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Isomorphism on Interval-Valued Complex Fuzzy Graphs

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Abstract

This research paper delves definition of isomorphism, weak isomorphism and co-weak isomorphism within the realm of highly edge improper and neighbourly edge improper interval-valued complex fuzzy graphs as well as their complementary counterparts. Additionally we establish the isomorphism of μ-complement of highly edge improper interval-valued complex fuzzy graphs.

Key words: Interval-valued complex fuzzy graph, order, size, degree of an edge, neighbourly edge improper, highly edge improper interval-valued complex fuzzy graph, complement, isomorphism.

AMS subject classification: $05C72$.

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1 Introduction

Isomorphism is one of the area in graph theory that many researchers focussing today.The notion of Interval valued fuzzy sets was introduced by Zadeh(9) in 1975, as an extension of fuzzy sets in which the values of the membership degrees are instead of the numbers, the intervals of numbers. The Interval valued fuzzy sets supply more satisfactory description of ambiguity than traditional fuzzy sets. Hence, the applications of Interval valued fuzzy sets are very much important to use in fuzzy control. Isomorphism on Fuzzy Graphs was discussed by A. Nagoor Gani, J. Malarvizhi(3). Isomorphism on Interval Valued Fuzzy Graph was studied by M. Naga Maruthi Kumari., R. Chandrasekhar(2). S. Ravi Narayanan and N.S. Santhi Maheswari discussed the isomorphic properties of irregular interval valued fuzzy graph (4). R.Sridevi., R. Venkateshwara was discussed the concept of edge improper interval-valued complex fuzzy graphs(8). In this article we are considering isomorphism on interval-valued complex fuzzy graphs. Though isomorphism on interval-valued fuzzy graph has already exist. This isomorphic concept is concentrated on these case which involve time periodic phenomena. We have discussed weak isomorphism, co-weak isomorphism, isomorphism and its related results.

2 Preliminaries

Definition 2.1. *(6)* A Interval-Valued Complex Fuzzy Graph(ivcfg) with an underlying set F is defined to be a pair $G:(S,T)$, where $S=[\mu_S^-,\mu_S^+]$ is an ivcf set on F and $T=[\mu_T^-,\mu_T^+]$ is a ivcf set on $I\subseteq F\times F$ such that
$\mu_T^-(jl)e^{i\alpha_T(jl)}\leq min\{\mu_S^-(j),\mu_S^-(l)\}e^{imin\{\alpha_S^-(j),\alpha_S^-(l)\}}$
$\mu_T^+(jl)e^{i\beta_T(jl)}\leq min\{\mu_S^+(j),\mu_S^+(l)\}e^{imin\{\beta_S^+(j),\beta_S^+(l)\}}$ for all $j,l\in F$.

Definition 2.2. *(6)* Let $G:(S,T)$ be an ivcf graph, where $S=\left(\mu_S^-,\mu_S^+\right)$ and $T=\left(\mu_T^-,\mu_T^+\right)$ be two ivcf sets on a non empty set F. The order of an ivcf graph is defined by $O_I(G)=(O_I^-(G),O_I^+(G))$, where $O_I^-(G)=\sum_{j\in F}\mu_S^-(j)e^{\sum_{j\in F}\alpha_S^-(j)}$ and $O_I^+(G)=\sum_{j\in F}\mu_S^+(j)e^{\sum_{j\in F}\beta_B^+(j)}$ for all $j\in F$.

Definition 2.3. *(6)* Let $G:(S,T)$ be an ivcf graph. The positive degree of a vertex is defined by $d_I^+(j)=\sum_{jl\in I}\mu_T^+(jl)e^{\sum_{jl\in I}\beta_T(jl)}$ and the negative degree of a vertex is defined by $d_I^-(j)=\sum_{jl\in I}\mu_T^-(jl)e^{\sum_{jl\in I}\alpha_T(jl)}$ for all $jl\in I$. Then degree of a vertex j is defined as $d_I(x)=(d_I^-(j),d_I^+(j))$.

