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Optical Sensing of Capillary Waves

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Abstract

I relay various topics I learned about this semester and summarize the specialties of Clemson's Micro-Photonics Laboratory. I report pertinent information for characterizing capillary waves with lasers and one recent method of characterizing them. Lastly, I outline a potential plan for mathematically modeling a laser's reflection off of capillary waves, and potential plans moving forward.

Background

Optics

The study of light and its interaction with its surroundings is called optics. The different methods of mathematical analysis for light, which differ based on the assumptions they make about light, include ray optics, wave optics, electromagnetic optics, and quantum optics, organized from least to most rigorous.

Basic Optics Terms

Light beams: Light beams are a special type of light, where the light mostly travels in the same direction.

Beam waist and divergence: After a light beam travels through a focusing (Fourier) lens, it gradually diminishes in diameter down to a minimum size, before it grows larger again. The minimum width is called the beam waist, and the angle at which it spreads out beyond the beam waist is called the beam divergence.

Near and far fields: These terms only have meaning after light has passed through a focusing, or Fourier, lens. The near field is the space along the beam path after the lens but before the beam has traversed the Rayleigh length. The far field is everything far beyond the Rayleigh range, in which the beam size increases linearly.

Rayleigh range: The Rayleigh range is the distance it takes for the beam's cross sectional area to double after reaching its minimum at the beam waist.

Paraxial approximation: The paraxial approximation exists in many math areas, and is the approximation of $\sin(x)$ and $\tan(x)$ as x (and possibly of $\cos(x)$ as 1 or $1\text{-}x^2$). In lasers this approximation can greatly simplify our math but is only accurate when the beam's direction of travel is almost parallel with the normal vectors of the components in our optical system.

Orbital Angular Momentum

The Micro-Photonics Laboratory at Clemson University specializes in light with orbital angular momentum, or OAM. OAM light has special properties and interactions with objects. Here we will focus on its basic properties.

The primary features of OAM light are caused by a "twisting" action. OAM light can be thought to twist around its axis of propagation. You can think about OAM light rotating around the central axis, like fan blades, at a frequency close to that of the light.

If you inspect the intensity profile of an OAM beam, you'll find no light at the center; the light travels in one or more rings. This is logical because any light exactly on the central axis cannot rotate around it. If you inspect the phase profile of an OAM beam cross section, you'll find a phase singularity at the center [3]; that is, the phase is undefined at the center of the beam, but that is acceptable because there is no OAM light there. In addition, the phase angle increases roughly linearly as you move in a circle around the center, and the phase angle at each point is ideally independent of its distance from the center. The number of times the phase angle increases by 2π as you traverse this circle is known as the OAM number.

The OAM number, or "topological charge," often represented by the variable m, is the predominant number used to characterize OAM beams. Typical types of OAM beams include Laguerre-Gaussian (LG) modes and Bessel-Gauss beams. [3] The radius of each of these beams is also important and is independent of the OAM number.

The HOBBIT System: A Tale of Light and a Ring

Many of the recent experiments at the Micro-Photonics Laboratory have employed an optical system called the HOBBIT system: Higher Order Bessel Beams Integrated in Time. [4] The system was designed and modified at the Micro-Photonics Laboratory, and explaining it fully is, of course, a paper of its own, but I would like to give a basic, non-mathematical description of its assembly here.

The HOBBIT system has gone through multiple designs, but it is always an optical system that takes in a Gaussian beam in one end and outputs a ring-shaped OAM perfect vortex beam out of the other end, or else chosen portions of that OAM beam. The HOBBIT is special because of the speed (MHz) with which it can change between transmitting these different portions of the ring and different OAM numbers. [4] In other words, the HOBBIT allows us two degrees of freedom in rapidly choosing what kind of beam we transmit and where we transmit it; between experiments, we can also choose the OAM ring size. It has been applied in experiments to find reliable communication channels through various media or to maximize power delivery through turbulent media.

