used English's (2013) conceptual framework for reflective and transformative learning together with Warshauer's (2015) Productive Struggle Framework, to identify the resolution to a learner struggle. We identified a struggle as resolving *productively* if it leads to the student's (a) reflection on the limits of his or her previously established knowledge and ability (English, 2013) and (b) perseverance with the activity towards understanding while remaining cognitively engaged in a challenging task (Warshauer, 2015). We identified a struggle as resolving *unproductively* if the student (a) reached a limit to knowledge or ability and got stuck, without reflection on the reasons for being stuck; (b) solved a task that had been reduced to a procedural task with lowered cognitive demand; or, (c) discontinued working towards understanding (Warshauer, 2015). We identified a struggle resolution as *destructive* when the student was overwhelmed or discouraged in the activity (English, 2013). Destructive struggle can involve negative emotion, such as fear or sudden aversion to learning the subject matter, or learning with the teacher and/or peers in the community.

Central to our framing is English's (2013) concept of 'the in-between realm of learning' which we used to demarcate struggle as a realm of experience in which learners are in a state of a searching that is no longer settled with established knowledge, but has not yet settled into having new knowledge. This realm of struggle opens up when students lean in to the uncertainty of arriving at a limit to knowledge or ability and begin to reflectively explore—individually or collectively—their uncertainty to find a problem, understand the nature of the found problem, and define its parameters.

2.2.1 Pedagogical listening

To develop our Pedagogical Listening Framework, we brought together, in an unprecedented way, the growing bodies of theoretical and empirical research on teacher listening mentioned above, that were operating largely in isolation from one another. Specifically, we drew on research in mathematics education and contemporary theories of listening in education, fields which have separately emphasised the

importance of teacher listening for attending to students' verbalised thinking during discussion. We use the term pedagogical listening as an umbrella term that encompasses five types of listening: *empathic; supportive; educative; self-reflective; and, generative* (described below). We view these five types of listening as interdependent, and understand that, in practice, different types of listening often work together to reciprocally support one another, and one type may lead to the emergence of another (Hintz & Tyson, 2015, p. 305 & 312). More broadly, we view the category of pedagogical listening as encompassing listening types that understand the teacher's listening as an *intentional* act of attending to learners as human beings, who deserve to feel heard (Murdoch et al., 2021). On this basis, our position is that pedagogical listening contrasts with *evaluative listening*. Evaluative listening is not focused on listening to the learner as a *person*, rather solely to the content of speech; it is focused on evaluating the correctness of learners' statements according to a learner-independent preconceived standard (Davis, 1996).

Empathic listening, following the work of Waks (2008, 2010), refers to listening openly to and for the learner's own understandings, feelings and perspectives around an idea or situation. Empathic listening requires the teacher to *actively* suspend her or his own categories, judgements, perspectives, feelings, and identity, such that the learner feels heard as a human being. Empathic listening can give teachers an understanding of features of expression of students who have unexpected, creative or even deviant behaviours, features of expression that are 'filtered out' through modes of what Waks calls 'cataphatic' listening (Waks, 2008, pp. 71–72). Thereby, empathic listening can allow students to feel 'really listened to' and experience 'profound appreciation,' enhancing their self-esteem and sense of belonging (Waks, 2008, pp. 71–72). Empathic listening is a way of being particularly attuned to how the learner is making sense of her or his whole world, without trying to classify the learner's expressed perceptions or experiences into pre-defined categories. This attunement to the whole person may be particularly evident when teachers attend to student affect, including fear or distress, or when they, by way of listening, become cognisant of how students may need other avenues for expression aside from verbalisation.

Supportive listening occurs when teachers are listening to and for ways to support learners to listen to one another, so that they learn to consider and learn from perspectives other than their own (Hintz & Tyson, 2015). Supportive listening is responsive to where each student is at in his or her thinking in a particular moment and supports links between students' thinking. During a classroom discussion, supportive listening is evident when teachers actively orient students to make sense of their peers' mathematical ideas. Supportive listening serves to build an inclusive classroom community, wherein students listen to, make sense of, and build on each other's thinking (Murdoch et al., 2021).

Educative listening occurs when teachers are listening to and for diverse student struggles with new ideas or interactions, and simultaneously, for ways to support the student to transform the struggle into a 'productive struggle', that is, a pathway for self-reflection, self-activity and new understanding (English, 2009, 2013, p. 134). Educative listening is evident when a teacher is asking a learner (or learners) how they are coming to understand something, what questions they have, or what they may find confusing or difficult. Educative listening has the purpose of cultivating the grey areas of student thinking between right and wrong, thus helping them to identify what it is they are

struggling with so that they can reflectively explore the nature of their struggle and persevere, rather than simply resolve or abandon it.

Self-reflective listening, following the work of Haroutunian-Gordon (2009, 2010), refers to listening to and for unexpected, challenging responses from students in a way that initiates the teacher's reflection and, in turn, promotes shifts in the teacher's thinking, values, beliefs and practices to better support learners' reflective learning processes. This type of listening is evident when a teacher openly expresses humility, admitting the limitations of her own knowledge or ability, and actively seeks to learn from interactions with students. During self-reflective listening teachers are attuned to what they otherwise may not wish to hear, what counters their own ideas, or what may challenge the dominant discourse, but may be valuable for developing learners' thinking and sense-making. The teacher's self-reflective listening instils in students a sense that their contributions are valued. It helps build relationships between teacher and student because it makes explicit the idea that the teacher is not the all-knowing, sole arbiter of what counts as knowledge.

