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Modeling repairable system failure data using NHPP realiability growth mode.

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ABSTRACT

MODELING REPAIRABLE SYSTEM FAILURE DATA USING NHPP
RELIABILITY GROWTH MODEL

by

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Stochastic point processes have been widely used to describe the behaviour of repairable systems. The Crow nonhomogeneous Poisson process (NHPP) often known as the Power Law model is regarded as one of the best models for repairable systems. The goodness-of-fit test rejects the intensity function of the power law model, and so the log-linear model was fitted and tested for goodness-of-fit. The Weibull Time to Failure recurrent neural network (WTTE-RNN) framework, a probabilistic deep learning model for failure data, is also explored. However, we find that the WTTE-RNN framework is only appropriate failure data with independent and identically distributed interarrival times of successive failures, and so cannot be applied to nonhomogeneous Poisson process.

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I would also like to specifically thank Dr. Christian Hansen, my thesis advisor, for introducing me to the thesis topic and the field of reliability and for his helpful comments, which led to improvements in the presentation of this thesis. Thank you, Dr. Xiuqin Bai and Mrs. Lynnae Daniels, for agreeing to serve on my thesis committee.

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1 Introduction

Tragedies like the Deepwater Horizon oil spill, the Boeing 737 Max airplane accidents, and the Chernobyl disaster brought reliability issues in design to light. Reliability is the ability of a system or component to perform its required functions under specified conditions for a specified period of time, according to the IEEE Standard Computer Dictionary.[29] It is a measure of the likelihood that a system will not fail. A system is deemed reliable if it satisfies the specified performance standards and operates faultlessly for a specified period. Probability and statistics serve as a good tool for improving the reliability. While probabilistic modeling and statistical analysis cannot directly improve reliability, they can be used to predict reliability using experimental data, test data, or field performance failure data.[31] Stochastic processes are the most powerful mathematical tools for studying models in reliability theory. This study presents the fundamental concepts in stochastic modeling for the repairable system. It's crucial to understand how frequently failures might happen in order to minimize failures in repairable systems. This involves predicting when failures will occur. As a result, reliability engineers and practitioners must be educated on stochastic processes and models that are important for system reliability.

For the purpose of this thesis, a system is defined as a collection of two or more parts which is designed to carry out one or more functions.[7] Systems can be classified into two categories; repairable systems and non-repairable systems. Systems that are non-repairable are those that are not repaired when they fail. They are discarded after failure. A light bulb is an example of a non-repairable system. A broken light bulb cannot be fixed and must be replaced. A satellite is also considered non-repairable because of its complexity and location in space. Once a satellite is launched into space, it is not easily accessible for repairs. Since most systems are, at least in principle, repairable in nature, non-repairable system are commonly referred to as a component or part. Typically, a repairable system is made up of component or a part that is discarded or replaced completely upon failure. Components are parts of the larger system that have a direct effect on the system's reliability.

As the name implies, repairable systems are restored to operation upon failure by means other than replacing the entire system. Asher and Feingold[7] in their book define a repairable system as "a system in which, after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method, other than replacement of the entire system." Repairable systems house components that are non-repairable. Common examples of repairable systems include automobile, computers, printers, etc. If a component or subsystem fails and renders an automobile inoperative, that component is typically repaired or replaced rather than purchasing a new vehicle. The engine, transmission, brakes, tires, and electrical systems are all examples of repairable parts of a car. When these parts break down or fail, they can be fixed to get the car back to working properly. Being able to repair an automobile can increase its lifespan and decrease the need for complete repairs, which can be advantageous from an economic standpoint. In other cases, repairing the system can be more expensive than replacing it entirely. An example is mobile devices. Repairing a broken smartphone or laptop, such as one with a cracked screen, a motherboard issue in laptops, or an issue with other internal components, can frequently be more costly than purchasing a new one. This is due to the high cost of replacement parts, as well as the specialized skills required to repair the complex system. It is important to understand the type of system being analyzed and use the appropriate reliability methods and tools. We'll use the terms

1. *Part or Component* refers to an item that cannot be repaired and is discarded after it fails.

2. *Socket* is an equipment position which, at any given time, holds a part of a given type.[7]

3. *System* is a collection of two or more sockets and their associated parts, interconnected to perform one or more functions.

In this study, when we refer to a system, we mean a repairable system. Ascher and Feingold presented the following example of a "happy", "sad" and "noncommittal" system from different sets of data: Figure 1 depicts failure data from three

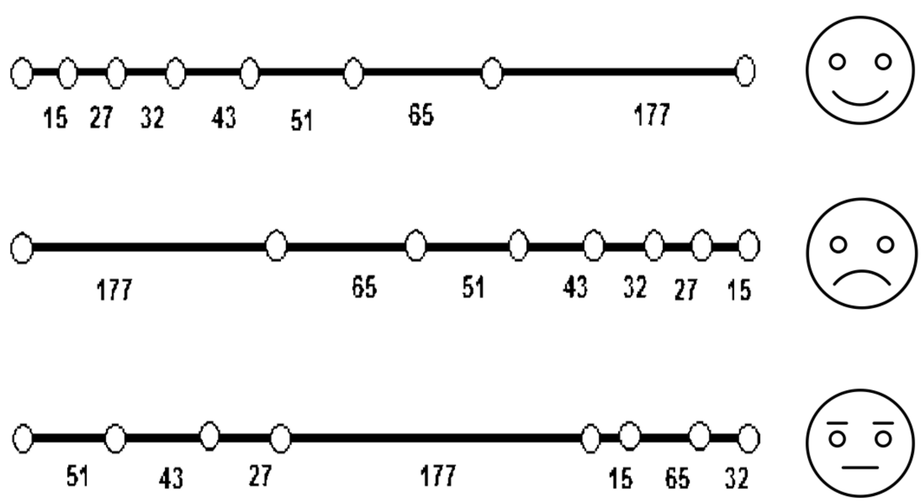


Figure 1: Times Between Successive Failures of Happy, Sad and Non-Committal Systems. (Source: Ascher & Feingold, 1984)

different systems. The first system, the "happy" system, fails less frequently. Failures occur more frequently in the "sad" system, but nearly equally frequently in the non-committal system. The happy system suggests that system reliability improves with age, whereas the sad system suggests that system reliability deteriorates with age. The non-committal system's reliability is neither improving nor deteriorating. According to Asher and Feingold, practitioners and reliability engineers are unaware of the different cases of interarrival times of failures in a system and mistakenly believe that all systems have independent and identically distributed (i.i.d) times between subsequent failures. They did, however, highlight the use of point process models to analyse data from repairable systems.

1.1 Objective of Study

The purpose of this thesis is to investigate and gain a better understanding of models utilized on failure data in repairable systems. Also, to understand the behavior of these systems.

1.2 A Brief History to Reliability

The word reliability can be traced back to 1816 and is first attested to by the poet Samuel Taylor Coleridge.[52] However, the modern field of reliability engineering did not emerge until the twentieth century. Dr. Walter A. Shewhart at Bell Labs promoted product improvement through statistical process control in the 1920s, around the time that Waloddi Weibull was working on statistical models for fatigue. W. Weibull's work later lead to the development of the Weibull distribution in the 1950s.[63] The modern use of the term reliability was defined by the United States military in the 1940s, as a product that would operate when expected and for a specified period.

During WWII, a group in Germany led by Wernher von Braun worked on developing the V-1 missile.[4] Following the war, it was revealed that the first ten V-1 missiles were all a failure. Despite efforts to supply high-quality parts and pay close attention to detail, all of the first missiles either exploded on the launch pad or landed "too soon" (in the English Channel). Throughout the 1940s and 1950s, poor field reliability of military equipment drew attention to the need for more formal methods of reliability engineering. Ad hoc studies were initiated under the leadership of the US Department of Defense (DoD), which eventually coalesced into a new discipline, reliability engineering. Several groups began formal research on reliability issues. These had a significant impact on how the statistical treatment of the area. The United States Department of Defense established the Advisory Group on the Reliability of Electronic Equipment (AGREE) in 1952, and it later produced widely used specification standards for the reliability of electronic equipment. The Professional Group on Quality Control of the IRE was formed in 1949 and became the IEEE Reliability Society in 1963. Also, the military handbook MH-217 was published around the 1960s. It was proposed to provide a standard for the prediction of failures of electronic military parts and systems in order to improve the reliability of the equipment being designed.

In the 1970s, interest in the risks and safety issues associated with the construction and operation of nuclear power plants grew in the United States and as well as other parts of the world. A large research commission led by Professor Norman Rasmussen was formed in the United States to investigate the problem. The multimillion-dollar project yielded the Rasmussen report, WASH-1400 (NUREG-75/014). Despite its flaws, this report is the first serious safety analysis of such a complex system as a nuclear power plant.[2] Similar research has been conducted in Europe and Asia. The oil crisis renewed interest in energy efficiency in Norway, particularly in the offshore oil industry. Engineers developed and used risk analysis and decision analysis techniques to improve the reliability of oil and gas pipelines while also supporting the reduction in asset costs.

The semiconductor industry began to expand in the 1980s, and the field of reliability engineering began to be applied to the software industry. Reliability engineering entered the digital age in the 1990s. As more industries began to use computers and software, having reliable software and hardware became increasingly vital.

In the twenty-first century, the field of reliability engineering has continued to evolve. Engineers are facing new challenges as a result of new technologies such as the internet of things (IoT) and big data. To meet these challenges, engineers are employing techniques such as predictive maintenance and artificial intelligence (AI). In recent years, Prognostic Health Maintenance (PHM) has emerged as a key enabler of reliable and efficient systems.[34] PHM is a multifaceted discipline that connects the study of failure mechanisms to product life cycle management. Health management is the technique of using diagnostic and prognostic information to intelligently manage the use and maintenance of a system. The end goal is enhanced reliability, safety, and minimized maintenance cost. According to the IEEE Reliability Society, PHM goes hand-in-hand with reliability, as PHM can directly improve effective reliability, availability, mission reliability, system safety, and maintenance by being able to provide on-condition health, prognose pending failure, and predict future health status. PHM was first introduced in the Department of Energy (DoE) and the DoD. The primary motivation for its implementation was to simultaneously reduce operating and support costs for the military and industries in the United States while increasing system availability. PHM gained popularity in both academia and industry. Over the last decade, it has seen growing research across multiple disciplines, such as machine learning and stochastic modeling in reliability engineering.[34]

1.2.1 Failure Data

The advent of the electronic age, accelerated by the Second World War, led to the need for more complex mass-produced component and parts with a higher level of variability in the parameters and dimensions involved. The 1940s and 1950s military equipment field reliability experience brought attention to the need for more formal reliability engineering approaches. This led to the gathering of failure data from the field as well as the analysis of test results. In the mid-1960s, efforts at the

UK Atomic Energy Authority (UKAEA), Royal Radar Establishment(RRE), and Rome Air Development Corporation, US (RADC) led to the creation of failure data-banks. Since the 1960s, failure data have been published.

1.3 Thesis Outline

The rest of this paper is organized as follows: Section 2 gives a brief review of the existing literature on modeling failures in repairable systems. Section 3 introduces basic reliability terms such as the hazard function, the mean cumulative number of failures, and the bathtub curve and points out the confusion with the term ”failure rate” in reliability literature. The third section also provides definitions and concepts of stochastic point processes that are relevant to the study, as well as point process models often used in repairable systems. The models used for the study, as well as the statistical tests and methods carried out in the research, are discussed. And lastly, an overview of the Weibull-Time-to-Event Recurrent Neural Network (WTTE-RNN) model. Section 4 focuses on a case study on modeling hardware failure data from an High-Performance computing (HPC) system. A description of the data used and how it was preprocessed. The results from the case study are also presented. Lastly, in the conclusion section, a brief summary of what the study entailed is provided.

2 Literature Review

Over the past 50 years, the theory and methods of repairable system reliability have been extensively developed and acknowledged in a number of publications. For this section of the study, we investigate related studies on statistical methods and mathematical models relevant to the reliability of repairable systems.

