Rational expectations and optimal monetary policy: estimates for Brazil and the U.S.

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RESUMO

A base teórica deste trabalho é o modelo macroeconômico de Taylor com contratos salariais justapostos, o qual permite estimar equações de preço e demanda agregada sob a hipótese de expectativas racionais. Utilizando o método generalizado dos momentos, o modelo é aplicado para as economias do Brasil e do Estados Unidos da América, para o período de 1973 a 1985. Então, a regra de política monetária ótima de Taylor é aplicada para as duas economias com o intuito de minimizar uma função de perdas advindas de inflação e desemprego, através da qual deriva-se uma curva de Phillips de segunda ordem, mostrando as diferentes variâncias de inflação e desemprego que são ótimas, dependendo dos pesos que cada variância representa nesta função de perdas. Os principais resultados são: existe uma interrelação entre as variabilidades da inflação e do produto; e, em ambas economias, as políticas monetárias não foram ótimas no período analisado.

Palavras-chave: curva de Phillips de segunda ordem, expectativas racionais, método generalizado dos momentos, política monetária ótima.

ABSTRACT

This paper uses Taylor's macroeconomic model with rational agents. The model is designed to allow the researcher to estimate aggregate demand and price equations under rational expectations and some inflexibility in wage adjustment arising from contracts. The generalized method of moments is applied to estimate the model for the Brazilian and U.S. economies in the period 1973-85. Government monetary policy functions based on losses arising from inflation and unemployment are then used to construct a second-order Phillips curve, which shows the different variances of inflation and unemployment that are optimal depending on loss function weights. The main conclusions are that there is a tradeoff between the variabilities of inflation and output in both economies, and that monetary policies were not optimal in either economy during the period.

Key words: second-order Phillips curve, rational expectations, generalized methods of moments, optimal monetary policy.

1. Introduction

The aim of this paper is to present and estimate a macroeconomic model that shows the relationship among output, money and inflation. The model uses the hypothesis of rational expectations. According to this hypothesis, it is possible to select macroeconomic policies that capture the reactions of agents to expectations about economic policy changes. The model is applied here to the economies of Brazil and the United States for the period 1973-85. The period was chosen because it excludes the fluctuations caused by Brazilian economic plans after 1985. The main aim is to analyze the reaction coefficients in accordance with Taylor's monetary policy rule and use the resulting standard deviations in inflation and output to build a second-order Phillips curve.

The paper comprises seven sections. Section 2 presents a short review of the literature. Section 3 discusses a specific macroeconomic model including uncertainty and rational expectations. The model is estimated in Section 4, which summarizes the results for Brazil and the U.S. Section 5 then deals with the question of an optimal policy to target levels of inflation and unemployment. Section 6 presents an empirical analysis of this monetary policy. The last section contains conclusions.

2. Review of the literature

This section outlines some of the important studies on the key issues relating to the model analyzed here: the Phillips curve and rational expectations. In the coming sections we examine the way in which an optimal monetary policy rule can minimize the loss from both components of a Phillips curve. The existence of rational expectations means that there is no first-order Phillips curve in the long run, only a second-order Phillips curve.

On the issue of employment and inflation, Phelps et al. (1970) develop the microeconomic foundations of employment and inflation theory. Product wage rates do not rise markedly and unerringly whenever unemployment rises. Nominal wages and prices are imperfectly flexible. Equally important in this study is its discussion of the Phillips curve relationship between employment and the rate of wage (or price) change. This discussion originated with Phillips (1958), who focused on the tradeoff between unemployment and wage changes. He showed the persistence of a nonlinear tradeoff in the United Kingdom for a century. Later on, many authors have followed up these points and developed the discussion further.

Lucas (1972, 1973) outlined a theory of monetary misperceptions, whose main proposition is that the relationship between prices and output, and hence employment, does not result from sluggish price and wage settings but from the incomplete set of information that individuals hold.

This incomplete information causes local misperceptions of pricing in the economy. Individuals do not know whether price changes arise from overall market changes in the economy or from changes in demand for their product. Thus the model implies that when actual inflation exceeds individuals' expectations, output increases. This theory has come under criticism in view of the evolution in communications. Information is now instantly available, and it is no longer acceptable to claim that pricing is influenced by imperfect information. The importance of the monetary misperception theory has therefore declined since Lucas propounded it.[1]

Before Lucas (1976), the discussion of the Phillips curve assumed adaptive expectations. For instance, Phelps et al. used this hypothesis and treated information like any other commodity. However, the rational expectations hypothesis was later substituted for adaptive expectations in the literature. Moreover, the so-called Lucas Critique showed that researchers were conducting policy analyses with large econometric models that would be logically invalid if expectations were formed rationally.[2] Rational expectations are in effect when a household forms expectations using available knowledge and information in order to avoid systematic errors. If an individual is systematically wrong, he or she will revise expectations so that they turn out to be accurate on average. This means errors will be randomly distributed around the correct value.[3]

In the debate between monetarists and non-monetarists, Taylor (1975) showed that monetary policy can influence real economic variables during the transition path. He used a simple momentary Phillips curve that depended on the rate of social utility. This curve relates the expected rate of inflation to unemployment using the natural rate hypothesis. Both the rational and adaptive expectations hypotheses are used to bear out the proposition. In the long-run, the public's optimal predictions of inflation converge to an equilibrium conforming with rational expectations. In contrast, on the transition path these predictions act as adaptive expectations with a time-varying coefficient of expectation depending on the precision of the policy, i.e. during the period in which inflationary expectations are transitional. More specifically, according to Taylor:

> *"...by choosing a suitable time path for policy, the monetary authorities can achieve desired levels of unemployment during the transition. The optimal path will depend on the policymakers' relative dislikes for unemployment versus a variable price level and on their rate of time preference."[4]*

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[1] See Lucas (1972, 1973) and McCallum (1989) about other contributions on this subject.

[2] The Lucas Critique was first presented by Lucas (1976). It has been widely referred to in the literature since then.

[3] Lucas & Sargent *et al.* (1981) published important papers on the role of rational expectations in macroeconomic models.

[4] Citation taken from a reprint of this paper in Lucas & Sargent et al. (1981, p. 1021). Fischer (1977) takes the same view on this issue.

Phelps & Taylor (1977) showed that monetary policy can stabilize fluctuations in output and employment in a stochastic model with rational expectations. The innovation here is that firms consider average profitability when pricing before goods are placed on sale. Thus the model features "sticky" prices and wages in the sense that they are predetermined. The model also implies that presetting of prices increases fluctuations of output around the normal employment level. This is when monetary policy feedback rules are useful, as they can reduce fluctuations even when expectations are rational, provided people know the policy rules.

In his most famous work, Taylor (1980) showed the influence of staggered contracts in business cycle fluctuations. Staggered wage contracts generate persistent unemployment and inflation. A diffusion of shocks across contracts causes unemployment to increase for several periods before diminishing. Taylor derived reduced-form contract equations from this. The main result was the persistence of unemployment generated by the model when contracts are three to four quarters long. The finding was similar to actually observed trends in the United States:

> *"Moreover, the model generates a persistence of wages and prices which gives rise to a statistical Phillips curve and presents a policy trade-off between price stability and output stability."*[5]

3. A model with uncertainty and rational expectations

The model here is based on Taylor (1981). In this model consumption and investment demands are reduced to a single demand equation, and wage and price decisions are reduced to an aggregate price determination equation. The estimated parameters are then used to find optimal policy rules. Parameters are considered fixed during policy rule changes. The policy-invariant behavior of most of the parameters results from the rationality of expectations. Thus according to Taylor (1975), these coefficients refer to the public's long-run perception of the economy.