Definition 2.4. *(6)* Let $G:(S,T)$ be an ivcf graph, where $S=\left(\mu_S^-,\mu_S^+\right)$ and $T=\left(\mu_T^-,\mu_T^+\right)$ be two ivf sets on a non empty set P. The size of an ivcf graph is defined by $S_I(G) = (S_I^-(G), S_I^+(G))$, where $S_I^-(G) = \sum_{j\in I} \mu_T^-(jl)e^{\sum_{jl\in I}\alpha_T(jl)}$ and $S_I^+(G) = \sum_{j\in I} \mu_T^+(jl)e^{\sum_{jl\in I}\beta_T(jl)}$ for all $jl \in I$.

3 Isomorphism on Edge Improper Interval-Valued Complex Fuzzy Graphs

In this section, homomorphism, weak isomorphism, co-weak isomorphism, isomorphism of two ivcf graphs are defined and some of its properties are discussed.

Definition 3.1. A homomorphism p of neighbourly edge improper ivcf graph(highly edge improper ivcf graph) G_1 and G_2 is a mapping $p\colon F_1 \to F_2$ which fulfills the following criteria
(i) $\mu_S^+(x)e^{i\beta_S^+(x)} \leq \mu_S^+(p(x))e^{i\beta_S^+(p(x))}$ and $\mu_S^-(x)e^{i\alpha_S^-(x)} \leq \mu_S^-(p(x))e^{i\alpha_S^-(p(x))}$
(ii) $\mu_T^+(xy)e^{i\beta_T^+(x)} \leq \mu_T^+(p(xy))e^{i\beta_T^+(p(xy))}$ and $\mu_T^-(xy)e^{i\alpha_T^-(xy)} \leq \mu_T^-(p(xy))e^{i\alpha_T^-(p(xy))}$.

Example 3.1. *Consider neighbourly edge improper interval-valued complex fuzzy graph on $G^*(F, I)$.*

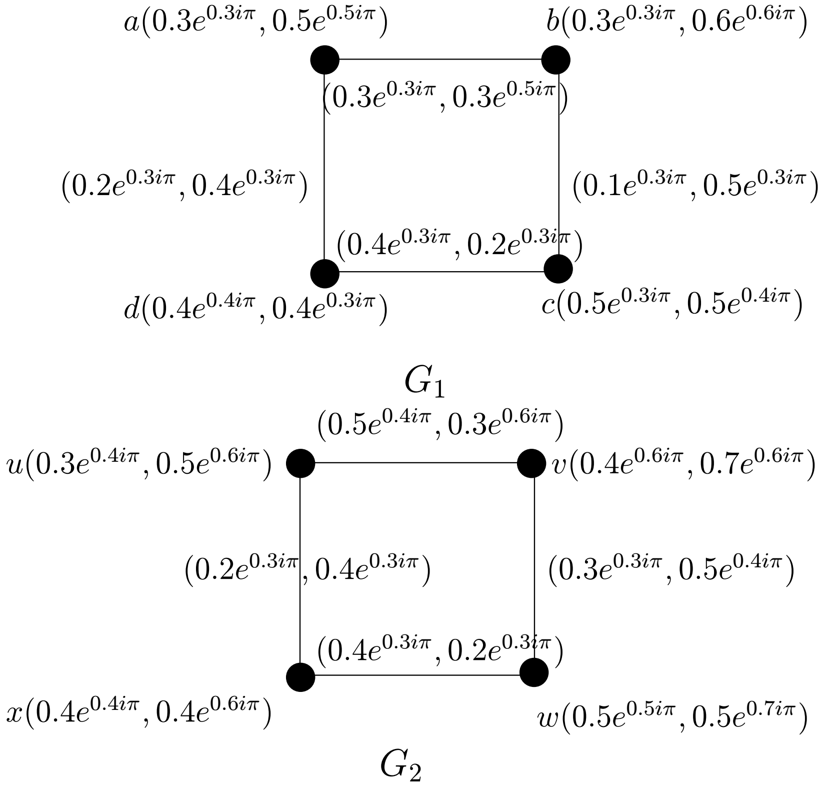


Figure 1:Homomorphism of neighbourly edge improper ivcf graph

In Figure 1, there is a homomorphism $p\colon F_1 \to F_2$ such that $p(a) = u$, $p(b) = v$, $p(c) = w, p(d) = x$.