One of the key areas of research on statistical methods for reliability in the 1960s and 1970s was concerned with drawing parametric inferences related to components using univariate life distributions considering both censored and uncensored observations. Much of the work in this area is reviewed in books by Bain (1978), Lawless (1982), and Nelson (1982).[9, 40, 48] The exponential, lognormal, gamma, and Weibull distributions are all commonly used for modeling component failure data, though the Weibull and exponential are more popular in reliability practice. Prior research on parametric statistical inference with both complete and censored data focused on the exponential distribution (e.g., Bartholomew 1957).[11] Up until the 1960s, almost all the studies on statistical analysis of the reliability of components assumed failure times to be exponentially distributed. However, the study by (Zelen et. al., 1961)[67], showed that the exponential distribution was not an appropriate model in many situations, while the Weibull distribution became more popular for life distributions. In recent years, the Lindley distribution has been used for modeling failure data and reliability.[39, 10] The Lindley distribution was developed by British statistician D. V. Lindley in a paper published in 1958.[42] The properties of the distribution itself remained relatively unstudied until a 2008 publication by Ghitany et al., but even so, the Lindley distribution has been used to model real-world data, including failure data.

Non-parametric estimation attempts to estimate an unknown function from a sample of data in the absence of any distribution assumptions. As a result, the non-parametric estimation approaches have no significant related parameters. Kaplan and Meier's work in 1958 [38] began the advancement in lifetime data analysis with incomplete (or censored) data using non-parametric techniques. Their work resulted in the development of the product-limit estimate, popularly known as the Kaplan-Meier estimator. This is a well-known method for calculating survival over time from lifetime data despite having censored data. Closely related to this is the Nelson-Aalen estimator. Since no distribution assumptions are required, one important application of the Nelson-Aalen estimator is to check the fit of parametric models graphically, which is why Nelson first introduced it in 1969 & 1972. Altshuler (1970) [5] developed the Nelson-Aalen estimator independently of Nelson in the context of competing risks animal experiments. Later, Aalen (1975, 1978b) utilized estimator on Markov chains and other event history models.[3] This estimator has become a widely used tool in reliability. Breslow and Crowley (1974)[15] studied the asymptotic behavior of the Kaplan-Meier and Nelson-Aalan estimators. They showed that the two estimators are consistent and asymptotically normal under certain conditions. Nelson W. (1982)presents hazard plots for multiple censored data.[47] Nelson W. (1982) and J.F. Lawless(1982)[30] have shown that the cumulative hazard plot can be useful for rough estimation of parameters in parametric models and distributions. In 2020, Jiang et al.[35] developed a non-parametric likelihood based estimation procedure for left truncated and right censored data using B-splines. This method is useful for dealing with data that was collected considerably later than the system's production or installation date.

Modeling the reliability of repairable systems caught the attention of professionals and researchers working in the field after the publication of the book by Ascher and Feingold (1984). The book has served as a primary reference for a large number of studies on the reliability of repairable systems.[62, 14, 64, 61, 69] The reliability models for repairable systems and non-repairable parts have some similarities. However, they cannot be modeled with the same reliability models. Doing so will lead to errors and inaccurate results.[6] The article by Basile et al. (2004)[13] focuses on the identification of reliability models for non repairable and repairable systems.
Non-repairable system reliability models employ the univariate probability distributions. The non-repairable system consists of only one component in operation. Upon failure, the component is discarded.[7] The renewal process is an appropriate model to use when the system is comprised of only one component. The study of non-repairable systems consists of determining the distribution of failure times. Of late, there have been some interesting works published on reliability models for non-repairable systems. Fang et al. (2021) in their work, "Reliability evaluation of non-repairable systems with failure mechanism trigger effect," proposed a method that integrated "Petri Net" and Monte Carlo to evaluate reliability.[26] Zhai et al. (2018) proposed a combinatorial model, named aggregated binary decision diagram (ABDD) for reliability analysis of non-repairable parallel phased-mission systems (PMS) subject to dynamic demand requirements.[68] Yañez et al. (2002) proposed a generalized renewal process (GRP) to address the disparities in renewal states whether the system is renewed to a "new" state (i.e., as good as new) or repaired to the condition it was in immediately before failing (i.e., as bad as old).[66]

In an HPC system, replacing components upon failure does not restore the system to "as good as new" condition, hence Crow's (1993) claim that the renewal process is not a suitable model.[22] There are two approaches that have been adopted in relevant studies to model the reliability of repairable systems: stochastic point processes and differential equations. Ascher and Feingold (1984) provide a brilliant survey and discussion of five stochastic point process models that are applicable to repairable systems. They emphasize the importance of the times between failures being independently and identically distributed for the Homogeneous Poisson Process (HPP). Çinlar (1975) defines a HPP as an orderly stochastic process with stationary, independent increments. HPP is the simplest

model for failures in a repairable system.[17] Many researchers have looked into the link between the nonhomogeneous Poisson process (NHPP) and the HPP including Brown and Proschan (1983), Bartoszyński et al. (1981) and Feigin (1979).[12, 16, 27] In NHPP models, the intensity function is assumed to depend on the cumulative system operating time, i.e. the age of the system, and not necessarily on the time of the most recent failure. (Ascher & Feingold, 1984). Modeling repairable systems that deteriorate or improve over time requires the use of a NHPP. A new model proposed by Ibrahimi (1993) on conditional NHPP provided valuable techniques for analyzing the reliability of complex repairable systems that are influenced by external factors. His work on conditional NHPP can also be utilized to model reliability growth.[25] The most commonly used model for reliability growth is proposed by Crow (1974). The Weibull Process (different from the Weibull distribution) is also known as the Power Law model. Other NHPP reliability growth models include The Cox-Lewis (Cozzolino) model proposed by Cox & Lewis (1966) and later by Cozzolino (1968)[19, 20]. This model also known as the log-linear model is utilized in situations where the Power Law model is rejected by a goodness-of-fit test. There is also a generalized version, Cozzolino's "Initial Defects" Model.
The differential equation reliability growth models is based on an approach quite different from the point processes approaches. These models can be very useful technique for reflecting known underlying mechanisms which contribute to reliability growth. For instance, if the rate of improvement is known to be inversely proportional to some power of time, this fact can be explicitly considered. Schafer et al. (1975) treat these models in detail in their article "Reliability Growth Study".[54] Differential equation reliability models include what has become known as the IBM model by Rosner (1961)[51], the exponential single-term power series model by Perkowski and Hartvigsen (1962), etc. Lloyd-Lipow model developed by Lloyd and Lipow (1962) estimates the reliability of a system comprised of a single failure mode.[44] Other models include the Aroef model by Aroef (1957) and the Simple Exponential Model.

Much research on repairable systems analysis have been conducted. Here are a few more notable works. In the study by Ascher and Hansen (1998) titled "Spurious Exponentiality Observed When Incorrectly Fitting a Distribution to Nonstationary Data" [8] , the authors stressed that ignoring the chronological ordering of interarrival times may lead to misleading results about the system's behaviour. They also refuted the notion that exponential distribution and HPP can be used interchangeably. Ascher and Hansen point out, "The close mathematical relationship between the HPP and the exponential distribution has led many practitioners to incorrectly-use the two concepts interchangeably, and many falsely believe that if the assumed "distribution" of interarrival times exponential, eg, when represented in a histogram, then it follows that the HPP model can be justified as an appropriate model for the system failures." In addition to failure time data, modern reliability databases typically also include information on the type of failure, the type of maintenance, and other factors. For recent literature Lindqvist (2007) reviewed basic modeling approaches for failure and maintenance data from repairable systems and presented a framework where the observed events are modeled as marked point processes, with marks labeling the types of events.[43]

There has been growing literature in the area of machine learning for reliability engineering. The rise in data availability and computer capacity in recent years has fostered significant progress in machine learning research. This will have a profound impact on academia and industry. Supervised learning algorithms have been used to estimate the remaining useful life (RUL), which is the number of remaining years that a component will function. Kang et al. (2021) propose a model that applies normalization and principal component analysis for predicting remaining life of the failure of equipment in continuous production lines.[37] Vanderhaegen et al. adopted a deep neural network model be adopted to predict a specific human car driving violation. [60] Recurrent neural networks (RNNs) are a type of neural network that are suited to handling time series data and other sequential. data[23] In his thesis, Egil Martinsson (2017) [45] developed an RNN model for predicting time to event.The model known as the Weibull Time to Event Recurrent Neural Network (WTTE-RNN) estimates the time to the next event in the case of discrete or continuous censored data and outputs parameters of the distribution of time to the next event.

3 Methodology

3.1 Basic Reliability Terms

Reliability Function: The reliability function $R(x)$, also known as the survival function $S(x)$, is the probability of an item operating for a certain amount of time without failure. The reliability function is the complement of the cumulative distribution function. If X is a random variable representing the time to failure of a component, the reliability function, $R(x)$ is defined as

$$\begin{aligned} R(x) &= Pr(X > x), \quad x \geq 0 \\ &= \int_t^{\infty} f(s)ds \end{aligned}$$

$R(x)$ represents the probability that the component is operating correctly at time x. $R(x)$ is a monotone non-increasing function of x. The item is assumed to be working properly or operational at time $x = 0$ and no item can work forever without failure: i.e. $R(0) = 1$ and $lim_{x\to\infty}R(x) = 0$. We can also define the probability of failure of component at or before time x, $F(x)$, the cumulative distribution function (CDF) as:

$$F(x) = P(X \leq x), \quad x \geq 0$$

The distribution function can also be defined in terms of reliability as:

$$F(x) = 1 - R(x)$$

F is a continuous and differentiable function "almost everywhere" with probability density function f defined as the derivative of the cumulative distribution function.

$$f(x) = \frac{dF(x)}{dx} \quad x > 0$$

Hazard Function: Also known as the hazard rate or force of mortality(FOM), given by $h(x)$. $h(x)dx$ is approximately the probability that a component fails in a small time interval $(x, x+dx)$ given that it has survived from time zero until the beginning of the time interval. The hazard function is the ratio of the probability density function to the survival function. It is applied to non-repairable items (component or part). x represents the time to failure of a component. The hazard function can be expressed as probability of failure between time x and $x + dx$, given that there were no failures up to time x. The probability expression is written as:

$$P(x < X \leq x + dx | X > x) = \frac{P(x < X \leq x + dx)}{P(X > x)}$$

The hazard function is derived below:

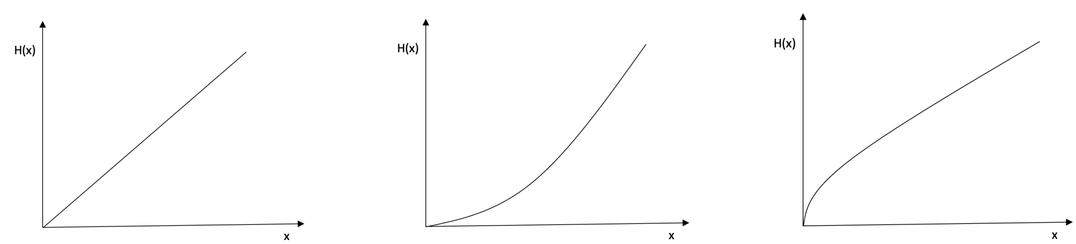
$$h(x) = \lim_{dx \to 0} \frac{P(x < X \leq x + dx)}{dx} \cdot \frac{1}{S(x)}$$
$$h(x) = \frac{f(x)}{1 - F(x)}$$

The simplest models, such as MIL-HDBK-217F(1991), assume a constant hazard rate. However, field data revealed that most systems do not have a constant hazard rate. Some probability distributions such as the Weibull distribution are often used in reliability engineering to represent time-dependent failure behavior. Other models, such as the "roller coaster" [65] model and mixed distribution models[33], have been proposed to model hazard rates.

Another measure of reliability is the **cumulative hazard function (CHF)**, defined by

$$H(x) = \int_0^x h(s)ds \quad x \geq 0$$

You can interpret $H(x)$ as the cumulative amount of hazard up to time x.