The aggregate demand equation comes from IS-LM relationships. Aggregate demand is made up of consumption, investment, and government. In turn, these are functions of the nominal interest rate, lagged values of income and real money, and the expected rate of inflation. By assumption, both IS and LM are log-linear relations. The investment and savings behavior — the IS relationship — can be represented as follows:

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[5] Taylor (1980, p. 21).

$$R_t = b_0 + b_1 y_t + b_2 y_{t-1} + b_3 y_{t-2} + E_{t-1}(p_{t+1} - p_t) + \eta_t \tag{1}$$

where: R_t is the log of the nominal interest rate in period t; y_t is the log of real output measured as a deviation from its potential level; p_t is the log of the price level; and η_t represents the stochastic shock to saving or investment behavior. It is relevant to note that Hall & Taylor (1986) defined potential real output as the steady upward trend underlying the behavior of real output. Moreover, in equilibrium the log of real output measured as a deviation from its potential level should be equal to the log of real expenditures measured as its deviation from trend.[6]

The parameter signs are $b_0 > 0$, and b_1, b_2, and $b_3 < 0$ because they reflect the negative correlation between output demand and the real interest rate. $E_{t-1}(p_{t+1}-p_t)$ in the above equation is the expected inflation rate between periods t and t+1. E_{t-1} is the conditional expectation operator based on all information available from the past. This is the rational expectation hypothesis. It should be recalled that this is a model of staggered price and wage determination. Thus product prices and wages are set at the start of each period at expected market-clearing levels based on the information available from the last period. E_{t-1} is therefore used instead of E_t.

The money demand behavior expressed by the LM relationship is a function of the nominal interest rate, aggregate demand and lagged real money balances, and is represented by the following equation:

$$(m_t - p_t) = c_0 + c_1 y_t + c_2 R_t + c_3(m_{t-1} - p_{t-1}) + \varepsilon_t \tag{2}$$

where ε_t is the stochastic disturbance term of the money demand function. The parameter signs are $c_0 > 0$, $c_1 > 0$, $c_2 < 0$ and $c_3 > 0$. Solving both equations for the nominal interest rate, plugging (1) into (2) and then solving for y_t gives

$$y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3(m_t - p_t) + \beta_4(m_{t-1} - p_{t-1}) + \beta_5 E_{t-1} \pi_t + \beta_6 T + \beta_0 + \upsilon_t \tag{3}$$

$$\upsilon_t = \eta_t - \theta_1 \varepsilon_{t-1} \tag{4}$$

where y_t is the log of real expenditures measured as a deviation from trend; m_t is the log of money balances during period t; p_t is the log of the aggregate price level during period t; π_t is the rate of

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inflation in period t $(p_{t+1}-p_t)$; T is a trend variable for the respective period; η_t represents a random shock to output; and ε_{t-1} is a random shock to inflation in period t-1.

The hypothesized values of the β coefficients have expected signs as follows: $\beta_1 < 0, \beta_2 < 0$, $\beta_3 > 0, \beta_4 < 0, \beta_5 > 0$. The coefficient β_0 could be above or below zero, depending on whether c_0 is lower or higher than $c_2 b_0$. It should be noted that there is a trend variable in equation (3), added by Taylor (1981) to the aggregate demand equation. If its coefficient (β_6) is positive, the deviation of real expenditures must be increasing, so that real expenditures must be departing from the trend. If β_6 is negative, real expenditures must be approximating to the trend. By assumption, the random vector (η_t, ε_t) is initially serially uncorrelated with mean zero and the variance-covariance matrix Ξ. The expectation is that θ_1 is negative and η_t is positive. All β coefficients are considered policy-invariant. The two lagged values of output can capture all sources of persistence. The lagged value of real money balances captures its adjustment to changes in the interest rate and income.

In this economy, price and wage decisions are staggered and multiperiod contracts overlap each other. This means prices and wages are set in advance (predetermined), not all firms set their wages and prices simultaneously, and they are maintained for more than one period. Firms choose to set their prices and wage rates one period in advance.

In this context, suppose there are two types of firms. Each type of firm sets its own price in different periods, and these remain during the following period.[7] In addition, let ψ_t represent a measure of market employment. Hence, ψ_t is equal to actual real output divided by potential real output. If the employment rate increases, the employment of some factors of production increases. Thus variable costs and total costs increase, and prices set as a markup over costs of production therefore also increase. Moreover, suppose the price behavior equation is log-linear and the market employment rate (ψ_t) is serially uncorrelated. Assuming rational expectations, Taylor concluded that the price determination equation has the following form:

$$\pi_t = \pi_{t-1} + \gamma_1 E_{t-1} y_t + \gamma_0 + \varsigma_t \tag{5}$$

$$\varsigma_t = \varepsilon_t - \theta_2 \varepsilon_{t-1} \tag{6}$$

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[7] A different explanation from Taylor's of this pricing process would be to assume that each type of firm, 1 or 2, sets its price in different periods but considers the other group of firms when doing so. For example, firms of type 1 could use the average of all type 2 firms' prices as their price when the number of type 2 firms is small. When the number of type 2 firms exceeds ten, type 1 firms use the average of the price of the ten highest potential competitors near themselves. In either case, type 1 firms consider this price setting plus a markup over production cost.

where ς_t is the stochastic structure of random shocks to inflation and ε_t is the random shock to inflation from changes in real money balances. The behavior of ε_t is more fully explained below. The variable y_t may be interpreted as the logarithm of the market employment rate, i.e. the log of actual real output minus the log of potential real output. The hypothesized value of γ_1 is expected to be positive. When the employment rate increases, the price level may increase because it is a markup over production cost.

An important characteristic of the above price equation is that it is perfectly accelerationist. If real output exceeds its potential level, then the inflation rate π_t will accelerate. The above price equation includes a lagged inflation rate, π_{t-1}, representing the variable $E_{t-1}\pi_t$. Thus the expectation is that the inflation rate will not increase from period t-1 to period t. Real output would therefore be permanently higher than potential real output only if, in equation (5), the expected inflation rate were permanently lower than the current inflation rate, which is not possible. Suppose y_t is permanently positive (i.e. the log of actual real output is always higher than the log of potential real output): then π_t is higher than $E_{t-1}\pi_t$. Recall that period t expectations about the inflation rate in period t+1, i.e. $E_t\pi_{t+1}$, are that π_{t+1} equals period t inflation rate, but the next period inflation rate (π_{t+1}) will again exceed period t inflation rate, and so on. Since expectations are rational, individuals will revise their expectations to adapt them this new process. Thus it is not possible to raise output *permanently* above its secular trend growth rate without *accelerating* rates of inflation. This implies a vertical Phillips curve in the long run against a non-vertical one in the short run. In the short run, actual real output can be higher or lower than potential real output. In the long run, actual real output tends to approach potential real output. When real output is equal to its potential level, the price adjustment equation above says that inflation in period t is equal to inflation in period t-1 because random errors also tend to zero in the long run. This explanation also stresses the importance of rational expectations in this model.

The stochastic structure is captured in the error term ς_t. This error term contains a first-order moving average term. So θ_2 is the transitory fraction of a given shock to the inflation rate, and $1-\theta_2$ is the persistent fraction in the subsequent period. This error structure can be interpreted as firms realizing that some firms make mistaken non-periodic price adjustments, and that they therefore should not fully incorporate these into their own prices.

The real money balances term in the demand equation suggests that whenever shocks affect prices in the price equation, the error structure of υ_t reflects these because it incorporates the element ε_{t-1}. The reason is that if there is a non-periodic shock to the price level, real balances will change as much as the periodic shock.

Estimation and control of this system requires suppression of the variables for expectations in equations (3) and (5). To solve equations (3) and (5) for their reduced form, p_t and m_t must be

taken as predetermined. Thus the conditional expectations of m_t and p_t ($E_{t-1}m_t$ and $E_{t-1}p_t$), given information in period t-1, are m_t and p_t respectively, because they are known by the end of period t-1. In other words, by the beginning of period t both m_t and p_t are known, but y_t and π_t (i.e. p_{t+1}) are not known, and nor are errors η_t and ε_t.