Definition 3.2. A weak isomorphism p of neighbourly edge improper ivcf graph(highly edge improper ivcf graph) G_1 and G_2 is a bijective mapping $p : F_1 \to F_2$ which fulfills the following criteria

(i) p is homomorphism (ii) $\mu_S^+(x)e^{i\beta_S^+(x)} = \mu_S^+(p(x))e^{i\beta_S^+(p(x))}$ and $\mu_S^-(x)e^{i\alpha_S^-(x)} = \mu_S^-(p(x))e^{i\alpha_S^-(p(x))}$.

Example 3.2. *A neighbourly edge improper ivcf graph is considered on $G^*(F, I)$.*

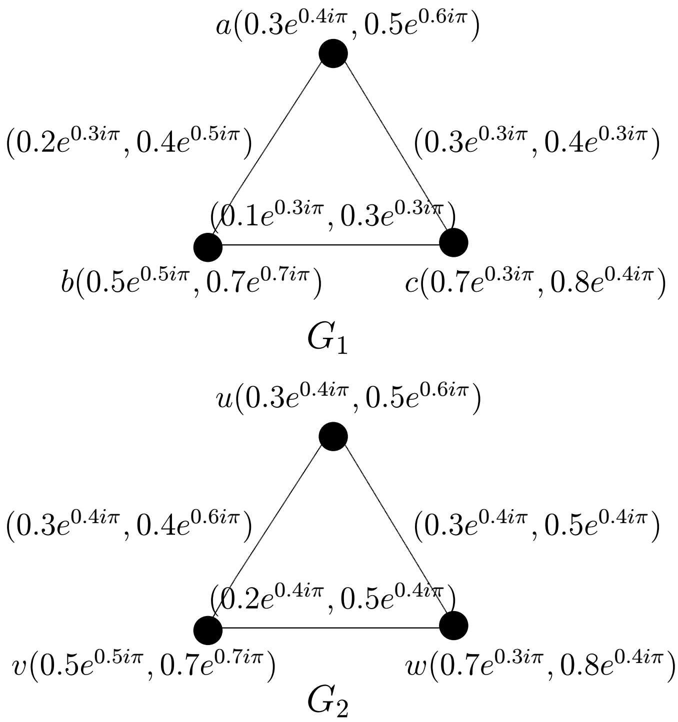


Figure 2: Weak isomorphism of neighbourly edge improper ivcf graph

In Figure 2, there is a weak isomorphism $p\colon F_1 \to F_2$ such that $p(a) = u$, $p(b) = v, p(c) = w$.

Definition 3.3. A co-weak isomorphism of neighbourly edge improper ivcf graph(highly edge improper ivcf graph) G_1 and G_2 is a bijective mapping $p\colon F_1 \to F_2$ which fulfills the following criteria

(i) p is homomorphism

(ii)$\mu_T^+(xy)e^{i\beta_T^+(x)} = \mu_T^+(p(xy))e^{i\beta_T^+(p(xy))}$ and $\mu_T^-(xy)e^{i\alpha_T^-(xy)} = \mu_T^-(p(xy))e^{i\alpha_T^-(p(xy))}$.