(a) Constant hazard rate (b) Increasing hazard rate (c) Decreasing hazard rate

Figure 2: Example of three types of cumulative hazard function, (a) constant hazard rate, (b) increasing hazard rate and (c) decreasing hazard rate

In figure 2, $H(x)$ is plotted against x. The plot of constant hazard rate, which is a linearly increasing function of time, implies that the hazard rate does not change with age. A concave-up plot of CHF against time implies an increasing hazard rate, whereas a concave-down plot of CHF implies a decreasing hazard rate.

Rate of Occurrence of Failures: For repairable systems, the intensity of failures is described by the rate of occurrence of failures (ROCOF) or intensity function. ROCOF is the probability of failure in a small interval divided by the length of the interval.[14] The estimation of ROCOF can become a very complicated procedure. [18] When $M(t)$ is differentiable, we defined ROCOF as

$$m(t) = M'(t)$$

The ROCOF is the derivative of the mean cumulative number of failures (MCNF). $M(t) = E[N(t)] =$ expected number of events (failures) in $(0, t]$

$$
\begin{aligned}
m(t) &= \lim_{\Delta t \to 0} \frac{M(t+\Delta t) - M(t)}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{E[N(t+\Delta t)] - E[N(t)]}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{E[N(t+\Delta t) - N(t)]}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{E[N(t, t+\Delta t)]}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{\text{expected number of failures in } (t, t+\Delta t)}{\Delta t}
\end{aligned}
$$

So expected number of failures in $(t, t+\Delta t) \approx m(t) \cdot \Delta t$
ROCOF is different from the hazard rate as it is a property of a sequence of failure times as opposed to a property of a single time to failure.

The ROCOF $m(t)$ is has some resemblance to the hazard rate $h(x)$.[32] The quantity $h(x)\Delta x$ is approximately the probability that a component will fail within the time interval $(x+\Delta x)$ given that the component is operating at time x. Similarly, the quantity $m(t)\Delta t$ is approximately the probability that a system will fail within the time interval $(t, t+\Delta t)$. Given these similarities, it is not surprising that the two functions are being confused with one another.[32]

The "Failure Rate" Confusion: There have been some misunderstandings regarding the use of terminologies to describe the system and its components, especially with the term "failure rate." Ascher (1984) argues in his book about the misuse of the term "failure rate." The term was frequently used interchangeably in literature for both force of mortality (FOM) or hazard rate of a non-repairable system and rate of occurrence of failures (ROCOF) of sequences of failures in a repairable system.[56, 57, 58] A subtle source of confusion about "failure rate" is the improper use of the term in an official publication, MIL-HDBK-217D (1982). Reliability engineers and researchers can make poor analysis decisions due to a lack of distinction.

The term has also been used to express the reliability of non-repairable components and non-repairable components functioning inside a repairable system. It also has been used to express the reliability of repairable systems. However, the meaning of the term failure rate is different in each of these contexts. Hence, the term "failure rate" must be avoided.[8]

Bathtub Curve: The ROCOF is generally a function of time and may follow the bathtub curve. Figure 3 depicts a typical bathtub curve. This is perhaps the most famous graphical representation in the field of reliability. It is named the bathtub curve based on the fact that its shape resembles a cross-sectional view of a bathtub. The curve is divided into three different sections. The burn-in period also known as early failures or infant mortality failures. The middle section is referred to as the useful life or random failures period, and it is assumed that failures occur at random, with a constant ROCOF. The wear-out failures are described in the latter section of the curve, and it is assumed that the ROCOF increases as the wear-out accelerates. The burn-in stage is characterized by

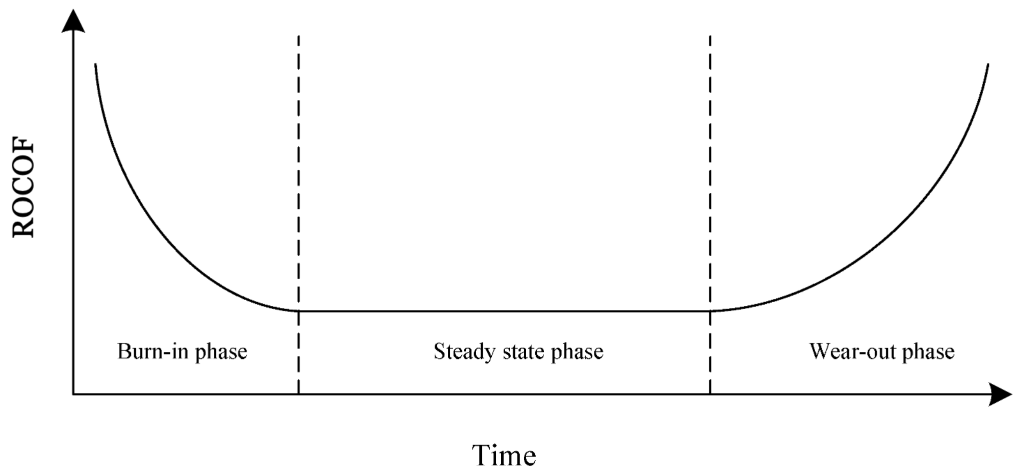
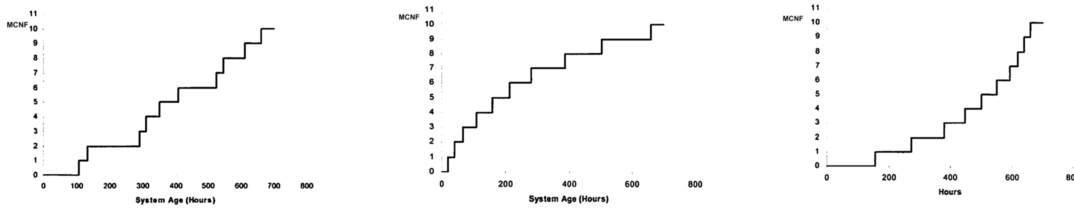


Figure 3: Bathtub shaped ROCOF

a high initial ROCOF that gradually decreases when defective parts fail early. This is followed by a slower, steady ROCOF dominated by randomly distributed failures. Failures during this time frame are usually covered by warranties. As parts wear out, the ROCOF rises again at the end of life. Failures in this stage can be attributed to aging, fatigue, wear-out, etc. and they are to be expected. Accelerated life testing[46] subjects the components or subsystems under test to extreme operating and/or environmental conditions, such as high temperatures, in order to induce failures in a short period of time. This enables the fast identification of weaker components.

Ascher (1984) also suggests that the bathtub curve for a repairable system substantially differs from that of a component or part. He claimed that the time between successive failures in the burn-in phase of the repairable system tends to increase, so the rate of occurrence of failures tends to decrease. However, failures of a part or component tend to occur more frequently, i.e., the interarrival times tend to become smaller in the initial phase of the bathtub curve. Even though he does not discuss the reasons for these types of behaviors, these differences in behaviors could be attributed to the lifetime of the systems. Typically, a repairable system has an unlimited lifetime, while a component has only one.

Mean Cumulative Number of Failures (MCNF) The cumulative plot is often used to visualize the mean cumulative number of failures for a repairable system over the age of the system. It is a great tool for identifying patterns in data as it reveals trends in the occurrence of failure. Figure below shows three different cumulative plots. The plot of MCNF reveals the evolution of failures



(a) Stable system (b) Improving system (c) Worsening system

Figure 4: MCNF plots for three different types of systems Source:[59]

with time. The shape of the curve indicates whether the number of failures in the system is increasing (or worsening), decreasing (or improving) or staying the same (or stable) over time. (see Figure 4) The system has a decreasing ROCOF and is improving if the MCNF curve is concave-down. A trendless (straight line) MCNF curve indicates that the system is staying the same. A concave-up or steep MCNF curve shows the system has an increasing ROCOF and is getting worse as there are more failures as time progresses.
The MCNF is calculated incrementally for each failure event, taking into account the number of systems at risk at the time. The MCNF must be adjusted for the presence of censoring i.e. when some observations do not have complete failure information. MCNF can be calculated based on parametric and non-parametric methods. These methods will be discussed in later sections.

Mean Time Between Failure (MTBF): MTBF is the mean operating time between failures, is a commonly used metric to represent the overall reliability of repairable systems. It represents how long a repairable system can operate without being interrupted by a failure. X_i for $i = 1, 2, 3, \ldots$ represent the time between failures or interarrival time, and the mean or expected time between failures is represented by $E[X_i]$.

The arithmetic mean value of the reliability function, $R(x)$, is another way to define MTBF, as shown below.

$$\text{MTBF} = \int_0^\infty R(x)dx$$

Assuming a constant rate of occurrence of failures, MTBF is defined as the inverse of the rate of occurrence of failure.

$$MTBF = \frac{1}{\lambda}$$

where λ is a constant rate of occurrence of failure.

There are numerous issues associated with the use of this metric and it also requires a number of assumptions. First, it assumes failures of a repairable system is a renewal process i.e., the system is "as good as new" after each repair. It also assumes the time between failures are independent and exponentially distributed with constant rate of occurrence of failure (i.e a HPP), meaning there is no early failures or wear-out. In their research[59], Trindade and Nathan argue that MTBF masks information and fails to account for trends in failure data. Different systems may have the same MTBF but very different failure behavior.

Mean Time To Failure (MTTF): This is the same as the mean time between failures but for non-repairable systems or components. MTTF is a maintenance metric that measures the average amount of time a component operates before it fails.

Censoring: This occurs when the precise failure time of the part or component is unknown. Data for which the exact failure time is known is referred to as "complete data." It is not always possible to collect data for lifetime distribution in a complete form. As a result, you may have to handle data that is censored or truncated. There are two common types of censored data in reliability; right censoring and left censoring.

1. Right Censoring. Observations may cease before the failure has occurred at time T. When a component's failure time is unknown but only known to exceed a certain point in time, it is said to be right censored. This can also be the case when the component is removed from the observation before it failed. C denotes the time at which observation ceases.

   $$T_i = min(T, C)$$

   Usually, 1 is used as an indicator for failure occurrence and 0 otherwise.

   Here's an example of right censoring: Suppose a study is conducted on the reliability of hard disk drives in a computer. We want to know how long the disks last before they fail. The computer has seven hard drives in their slots and we observe them over a one-year period. During this period, three disks failed and were replaced with new ones. At the end of the one-year period, we had seven hard disk drives that did not fail. We do not know the failure times beyond the one-year period because we stopped observing. The failure times of the four disk drives that did not fail during the observation period are known to be right censored. The three replacement disks are also censored.

2. Left Censoring. This happens when a component of a repairable system has been working for an unknown amount of time before we start observing it. This type of data is uncommon in the study of systems in reliability.

Failure Truncation and Time Truncation: The data are considered to be failure truncated if the monitoring of the repairable system ends after a specified

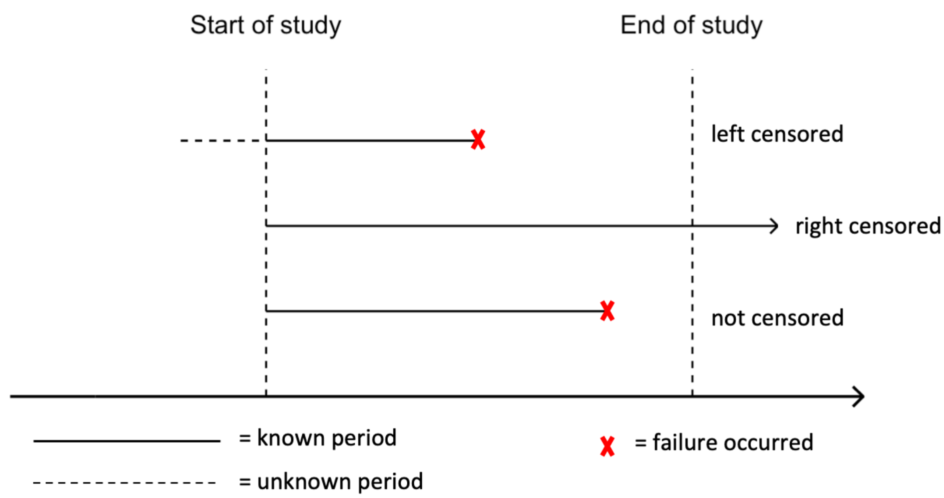


Figure 5: Types of Censoring

number of failures, say n. If monitoring stops at a predetermined time T, the data is said to be time truncated.[14] The data used for the case study in this study (see 4.1) is time truncated.