The procedure for obtaining reduced forms for the aggregate demand and price determination equations consists of three steps. First, take expectations for (3) and (5) in period t-1 using (4) and (6). Then solve for $E_{t-1}y_t$ and $E_{t-1}\pi_t$. Finally, substitute the respective solutions in (3) and (5). The respective reduced form equations are as follows:

$$y_t = a[\beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3(m_t - p_t) + \beta_4(m_{t-1} - p_{t-1}) + \beta_5\pi_{t-1} + \beta_6 T + \beta_5\gamma_0 + \beta_0 - (\beta_5\theta_2 + \theta_1)\varepsilon_{t-1}] + \eta_t \quad (7)$$

$$\pi_t = a[\gamma_1(\beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3(m_t - p_t) + \beta_4(m_{t-1} - p_{t-1})) + \pi_{t-1} + \gamma_1\beta_6 T + \quad (8)$$
$$+ \gamma_1\beta_0 + \gamma_0 - (\gamma_1\theta_1 + \theta_2)\varepsilon_{t-1}] + \varepsilon_t$$

where $a = 1/(1 - \beta_5\gamma_1)$.

Since the parameters of the two structural form equations are policy-invariant, so are the parameters of the reduced form equations.

4. Empirical analysis

This section deals with estimation of the aggregate model under uncertainty and rational expectations for the economies of Brazil and the U.S. For the Brazilian economy the model is applied to total and industrial product. In both cases, the estimation covers the period 1973-85 in quarterly terms. This period reflects fluctuations in output and inflation without the influence of the economic plans introduced after 1985. The choice of period is therefore designed to reduce possible effects of these plans on estimates and enhance the convergence of the model. The model is also applied to the U.S. economy in order to compare the results of optimal monetary policy for a less stable economy (Brazil) and a more stable economy. The comparison also provides knowledge about possible changes in the estimated parameters for a more stable condition of the Brazilian economy in future.

With respect to total and industrial production, the goal was to find out if there is any difference between the estimates using these two aggregates for the same economy. In previous work, Dias (1993a, 1994) used industrial production as a proxy for total production. The main

reason was to reduce the influence of business-cycle fluctuations, which are less present in industrial production than total production. Total production also includes agriculture and services.

If expectations are to be consistent with the model presented in last section and with the effects of economic policy, the restrictions on the parameters of equations (7) and (8) formed by a vector autoregressive moving-average model must be satisfied. The statistical method used to satisfy these restrictions was Minimum Distance Estimation.[8] This method is used when the independent variables are not stochastic and the regression error is not normal. Taylor (1977) showed that to guarantee a unique solution in stochastic macroeconomic models with rational expectations, the condition of finite variance of equilibrium price distribution is not sufficient. This condition restricts the expected price path to a stable path. The finite variance (stability) condition must be extended to a minimum variance condition in order to guarantee uniqueness. This eliminates all but one equilibrium price distribution. The exogenous variables are y_{t-1}, y_{t-2}, $m_t - p_t$, $m_{t-1} - p_{t-1}$, p_{t-1} and time trend T, while the endogenous variables are y_t and π_t.

The deviation between the log of GNP and the log of potential GNP is represented by the log of the ratio between actual real output and potential real output. This is the employment variable required in the price-adjustment equation (5). For consistency, this variable is also used to estimate the demand equation (3). With respect to the U.S. economy, natural real GNP is used here as a proxy for potential real GNP, as in Gordon (1990). Potential real GNP and potential industrial production were not available for Brazil, and the natural product was therefore used in both calculations. For Brazil, real gross domestic product (GDP) and the real industrial production index (IPI) were used, considering general industry, based on Contador & Santos (1987). The respective natural products were calculated as in Gordon (1990). The calculation of natural products was based on geometric interpolation among quarters representing peaks during the time period analyzed. The peaks were found by analyzing the data graphically in logarithmic terms.[9] Finally, the data are the deviation of the log of production from the log of natural production.

The price series is given by the log of the consumer price index for Brazil, and the GNP deflator for the U.S. The nominal money series used is the log of M1. For the Brazilian economy, seasonal adjustment of M1 and the consumer price index series was performed using the moving-average method. These series for the Brazilian economy were produced by IBGE, the Brazilian Office of Statistics. For the U.S. economy, data were taken from Gordon (1990). Table 1 gives the estimates for both economies and cases.

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[8] Malinvaud (1970).

[9] For example, between benchmark quarters 1980:4 and 1985:4, the one-quarter growth rate (q^n) of the nth quarter (Q^n) was calculated as follows: $q^n = [(Q^n\ 1985:4/Q^n\ 1980:4)^{1/20}]\ 1$.

Table 1

Resulting Estimates for Brazil and the U.S. 1973-85

Economy	US	BRAZIL	
Case	Gross National Product	Gross Domestic Product	Industrial Product (IPI)
β_0	-0.055 (1.64)	-0.014 (2.29)	-0.029 (2.93)
β_1	1.289 (9.39)	0.459 (3.16)	1.00 (6.90)
β_2	-0.502 (4.35)	-0.035 (0.28)	-0.30 (2.19)
β_3	0.271 (2.54)	0.172 (2.21)	0.074 (0.76)
β_4	-0.200 (1.82)	-0.081 (1.01)	0.076 (0.79)
β_5	0.525 (2.38)	0.259 (2.20)	0.280 (1.66)
β_6	-0.0000037 (0.08)	-0.00037 (1.47)	0.0006 (1.48)
γ_0	0.00075 (0.56)	0.0028 (0.96)	0.0017 (0.80)
γ_1	0.059 (0.83)	0.036 (0.28)	0.00033 (0.005)
θ_1	-0.231 (1.70)	0.117 (0.87)	0.145 (0.88)
θ_2	0.728 (6.90)	0.159 (1.03)	0.259 (1.84)
ζ_η	0.0041	0.0076	0.0108
ζ_ε	0.0054	0.0142	0.0151
φ	-0.0573	-0.1058	-0.1166

Note: Numbers in parentheses are absolute asymptotic t-ratios. ζ_η and ζ_ε are the estimated standard deviations of the equations. The cross-correlation of estimated residuals is $\rho(\varepsilon_t,\eta_t)=\rho(\eta_t,\varepsilon_t)=\varphi$.

4.1. The US economy

The second column of Table 1 shows estimates of the parameters of equations (3) to (5) for the U.S. economy. Considering that all variables are constant in equation (3) except y_{t-1} and y_{t-2}, the acceleration component can be calculated for this economy as approximately $0.8y_{t-1}+0.5(y_{t-1}-y_{t-2})+C$, where C represents the constancy of other variables. It follows that investment in period t-1 is approximately 50 percent of the change of output from period t-2 to period t-1. The coefficient β_1 also tells us that the rate of change in the growth of output is positive and about 0.29 percent.

The estimates of β_3 and β_4 are both significant and consistent with a partial adjustment hypothesis for the money demand equation. They are opposite in sign and β_4 is smaller than β_3 in absolute value, as expected. This means the presence of lagged value for real money balances in the money demand equation is sufficient for partial adjustment of these balances to changes in interest rates and income. β_5 is also significant. Its positive sign tells us that if the expected inflation rate in the future is higher than inflation in the current period, expenditures should increase because rational households know that expansion in prices in the next period will produce a contraction in their expenditures. The coefficient of the time trend is negative and not significant. The intercept β_0 has a negative sign and is significant to a 90 percent confidence level.

However, in this model neither estimate of the parameters in the inflation equation is significant. Households in this economy do not base changes in expectations about the future inflation rate on changes in expectations about the value of future output. Nonetheless, estimates of the price equation can be used to calculate the non-accelerating inflation point and explain deviations of output from natural output.