Generative listening occurs when teachers are listening to and for opportunities for the students' dialogue to *generate* new ideas and directions for the discussion, such that new, previously unforeseen, understandings, educational opportunities, norms and goals can emerge. This concept builds directly on Davis' (1996, 1997) conceptualisation of 'hermeneutic listening', and Yackel et al.'s (2003) idea of 'generative listening'. Such listening involves teacher and student roles being conflated as multiple perspectives are taken to explore, make sense of, and build on the ideas being discussed, such that the interactions generate something new. Generative listening is evident when a teacher leans into a learner's insight as a means for opening up previously unforeseen avenues for exploration which serve to develop new collectively established interpretations, knowledge and understandings. Through generative listening teachers attend to what is fruitfully emerging within the learners' dialogue and interactions, rather than being oriented on a fixed, prescribed endpoint.

Summary of pedagogical listening types

Listening Type	What the listening attends to during a discussion	Classroom Indicators: Examples of teacher-listener response when this type of listening is enacted
Empathic Listening	*Empathic listening* refers to listening openly to and for the learner's own understandings, feelings and perspectives around an idea or situation (Waks, 2008, 2010).	● 'I am hearing you say you need more support, am I understanding you?' ● 'You can tell me your concerns, I am hear to listen'
Supportive Listening	*Supportive listening* occurs when teachers are listening to and for ways to support learners to listen to one another, so that they learn to consider and learn from perspectives other than their own (Hintz & Tyson, 2015).	● 'Let's listen to __ tell us about his ideas' ● "Let's think about __'s solution. What questions do we have for her?"
Educative Listening	*Educative listening* occurs when teachers are listening to and for diverse student struggles with new ideas or interactions, and simultaneously, for ways to support the student to transform the struggle into productive sturggle, i.e. a pathway for student self-reflection, self-activity and new understanding (English, 2009, 2013).	● 'Let's talk about what people found challenging in this problem' ● 'You two have different answers. Let's work to understand how you each got here.'

(Continued)

Listening Type	What the listening attends to during a discussion	Classroom Indicators: Examples of teacher-listener response when this type of listening is enacted
Self-reflective Listening	*Self-reflective listening* refers to listening to and for unexpected, challenging responses from students in a way that initiates the teacher's reflection and, in turn, promotes shifts in teacher thinking, values, beliefs and practices to better support learners' reflective learning processes (Haroutunian-Gordon, 2009, 2010).	• 'You're seeing something I did not see, let's talk about it' • 'I had not thought of that approach, you are challenging me to think about this in a new way'
Generative Listening	*Generative listening* is listening to and for opportunities for the students' dialogue to generate new ideas and directions for the discussion, such that new, previously unforeseen, understandings, educational opportunities, norms and goals can emerge (Davis, 1996, 1997; Yackel et al., 2003).	• 'I'm curious about the ideas emerging in this group, as well as those from this other group, together what do they make you wonder?' • 'Building on your ideas from how you use and see this math at home, and some of the patterns we've been exploring, what insights are emerging for you all?'

3. Methods

The goal of our exploratory study is to gain insight into the types of listening teachers enact when students are verbalising struggle during mathematical sense-making discussions. Our conceptual framework informed our methodology and analysis. We first identified episodes of struggle, and then coded the data to identify types of teacher listening occurring within each episode. We sought to confirm and disconfirm our analysis by interpreting transcripts, reviewing corresponding field notes, videorecorded lesson segments, and focal teachers' narrations and observations gathered through recall interviews and written journals. Throughout the data collection and analysis phases, the research team held weekly meetings (via video conferencing) to define, discuss, review, and select episodes of student struggle and the coding of listening types.

3.1. Contexts

Within our two contexts, the United States and Scotland (UK), we purposefully selected one school in three different regions (one elementary school in the Southwest and one in the Northwest, USA, and one primary school in Scotland, UK) for the following two central reasons: 1) the schools were committed to supporting teachers' implementation of inquiry-oriented mathematics classrooms; 2) the schools had a student population representative of the larger regional context of the school. In the Southwest, USA, the elementary school was a public charter school that serves a diverse population of students, who come to the school from all over the city, with approximately 70% of students LatinX, 25% White, under 5% African American, and under 5% Native American. In the Northwest, USA, the elementary school was a public school serving a tribal reservation community with a predominantly Native American student body. In Scotland, the primary school we worked with was a non-denominational, mainstream school located in a small urban setting with the majority of students from a White-European background and 10% from a minority ethnic background.

3.2. Participants

In order to learn about the teachers' listening practices, we needed to identify which teachers engaged in mathematical sense-making discussions and to what extent they attended to student struggle during these discussions. At each school, we conducted a survey of all teachers. The survey focused on the methods they used when teaching mathematics, with specific attention to their approach to responding to students' expressed confusion, error and the like. Using this survey data, we then selected four to five teachers in each of the three schools to observe during one mathematics lesson. As an observation protocol, we used 'The Democracy Empowerment Rubric' (Mardell & Hanna, 2016), which provides criteria for the evaluation of classroom talk as part of democratic, collaborative knowledge building; we adapted this protocol by adding corresponding criteria for the evaluation of classroom listening. We used the combined survey data and observation data to identify a subset of teachers, three in each school (equalling nine focal teachers in total in the study) who had (a) experience with leading mathematical discussions, (b) self-assessed mid-high level of mathematical knowledge, (c) an explicit interest in developing their inquiry-oriented approaches to teaching mathematics, and, (d) an observed ability to attend to students' verbalisations of struggle through listening.