3.2 Basic Concepts for Stochastic Point Processes

Definition 1 *Let* $(\Omega, \mathcal{F}, P)$ *be a probability space and let* T *be an arbitrary set (called an index set). Any collection of random variables* $X = \{X_t : t \in T\}$ *defined on* $(\Omega, \mathcal{F}, P)$ *is called a stochastic process with index set* T*.*

Every $t \in T$ corresponds to some random variable X_t. A random experiment has the outcome $\omega \in \Omega$ according to the probability measure P. A realization (or sample path) of a stochastic process corresponds to the outcome ω. The set of all possible realizations of a stochastic process is called ensemble. Stochastic processes are classified according to state space and time domain. The state space can be either discrete or continuous, and the time domain can also be either discrete or continuous.

An important class of stochastic processes have the Markov property. A stochastic process has the Markov property if its future evolution depends only on its current state, and does not depend on past history.

A point process is a stochastic model that describes the occurrence of events over time. A model of a repairable system must describe the occurrence of events over time. In the context of reliability, events are typically referred to as "failures". In repairable systems analysis, the two main models used are stochastic point processes and differential equations. This thesis will focus on only stochastic point processes for modeling failures in a repairable system.

Definition 2 (Ascher and Feingold, 1984) *A stochastic point process is a mathematical model for a physical phenomenon characterized by highly localized events distributed randomly in a continuum.*

A stochastic point process is simply a random collection of points that fall into some space. The continuum is time in our application, even though (Cox 1966) labels this an oversimplification. Hence, we are dealing with temporal point processes. Temporal point processes represent a set of observed events that occur at various points in time.

Before we discuss some basic concepts in stochastic point processes, we introduce the counting process. Suppose a component in a repairable system is put into operation at time T_0. The first failure of the component will occur at time T_1. The faulty component will be replaced, and the system will be restored to normal operation. The second and third failure will occur at T_2 and T_3 and so on. We thus get a sequence of failure times $T_1, T_2, T_3, \ldots$. Let X_i be the time between failure $i-1$ and failure i for $i = 1, 2, 3, \ldots$. The counting process is used to model a sequence of events failures. The sequence of interarrival times, $X_1, X_2, X_3, \ldots$ will generally not be independent and identically distributed - unless the system is restored to "as good as new" condition.

Definition 3 (Ross 1996) *A stochastic process $\{N(t), t \geq 0\}$ is said to be a counting process if $N(t)$ satisfies:*

1. $N(t) \geq 0$
2. *$N(t)$ is integer valued*
3. *If $s < t$, then $N(s) \leq N(t)$*
4. *For $s < t$, $[N(t) - N(s)]$ represents the number of failures that have occurred in the interval $(s, t]$*

A counting process may be represented as either a sequence of failure times or a sequence of interarrival times, as both representation contains the same information about the counting process.[49]

Arrival and Interarrival Times
$T_i, i = 1, 2, 3, \ldots$ measures the total time from the start of operation T_0 to the ith failure and is called the arrival time to that failure. T_i is a random variable. $X_i, i = 1, 2, 3, \ldots$ is the interarrival time between failure $i-1$ and failure i. X_i is a random variable. Since the origin for X_i is the arrival time of the failure at $i-1$, we say that the X_i's are chronologically ordered.[7] $T_k = X_1 + X_2 + X_3 + \ldots + X_k$

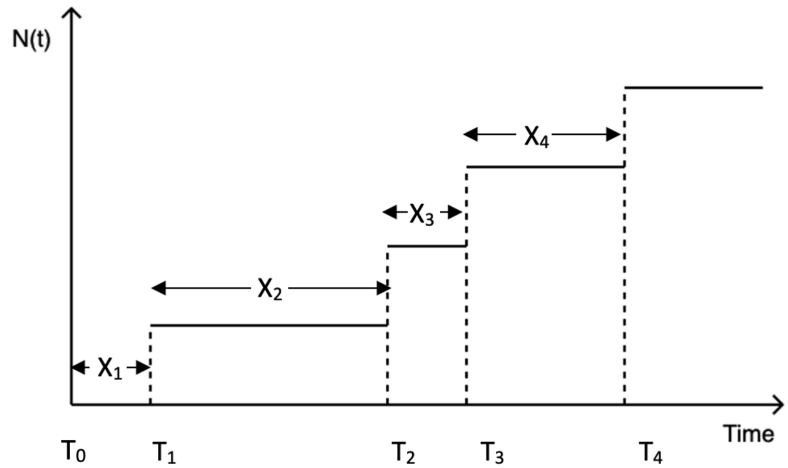


Figure 6: Relationship between the number of failures $N(t)$, the interarrival times, X_i and failure times, T_i.

The random variable, $N(t)$, is the number of failures which occur during $(0, t]$. $\{N(t), t \geq 0\}$ is the integer valued counting process which includes both the number of failures in $(0, t], N(t)$, and the moment in time $T_1, T_2, T_3, \ldots$ at which they occur. $N(t)$ represents the cumulative number of failures. The expected value of $N(t)$ is denoted at $M(t)$, i.e. $M(t) = E[N(t)]$, is the MCNF.

Independent Increments
If a counting process has independent increments, then there is no dependence between the number of failures in an interval and the number of failures in another interval. Mathematically, a counting process $\{N(t), t \geq 0\}$ is said to have independent increments if for all $t_0 \leq t_1 \leq \ldots \leq t_k, k = 2, 3, \ldots, N(t_1) - N(t_0), \ldots, N(t_k) - N(t_{k-1})$ are independent random variables.

Stationary Increments
A counting process $\{N(t), t \geq 0\}$ has stationary increments if for any two points $t > s \geq 0$, and any $\Delta > 0$ the random variables $N(t) - N(s)$ and $N(t + \Delta) - N(s + \Delta)$ are identically distributed.

Synchronous and Asynchronous sampling
The process is sampled by an asynchronous sampling when arrival times of failures are observed after the time the system was placed into operation, without prior knowledge of the failures before the observation began, i.e. system starts operation at $T_{-\infty}$ but observation begins at T_0. Sampling is synchronous when the system is put into operation simultaneously with the start of observation.[7]

Stationary point process
A stochastic point process is said to be stationary if its increments are stationary.

The renewal process (see section 3.3), has independent and identically distributed interarrival times, so under synchronous sample, it is a stationary sequence of interarrival times. However, the process is still not stationary because it does not have stationary increments.

Rate of occurence of failures for stationary, transient and nonstationary process
ROCOF can be stationary, transient or nonstationary. It might be constant but Cox and Lewis (1966) point out that "the possibility of a constant ROCOF is usually ignored...." It is shown (in Cox and Lewis, 1996) that for a stationary process

$$m(t) \equiv M'(t) \equiv \frac{d}{dt}E[N(t)] \equiv \frac{1}{E[X_i]} = \frac{1}{E[X]} = m,$$

the ROCOF is the reciprocal of the mean time between failure or interarrival times. The ROCOF of an asynchronously sampled point process, such as the renewal process, is just the reciprocal of the mean of each interarrival times. The time invariance of the asynchronous ROCOF characterizes a stationary process, i.e. events of the process occur at a constant rate.[8] The ROCOF for a transient process is time dependent at the beginning of processes but eventually approaches the constant $m = \frac{1}{E[X]}$.[7] A nonstationary point process has a time dependent ROCOF which could asymptotically approach a constant. It can approach an asymptote regardless of whether the process is sampled synchronously or asynchronously.

Intensity Function
The intensity function of a stochastic point process, $\rho(t)$, is the same as the ROCOF associated with a repairable system.

Improving and deteriorating properties of a stochastic point process
A repairable system is said to be deteriorating if the time between failures have a tendency to become shorter as it ages. When the times between failures have the tendency to increase, then the system is improving.[14]

Chronological ordering of component failure times
This is when component failure times are chronologically ordered. A key point to remember is that if data exist in a specific order (chronologically or otherwise), the data should be initially evaluated in that order.[36]

Typically, the assumptions we make about how a system ages and how failure and repair affect it will influence our choice of a repairable system model.

Renewal (or Perfect) Repair
A renewal repair presumes that the system is restored to like-new condition following the repair. If every repair is a renewal, then the time between failures is independently and identically distributed. The renewal process (see section 3.3.4) is a suitable model for the system.

Minimal Repair

Minimal repair means that the repair done on a system leaves the system in exactly the same condition as it was before the failure.[14] The assumption of minimal repair leads to the nonhomogeneous Poisson process (NHPP). The NHPP is often a good model for repairable systems because it can model systems that are deteriorating or improving.

3.3 Models Applicable to Repairable Systems

This section is a brief discussion of commonly used point processes which have been applied to model repairable systems.

3.3.1 Probabilistic Models: The Poisson Process

Poisson processes are the most commonly used probabilistic models in a counting process.

Definition 4 (Poisson Process) *A counting process $N(t)$ is said to be a Poisson process if*

1. *The cumulative number of failures at time $t = 0$ is 0, i.e. $N(0) = 0$.*
2. *$\{N(t)\ t \geq 0\}$ has independent increments.*

Consider a process of point events occurring on the real axis. Let $N(t, t + \Delta t)$ denote the number of in a small time interval $(t, t + \Delta t]$

$$Pr[N(t, t + \Delta t) = 0] = 1 - \rho \Delta t + o(\Delta t) \tag{1}$$
$$Pr[N(t, t + \Delta t) = 1] = \rho \Delta t + o(\Delta t) \tag{2}$$
$$Pr[N(t, t + \Delta t) > 1] = o(\Delta t) \tag{3}$$

where ρ is called the intensity function or the rate of the Poisson process.

The properties above of the Poisson process imply that

$$P[N(t) = n] = e^{-\rho t} \frac{(\rho t)^n}{n!}$$

is a Poisson distribution with mean ρt. The most commonly used probabilistic models in the counting process are homogeneous and nonhomogeneous Poisson processes.

3.3.2 Homogeneous Poisson Process (HPP)

HPP is defined as a sequence of independent and identically exponentially distributed X_i's. Çinlar (1975) defines HPP as the orderly stochastic process with stationary, independent increments.

Definition 5 (Ascher and Feingold, 1984) *The counting process $\{N(t), t \geq 0\}$ is said to be an HPP if*

(a) *The number of failures in any interval of length $t_2 - t_1$ has a Poisson distribution with mean $\rho(t_2 - t_1)$. That is, for all $t_2 > t_1 \geq 0$,*

$$Pr[N(t_2) - N(t_1) = j] = \frac{e^{-\rho(t_2 - t_1)}[\rho(t_2 - t_1)]^j}{j!}$$

for $j \geq 0$. From condition (a) it follows that

$$E[N(t_2 - t_1)] = \rho(t_2 - t_1)$$

where the constant ρ, is the rate of occurrence of failures. Since the HPP has stationary, independent increments, $m(t) = m_f(t) = \rho = 1/E[X]$, i.e. the ROCOF is a constant. From the definition, the reliability function, $R(t_1, t_2)$ is

$$R(t_1, t_2) = e^{-\rho(t_2 - t_1)}$$

The time between failures of HPP are exponentially distributed with mean $1/\rho$ and the time to the n^{th} failure, T_n from a system modeled by an HPP has a gamma distribution. A system that fails in accordance with an HPP, has no memory of its age. This is the most basic model for repairable systems, however it should be used with caution.[14] The HPP cannot be used to describe systems that deteriorate or improve because it has a constant intensity function or ROCOF.

3.3.3 Nonhomogeneous Poisson Process (NHPP)

NHPP is a direct generalization of HPP. The rate of occurrence of failure (ROCOF) for NHPP is assumed to vary with time, $m(t) = \rho(t)$, rather than being constant.