Non-accelerating inflation occurs where the change in the inflation rate from one period to the other is equal to zero. This occurs when real GNP falls short of current natural real GNP by approximately 1.3 percent.[10] At this point, the unemployment level is one at which the inflation rate neither accelerates nor decelerates. This result coincides with the quarterly sample mean of the variable y_t. During the period analyzed here, the quarterly sample mean of the GNP gap, i.e. the deviation of actual real GNP from its natural level in logarithmic terms, is equal to -0.013. The sample mean unemployment rate of the GNP gap is 1.3 percent. This means the inflation rate was not accelerated by the unemployment level during the period 1973-85.

As stated above, the variable y_t in the price determination equation represents the deviation of actual real GNP from its natural level in logarithmic terms. This is a good proxy for the unemployment rate. Suppose actual real GNP is *above* the non-accelerating inflation point. This means unemployment is *lower* and *below* the non-accelerating inflation point. Thus the inflation rate should increase. The coefficient of the expected deviations in output from its natural level, 0.059, implies that inflation will *increase* by 0.94 percent of its annual rate for each year that real GNP is 1 percent *above* the non-accelerating inflation point.[11] By the same method, when the annual unemployment rate is 1 percentage point *above* the non-accelerating inflation point, the rate of inflation will *decrease* by 2.82 percentage points.[12]

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[10] For $\Delta\pi_t = \pi_t - \pi_{t-1} = 0$, $E_{t-1}y_t$ must be equal to -0.013.

The error structure shows that approximately 73 percent of any shock to inflation is temporary. Also, 27 percent of any shock to the inflation rate will persist into the subsequent period. Since θ_1 is negative, real money balances do not fully adjust to every shock in the price level. The correlation of the two error terms of the two regressions is low, 0.057 This means little serial correlation remains in the regressions, as expected from the presence of the two lagged dependent variables in equation (3).

4.2. The brazilian economy

Two cases were considered for the analysis of Brazil: one used deviations of real gross domestic product from natural product to estimate y_t, and the other used deviations of real industrial general product from its natural product. This permits investigation of the effects of monetary policies on the industrial sector.

4.2.1. First case: considering gross domestic product

Considering the estimates of β_1 and β_2 in Table 1, column 3, the acceleration component for the Brazilian economy is equal to 3.5 percent. Thus for the economy as a whole, investment in period t-1 is approximately 3.5 percent of the change in output from t-1 to t-2. In addition, β_1 implies that output is expanding at a decreasing rate in Brazil.

The estimates for β_3 and β_4 have the expected sign and relationship to each other. However, β_4 is not significant. This is similar to the result for the U.S. economy. The main difference in this case is that the size of the estimates is smaller for the Brazilian economy. The significance of β_5 is in line with the intertemporal substitution effect. This means higher future prices relative to current prices should stimulate expenditure in the current period. The time trend variable is not highly significant and its negative sign tells us that actual real output tends to approximate to natural real output in the period analyzed. The intercept is negative and significant.

In the inflation equation both coefficients are positive and not significant. Thus there is no significant evidence that a higher employment level increases inflation in the Brazilian economy.

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[11] $0.059 \times 4 \times 4 = 0.94$.

[12] According to Okun's law, for each percentage point that real GNP changes in relation to its natural level, the inflation rate changes by 0.94. Thus, using Okun's multiplier of 3, $0.94 \times 3 = 2.82$.

One possible explanation is that the inflation rate may follow changes in the money supply more closely than expectations about the employment level.[13]

As Table 1 also shows, when inflation growth is equal to zero, variations in output around its natural level give a non-accelerating inflation point of about 7.87 percent. Thus the non-accelerating inflation point occurs when output departs 7.87 percent from its natural level. Compared with the results for the U.S., this non-accelerating inflation point indicates that unemployment must be high in order to decrease inflation in the Brazilian economy. The history of the Brazilian economy shows that the inflation rate has been high and persistent. Moreover, the quarterly sample mean of the unemployment rate measured as the deviation of actual from natural product is 2.2 percent. Thus actual unemployment was an inflation accelerator in the Brazilian economy during the period.[14]

The coefficient of expected output deviations in the inflation equation (5) indicates that the annual rate of inflation *increases* by 0.58 percentage points for each year in which the real production is 1 percent *above* the non-accelerating inflation point. If the unemployment rate is 1 percent *above* the non-accelerating inflation point in a particular year, the annual inflation rate *decreases* by 1.74 percent. Changes in inflation and unemployment with respect to each other in the Brazilian economy are more rigid than in the US economy.

Estimates of error structures θ_1 and θ_2 indicate the behavior of shocks. Despite its significance, θ_2 suggests that about 16 percent of any shock to the inflation equation is temporary. Thus 84 percent of the shock persists into the subsequent period. This is in accordance with the presence of inertia in the Brazilian economy, as pointed out by many authors. The positive sign of θ_1 indicates that nominal balances fully adjust to every shock to the price level. This reflects the fact that the effects of price shocks in the Brazilian economy tend to be immediately incorporated into the inflation equation.

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[13] Simonsen & Dornbusch (1987) show that indexation of wages and prices significantly contributed to the high inflation rate observed after 1979 in Brazil. Subsequently, this same indexation improved inflation with no direct relation to employment.

[14] Cavalcanti (1990) used a macroeconomic model with adaptive expectations to emphasize the effects of interest rates on a Phillips curve for Brazil, considering that agents are risk averse. This Phillips curve is different from ours. It assumes three solutions: the deviation of output from its quasi-potential level, the nominal interest rate and the inflation rate. Cavalcanti's estimations use the two stages least squares method for the period 1976-89. It should also be mentioned that the variable representing output deviation was calculated using capacity utilization in intermediate industry. The main conclusion is that it is impossible to reject the hypothesis that the expected real interest rate and the inflation variation coefficient significantly affected output deviation in the period.

4.2.2. Second case: considering general industrial production

The fourth column of Table 1 gives estimates for the case which considers the deviations of real product from its natural level in the industrial sector of Brazil. As before, assuming that all variables of equation (3) except output are constant, β_1 and β_2 indicate a stable aggregate output function. Here, investment in period t-1 is approximately 30 percent of the change in output from t-2 to t-1. This is much higher than the impact of output change on investment when considering the Brazilian economy as whole. Yet β_1 gives a constant rate of change in the growth of industrial production. Thus some other sector must be expanding at a decreasing rate that exceeds the constant rate of increase in the industrial sector, causing this effect on the overall economy.

β_3 and β_4 both have positive signs, thus also contradicting the predicted estimates in absolute value. This is not in line with expectations about partial adjustment of real money balances. However, these variables are not significant, indicating that changes in real money balances are not relevant to changes in real industrial production as regards its deviations from the natural level in this period. β_5 is positive and significant, which is consistent with expectations about the parameter. This means aggregate demand from the industrial sector also follows the theory of intertemporal substitution effects. The time trend variable is relatively insignificant and positive. The intercept is negative and significant.

As in the overall economy, the inflation equation does not show a strong correlation between changes in inflation and deviations from the natural level of industrial production. The estimates in Table 1 show that the relationship between inflation and output is more clearly evidenced in the aggregate demand equation. The non-accelerating inflation point is not considered in this case because the significance of γ_1 (the "t" statistic in question is close to zero).

θ_2 is significant but θ_1 is not. θ_2 suggests that about 26 percent of the effect of any shock to the inflation equation is temporary in the industrial sector. As for the economy as a whole, most of the shock will persist into the subsequent period. The sign of θ_1 again follows that of the overall economy, indicating that the effects of price shocks tend to be incorporated into the inflation equation immediately.

5. Optimal monetary policy rule

The specific model for uncertainty and rational expectations presented and estimated above is used next to study an optimal monetary policy, based on Taylor (1981). The purpose is to minimize the variability of output and inflation around their target levels.