The latest version of the HOBBIT system is composed of two AODs (acousto-optical deflectors), two log-polar optics, and a Fourier lens. The two AODs control the OAM state and the positions along the ring transmitted in a given moment, and the HOBBIT obtains its speed from their rapid transmission rate. The log-polar optics wrap the linear AOD outputs into the ring shape.

Capillary Wave Basics

Capillary waves are waves whose characteristics are predominantly controlled by the capillary effect, that is, by surface tension, adhesion, and cohesion; however, we will treat these forces all as surface tension.

To note, capillary waves are not the only type of wave. Gravity waves are dominated by the effects of gravity's restoring force on the peaks of the water, but at small enough wavelengths, gravity's effect is negligible, outweighed by the capillary effect. Sometimes recognized is their area of overlap, called the gravity-capillary wave. For the moment we will ignore this region where both have an effect and instead focus on the boundary between waves where the capillary effect is the strongest and those where gravity is the strongest. This boundary is best characterized as the place where the dispersions of gravity and the capillary effect cancel out. [1]

Dispersion describes the way that wave crests disperse as they travel; that is, the way that waves travel in groups, and their crests and troughs may travel at a speed (the phase velocity) slower or faster than that of the group of waves (the group velocity). If the phase velocity is faster than the group velocity, crests and troughs will seem to disappear as they reach the front of a group of waves, and new ones will appear out of the back of the group. If the group velocity is faster than the phase velocity, the waves proceed the other way around.

At this boundary between gravity and capillary waves, the two dispersions cancel out, so that the phase velocity equals the group velocity, with no waves moving through the group. This equilibrium occurs at the capillary wavelength and critical velocity [1]:

$$\lambda_c = 1.73 \text{ cm}, \quad v_c = 23.1 \text{ cm/s}$$

Near this boundary between gravity and capillary waves, both forces must be taken into account, yielding the dispersion relation [1]:

$$\omega^2 = g k + \frac{\gamma k^3}{\rho} \tag{1}$$

However, as long as the wavelength is much less than λ_c, then gravity's effect is negligible, the phase velocity is less than the group velocity, and the wave is a capillary wave. Then we can make the following approximations. The dispersion relation reduces to [1]:

$$\lambda = 2\pi \left(\frac{\sigma}{\rho} \right)^{1/3} \omega^{-2/3} \tag{2}$$

Capillary waves have a wavelength-phase velocity relationship of [1]:

$$\frac{v_p}{v_c} \simeq \left(\frac{k\, l}{2} \right)^{1/2} \tag{3}$$

and approximately a constant phase velocity-group velocity ratio of [1]:

$$\frac{v_g}{v_p} \simeq \frac{3}{2} \tag{4}$$

Characterizing Capillary Waves

Capillary waves are affected by the properties of the water they travel through. Of special interest to us is the concentration of surfactants in the water. Surfactants are chemicals who decrease the surface tension at the boundary between two mediums; I am yet unsure if surfactants affect elasticity. However, as a first step, it should suffice for us to measure the surface tension of our wave, which requires we know how surface tension affects the form of the wave.

Capillary waves are approximately sinusoidal in shape [2], and their height decays exponentially as the waves move away from their source. Therefore, there are three variables we need to characterize a capillary wave. We must know two of the following three: its frequency, wavelength, and phase velocity; we must also know its rate of decay, most easily measured as decay length- the distance the wave travels before it decays by a factor of $^1/_e$. To note, all writing beyond this point is concerned with capillary waves of the Laplace (transverse) mode, not the longitudinal and fast-decaying Marangoni modes.

It is tempting to attempt to estimate the surface tension using only the surface tension of a known-frequency capillary wave on water, and as an approximation, this is valid, using the dispersion relation, equation 2 above.

However, according to A. Said Ismail [2], elasticity also has a small effect on wavelength, as large 2% for aqueous octanol solutions. We would be left without enough information to precisely calculate the surface tension, except that decay length is mostly determined by elasticity, so that we can use decay length to determine elasticity, and then both elasticity and wavelength to determine surface tension. To determine whether or not we truly need decay length information, further research must be done into the precision needed on our surface tension measurements in order to determine surfactant concentration, i.e., how much do the surfactants of interest change the surface tension of water? For example, if they only change water's normal surface tension by 10%, then accounting for elasticity will be necessary.