Definition 6 (Ascher and Feingold, 1984) *The number of failures in any interval (t_1, t_2) has a Poisson distribution with mean $\int_{t_1}^{t_2} \rho(t)dt$. That is, for all $t_2 > t_1 \geq 0$*

$$Pr[N(t_2) - N(t_1) = j] = \frac{e^{-\int_{t_1}^{t_2} \rho(t)dt} \{\int_{t_1}^{t_2} \rho(t)dt\}^j}{j!}$$

for $j \geq 0$. From the above, it follows that

$$E[N(t_2 - t_1)] = e^{-\int_{t_1}^{t_2} \rho(t)dt}$$

From the definition, the reliability function, $R(t_1, t_2)$ is

$$R(t_1, t_2) = e^{-\int_{t_1}^{t_2} \rho(t)dt}$$

For NHPP, the interarrival times X_i's are neither independent nor identically distributed. The interarrival times are not independent samples from any single distribution, including the exponential distribution. However, the independent increment property still holds. NHPP is characterized by the minimal repair assumption, for which states that system after repair is only as good as it was immediately before the failure. A popular case of the NHPP is the Power Law Process (see Section 3.4.1)

3.3.4 Renewal Process

The renewal process is also a generalization of HPP. The time between successive failures, like the HPP, is independently exponentially distributed but also independently and identically distributed, with PDF, $f(x)$. When sampled synchronously, the renewal process is an example of a transient point process[7] and the ROCOF is asymptotically constant. The term "good as new" has been used to describe renewal. The ROCOF for the renewal process can be derived from the PDF as[18]

$$m^*(s) = \frac{f^*(s)}{1 - f^*(s)}$$

where $f^*(s)$ and $m^*(s)$ denote the Laplace transform of the PDF and ROCOF.

3.3.5 Superimposed Renewal Process (SRP)

Suppose that there are n renewal processes operating independently of each other. Then the stochastic process formed by the union of all events is known as the superposition of n renewal processes or a superimposed renewal process (SRP). Çinlar (1972) presents a thorough review of the SRP. In terms of reliability, SRP can be explained as follows. Suppose a system is made up of n sockets, each of which contains a component. When a component fails, the entire system fails, and the failed component is replaced with a new, identical component. The socket is considered "as good as new" immediately after the component is replaced. The replacement can be regarded as a renewal. Note that SRP does not have independent and identically distributed interarrival times. (See figure 7) In figure 7, an SRP is formed by failures observed from n sockets. The failures in each

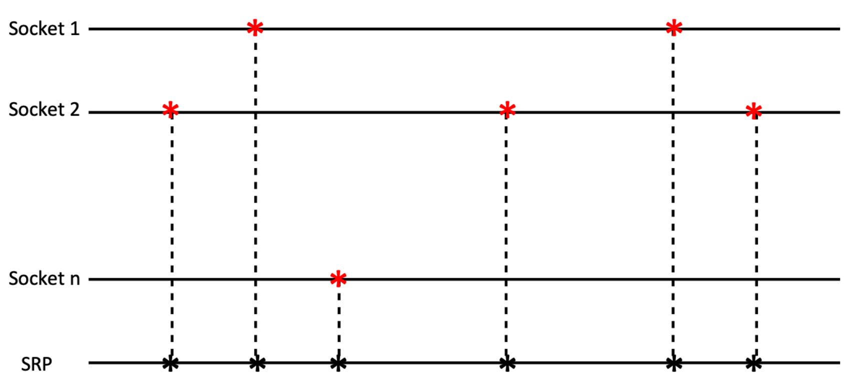


Figure 7: Superposition of Renewal Processes

of the n sockets are depicted by the red crosses and the failures of the system, depicted by the black crosses on the bottom horizontal line, are formed by the union of all the failures of the n sockets.

3.4 NHPP Reliability Growth Models

The most commonly used stochastic process for modeling reliability growth is the nonhomogeneous Poisson process (NHPP). The Power Law model and the Log-linear model are discussed in this section.

3.4.1 Power Law model

A number of reliability growth models have been proposed in the literature for estimating system reliability. The power law model, often known as the Weibull process, is the most commonly discussed NHPP model in literature. Perhaps the popularity of this model could be due to the study conducted by Duane (1964). In this study, Duane[24] noticed that the cumulative rates of failure plotted against the cumulative operating time were close to a straight line on a ln-ln scale. Crow (1974)[21] noted that the Duane postulate could be stochastically represented as a Weibull process. More studies were carried out on the power law model after Duane's study. Rigdon and Basu (1989)[50] provided a thorough review of the model. Lee and Lee (1978)[41] and Bain and Engelhardt (1980)[9] addressed point estimates and proposed tests for the model's parameters. Thompson (1988) and Ascher and Feingold (1984) examine the applications of this model and present various inference tools.

Suppose a system is put into operation at time $T_0 = 0$, let $T_1 < T_2 < \ldots < T_n$ be the first n arrival times of a random point process. The power law process is defined by the intensity function (or ROCOF). The Crow/AMSAA reliability model is as follows

$$\rho(t) = \lambda\beta t^{\beta-1} \quad \lambda > 0, \beta > 0, t > 0 \tag{4}$$

t is the age of the system. The process in Equation (4) can be expressed through the mean cumulative number of failure function

$$M(t) = E[N(t)] = \lambda t^{\beta} \tag{5}$$

When $\beta = 1$, interarrival times, X_i's follow an exponential distribution. The process reduces to a HPP in this case. In the presence of reliability growth, however, the interarrival times should be stochastically increasing. This occurs for the Weibull process when $0 < \beta < 1$, i.e. when $\rho(t)$ is decreasing.[22] Finkelstein (1976) reparameterized the intensity function as [28]

$$\rho(t) = \left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1} \qquad \alpha > 0, \beta > 0, t > 0 \tag{6}$$

$$M(t) = \left(\frac{t}{\alpha}\right)^{\beta} \tag{7}$$

where $\alpha = \left(\frac{1}{\lambda}\right)^{\frac{1}{\beta}}$
α and β represent the process's scale and shape parameters, respectively.

3.4.2 Parameter Estimation of the Power Law Model

The values of λ and β will be estimated based on failure data from the k systems. Assuming we have k systems, whose operation starts at times S_q and ends at T_q for $q = 1, \ldots, k$. N_q is the total number of failures for the qth system and X_{iq} is the age of the system at the ith occurrence of failure. The maximum likelihood estimates for λ and β are given by

$$\hat{\lambda} = \frac{\sum_{q=1}^{k} N_q}{\sum_{q=1}^{k}(T_q^{\hat{\beta}} - S_q^{\hat{\beta}})} \tag{8}$$

and

$$\hat{\beta} = \frac{\sum_{q=1}^{k} N_q}{\hat{\lambda}(T_q^{\hat{\beta}} \ln T_q - S_q^{\hat{\beta}} \ln S_q) - \sum_{q=1}^{k} \sum_{q=1}^{N_q} \ln X_{iq}} \tag{9}$$

Eqns (8) and (9) must be solved by an iterative procedure. If all the systems have the same start time at zero, i.e. $S_q = 0$ and all end at the same time at $T_q = T$ then these equations simplify to Eqns. (10) and (11) below.

$$\hat{\lambda} = \frac{\sum_{q=1}^{k} N_q}{kT^{\hat{\beta}}} \tag{10}$$

$$\hat{\beta} = \frac{\sum_{q=1}^{k} N_q}{\sum_{q=1}^{k} \sum_{q=1}^{N_q} \ln\left(\frac{T}{X_{iq}}\right)} \tag{11}$$

In Eqns. (10) and (11) the maximum likelihood estimates are in closed form. Also when $k = 1$, the estimates for λ and β are,

$$\hat{\lambda} = \frac{N_1}{T_1^{\hat{\beta}}} \tag{12}$$

$$\hat{\beta} = \frac{N_1}{\sum_{q=1}^{N_1} \ln\left(\frac{T_1}{X_{i1}}\right)} \tag{13}$$

3.4.3 Test of Significance for $\hat{\beta}$ - MIL-HDBK-189 test (1981)

This test is for tracking reliability growth developed by the US Army Materiel Systems Analysis Activity (AMSAA) (Unkle and Venkataraman, 2002). It assumes that the ROCOF of failure events is $\rho(t) = \lambda\beta t^{\beta-1}$.

The hypotheses to be tested are:

$$H_0 : \beta = 1$$

i.e. no trend in data (homogeneous Poisson Process).

$$H_1 : \beta \neq 1$$

The alternate hypothesis implies that there is trend in data (nonhomogeneous Poisson process). The MIL-HDBK 189 test is based on the test statistic

$$\mathcal{U} = 2\sum_{i=1}^{m-1} \ln\left(\frac{T_m}{T_i}\right) \tag{14}$$

$\mathcal{U}$ is distributed with the Chi-square distribution, χ^2 with degrees of freedom $2(m-1)$. Interpretation:

- If the null hypothesis is rejected, you might conclude that your data has a trend and should be modeled with a nonhomogeneous Poisson process, such as the power-law process.
- If you fail to reject the null hypothesis, there is not sufficient evidence to reject the null hypothesis of homogeneous Poisson process model. HPP is an appropriate model to use.

In the case of no growth, β is equal to 1; when $\beta < 1$, the process indicates reliability growth, when $\beta > 1$, the process indicates reliability deterioration.

3.4.4 Cox-Lewis Log-Linear Model

Another NHPP reliability growth model is the Log-Linear model proposed by Cox and Lewis (1966). If the Power Law model in section 3.4.1 with intensity function $\rho(t) = \lambda\beta t^{\beta-1}$, is rejected by a goodness-of-fit test, the Log-linear model can be fitted. The failure intensity function of the log-linear model is given as

$$\rho(t) = e^{\alpha_0+\alpha_1 t}, \quad -\infty < \alpha_0, \alpha_1 < \infty, t \geq 0 \tag{15}$$

The intensity function, $\rho(t)$, of this model has the advantage that since $\hat{\rho(t)}$ will be non-negative, so no nonlinear constraints need to be placed on parameter estimates. The parameters α_0 and α_1 can be estimated from the failure data. Despite the advantages of the log-linear model's intensity function, estimating the parameters could be difficult.

$$M(t) = \frac{e^{\alpha_0+\alpha_1 t} - e^{\alpha_0}}{\alpha_1} \tag{16}$$

3.4.5 Goodness-of-Fit Test for NHPP models

The Cramer-Von Mises Statistics [21], for the goodness of fit test is

$$C_R^2 = \frac{1}{12m} + \sum_{i=1}^{m}\left(\hat{R}_i - \frac{2i-1}{2m}\right)^2 \tag{17}$$

where

$$\hat{R}_i = \left(\frac{T_i}{T}\right)^{\hat{\beta}}$$

The observation on the system ends at time T and the ith arrival time is given as T_i. N is the number of system failures.

C_R^2 has its own critical values for various values of n. A value of C_R^2 larger than the critical value leads to the rejection of the null hypothesis

$$H_0 : \text{the failure times were governed by a power law process.}$$

and the conclusion that the model does not fit adequately.

An information-based model selection procedure is another method for evaluating the model's goodness-of-fit. This enables for the comparison of several models. Two widely used information criteria for assessing model fit are Akaike's information criterion (AIC) and Bayesian information criterion (BIC). The AIC and BIC are computed as follows:

$$AIC = -2\text{Loglikelihood} + 2k$$

$$BIC = -2\text{Loglikelihood} + klog(n)$$

where k denotes the total number of parameters in the model and n denotes the total number of observations (i.e. failures). The model with the lowest AIC/BIC values is preferred.