The parameters in equations (3) and (5) are policy invariant, since expectations are rational. Thus they are policy invariant to the mechanism generating the money supply. Together with the fact that the level of nominal money balances appears explicitly, this means monetary feedback rules can be calculated using optimal control techniques. The goal of this monetary policy is to reduce fluctuations in real output and inflation around their average target levels. Since the parameter estimates can be considered true values given rational expectations the target level of real output can be the non-accelerating inflation point. Determination of the target level of inflation requires more complicated analysis involving an investigation of the welfare costs and benefits of alternative average inflation levels. One possible inflation target (π^*) could be the rate at which seigniorage (the government's revenue from printing money) is maximized. This would reduce the welfare loss due to inflation. Seigniorage and the welfare costs of inflation are discussed by Dias (1993b).

The variable used as instrument in this monetary policy is the monetary base. In real terms, this is the same as real money balances. Real output and the inflation rate are the target variables in Taylor's "loss to social welfare function" (1981):

$$\mathfrak{S} = \gamma_1(\chi_t - \chi^*)^2 + \gamma_2(\pi_t - \pi^*)^2 \tag{9}$$

where χ_t and π_t are now actual level of output and inflation and χ^* and π^* are the target levels of output and inflation respectively. All elements are expressed as logarithms. λ_1 and λ_2 are the weights associated with the costs of fluctuations around the target levels of output and inflation respectively. Their values are $0 \le_1 \le \lambda_1$ and $\lambda_2 = 1 - \lambda_1$. These weights may be taken as exogenously chosen by nature for the policy maker. The presence of χ^* and π^* in the above objective function implies that the targets for output and inflation may be different from zero. Moreover, they are not required to be constant for ever. The consequences of variations in λ_1 and λ_2 are discussed below.

The optimization procedure is equivalent to monetary feedback rules designed to minimize the expected value of loss function $\mathfrak{S}$ for the steady state distribution. This is the same as limiting the fluctuations of unemployment around the natural rate. That is, if λ_1 is approximately equal to 1, λ_2 is approximately equal to zero and inflation does not count. The procedure requires matrix notation to summarize the autoregressive and moving-average dynamics, and the impact of money supply on these dynamics. In the following equation, d_t is the deviation of real money balances from some trend:

$$d_t = m_t - p_t - d_1 T - d_0 \tag{10}$$

The matrix notation of the model, substituting d_t for $m_t - p_t$ and omitting the time trend and the constants, is as follows:

$$Y_t = BY_{t-1} + cd_t + s_t \qquad (11)$$

where:

$Y_t = (y_t, y_{t-1}, d_t, \Pi_t, \varepsilon_t)'$
$s_t = (\eta_t, 0, 0, \varepsilon_t, \varepsilon_t)'$
$c = a(\beta_3, 0, a^{-1}, \gamma_1\beta_3, 0)'$

$$B = a \begin{bmatrix} \beta_1 & \beta_2 & \beta_4 & \beta_5 & -(\beta_5\theta_2+\theta_1) \\ a^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \gamma_1\beta_1 & \gamma_1\beta_2 & \gamma_1\beta_4 & 1 & -(\gamma_1\theta_1+\theta_2) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where y_t and Π_t represent the *deviations* of output and inflation from their respective target levels χ^* and π^*

The loss function can now be represented as

$$Y_t' \Lambda Y_t \qquad (12)$$

where Λ is a square matrix. It has a first diagonal λ_1, a fourth diagonal λ_2, and zero is elsewhere.

Since d_t can serve as a control variable, Taylor's feedback rule is as follows:

$$d_t = g_1 y_{t-1} + g_2 y_{t-2} + g_3 d_{t-1} + g_4 \Pi_{t-1} + g_5 \varepsilon_{t-1} \qquad (13)$$

According to this equation, real money balances are set from the state vector Y in period t-1, Y_{t-1}. Thus they can be chosen by the monetary authorities. Recall that Π_{t-1} is equal to the inflation rate in period t-1 minus its target level. ε_{t-1} is the random shock to inflation in period t-1 displayed in equation (4).

The optimal control technique is from Chow (1975). The aim is to find the circumstances under which vector $g = (g_1, g_2, g_3, g_4, g_5)$ is time invariant, thus giving a steady state for the optimal control equation (13). Accordingly, the vector g that minimizes the expected value of (12) in the steady state is given by

$$g = -(c'Hc)^{-1}c'HB, \qquad \text{where} \tag{14}$$

$$H = \Lambda + (B + cg)'H(B + cg)^{15} \tag{15}$$

Assuming policymakers do not change plans for the future, the optimal control rule with rational expectations will have time consistency.

Because the price equation is perfectly accelerationist, the Phillips curve is vertical in the long run. Thus it is not possible to analyze the long-run tradeoff between output and inflation using this model. To do this, it is necessary to consider a "second-order" Phillips curve and examine the long-run tradeoff between *fluctuations* in output and *fluctuations* in inflation. This requires finding the steady-state values of the optimal standard deviation of real output (σ_y^2) and inflation (σ_π^2) respectively, corresponding to various values of λ_1 and for the feedback vector g.[16] The steady-state variance-covariance matrix of Y_t is given by the following matrix:

$$\Sigma = V + (B + cg)\, \Sigma\, (B + cg)' \tag{16}$$

where V is the variance-covariance matrix s_t.[17] Below this monetary policy rule is applied for the Brazilian and U.S. economies, together with Taylor's optimal monetary feedback rule distinguishing overall production from industrial production in Brazil. The parameters estimates from the last section are used as true parameters to solve the system defined in equations (9) to (16).

6. Empirical analysis of the optimal monetary policy

The estimated coefficients from the equations for output and price determination (10) and (12) are assumed to be true parameters. Tables 2, 3 and 4 display different values of λ_1 and hence λ_2, together with the resulting reaction coefficients for equation (13) and the optimal standard deviations of output and inflation for both economies and cases analyzed here.

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[15] The proof of this system is set out in Chow (1975, p. 170).

[16] Note that $\sigma_y^2 = E(\chi_t - \chi^*)^2$ and $\sigma_\pi^2 = E(\pi_t - \pi^*)^2$

[17] $V = E(s_t s_t')$. Thus V is a square matrix with five rows and columns. It has a first diagonal $(\zeta_\eta)^2$, fourth diagonal $(\zeta_\epsilon)^2$, and fifth diagonal $(\zeta_\epsilon)^2$. By definition, the policy problem described by equations (9) to (16) minimizes E tr $\Lambda\Sigma$. Where Σ is a square matrix with five rows and five columns. It has a first diagonal σ_y^2 and fourth diagonal σ_r^2

6.1. The US Economy

The reaction coefficients not shown in Table 2 (g_i, i = 1,2,3) have fixed values for all values of λ_i. They are $g_1 = -4.76$, $g_2 = 1.85$, and $g_3 = 0.74$. Thus the difference between feedback rules that weight fluctuations in output and those that weight fluctuations in inflation are represented in policy reactions g_4 and g_5. The invariability of the coefficients of lagged values of output and real money balances can be explained as follows: since fluctuations in output are directly related to fluctuations in inflation by the influence of aggregate demand on prices, "policy will reduce the variability of both output and inflation by reducing the *own-persistence* of business cycle fluctuations."[18]

Table 2

Optimal Policy Reaction Functions and
Resulting Output-Inflation Variations (U.S.)[19]

λ_1	$g_4(\Pi_{t-1})$	$g_5(\varepsilon_{t-1})$	σ_y (%)	σ_π(%)
0.01	-28.57	19.95	1.38	2.19
0.10	-11.76	7 71	0.84	2.33
0.20	-8.68	5.47	0.73	2.43
0.45	-5.76	3.34	0.60	2.66
0.50	-5.41	3.09	0.59	2.71
0.70	-4.23	2.23	0.53	2.98
0.90	-3.12	1.42	0.47	3.63

Table 2 shows the reaction coefficients to inflation, measured as its deviations from the target level, and to lagged price shocks (g_4 and g_5) in columns 2 and 3 when λ_1 varies from 0.01 to 0.90. If the value of λ_1 is large, the feedback rule is output-regarding. If λ_1 is small, the weight on inflation fluctuations (λ_2) is large and the feedback policy is monetary policy by decreasing the

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[18] Taylor (1981, p. 673).