   We worked with school administrators and teachers to communicate with students and their families about the study and received consent and assent forms from every teacher and child, and the children's parents or guardians. One child opted out after consenting, and their wishes were respected. All teacher and student participants were assigned pseudonyms. Of our nine focal teachers, there were seven female and two male teachers, and all nine teachers identified as White, native English speakers. Similarly, as researchers, we identify as White, native English speakers. Going into schools with diverse populations of students, including students from minoritized backgrounds, we made efforts to enter each classroom, and engage in analysis, with cultural humility and curiosity (Yeager & Bauer-Wu, 2013) and ever-evolving critical consciousness (Freire, 1970/2000).

3.3. Data collection

The selected nine focal teachers each engaged in a preliminary semi-structured interview (approximately 40 minutes in length) focused on exploring their attitudes and beliefs around: listening; student struggle; mathematics teaching; and, mathematical thinking and learning. Following this, in each focal teacher's classroom, we observed and video-recorded five consecutive mathematics lessons (each ranging from 45 to 75 minutes in length) over the course of one week. Two cameras were used to capture teacher-student and student-student interactions. While videorecording, each researcher observed the lessons and took field notes, noting interactions during discussions and potential indicators of student struggle. In addition, to gain further insight into teacher thinking and practice beyond the observed lessons, we asked each teacher to keep a mathematics teaching journal to document their reflections on their teaching and their students' learning. Each teacher made one journal entry after each mathematics lesson for three weeks (the week before, the week during, and the week following our classroom

observations and videorecording). We also collected teacher lesson plans and student work to help us understand the instructional goals and student engagement with the activities we were observing. Finally, we conducted semi-structured stimulated recall interviews (Gass & Mackey, 2000) with each focal teacher on selected video-recorded lesson segments (identified through our analysis as 'struggle episodes,' detailed below) to gain insight into teachers' intentions and into what was surfacing for them *in-the-moment* (e.g. met or failed expectations; concerns; challenges; new insights) when hearing and responding to students' verbalising struggle during discussions. The recall interviews were video recorded (approximately 60 minutes in length) and occurred within a week following observation and videorecording in the classroom.

3.4. Analysis procedures

The aim of our analysis was to identify, examine and describe the types of listening teachers enact when students were verbalising struggle. We triangulated patterns of teacher listening within what we first identified as student 'struggle episodes'. We used our Pedagogical Listening Framework to inform our analysis of types of teacher listening. We recognised that the list of pedagogical listening types in our framework was not exhaustive, and accounted for this in our analysis procedures. Through interaction analysis we assessed our Pedagogical Listening Framework's empirical validity and modified it accordingly using an iterative process between the framework, the data and analysis (Miles & Huberman, 1994).

3.4.1. Identifying struggle episodes

Identifying struggle episodes was a multiphase process supported by English's (2013) conceptual framework for reflective and transformative learning and by Warshauer's (2015) Productive Struggle Framework, described above. For phase one, during observation and videorecording of the mathematics lessons in a focal classroom, researchers noted (in field notes) student verbalisation of struggle during sense-making discussions that could potentially constitute a struggle episode. Phase two focused on confirming or disconfirming initial identification of broadly construed struggle episodes (from phase one) by reviewing researcher field notes alongside the videorecorded lesson at the end of each classroom observation day. At this point, the lead researcher working within the focal school identified all the roughly estimated struggle episodes that had occurred in that day's observed lesson. Struggle episodes were observed, noted and recorded in each of the nine focal classrooms. In total, 59 preliminary struggle episodes were initially identified across the nine focal classrooms; the episodes varied in length and in number per teacher (number of episodes per teacher: 10, 8, 8, 7, 7, 6, 5, 5, 3).

In a third phase, at the end of the week of videorecording in a focal classroom, the researchers in the given focal context narrowed on a representative sample of preliminary struggle episodes (identified in phase two) from the video data of each focal classroom that were then shared for discussion with the focal teacher during the recall interview. As much as possible, we included episodes in our selection that involved one or more students struggling with complex conceptual understanding of the mathematics, and also that illuminated the social-emotional entailments of such struggles. The teacher's recall interviews and journal notes helped us learn more about his or her thinking during

the selected episodes of student struggle, which informed our later analysis of teacher listening.

In phase four, informed by the cumulative insights gathered during phases one through three, the researchers worked collaboratively and iteratively across the three contexts to determine the video segment that would constitute a *struggle episode* for analysis by the team. First, we agreed on a starting point of the struggle as the point at which there was an identifiable 'discontinuity in learning', i.e. the student(s) verbalised a limit to knowledge, ability or understanding (e.g. 'I'm confused'); and, we agreed an ending point as the point at which the struggle resolved as either 'productive', 'unproductive', or 'destructive'. To constitute an 'episode', the student verbalisation of struggle needed to be taken up by the teacher. In addition, as the boundaries for an episode, we included parts of the teacher-student interactions that provided context to the student struggle, in that they showed what content and/or set of interactions led to the student verbalisation of struggle and the struggle resolution. As a team, we identified three struggle episodes from each focal classroom (equalling a total of 27 struggle episodes across the nine focal classrooms), which appeared to be the most viable for sustained analysis of teacher listening types.