3.5 Nonparametric Methods

3.5.1 Nelson-Aalen Estimator

The Nelson-Aalen Estimator provides a nonparametric estimate of the Cumulative hazard function $H(x)$ and the Mean Cumulative number of failures , $M(t)$. Without any parametric assumptions, the hazard rate $h(x)$ might be any nonnegative function, making estimation problematic. [3] Yet, it turns out that estimating the cumulative hazard function is simple

$$H(x) = \int_0^x h(s)ds \tag{18}$$

without making any assumptions about the distribution of $h(x)$. This is analogous to estimating the cumulative distribution function, which is significantly simpler than estimating the density function. The result is the Nelson-Aalen estimator, which is given as (Nelson, 1969)

$$\widehat{H(x)} = \sum_{T_j \leq x} \frac{1}{Y(T_j)} \tag{19}$$

where $Y(t)$ is the number of individuals at risk at time t (in survival analysis terms). Eqn. 19 is an increasing step function, which may look like a smooth curve with a large sample of data. Eqn. 19 can also be defined as

$$\widehat{H(x)} = \sum_{x_i \leq x} \frac{d_i}{n_i} \quad x \geq 0 \tag{20}$$

$d_i = 1$ when failures occur at distinct times and there are no multiple failures occurring at the same time.

Nelson (1988) developed a nonparametric estimate for the mean cumulative number of failures, $M(t)$, given by

$$\widehat{M(t)} = \sum_{t_i \leq t} \frac{d_i}{n_i} \quad t \geq 0 \tag{21}$$

where t_i stands for the observed failure times, d_i is the number of failures observed at these times and n_i the number of systems in operation at these times. As discussed in section 3.1, the shape of the MCNF curve reveals the system's behavior. Nelson-Aalen estimator has the advantage of being able to be used on both complete and censored data.

3.5.2 Laplace Trend Test

For a counting process, the times between successive failures may tend to get longer or shorter. The trend test is performed to determine if the system is improving or deteriorating. It is essential to first analyze the X_i's in chronological order to test for trend since the existence of trend shows that the data is non-stationary. Plot the interarrival times in chronological order. If there's a trend, check for NHPP or other nonstationary models. No trends imply that the X_i's may be identically distributed but not necessarily independent. You can check for dependence and use the appropriate models. For an improving (deteriorating) system, successive inter-arrival failure times will likely become larger (smaller).

The Laplace Test can also be used to determine whether or not an observed series of events has a trend. The hypothesis test is

$$H_0 : \text{No trend}$$
$$H_a : \text{Trend}$$

The test uses chronologically ordered arrival times $T_1, T_2, \ldots T_m$.[8] The Laplace test statistic is

$$U_L = \frac{\frac{\sum_{i=1}^{m-1} T_i}{m-1} - \frac{T_m}{2}}{T_m \sqrt{\frac{1}{12(m-1)}}} \tag{22}$$

The null hypothesis is rejected and there is an evidence of trend if:
$U_L > Z_{\alpha/2}$ (reliability deteriorating)
$U_L < Z_{\alpha/2}$ (reliability improving)

Interpretation:

- Negative values of U_L less than z-score means that there is downward or decreasing trend, this indicates a decreasing rate of occurrence of failures.
- Positive values of U_L greater than z-score means that there is upward or increasing trend, this indicates an increasing rate of occurrence of failures.

3.6 Probabilistic Deep Learning Model for Failure Data

This section introduces a recurrent neural network based on survival analysis. The Weibull Time To Event Recurrent Neural Nekwork by Egil Martinsson (2016) [45]. In contrast to one of the main assumptions in survival analysis, which is the occurrence of a single event or failure, WTTE-RNN can model recurring events or multiple failures. We will describe the WTTE-RNN model using a general framework for censored data, but before then, we present some fundamental deep learning concepts.

3.6.1 Brief Introduction to Deep Learning

Deep learning is a subset of machine learning inspired by the structure of the human brain. Deep learning is built on artificial neural networks, which were influenced by biological neurons.

A neural network consists of an input layer, an output layer and, in between, hidden layers. The layers are connected via nodes, and these connections form a "network" of interconnected nodes. An output of a neural network is computed through a series of calculations. First, computing the dot product between the inputs and their respective weights. Then add the bias term and apply the activation function to the result from the input layer. The activation function decides whether the neuron should be activated if the output of each node exceeds a certain threshold value. This results in the output of one node becoming the input of the next node. This process of passing information from data from one layer to the next layer defines this neural network as a feedforward network. Neural networks learn through a feedback process known as backpropagation. This involves comparing the output a network produces with the actual output and using the difference between them to modify the weights of the connections between the units in the network, working from the output units through the hidden units to the input units going backward, in other words. Figure 8 demonstrates how a recurrent neural network moves information between layers.

3.6.2 Recurrent Neural Networks (RNN)

RNNs are a class of neural networks that utilise sequential or time series data. RNNs are commonly used in natural language processing, time series analysis, and machine translation. The internal memory of RNNs help them remember important things about the input they received, which allows them to anticipate what will happen next with great accuracy. Figure 8 demonstrate how RNNs transfers information between layers.

3.6.3 Censoring and The Likelihood Function

Let $\mathcal{L}(x, \theta)$ be the likelihood function known as the joint PDF of the sample $Z = Z_1, ..., Z_n$. WTTE-RNN uses a likelihood function as its loss function. By

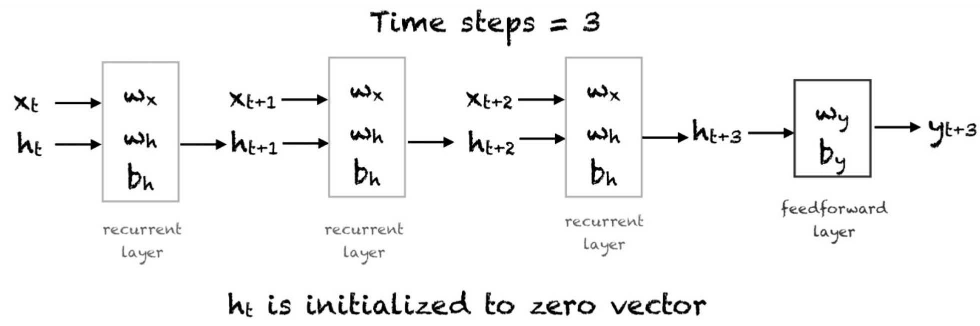


Figure 8: Layers of A Recurrent Neural Network

constructing the likelihood function, we use it as a loss function for the model, depending on θ and to maximize the probability of $X = x|\theta$, which will be the same as minimizing the log-likelihood log(L(x, θ)).

3.6.4 WTTE-RNN

WTTE-RNN is a framework for predicting the time until the next event as a discrete or continuous Weibull distribution, with two parameters of the Weibull distribution being the output of a recurrent neural network. The Weibull distribution is used in the model because it has some good attributes, such as a closed-form PDF, CDF and hazard function, and it can be used as a good approximation for many distributions such as the exponential distribution. The model is trained using a special objective function that allows use of censored data by constructing a likelihood function for censoring.

Given $t \in [0, \infty)$, scale parameter, $\alpha \in (0, \infty)$ and shape parameter, $\beta \in (0, \infty)$, a random variable $X \sim Weibull(\alpha, \beta)$ (continuous case) has cumulative hazard function given as

$$H(x) = \left(\frac{x}{\alpha}\right)^{\beta} \tag{23}$$

and hazard function

$$h(x) = \left(\frac{x}{\alpha}\right)^{\beta-1} \frac{\beta}{\alpha} \tag{24}$$

The loss function of WTTE-RNN is

$$\mathcal{L}(x, \theta) = \prod_{i}^{n} Pr(X = x_i)^{u_i} \cdot Pr(X > x_i)^{1-u_i} \tag{25}$$

$$log(\mathcal{L}(x, \theta)) = u_i \sum_{i}^{n} log(e^{-H(x_i)} h(x_i)) + (1 - u_i) \sum_{i}^{n} log(e^{-H(x_i)}) \tag{26}$$

$$= \sum_{i}^{n} [u_i \cdot log(h(x_i)) - H(x_i)] \tag{27}$$

where u_i is 0 if timestep is right censored or 1 otherwise.

The framework used an exponential activation function for the scale parameter, $\alpha > 0$

$$f(x) = x, \text{if } x > 0 \quad \text{and} \quad a(e^x - 1), \text{if } x \leq 0 \tag{28}$$

and the softplus activation function for shape parameter, β.

$$f(x) = log(1 + e^x) \tag{29}$$

The activation function is used to ensure that the outputs of the parameters of the Weibull distribution, α and β are positive.

The WTTE-RNN model, in the context of reliability, assumes that interarrival times are i.i.d. For this reason, the model is not appropriate for modeling reliability improvement or deterioration. It can be a good machine learning model for renewal processes and HPP where interarrival times are i.i.d.

4 Case Study

High-Performance Computing, (or HPC) uses supercomputers and computer clusters to solve advanced computational problems that require computing power and performance that is beyond the capabilities of a typical desktop computer. These large computational problems exist in numerous fields such as science and engineering. HPC clusters have three main components: compute, network, and storage components. HPC clusters have multiple compute servers (or computers) networked together into a cluster. Each cluster's nodes work in parallel with one another, boosting processing speed to achieve high-performance computing. To capture the output, the cluster is networked to a data storage system. All HPC cluster nodes have the same components as a laptop or desktop: CPU cores (also known as processors), memory (or RAM), and disk space. What distinguishes a personal computer from a cluster node is the quantity, quality, and power of the components.

The rise of big data and artificial intelligence has contributed to an increase in demand for high-performance computing systems in both industry and academia. The growing demands have resulted in frequent HPC failures. These failures may cause system outages, which can be costly and disruptive to institutions and the people who rely on them. This emphasizes the significance of ensuring the reliability of HPC systems. As a result, a detailed understanding of failure characteristics can better guide HPC management and thereby limit the occurrence of failures, improving system performance and reliability. The goal of this case study is to model the reliability of HPC systems based on failure data obtained during the first seven years of operation using stochastic point process models.

4.1 Data

The case study uses data on the hardware failures of HPC computing systems from the Computer Failure Data Repository (CFDR).[55] This data set is a record of hardware failures recorded on the High Performance Computing System-2 (MPP2) operated by the Environmental and Molecular Science Labratory (EMSL), Molecular Science Computing Facility (MSCF) at Pacific Northwest National Laboratory (PNNL) from November 2003 through to September 2007. Below is a description of the HPC system from PNNL provided by CFDR.[55]

The MPP2 computing system has the following equipment and capabilities:

- HP/Linux Itanium-2
  - 980 node/1960 Itanium-2 processors (Madison, 1.5 GHz) configured as follows:
  - 574 nodes are "fat" compute nodes with 10 Gbyte RAM and 430 Gbyte local disk

- 366 nodes are "thin" compute nodes with 10 Gbyte RAM and 10 Gbyte local disk
  - 34 nodes are Lustre server nodes (32 OSS, 2 MDS)
  - 2 nodes are administrative nodes
  - 4 nodes are login nodes

- Quadrics QsNetII interconnect
  - 11.8 TFlops peak theoretical performance
  - 9.7 terabytes of RAM
  - 450 terabytes of local scratch disk space
  - 53 terabytes shared cluster file system, Lustre

The applications running on this system are typically large-scale scientific simulations or visualization applications. The data contains an entry for any failure that occurred during the 5-year time period and that required the attention of a system administrator. For each hardware failure, the data set includes a timestamp for when the failure happened, the node affected, what failed in the node affected, a description of the failure, and the repair action taken. Table 2 shows the first five rows of the raw data from PNNL.

Date	HardwareID	What Failed	Description of Failure	Action
003-11-29 00:00:00	node 13	DISK	I/O error on Drive SDG	REPLACE
2003-11-29 07:00:00	node 25	DISK	I/O error on Drive sdf	REPLACE
2003-11-29 07:00:00	node 30	DISK	I/O error on Drive sdb and sdc	REPLACE
2003-11-29 07:00:00	node 63	DISK	I/O error on Drive sdh	REPLACE
2003-11-29 08:00:00	node 380	DISK	I/O error on Drive sdf	REPLACE

Table 1: Sample of hardware failure data from PNNL

4.1.1 Data Preprocessing

There were several steps of preprocessing the dataset used before we arrived at a dataset suitable for the analysis used in the research. we selected a subset of the dataset that had been replaced after the failure. The system was placed into operation at the same time that hardware failure record keeping began.[1] There is no information given on the exact start and end dates of the study. All that is known is that the study began in November 2003 and ended in September 2007. The dates November 1, 2003, and September 1, 2007 were selected for the start and end of the study, respectively. This way we could calculate the arrival times of failures, T_i and the interarrival times, or the time between successive failures, X_i. Failure arrival times, T_i and interarrival times, X_i were both measured in days for the study. Our data is censored. Since we are dealing with a repairable system, there could be more than one failure in the system during the observation period (Nov. '03 - Sept '07). Components in the system are replaced upon failure. In the study, we treat censored age as the time between the last failure for each component and the time the study ends. See figure 9.