Example 3.3. *A highly edge improper ivcf graph is considered on $G^*(F, I)$.*

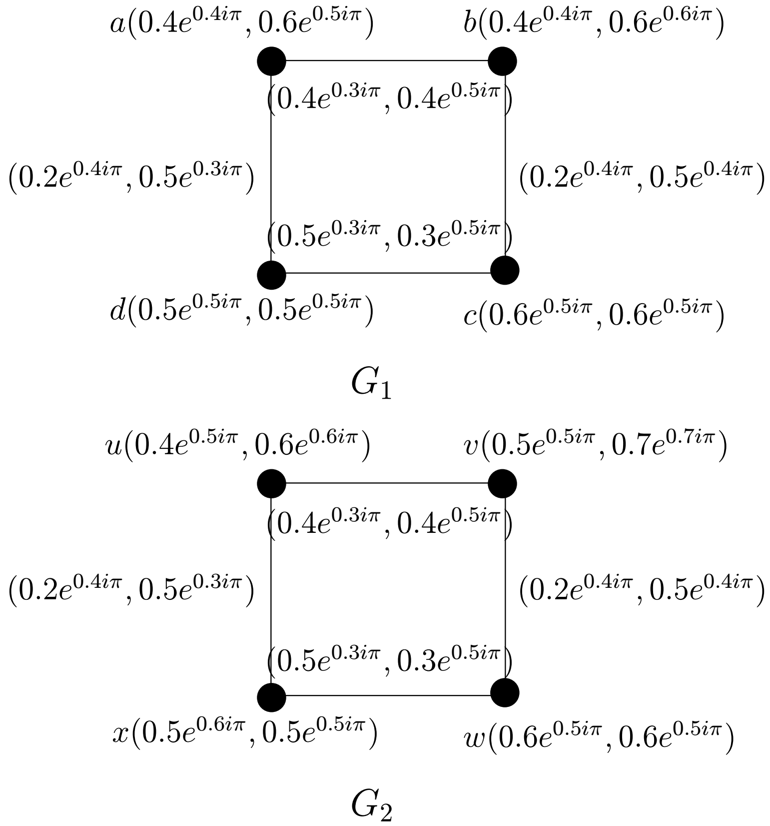


Figure 3:Co-weak isomorphism of neighbourly edge improper ivcf graph

In Figure 3, there is a homomorphism $p\colon F_1 \to F_2$ such that $p(a) = u$, $p(b) = v$, $p(c) = w, p(d) = x$.

Definition 3.4. An isomorphism p of neighbourly edge improper ivcf graph(highly edge improper ivcf graph) G_1 and G_2 is a bijective mapping $p\colon F_1 \to F_2$ which fulfills the following criteria
(i) $\mu_S^+(x)e^{i\beta_S^+(x)} = \mu_S^+(p(x))e^{i\beta_S^+(p(x))}$ and $\mu_S^-(x)e^{i\alpha_S^-(x)} = \mu_S^-(p(x))e^{i\alpha_S^-(p(x))}$.
(ii) $\mu_T^+(xy)e^{i\beta_T^+(x)} = \mu_T^+(p(xy))e^{i\beta_T^+(p(xy))}$ and $\mu_T^-(xy)e^{i\alpha_T^-(xy)} = \mu_T^-(p(xy))e^{i\alpha_T^-(p(xy))}$.

Theorem 3.1. *If any two neighbourly(highly) edge improper ivcf graphs are isomorphic then their order and size are same.*

Proof. If $p\colon G_1 \to G_2$ be an isomorphism between neighbourly edge improper ivcf graph then,
$\mu_S^+(x)e^{i\beta_S^+(x)} = \mu_S^+(p(x))e^{i\beta_S^+(p(x))}$ and $\mu_S^-(x)e^{i\alpha_S^-(x)} = \mu_S^-(p(x))e^{i\alpha_S^-(p(x))}$.
$\mu_T^+(xy)e^{i\beta_T^+(x)} = \mu_T^+(p(xy))e^{i\beta_T^+(p(xy))}$ and $\mu_T^-(xy)e^{i\alpha_T^-(xy)} = \mu_T^-(p(xy))e^{i\alpha_T^-(p(xy))}$.

$$O_I(G_1) = (\sum \mu_S^+(u)e^{i\beta_S^+(u)}, \sum \mu_S^-(u)e^{i\alpha_S^-(u)})$$
$$= (\sum \mu_S^+(p(u)e^{i\beta_S^+(p(u))}), \sum \mu_S^-(p(u)e^{i\alpha_S^-(p(u))}))$$
$$= (\sum \mu_S^+(w)e^{i\beta_S^+(w)}, \sum \mu_S^-(w)e^{i\alpha_S^-(w)}) = O_I(G_2)$$
$$S_I(G_1) = (\sum \mu_T^+(uv)e^{i\beta_T^+(uv)}, \sum \mu_T^-(uv)e^{i\alpha_T^-(uv)})$$
$$= (\sum \mu_T^+(p(uv)e^{i\beta_T^+(uv)}), \sum \mu_T^-(p(uv)e^{i\alpha_T^-(p(uv))}))$$
$$= (\sum \mu_T^+(wz)e^{i\beta_T^+(wz)}, \sum \mu_T^-(wz)e^{i\alpha_T^-(wz)}) = S_I(G_2).$$

□

Remark 3.1. *The converse of above Theorem 3.1 need not be true for both neighbourly edge improper ivcf graph and highly edge improper ivcf graph.*

Fact 3.1. *If neighbourly edge improper ivcf graph(highly edge improper ivcf graph) are weak isomorphic then their order are same.*

Remark 3.2. *The converse of the above fact is need not be true.*

Fact 3.2. *If neighbourly edge improper ivcf graph(highly edge improper ivcf graph)are co-weak isomorphic then their size are same.*