3.4.2. Identifying types of teacher listening during struggle episodes

Using interaction analysis (Jordan & Henderson, 1995), we analysed the selected student struggle episodes to identify types of teacher listening occurring during student verbalisation of struggle. We coded for listening types by examining how student talk was responded to by the teacher in each episode. Instead of coding line by line, we identified and coded each interaction set (what was said by a student or teacher, and how it was responded to by a student or teacher) in the episode.

Using our Pedagogical Listening Framework, each researcher coded data separately in Dedoose, a qualitative analysis software. We also had a category for 'evaluative listening' and 'other type of listening' to account for types not identified within our pedaogical listening framework. Each struggle episode was coded for teacher listening types by at least two researchers, one working within the focal school, and one outside of that school context. To establish intercoder reliability, we compared our coding, discussing inconsistencies, and through this iterative process sought to determine what types of teacher listening were evident when student struggle was verbalised. We conducted open coding to confirm or disconfirm our emergent findings and to identify patterns. Using constant comparison (Glaser & Strauss, 1965) across the different types of data we collected, we triangulated data and developed vignettes to illustrate types of pedagogical listening.

4. Findings

Overall, our analysis revealed that a constellation of pedagogical listening types from our framework could be consistently documented in seven of the nine focal teachers' classrooms when students verbalised struggle during mathematical sense-making discussions. While each of the seven teachers enacted different subsets of the pedagogical listening types, all the types were reflected in the data. For the two teachers where pedagogical listening types were not consistently documented, we identified that the predominant mode of listening during student verbalisation of struggle was

'evaluative' (e.g. Crespo, 2000; Davis, 1996). [As an example, we identified the focal teacher's listening as 'evaluative' in the following excerpt: Teacher: 'So nine, ninety plus ten is what?' Student: 'Ten.' Teacher: 'It isn't. Ninety plus ten is not ten. Ninety plus ten?'] The struggle episodes in these two classrooms were predominantly categorised as resolving in 'unproductive', or in one case 'destructive' struggle. Each of the two focal teachers from these two classrooms shared separately in their recall interviews that (despite their initial interest in supporting sense-making discussions) during the time period between our pre-interview and our filming, their focus had shifted onto getting through the prescribed lessons and getting students to right answers due to curricular expectations and pressure to increase students' proficiency on standardised tests.

We selected the following three vignettes, one from each focal school, as representative of the types of pedagogical listening, and associated practices, that could be documented, to varying degrees, in seven of the nine focal teachers' classrooms when *struggle* was expressed verbally by students during sense-making discussions. We deemed these three cases 'revelatory' because they contain salient examples of the listening type(s) present (Engle et al., 2014). Each of the vignettes were categorised as resolving in productive struggle. The vignettes are not meant to indicate that the listening types happen in isolation, and the status of the vignette is not to be treated exhaustively.

Vignette 1: supportive listening

The struggle episode: In Mr. Elliot's class (with students around age 7), we observed Ky express struggle in the form of 'why' questions around how to understand his classmate Layla's representation of her thinking.

Mr. Elliott and his students are engaging in sense-making about a double-digit addition problem (31 + 29). As students work independently, many use tools such as paper and pencil, cubes, sticks and straws, and a place value chart on the wall. Mr. Elliott walks around the classroom kneeling beside students to ask them about their ideas and how they are thinking about the problem. He asks them to record their thinking so that their classmates can understand their strategy, stating 'show your thinking in a way that another person could understand'. *After students have time to think individually about the word problem, Mr. Elliott brings the whole group back together for a strategy sharing discussion*:

> Mr. Elliott: We're going to start by hearing and understanding Layla's thinking. Before we hear her ideas, let's take a few moments to study her strategy and see if we can make sense of, or have questions about, her thinking. As you study her strategy and math drawing, what is our job?
>
> Students (*a few students offer-up ideas and call them out*): 'to understand her work', 'to listen', and 'to ask questions about her thinking'.

Mr. Elliott gives the students time to study Layla's work before discussing their observations and questions with a partner, and then, as a whole class.

Mr. Elliott: Ask questions if you're not sure what another mathematician is sharing. Ky, you look like you have a question. What do you want to ask Layla?

In this passage, Mr. Elliot's statements and question indicate the importance of supporting young mathematicians to ask each other questions to understand another's thinking. Mr. Elliot's question to Ky creates an opening for Ky to share what he is working to understand.

Ky: Yeah. Why did she write, 'because 9 is close to 10?'

Mr. Elliott: Well, why don't you ask *her* that?

In this moment, Mr. Elliott orients Ky towards Layla, and supports student to student discourse in sense-making discussion.

Layla (*jumping in to respond to Ky's question*): I tried to erase it, but then. . .

Mateo: You ran out of time?

Layla: Yeah.

Ky: But what is it for?

Layla: Because I was gonna say 'cause first I didn't really draw the [inaudible] for the ones. That's why I drew that, '9 is closer to 10' so that they knew that those were ones—they weren't like a number sentence or something else.

Ky: Okay.

They end the discussion of Ky's question.

We call the teacher's primary mode of attending in this episode *supportive listening*, because the focus of the listening was around attending to how to support students to listen to each other, so they learn to consider, and learn from, perspectives other than their own. Mr. Elliott set up the discussion in a way that created opportunities for different students' mathematical strategies to be heard by one another, in particular, when he states that the aim is to '*hear and understand*' Layla's thinking, and then directs students to '*take a few moments to study her strategy and see if we can make sense of, or have questions about, her thinking*'.