A Successful Method of Characterization

Researchers at Queen Mary University of London, including Dr. A Said Ismail, conducted research on a new optical method for characterizing capillary waves. Their 2021 paper [2] gives an extensive review of the current state of the research on analyzing, generating, and characterizing capillary waves.

Their work uses a 0.255 mM solution of 1-octanol, because it has low volatility and negligible barrier resistance, and its surface tension and equation of state are well-known from current literature. They chose this medium to have a known reference against which to measure the reliability of their technique. The chosen concentration puts the surface near maximum damping, so that changes in elasticity will have little effect on the decay length L_d .

After extensive research and reporting of the most effective methods to generate capillary waves, the group chose to use a mechanical method to produce the waves. [2] Mechanical methods are simple to set up and vary and can generate different wave symmetries, but they produce non-linear oscillations near the resonant frequency of the mechanical setup. A speaker generates the desired frequency and is attached by an arm to a glass slide which slightly penetrates the water and generates flat wave fronts.

Their work uses a new method of characterizing capillary waves. Namely, they place a light under the water, and place a camera relatively close above the surface. The capillary waves act as small lenses, and because the camera is closer than the focal length of the effective lenses, the amount of light received at a given pixel is linear with respect to the curvature of the wave there. The curvature of each crest is approximately proportional to the height, so that using a simple sinusoidal fit, the wavelength and relative height can be easily detected.

This experiment yielded good accuracy- on average 0.2% away from the theoretical wavelength and 5% away from the theoretical decay length. Neither of these variations were statistically significant relative to the experiment's standard deviations of 0.5% and 5%, respectively. From these values, elasticity and surface tension were determined within 1 mN/m and 0.3mN/m, respectively. They claim this is noteworthy, especially in light of the fact that they approached critical damping, where elasticity has the smallest effect on decay length.

Method

Our aim is to use the strengths of our laboratory to characterize the waves in a unique and practical way. As an independent check of any experimental findings, it is helpful to perform a parallel simulation; below is explained a potential modeling approach that shows promise. This analysis would be performed in MATLAB and would build off of the simulation code for similar problems used in the Micro-Photonics Laboratory.

A Potential Mathematical Analysis Method: Superposition of Plane Waves

We considered the following mathematical method to model the interaction between an OAM beam and a capillary wave:

Fourier optics is built on the assumption that any picture can be represented as the sum of an infinite number of 2-dimensional sinusoidal functions. A beam's intersection with a given planar surface is a 2-dimensional picture, and plane waves incident on this surface have cross-sections that appear as sinusoids. Therefore, any beam can be represented at a surface as the sum of an infinite number of plane waves, and because the following interactions are linear, these same plane waves define the beam away from the surface. The method to implement this representation is called angular spectrum decomposition. This plane wave decomposition has been performed on LG beams with OAM [6], but not on portions of a perfect vortex beam.

Therefore, to model the OAM beam-capillary wave interaction, we plan to first decompose our perfect vortex into plane waves. Then, at the sinusoidal interface, we will account for the plane wave's angle from the normal vector during the following: We will account for the Fresnel reflection and transmission coefficients as point-by-point amplitude scaling; we will account for horizontal displacements of different portions of the plane wave, experienced because of the off-axis incident plane waves and the differences in distance traveled; and finally, we will account for phase delays experienced due to these different distances to the surface. Further research is required before simulating beam propagation away from the approximately sinusoidal surface of the capillary waves.

Conclusion

In conclusion, we have summarized some basic optics knowledge, with an emphasis on those topics most used in Clemson's Micro-Photonics Laboratory. We have summarized capillary waves and their properties pertinent to characterizing them with lasers. Finally, we have outlined one successful method of characterizing them and one approach to modeling their interactions with light beams.

References

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