[19] Standard deviations are at annual rates.

growth rate of real money balances. Part of this effect is offset by a positive reaction of ε_{t-1}. This is explained by the fact that the rise in inflation is expected to be temporary. However, if a monetary policy occurs, initially the inflation rate will change substantially, followed by a change in the growth rate of real money balances.[20]

In other words, if the weight term λ_1 grows larger, the attention moves toward fluctuations in output. From Table 2, the reaction coefficients g_4 and g_5 become smaller in absolute values, tending toward zero. Since the signs of g_4 and g_5 do not change, the direction of their effects remains. A monetary policy now has lower effects on the fluctuations of output around its target level. Thus the policy is more accommodating to changes in the inflation rate. Based on Table 2, if λ_1 is equal to 0.90 and the money supply increases by 1 percent, the reaction coefficients of inflation and lagged price shock cause a 1.7 percent *net* decrease in the growth rate of real money balances.[21] This last result is much lower than when the weight on output fluctuations (λ_1) is low.

Estimation of (16) follows the procedure for equations (9) through (16). The fourth and fifth columns in Table 2 represent the optimal standard deviations of quarterly output and inflation stated in annual rates (σ_π is multiplied by 4) and percentages respectively. They are plotted in Figure 1, which traces the tradeoff between fluctuations in output and inflation. In Table 2, as λ_1 increases, the optimal standard deviation of output (σ_y) becomes lower. This is shown in the vertical axis of Figure 1. In turn, when the weight on inflation fluctuations λ_2 increases, the optimal standard deviation of inflation (σ_π) falls. This is shown in the horizontal axis of Figure 1. An inflation-regarding policy decreases the variability of the inflation rate around its target level, while an output-regarding monetary policy decreases the fluctuations of output around its target level. Figure 1 gives the efficiency locus of the curve. Since this is a downward-sloping curve, there is a tradeoff between the variability of inflation and output. The authorities must choose between output-regarding and inflation-regarding policies.

If the minimum variability of inflation is attained, the implication is that monetary policy is fully inflation-regarding. This is when λ_2 is equal to 1 (consequently λ_1 is zero).[22] The minimum value for σ_π is 2.15, which is reached when σ_y is 2.51 percent. The minimum value of σ_y is 0.41 percent (λ_1 is one and λ_2 is zero) and is reached when σ_π is 18.06 percent. Thus the tradeoff becomes vertical when the output fluctuation reaches a standard deviation of 2.51 percent.

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[20]Using the results from Table 2, if the money supply increases by 1%, growth in real money balances decreases by 28.6% through deviations of inflation from its target level. Part of this effect — about 20% — may be offset by a reaction caused by monetary policy in the lagged price shock coefficient.

[21]-3.12 + 1.42 $\cong$ 1.7.

[22] Not shown in Table 2.

Figure 1
Output-Inflation Variation Tradeoff (US)
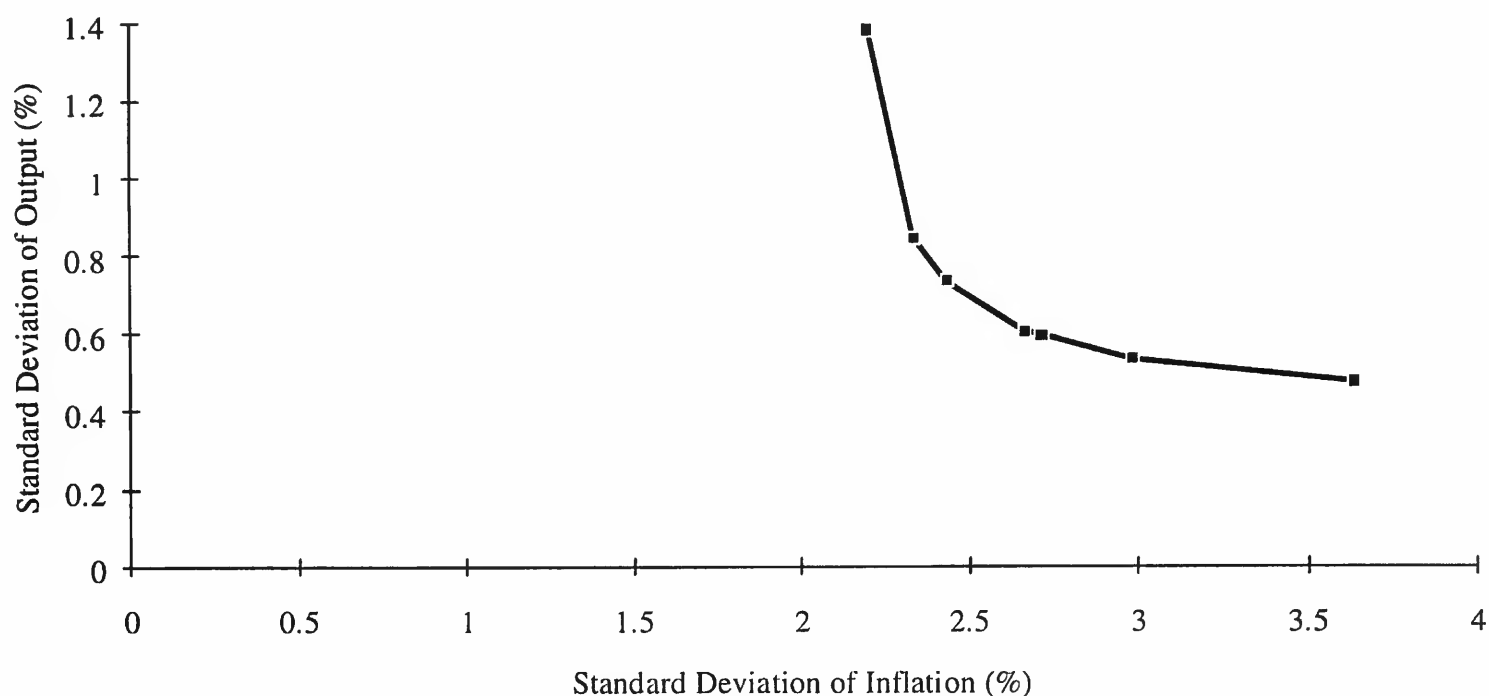
The data show that the sample means of the standard deviations of inflation and output are 4.6% and 1.03% respectively. A comparison of these sample means to the efficient locus shown in Figure 1 shows that the actual of U.S. performance was not optimal.

6.2. The brazilian economy

6.2.1. First case: considering gross domestic product

In the case of Brazil, monetary policy is also able to reduce the own-persistence of business cycle fluctuations, since $g_1 = -2.67$, $g_2 = 0.20$, and $g_3 = 0.47$ are constant in the period 1973-85. Table 3 shows the remaining reaction coefficients g_4 and g_5 for this first case. Both coefficients decrease in size as monetary policy changes from inflation-regarding to output-regarding. This means the coefficients of the deviation of inflation from its target level and the price shock coming from period t-1 have a lower effect on growth in real money balances when monetary policy becomes output-regarding.

If the feedback policy is inflation-regarding, g_4 is large in absolute value and negative. This optimal coefficient reacts to increases in the inflation rate *above* its target level by *reducing* the growth rate of real money balances by 49.5 percent.[23] According to Table 3, the effect of the inflation reaction coefficient is partially reduced by the offsetting positive reaction coefficient of ε_{t-1}.[24]

Comparing the results for the Brazilian and U.S. economies, for all values of λ_1, the reaction coefficient of Π_{t-1} is larger for the Brazilian economy. Thus in this economy the deviations of inflation from its target level are more sensitive to monetary policies. This can also be seen from the higher values of optimal standard deviations of inflation in Brazil than in U.S. (column 5 in Tables 2 and 3). Moreover, the optimal standard deviations of output are also higher in Brazil than in the U.S. (column 5 in Tables 2 and 3).