When Mr. Elliot asks Ky to verbalise his uncertainty around Layla's thinking ('*what do you want to ask Layla?*'), we consider Mr. Elliot to shift towards active engagement in supportive listening. Mr. Elliot's supportive listening continues and is reinforced when he states '*why don't you ask her that?*,' thereby removing himself—as the teacher—from being the mediator of the interaction. Instead, he encourages Ky and Layla to engage directly with each other in the practice of listening to and responding to one another's ideas and reasoning. More broadly, we view Mr. Elliot's listening as oriented on the process of thinking collectively with and within a community as a way of understanding the complex thinking of another human being, and not on getting students to simply state the right answer.

Vignette 2: educative listening

The struggle episode: In Ms. Moreau's class (with students around age 11), we observed Ray articulating his struggle as 'confusion' in relation to understanding how his small group had solved a problem of finding weight using a two-pan balance.

Ms. Moreau engages students in a targeted discussion (Kazemi & Hintz, 2014) focused on sense-making through reflection on mathematical difficulties. The day before the discussion, students had engaged in group work at hands-on activity stations designed around problems of measuring the weight of various objects. The group work was followed by a reflective writing task, in which each student was asked to document his or her experiences with the measuring activities and to pay special attention to what went awry and how their group responded when things did not work. At the start of the targeted discussion (excerpted below), Ms. Moreau asks students to share their written reflections, stating the focus as follows: explain how 'you tried doing something first but then it didn't work, so then you tried to do something else.' In this short excerpt, they discuss one of the previous day's activity stations, in which groups had the task of using a 2-pan balance, a twenty-gram weight, and a bag of cubes to determine how to weigh out fifty grams of cubes:

> Ms. Moreau: So, I think most of us got that but what I'm interested in is did somebody do something differently and they were, and then they thought, 'wait a minute this is just not working. We're not doing this right'. […] Ray? […]

Ms. Moreau's question to the class around reflecting on what was 'not working,' indicates the importance of creating opportunities to reflect on, and listen to, moments of uncertainty, confusion or self-identified error during sense-making.

> Ray: Well, first of all, erm, we put the twenty grams on. And then we measured, and we, and we put on the counters for the other side. And then we, then we found out how we counted them to find out [inaudible]. And then after that we had to, erm, me and Sarah were confused what to do. And then Lenny told us like, eh, what we should do. And so, I think it was when we had to times it by eight and like divide it by four, or something.
>
> Ms. Moreau: Was it? Think again, think again. Do you? […] Well let's talk it through. Twenty, erm, grams, when you worked out twenty grams Ray, how many cubes was it? Can you remember?

When Ms. Moreau asks Ray to '*think again*' and verbalise his thinking process, she opens up an opportunity for him to reflectively explore the nature of his confusion.

> Ray (*sounding hesitant*): Erm, I think it was about eight.
>
> Ms. Moreau: It was, wasn't it, eight. So, twenty grams were eight. So, Ray is maybe still not a hundred percent sure why then what, what happened next? What, why did you, how did you then work it out? So, who can explain to Ray? And maybe for anybody else that's sitting thinking, 'I never really fully understood what was going on at that 2-pan balance'. Who thinks they can give a good explanation of that? Right Mila, go for it …

Here, when Ms. Moreau invites other students to share their thinking with Ray, she is supporting collective sense-making around an individual classmate's struggle as a means of transformation of struggle towards new understandings.

Mila: Erm, well once we measured out what erm twenty grams was [inaudible] counter. And then we doubled it to get forty which was . . .

Ms. Moreau: Why did you double it though?

Mila: (*sounding hesitant, at times pausing*): because we knew it was fifty. And so, we knew that it would get us closer to fifty. We doubled it to get forty. And then we thought about what ten was. . .and ten, we had twenty erm cubes and it was four more to get us to erm fifty.

Ms. Moreau: So, good, and so where did the four cubes come from? Lenny? [. . .]

Lenny: [. . .] we needed another four because erm half of eight is four, so we had to do half of twenty, so four cubes is ten.

They end the discussion with Ray verbally confirming a better grasp.

We call the teacher's primary mode of attending in this episode *educative listening*, because the focus of the listening was around attending to precise moments of students' confusion, uncertainty and the like, associated with making effort to understand the mathematics (i.e. struggling); in addition, the teacher was simultaneously attending to how to support students to reflect on the nature of their struggle and persevere towards new understanding. We consider Ms. Moreau to be setting up the discussion to listen *for* mathematical struggle, because she asked students to verbalise 'what didn't work,' that is, what had gone awry mathematically during the hands-on activities.

This form of sense-making discussion provided students with the opportunity to express their 'negative knowledge,' i.e. knowledge of what is wrong and should be avoided (Gartmeier et al., 2008), while providing the teacher, Ms. Moreau, the opportunity to listen to what students believe they do not yet understand and are still struggling to understand. When Ms. Moreau asks Ray to '*think again*' and talk about the process of his mathematical thinking in relation to the number of cubes needed to get twenty grams, we consider her to become actively engaged in educative listening, in listening directly to Ray's struggle. Although Ray gives the right answer of eight, Ms. Moreau appears to tune into Ray's uncertainty around why the answer was eight. In that moment, we consider her deepening her enactment of *educative listening* (she asks him, '*why then . . .?*'), since she remains focused on attending to how to help Ray explore the nature of his struggle. When she states that there may be others, aside from Ray, who did not understand the two-pan balance task, (she states, '*there may be others sitting thinking, "I never really fully understood what was going on at that 2-pan balance"*') and invites students to help one another by sharing their thinking process around the task, we view Ms. Moreau as still actively engaged in educative listening. In this moment, she is listening for ways to support students to not merely remain stuck (thus, landing in an unproductive struggle), nor to be satisfied with knowing the right answer, but rather to reflectively transform their struggle by deepening understanding through collective troubleshooting and revising of strategies.