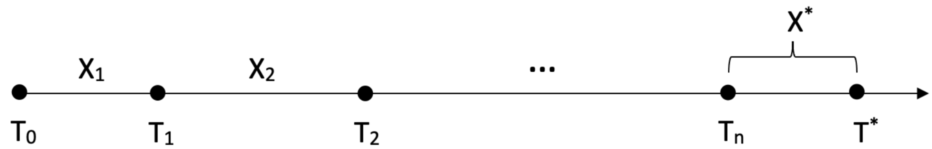


Figure 9: X^* represents censored age of component.

4.2 Results

This section is a report on the results from the models used in this study. JMP,[53] a software built on SAS mainly for reliability analysis, was used for the analysis.

4.2.1 Nelson-Aalen MCNF Plot for HPC system

We start our analysis by plotting the mean cumulative number of failures as a function the system's age. If the time interval between successive failures becomes longer, the system's reliability improves. Conversely, if the interval between failures is decreasing, the system's reliability is deteriorating. (see figure 10)

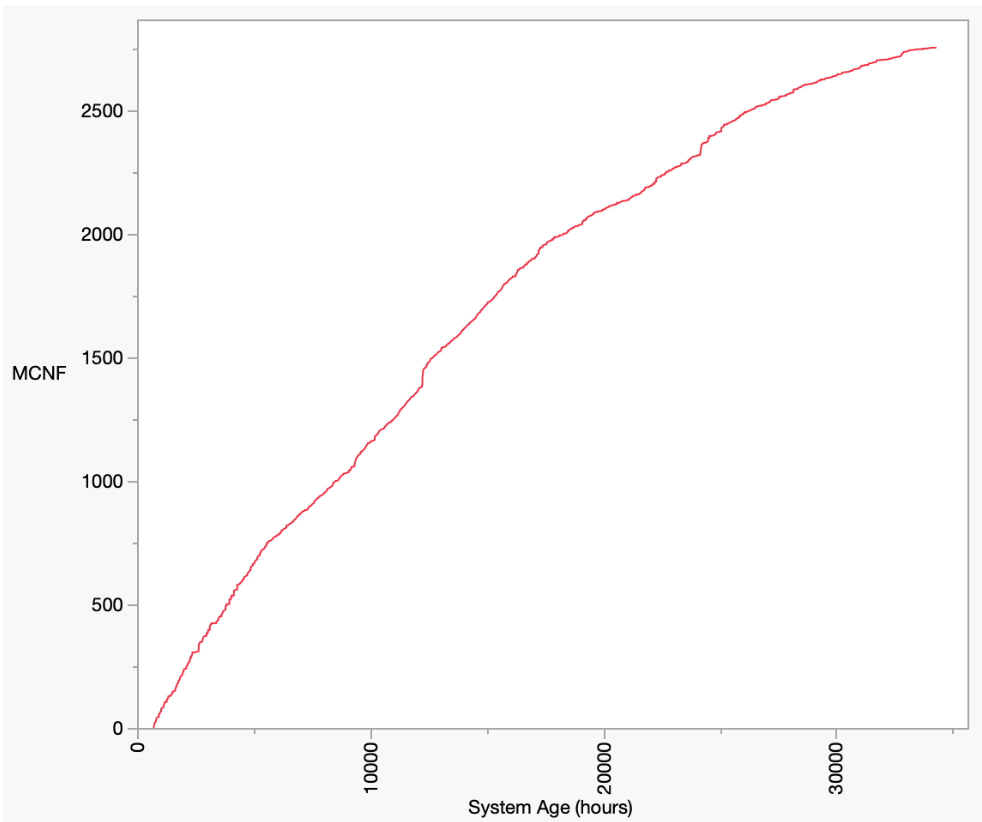


Figure 10: Mean cumulative number of failures from the HPC System

From the plot of the MCNF, there is no evidence of the system getting worse with age. We will conduct the trend test to determine whether or not the system is improving.

4.2.2 Result of Laplace Trend Test

The test statistic of the Laplace trend test for testing the null hypothesis of no trend vs the alternate hypothesis of trend in data at 95% confidence level is $U_L = -8.9837 < Z_{\alpha/2} = -1.96$. There is enough evidence to reject the null hypothesis of no trend. We can conclude that the reliability of HPC system under study is improving. The interarrival time of failures is getting longer as the system ages.

4.2.3 Results from parametric models

Despite the fact that nonparametric estimations reveal that the HPP is not a good model for our system due to evidence of trend, let's see how an HPP model fits and performs with our data.

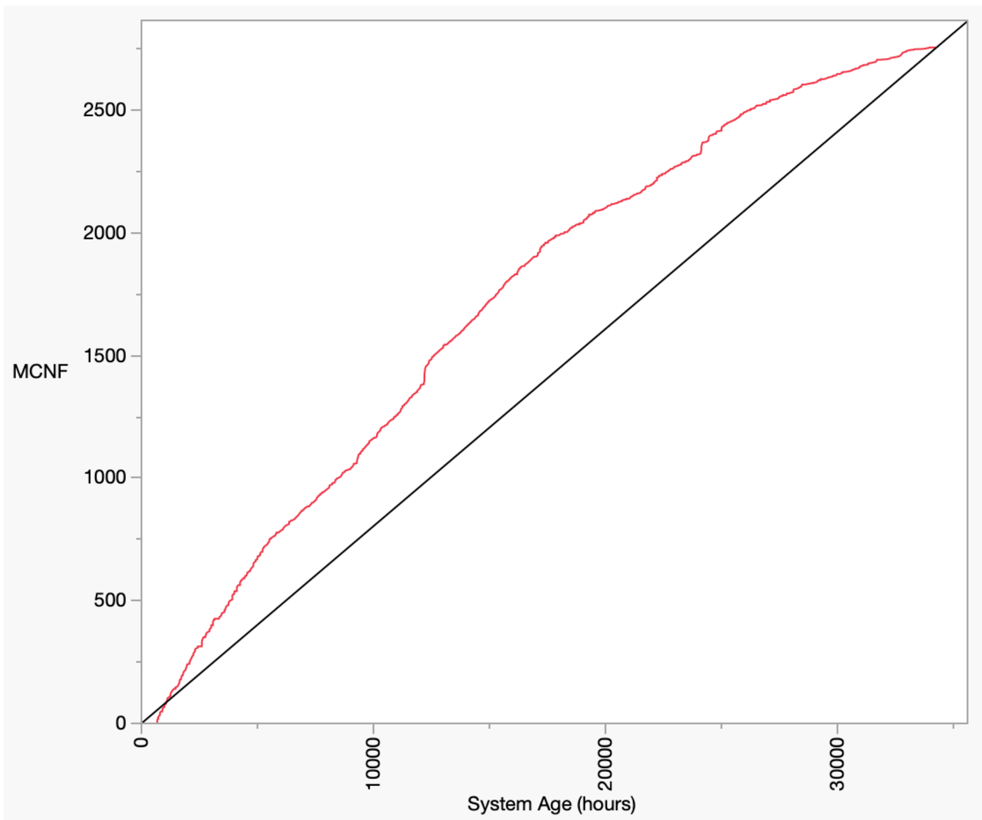


Figure 11: Fit of HPP Model

We can see that HPP may not be a good fit for our failure data. We now check with two NHPP reliability growth models, the Power Law process and the Log-Linear model.

Parameter	Estimate	Std. Error	95% C.I. (Lower)	95% C.I. (Upper)
α	1.14683	0.22692	0.70206	1.59159
β	0.76865	0.01464	0.73995	0.79735

Table 2: Parameter Estimates of Power Law Process

The intensity function and mean cumulative number of failures of the Power law process is

$$\widehat{\rho(t)} = 0.67024 \left(\frac{t}{1.14683}\right)^{-0.23135}$$
$$\widehat{M(t)} = \left(\frac{t}{1.14683}\right)^{0.76865}$$

The MIL-HDBK 189 test: $\mathcal{U} = 208.700$, p-value < 0.0001. The test rejects the null hypothesis of $\beta = 1$ (or constant MTBF) in favor of the Nonhomogeneous Poisson process as a model for this data.

Figure 12 fits a Power law process NHPP reliability growth model to the arrival of failures data. The dotted line in figure 15 is a pointer that shows the value at the selected point of the graph. The failure intensity is a decreasing function of time. This demonstrates that our system is improving.

Parameter	Estimate	Std. Error	95% C.I. (Lower)	95% C.I. (Upper)
α_0	-1.863074	0.0325852	-1.926940	-1.799207
α_1	-4.375e-5	2.0318e-6	-4.773e-5	-3.977e-5

Table 3: Parameter Estimates of Log-Linear Model

Figure 13 shows the fit of the log-linear model on the failure data. The log-linear model seems to have a better fit as compared to the power law process. Fig. 14 shows that the log-linear model also has a decreasing ROCOF.

4.2.4 Results from Goodness-of-fit tests

The goodness-of-fit test: $C_R^2 = 2.303485$, p-value < 0.01 The computed test statistics corresponds with p-value that is less than 0.01. We conclude that the Power Law model does not provide an adequate fit to the data.

The log-linear model fits the failure data from the MPP2 HPC system better than the Power Law process, according to the AIC/BIC values from table 4.

To select the most appropriate NHPP model for failure data from our HPC system, we calculated the mean square errors (MSE) between the observed mean cumulative number of failures, $M(t)$, and the estimated mean cumulative number of failures, $\widehat{M(t)}$ for each NHPP reliability growth model. Because the log-linear

Model	−2Loglikelihood	AIC	BIC
HPP	19409.121	19413.121	19425.54
Power Law Process	19200.421	19204.421	19216.846
Log-Linear Model	18919.436	18923.436	18935.861

Table 4: Goodness-of-fit: AIC & BIC

model proposed by Cox and Lewis has a much lower MSE, it is chosen.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(M(t_i) - \widehat{M(t_i)}\right)$$