Table 3

Optimal Policy Reaction Functions and
Resulting Output-Inflation Variations (Brazil)[25]

| λ_1	$g_4(\Pi_{t-1})$	$g_5(\varepsilon_{t-1})$	$\sigma_y(\%)$	$\sigma_\pi(\%)$
0.01	-49.46	8.54	13.75	7.26
0.10	-17.88	3.52	7.63	11.11
0.20	-12.62	2.69	6.25	13.21
0.45	-7 75	1.91	4.68	17.29
0.50	-7.16	1.82	4.45	18.12
0.70	-5.23	1.51	3.63	22.16
0.90	-3.41	1.22	2.64	30.73

Based on columns 4 and 5 in Table 3, it can be seen that there was a long-run tradeoff between the variability of inflation and the variability of output in the period 1973-85. This tradeoff is represented in Figure 2.

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[23] Rocha (1992) showed for the period 1968-85 that the real and monetary sectors in the Brazilian economy are dichotomized when money is anticipated by the policy rule. The relationship between inflation in period t-1 and money growth in period t-1 is positive and has a coefficient of 0.83 in the period, but this does not contradicts our results because we are considering Π_{t-1} instead of π_{t-1}.

[24] For example, if M1 increases by 1%, the lagged deviation of inflation from its target level reacts so as to decrease growth of real money balances by 49.5%. The positive effect of the lagged price shock reaction coefficient causes a reduction of approximately 8.5%.

[25] Standard deviations are at annual rates. BR stands for Brazil.

Figure 2

Output-Inflation Variation Tradeoff (BR-GDP)
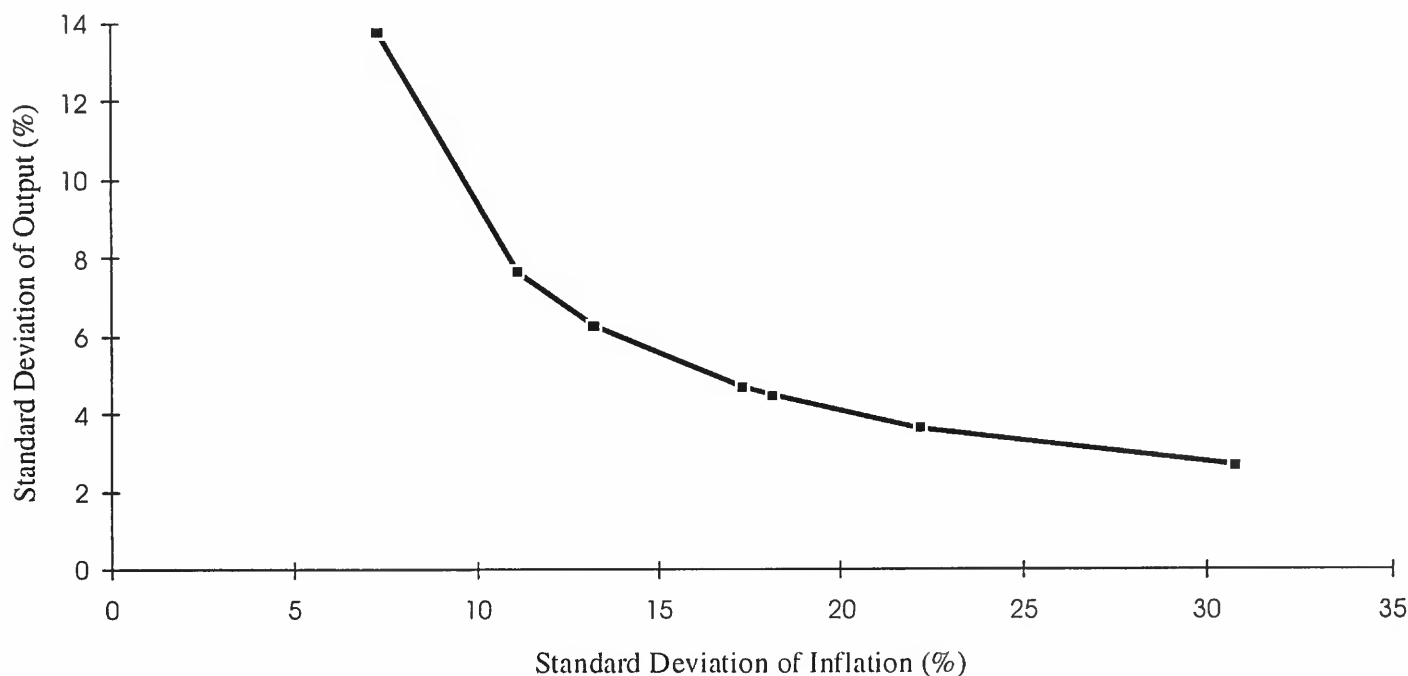
A fully output-regarding policy can achieve a minimum variability of output (σ_y). The minimum value for σ_y is 0.76 percent per annum. This is reached when the optimal standard deviation of inflation is high, and is 147.54 percent.[26] Thus a fully output-regarding policy may not be considered as a choice. The minimum value for σ_π is 5.6%. This is related to a σ_y value of 32.71%. According to Table 3, this economy had higher inflation rates than the U.S. economy during the period analyzed. Moreover, the variability of inflation is higher than the variability of output. In order to change the variability of output, inflation has to vary more. Thus changes in the inflation rate are more rigid in the Brazilian economy.

Moreover, the actual performance of inflation and output variability in Brazil gives a σ_y of 4.65% and a σ_π of 37.9% in the period. This implies a point outside and to the right of the efficient locus. Thus the actual performance of the Brazilian economy was not optimal.

6.2.2. Second case: considering general industrial production

According to Table 4, when λ_1 goes from 0.01 to 0.90, i.e. from an inflation-regarding policy to an output-regarding policy, the reaction coefficients of g_4 and g_5 decrease in absolute value but their signs remain the same. The remaining reaction coefficients are $g_1 = -13.00$, $g_2 = 4.00$, and

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[26] This is not shown in Table 3.

$g_3 = -1.00$. Although the coefficients of y_{t-1}, y_{t-2} and d_{t-1} are rigid to changes in monetary policy, they are much larger in this case than in the first one. This is also true for the reaction coefficients of Π_{t-1} and ε_{t-1}, and for the optimal deviation of inflation and of output. These tell us that the industrial sector is more sensitive to this rule of monetary policy than the overall economy. Another difference is the sign of g_3. Growth in real money balances is negatively related to itself in different periods. This could be explained by the larger variability of inflation.

In this case, an inflation-regarding policy also causes g_4 to be large in absolute value and negative. If λ_1 is equal to 0.01, increases in the inflation rate *above* its target level *reduce* the growth of real money balances by 134.07 percent. This reduction will be partially offset by the positive reaction coefficient of ε_{t-1}.

Table 4

Optimal Policy Reaction Functions and Resulting
Output-Inflation Variations (Industrial Sector - Brazil)[27]

λ_1	$g_4(\Pi_{t-1})$	$g_5(\varepsilon_{t-1})$	σ_y (%)	σ_π (%)
0.01	-134.07	36.64	136.19	55.14
0.10	-33.64	10.63	57.50	101.29
0.20	-18.97	6.83	34.03	117.66
0.45	-8.83	4.20	12.81	132.15
0.50	-7.93	3.97	10.69	133.58
0.70	-5.57	3.36	4.91	137.48
0.90	-4.22	3.01	1.69	139.81

For large values of λ_1, the optimal control policy is output-regarding. Table 4 shows that the signs of g_4 and g_5 do not change and the direction of their individual effects remains the same. When the weight on output is large and less concern is given to inflation, growth of real balances tends to decrease at a slower rate. However, the size of the reaction coefficients cannot be overlooked. Thus, when considering production in the industrial sector, monetary policy is not accommodating to changes in the inflation rate. This is also true for the economy as a whole, albeit on a lesser scale. Thus in the Brazilian economy, monetary policies may be as concerned with keeping output at its potential level as with the variability of inflation around its target level.[28]

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[27] Standard deviations are given as annual rates.