Vignette 3: empathic listening, self-reflective listening and generative listening

The struggle episode: In Mr. Lewis' class (with students around age 9), we observed Sam express struggle in the form of emotional distress after getting tripped up trying to solve a mixed fraction problem at the board during a whole-class discussion.

Mr. Lewis and his students are engaging in a strategy sharing discussion about fractions. During the discussion, Mr. Lewis invites several different students to work through a problem on the board, each student coming up one at a time and building on the written work of the previous student. Sam volunteered to take over solving a problem that a classmate had begun working out on the board. As Sam is trying to figure out the next steps, his face becomes very expressive (at times, appearing to crack a smile) as he tried to think what 'W' (whole number) meant and classmates start to laugh. Mr. Lewis encourages Sam to stay focused, take a pause, and asks Sam if he wants more time to think or wants help from a 'coach' (a term used in Mr. Lewis' class to refer to a peer who can help you when you are struggling). Sam asks for a coach, chooses a classmate to take over solving the problem and sits down. After students leave the whole-class discussion on the carpet and begin to work individually at tables, Mr. Lewis notices Sam has his head down and discreetly calls him over to the teacher desk:

Vignette 3–Part 1: Empathic Listening and Self-reflective Listening

Mr. Lewis (*to Sam*): what's going on?

Sam (*crying*): Everybody was laughing at me, you know […] I wasn't trying to make them laugh.

Mr. Lewis: would you feel more comfortable chatting outside?

Sam nods and they move outside the classroom to speak one on one.
    When Mr. Lewis asks Sam what is wrong and then sits with Sam outside the classroom, he is providing the space to hear and understand Sam's feelings.

Sam (*crying*): I was trying to do math, people [inaudible] and laughed at me

Mr. Lewis encourages Sam to take some time and take some deep breaths. Sam, still trying to catch his breath, continues to be upset, while Mr. Lewis is silent before offering Sam some insight.

Mr. Lewis: It's never a problem, coming up to the board and forgetting something, but I think what [the students] were reacting to, is the same thing I saw. Like when you were coming up there, like you were a different person than you normally are, and they were reacting to that. […] Now I think that when they laughed, you kind of lost your balance a little bit and *did* forget what 'W' was. But by that point, they didn't recognize that you weren't being goofy anymore, they didn't see that you were in a place where you had forgotten.

Mr. Lewis suggests that they talk to the whole class about what happened, but at first Sam does not want to.

Mr. Lewis (*explaining his reasoning for wanting to speak with the whole class*): And I just want to have them understand what you're going through right now as a team. And I think that by them understanding that, you'll see them be more compassionate toward you. Can we do it? Let's give it a try …

Sam: Okay

Back in the classroom, all students are asked to gather on the carpet again:

Mr. Lewis (*to the whole class*): So, the reason we're coming back to the carpet is because we have somebody that's upset right? And we have to talk it out... And, the thing about math is that you're not always going to be successful [...] Now what I saw is when he came up to the board he was being goofy, and I saw you guys respond to that energy right? [...] But I think through that exchange [...] he lost his balance a little bit and truthfully didn't remember what the 'W' stood for. However, we, as his audience, we couldn't see that. We couldn't see that transition happen – from him demonstrating his knowledge and his uniqueness to 'wait a minute, I'm not sure anymore'. So when we continued laughing, then what do you think his feeling was inside? Like, it's one thing if I am coming up here and being goofy. We're all human, we do this from time to time. But let's say ... all of a sudden I lose my math balance a little [...] and I look at my audience and everybody's still kind of laughing a little bit. Not your fault, we just couldn't see that transition happen – I didn't see it either. I interpreted, when [Sam] asked for 'coach', as he was being goofy. Do you think he was in a safe place anymore?

Students: [*verbal gesture indicating 'no'*]

In Part 1 of this vignette, we consider two modes of attending to be present: *empathic listening* and *self-reflective listening*. In providing space for Sam to express his emotion and his feeling of being laughed at, Mr. Lewis' focus was on listening to Sam's feelings which indicated to us that his mode of attending is *empathic listening*. We consider this listening empathic as it appeared that Mr. Lewis was able to attend closely to Sam as a human being with valid feelings, aims and expectations. Although Mr. Lewis explained to Sam the reasons the other students may have been laughing, the dialogue indicates that Mr. Lewis came to suspend his own judgement and tried to understand how Sam was making sense of his own world. We consider this suspension of judgement to be revealed indirectly when Mr. Lewis indicates to Sam that the aim of talking to the whole class was for the other students to understand Sam's emotion (what he was '*going through*') so that they can come to have sympathy for him (become '*more compassionate*' towards him). These statements suggest that Mr. Lewis had become more understanding of Sam's feelings and perspectives by having listened to how Sam was genuinely hurt, and thus we view that he was *listening empathically* to come to that stance.