Model	MSE
Power Law Process	37600.699
Log-Linear Model	3387.614

Table 5: Mean Squared Error

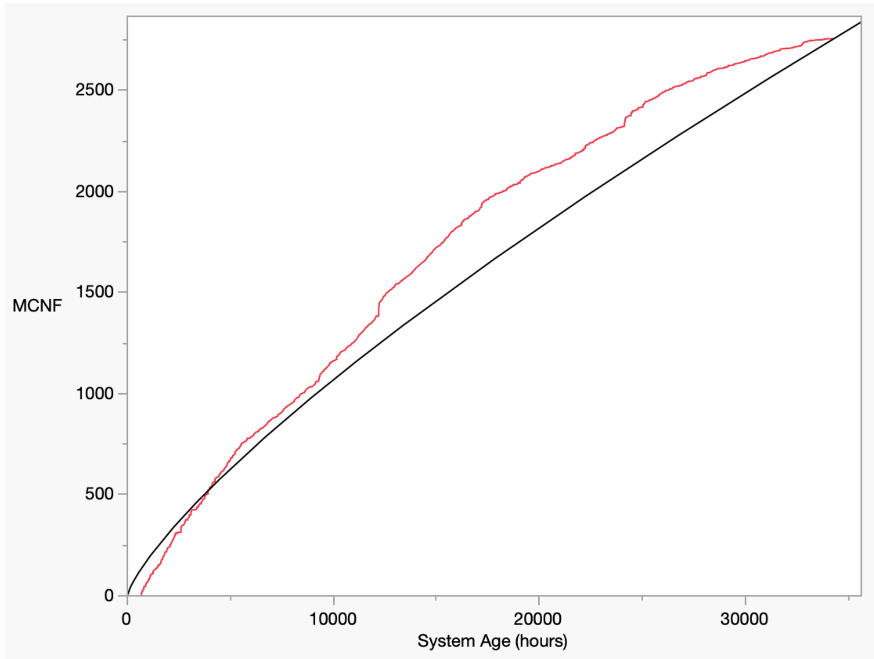


Figure 12: Fit of Power Law Process

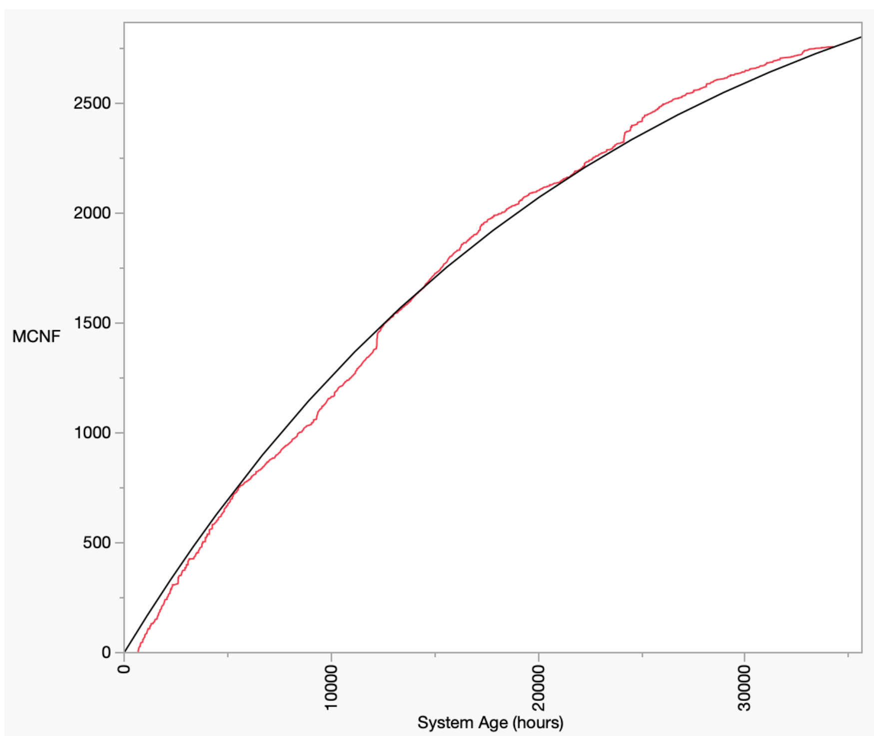


Figure 13: Fit of Cox-Lewis Log-Linear Model

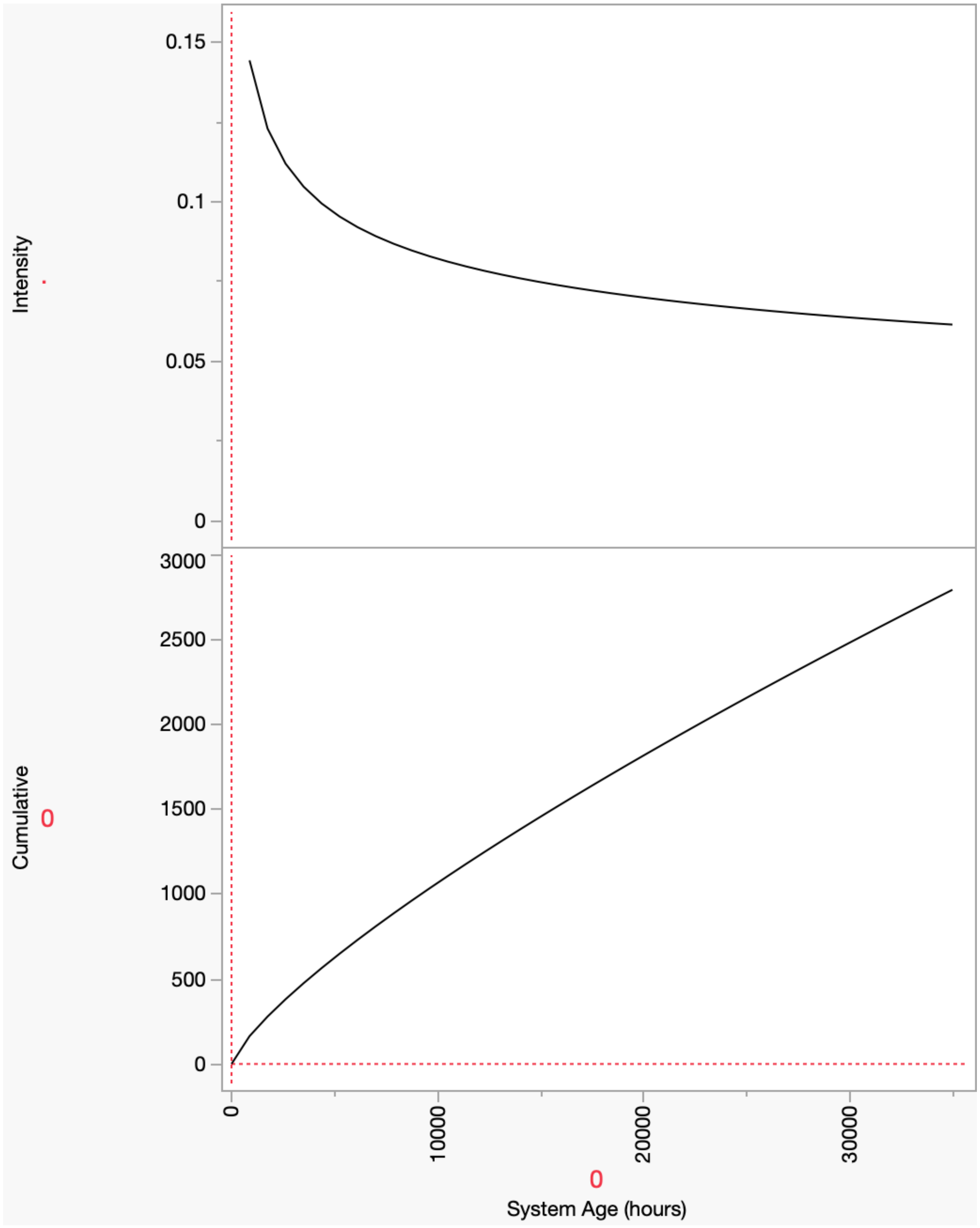


Figure 14: Plot of Failure Intensity Function and Cumulative Failures for Power Law Process

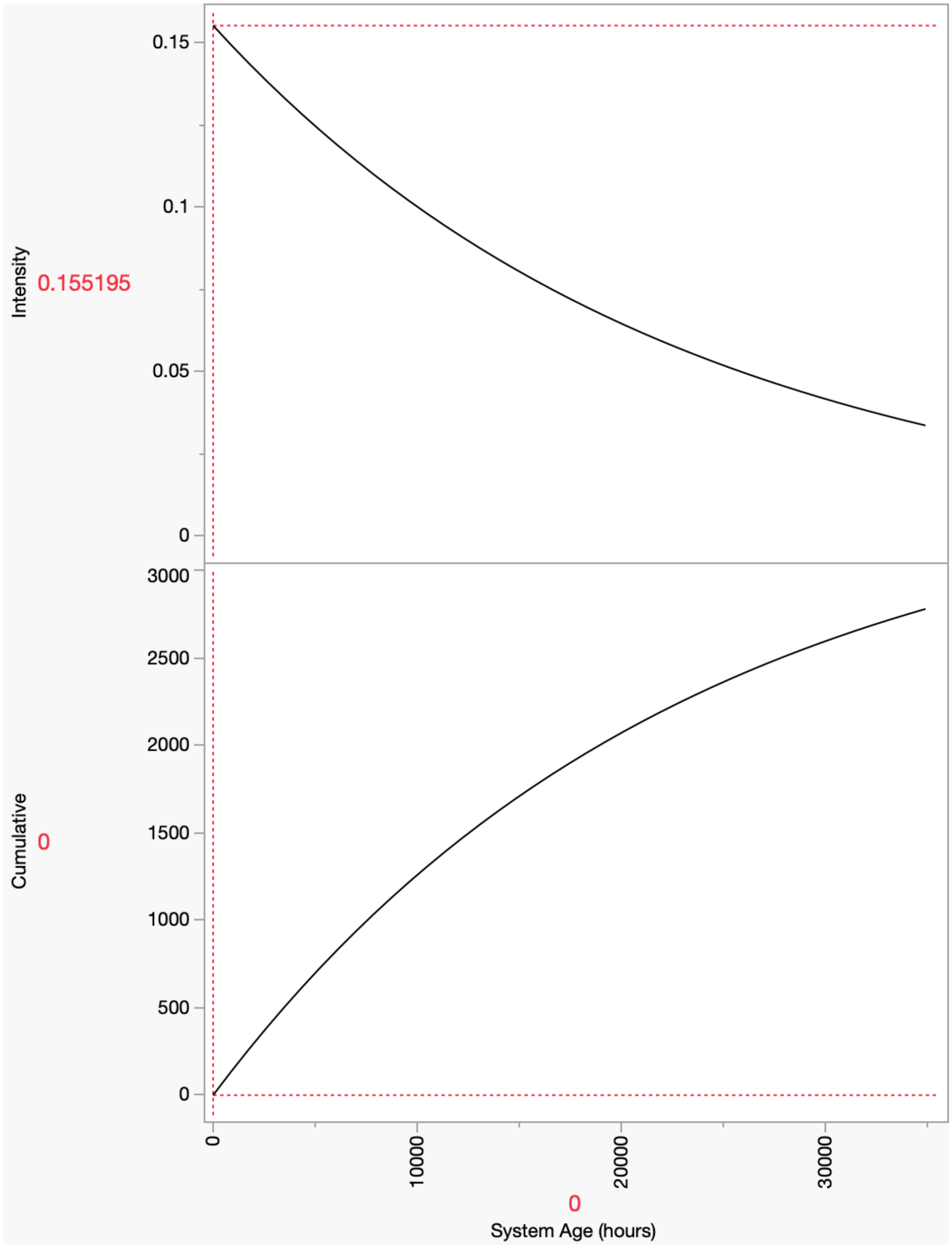


Figure 15: Plot of Failure Intensity function and Cumulative Failures for the Log-linear model

5 Conclusion

Several studies have highlighted the significance of studying failure data from repairable systems. The thesis discusses the fundamental concepts of reliability engineering for repairable systems. It also looks at stochastic point processes as they apply to repairable systems. A case study is undertaken to see how the reliability models discussed apply to failure data.

In the case study we study of failure data that was collected over the past two decades at Pacific Northwest National Laboratories. We find that the interarrival times of failure of the HPC system studied are not independent and identically distributed. For this reason, the HPP is not an appropriate model to use. After conducting the Laplace trend test, we discovered a trend in the data. The decreasing rate of occurrence of failures as the system ages indicates that we have an improving system. The plot of the intensity function indicates that the system is in the burn-in stage, where there is an initial high number of failures due to defective parts in the system. These parts are replaced, and the occurrence of failures decreases as the system ages. The log-linear model is chosen as an appropriate model for the failure data used in the case study, with results from the goodness-of-fit tests and mean squared error (MSE).

5.0.1 Future Work

The WTTE-RNN model can be extended with other distributions. A future work will be to develop a variant of the WTTE-RNN that is appropriate for reliability growth models.

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Honors and Awards:	Graduate Service Appointment, Mathematics Department, 2021-2023, Eastern Washington University
	First Class Honors, 2019, Kwame Nkrumah University of Science and Technology
Professional Experience:	Internship, Carnegie Mellon University, Pittsburgh, Pennsylvania, 2019
	Internship, University of Kiel, Kiel, Germany, 2017