Figure 3

Output-Inflation Variation Tradeoff (BR-IPI)
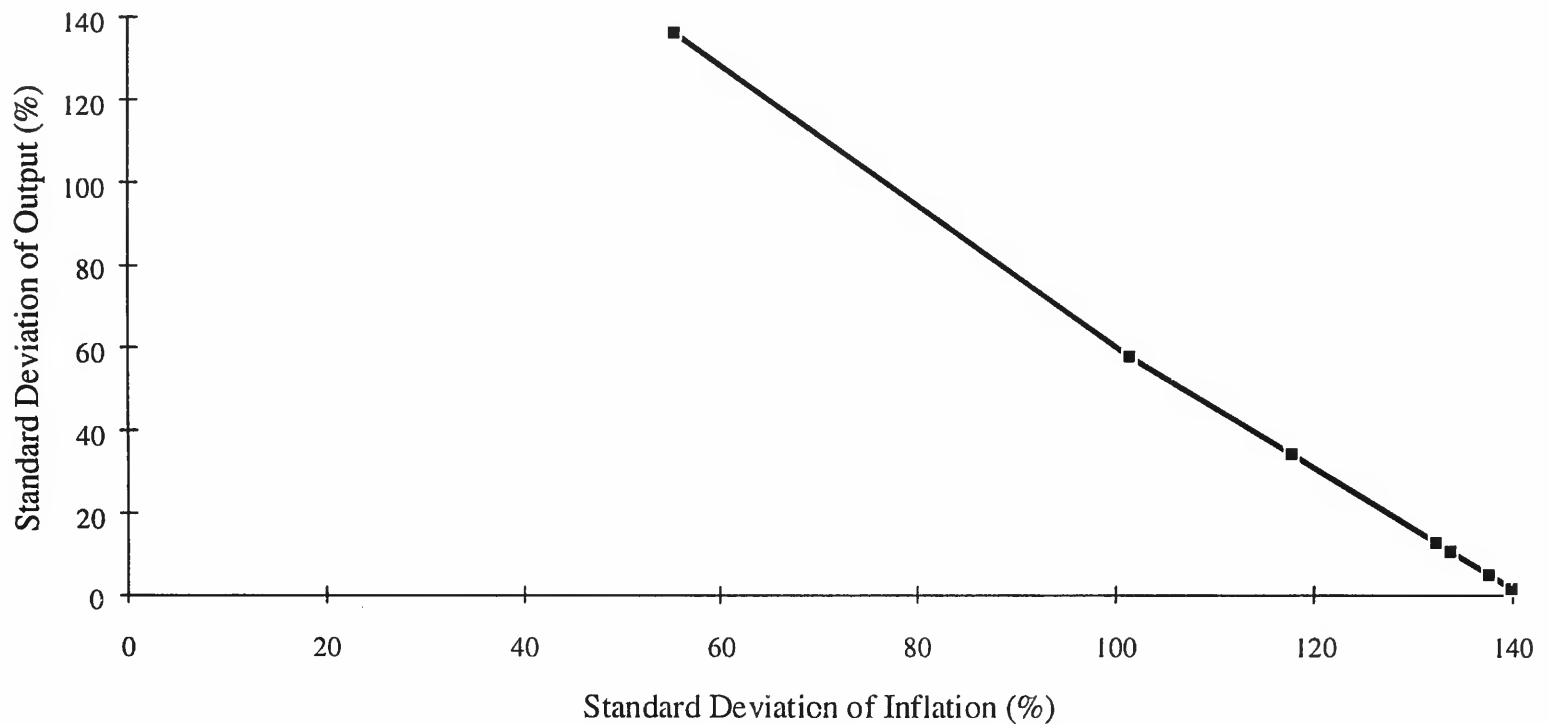
In this case, the second-order Phillips curve is downward sloping and nearly linear, as shown in Figure 3. The output deviation would have to be 3368.18 percent higher than its target level to reach a minimum inflation rate of 6.02 percent. Thus a fully inflation-regarding monetary policy may not succeed. The minimum deviation of output from its target level, 1.11 percent, requires a 140.65 deviation of the inflation rate from its target level.

Moreover, the results for inflation-regarding policies in both cases imply that the variability of inflation increases considerably when fully output-regarding policies are pursued. Finally, the study suggests that monetary policies should be as concerned with keeping output around its natural level as with variability of inflation around its target level in the Brazilian economy.[29]

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[28] Cardoso (1983) showed empirically that a macroeconomic model with wage and price indexing implies an inflation rate that follows a random walk. She estimated this hypothesis using Sims's causality test for the period 1968-82. This hypothesis is justified by the fact that wages and the exchange rate were indexed to inflation and monetary policy was fully accommodating since 1968. Moreover, in the Brazilian economy as a whole, monetary accommodations were not able to remove inflationary inertia. If we accept Cardoso's results, monetary policy might not have been output-regarding in the period 1968-82.

[29] This conclusion is in line with Issler (1991), who estimated the relationship between the inflation level and the conditional variance of inflation in Brazil during the period 1971-85. His results show that economic agents preferred low inflation to high inflation in the period.

7. Conclusions

This paper uses an optimal monetary policy rule to minimize the variability of inflation and output around their target levels. The model considers an equilibrium in the interaction between aggregate demand and price equations with rational expectations. In general, the analyses applied to the period 1973-85 for both economies and cases imply the following conclusions:

1. Investment in period t-1 is approximately equal to 50% of the change in output from period t-2 to t-1 in the U.S. economy, 3.5% in the Brazilian economy, and 30% when considering Brazilian industrial production. Thus, in Brazil the accelerator component is much lower than in the U.S.

2. The influence of output in t-1 on output in t is about 79% in the U.S., 42% in Brazil, and 70% in the Brazilian industrial sector.

3. As expected, higher future prices of goods relative to current prices should stimulate expenditure in the current period in both economies and cases. The observed significance of the inflation coefficient in the aggregate demand equation is important because it implies a relationship between inflation and output measured in terms of the deviations from their natural level.

4. The error structure suggests that about 16 percent of any shock to the inflation equation is temporary in Brazil. Hence, 84 percent of the shock will persist into the subsequent period. For industrial production these proportions fall to 26% and 74% respectively. Thus most of the shock will persist into the subsequent period. These results are opposite to the ones found for the U.S.

5. The *own-persistence* of business cycle fluctuations in output are reduced by Taylor's monetary policy rule in both economies and cases.

6. Whether policy is inflation-regarding or output-regarding, optimal monetary policies tend to reduce the growth of real money balances.

7. If monetary policy is concerned with inflation, the optimal coefficient of lagged inflation reacts to increases in the inflation rate *above* its target level by *reducing* the growth rate of real money balances by 49.5 percent, when considering the overall Brazilian economy. This reaction is larger then the U.S. coefficient and smaller than the reaction of the Brazilian industrial sector.

8. There is a tradeoff between the variability of inflation and the variability of output.

9. Thus in order to change inflation in Brazil, the variability of employment must be higher than in the U.S., indicating higher losses to Brazilians if they are to stabilize inflation.

10. Finally, in both Brazil and the U.S., monetary policies were not optimal in the period. Thus monetary policy was not efficient in keeping the variabilities of inflation and employment at their minimum in both economies.

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