We also view Mr. Lewis to be enacting *self-reflective listening*, in that he was listening to an unexpected response (from Sam) that challenged Mr. Lewis' previously held perspectives. This, in turn, led Mr. Lewis to reflect in a way that shifted his thinking and practice. Mr. Lewis' response to Sam, when he says ('*I think what they were reacting to, is the same thing I saw ... you were a different person than you normally are*') indicates that Mr. Lewis had held a particular view of Sam and Sam's behaviour prior to their one-to-one talk. Mr. Lewis' view was being challenged by Sam's emotional explanation ('*I was trying to do math*'). Later in part 1 of the vignette, when Mr. Lewis is speaking to the whole class, his statement, '*I didn't see it either. I interpreted, when [Sam] asked for "coach", as he was being goofy*' indicates that he recognises he was wrong about his interpretation of Sam. We consider these combined statements of Mr. Lewis to indicate that he reflected on and shifted his thinking about Sam. Further, Mr. Lewis' use of metaphors to help the class understand Sam's emotional experience (when Mr. Lewis refers to '*losing your math balance*' and '*not being in a safe place*') indicate that Mr. Lewis had come to a significantly revised understanding of Sam on account of his one-to-one conversation with Sam. Specifically, he no longer sees Sam as a student who was just '*being goofy*', but as a person who is vulnerable (who can '*lose [their] math balance*' and

not feel like they are in '*a safe place*') when actively making effort, i.e. struggling, to do mathematics. On this basis, we view that during the one-to-one conversation with Sam, Mr. Lewis was actively engaged in *self-reflective listening*.

Vignette 3–Part 2: Generative Listening

Mr. Lewis: [. . .] And so the reason we have to come and talk about it is because it has to be a safe place for everybody in the room, because . . . we have to make sure that when [Sam] goes back to the board, he's willing to take a chance [. . .] So how can we support him? Let's start with me. How can I support him? Brutal honesty, go ahead [. . .] what mistakes do you think I made in talking with Sam?

After one student's broad suggestion that the teacher should help Sam, another suggestion comes:

Laura: Maybe you could have umm, maybe ask him if he really doesn't know it or . . . if he's being goofy.

Mr. Lewis (*to Laura*): I think that's a great suggestion,

Mr. Lewis (*to Sam*): If we see this happen again, what I'm going to do is come up to you, and I'm just going to ask you silently, 'Are we in a good place?', Okay? And that will be just what we say to each other.

Sam nods, confirming that he agrees

Mr. Lewis (*to the class*): And that will be the same for everybody else.

In Part 2 of the vignette, Mr. Lewis creates an opportunity for the students to generate ideas around how Mr. Lewis could change his behaviour when facilitating discussions (he asks the class, '*How can I support him?*'). At that moment, we view Mr. Lewis' primary mode of attending to shift to *generative listening*, in that he appeared to be listening to and for ways that the students' ideas could generate ideas for creating new norms around teacher-student interaction in order to maintain a sense of the classroom as a safe space to risk sharing struggle. Rather than deciding for himself (as the teacher) how to modify his responses to students, Mr. Lewis decided to engage the class in a group thinking activity on the necessary changes he needed to make in order to better support Sam, and the other students, going forward. Mr. Lewis' response to Laura, showing how he was taking up her suggestion, further indicated his engagement in *generative listening* as an open form of listening that allows new ideas to emerge from collective thinking. On the basis of these students coming together as a community to address one student's struggle, a new norm for future teacher-student interactions, which applied to all students, was established (namely, that if the teacher is not understanding a student's behaviour, the teacher will quietly ask the student '*Are we in a good place?*').

Analysing the vignette as a whole, we view this an example of how listening is complex in that different types of listening work together, not in isolation as discrete units, within one dialogic interaction, and that boundaries between categories are fluid rather than rigid (Davis, 1997; Hintz & Tyson, 2015). Mr. Lewis' listening and responses to his students indicate he made a deep shift in his beliefs about students, involving his recognition that he may not have been aware of the vulnerable work students do when asked to publicly share their emergent thinking and struggle.

5. Discussion

In this study, we focused on defining, identifying, and describing the types of listening teachers enact when students verbalise struggle as an expression of students' effort to make sense of mathematics. We developed the Pedagogical Listening Framework, which differentiates five types of teacher listening (empathic, supportive, educative, self-reflective, and generative). Our study brought together these five types of teacher listening and identified them in the context of mathematics classrooms when students were verbalising struggle during mathematical sense-making discussions. We found all five types to be empirically salient.

Our research advances understandings of pedagogy that fosters more equitable mathematics classrooms (Carpenter et al., 1999; Turner & Celedón-Pattichis, 2011) by indicating that *how* teachers listen to their students' verbalised struggle during sense-making discussions matters. In particular, in our study, it appeared that, in moments of student struggle, the teacher's pedagogical listening provided students with opportunities to explore their 'learner's rights'—the rights 'to be confused', 'to claim mistakes', 'to speak, listen and be heard', 'to write, do and represent only what makes sense' (Kalinec-Craig, 2017, p. 1)—in ways they otherwise would not have had. The third vignette, in particular, highlights the complex work of listening involved in maintaining what is considered the foundational right of the learner: 'the right to feel safe and have their ideas respected' (Kalinec-Craig, 2017, p. 9; see also Hintz et al., 2018). Thus, we believe that pedagogical listening may be a significant factor in supporting teachers' development of listening as a rehumanising practice (Aikenhead, 2017; Gutiérrez, 2018), which ensures that children are fully seen, heard and valued (Civil, 2014) and interrupts the dehumanising habits of schooling (Freire, 1970/2000).

Prior theoretical work has focused on clarifying conceptual distinctions between individual types of teacher listening that centre students' thinking, but largely without attention to how different types may work in concert with one another (e.g. Haroutunian-Gordon & Laverty, 2011). Prior empirical work in mathematics teaching (especially Davis, 1996, 1997 and studies drawing on his work) have identified types of teacher listening occurring in the context of mathematical discussions, but without particular attention to how teachers listen when students are verbalising struggle during discussions. Beyond this, prior empirical work on teacher response to student struggle (Warshauer, 2015) and theoretical work on teacher listening (English, 2013) have indicated the importance of attending to how teachers listen to student struggle, without investigating how such listening takes place in practice. Our study extends each of these areas of prior research by identifying and describing five types of pedagogical listening that teachers enact when students verbalise struggle during mathematical sense-making discussions. Additional types may be identified by future research.

Through our analysis, we triangulated that pedagogical listening describes the types of listening teachers engage in when listening *to*, *for* and *with* a learner expressing struggle. It appeared that teachers actively engaged in such listening *not for the purpose* of measuring the child's thinking against an external standard (as evaluative listening does), but rather, *for the purpose* of learning about the child and, simultaneously, about the environment the child needs to support his or her process of building meaningful connections to self, others, and to the subject matter (in this case, mathematics). The overarching question that seemed to pervade the teacher's curiosity, reflective thinking

and decision-making when listening pedagogically appeared to be, '*Do the conditions of the classroom environment support the learner to safely express struggle and fully engage in reflective, collaborative inquiry into struggle*?'

In our study, teachers' practices of pedagogical listening appeared to be supported by a cultivated disposition to listen to all children's expression of struggle. We call this a 'listening stance' (Schultz, 2003). Extending Schultz's (2003) definition, we consider a listening stance to mean being prepared to listen to, respond responsibly to, and *be with*, all learners as their thinking and struggle emerges.

The current study explored the listening approaches and practices of a small sample of teachers with an interest in inquiry-oriented mathematics teaching. While we considered diversity of contexts, taking account of practicing teachers approaches to reforms within two separate countries (US and Scotland), and three separate schools with differing populations of students, the scope of the study was limited in racial and linguistic diversity of the focal teachers. In addition, the scope of the study was limited in racial and linguistic diversity of the researchers. Future work should attend to the enactment of listening by a more racially and linguistically diverse range of educators and researchers to support deeper understanding of the theory and practice of pedagogical listening.

Our study illuminates a need for more research around the relationships among pedagogical listening types (Davis, 1996; Hintz & Tyson, 2015). For example, why might some types of listening occur more than others? Is the occurrence of some listening types required for others to occur? Indeed, more research is needed to learn if any, or all, of the pedagogical listening types are necessary for fostering student struggle to resolve as 'productive'; this is particularly important, since productive struggle has been found to be a vital component of students' learning with conceptual understanding in mathematics, and beyond (e.g. Hiebert & Grouws, 2007; NCTM, 2014; Warshauer, 2015; Warshauer et al., 2021).

Important questions emerged from our study around students' experiences of and perspectives on being listened to when verbalising struggle: Do students feel valued when their struggle is being listened to pedagogically by a teacher, and if so, how? In classrooms where teachers are listening pedagogically to student struggle, do students feel they are developing identities that help them see themselves as the mathematicians that they are (Aguirre et al., 2012; Wortham, 2010)?, and Are students more empowered to co-construct socio-mathematical norms (Yackel & Cobb, 1996) around the importance of expressing struggle (and associated experiences of uncertainty, confusion, felt difficulty) as part of the process of constructing mathematics? Or, how and why do students who choose to be silent or who do not want to express their struggle verbally experience a teacher's pedagogical listening (Hintz, 2011; Schultz, 2010; Tyson et al., 2022)? Do any of the pedagogical listening types help describe the listening students engage in when hearing their classmates verbalise struggle (Aljarrah & Towers, 2022) and share their 'rough draft thinking' (Jansen, 2020)? We view these as areas for our future research.

Our study brings to light a need for further research around the contexts which hinder teachers from engaging in pedagogical listening. For instance, historically, mathematics in schools has tended to pose tasks and problems with an emphasis on one solution path that leads to the 'right' answer, laying the groundwork for evaluative listening to take a deep hold in mathematics teaching and learning. In addition, testing regimes and accountability measures, which do not measure

learning, but measure correctness according to a pre-defined standard (Boaler, 2003), put pressure on teachers to get students to produce right answers. There is indication in our findings that this may give teachers limited opportunities to move beyond evaluative listening in mathematics teaching.

We believe that as teachers learn to create opportunities for *all* students to productively struggle (Lynch et al., 2018), teacher education and professionalisation must support teachers in cultivating complex modes of listening that we have brought together under the concept of pedagogical listening. Our study indicates the ways in which the teaching profession can be considered a 'listening' profession. Specifically, it points to the need to know more about how *learning to teach* in ways that centre student sense-making and struggle may be linked to *learning to listen* pedagogically. Our framework can contribute to teacher education and professionalisation, as a means of supporting new and experienced teachers to reflect on the ways they may be enacting listening in how they support student conceptual learning, as well as how they ensure children feel heard. Our findings indicate that the Pedagogical Listening Framework is a useful tool for identifying the complex ways that a teacher listens as students verbalise struggle while making sense of mathematics—thereby making mathematics their own.

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ORCID

Andrea R. English http://orcid.org/0000-0002-1351-0507
Kersti Tyson http://orcid.org/0000-0002-5649-0518

Ethics statement

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