Remark 3.3. *The converse of the above fact is need not be true.*

Theorem 3.2. *If G_1 and G_2 are neighbourly(highly) edge improper ivcf graphs which are isomorphic then the degrees of corresponding vertices u and $p(u)$ are preserved.*

Proof. If $p\colon G_1 \to G_2$ is an isomorphism between neighbourly edge improper ivcf graph then

$\mu_T^+(uv)e^{i\beta_T^+(uv)} = \mu_T^+(p(uv))e^{i\beta_T^+(p(uv))}$ and
$\mu_T^-(uv)e^{i\alpha_T^-(uv)} = \mu_T^-(p(uv))e^{i\alpha_T^-(p(uv))}$.

$$d_I^+(u) = \sum \mu_T^+(uv)e^{\sum i\beta_T^+(uv)} = \sum \mu_T^+(p(uv))e^{\sum i\beta_T^+(p(uv))} = d_I^+(p(u)).$$

$$d_I^-(u) = \sum \mu_T^-(uv)e^{\sum i\alpha_T^-(uv)} = \sum \mu_T^-(p(uv))e^{\sum i\alpha_T^-(p(uv))} = d_I^-(p(u)).$$

Thus the degrees of corresponding vertices of G_1 and G_2 are the same. □

Remark 3.4. *The converse of Theorem 3.2 need not be true.*

Theorem 3.3. *Any two highly edge improper ivcf graphs are isomorphic if and only if their complement are isomorphic.*

Proof. Let G_1 and G_2 be two highly edge improper ivcf graphs.
Assume that G_1 and G_2 are isomorphic. There exists a bijective map $p\colon F_1 \to F_2$ satisfying

$$\mu_S^+(x)e^{i\beta_S^+(x)} = \mu_S^+(p(x))e^{i\beta_S^+(p(x))}, \mu_S^-(x)e^{i\alpha_S^-(x)} = \mu_S^-(p(x))e^{i\alpha_S^-(p(x))},$$

$$\mu_T^+(xy)e^{i\beta_T^+(xy)} = \mu_T^+(p(xy))e^{i\beta_T^+(p(xy))}, \mu_T^-(xy)e^{i\alpha_T^-(xy)} = \mu_T^-(p(xy))e^{i\alpha_T^-(p(xy))},$$

$$\bar{\mu}_T^+(xy)e^{i\bar{\beta}_T^+(xy)} = \mu_S^+(x)e^{i\beta_S^+(x)} \wedge \mu_S^+(y)e^{i\beta_S^+(y)} - \mu_T^+(xy)e^{i\beta_T^+(xy)}$$

$$= \mu_S^+(p(x))e^{i\beta_S^+(p(x))} \wedge \mu_S^+(p(y))e^{i\beta_S^+(p(y))} - \mu_T^+(p(xy))e^{i\beta_T^+(p(xy))}$$

$$\bar{\mu}_T^-(xy)e^{i\bar{\alpha}_T^-(x)} = \mu_S^-(x)e^{i\alpha_S^-(x)} \wedge \mu_S^-(y)e^{i\alpha_S^-(y)} - \mu_T^-(xy)e^{i\alpha_T^-(xy)}$$

$$= \mu_S^-(p(x))e^{i\alpha_S^-(p(x))} \wedge \mu_S^-(p(y))e^{i\alpha_S^-(p(y))} - \mu_T^-(p(xy))e^{i\alpha_T^-(p(xy))}$$

Hence $\bar{G}_1 \cong \bar{G}_2$.
Conversely assume $\bar{G}_1 \cong \bar{G}_2$
There exists a bijective map $p\colon F_1 \to F_2$ satisfying

$$\mu_S^+(x)e^{i\beta_S^+(x)} = \mu_S^+(p(x))e^{i\beta_S^+(p(x))}, \mu_S^-(x)e^{i\alpha_S^-(x)} = \mu_S^-(p(x))e^{i\alpha_S^-(p(x))},$$

$$\bar{\mu}_T^+(xy)e^{i\bar{\beta}_T^+(xy)} = \bar{\mu}_T^+(p(xy))e^{i\bar{\beta}_T^+(p(xy))}, \bar{\mu}_T^-(xy)e^{i\bar{\alpha}_T^-(xy)} = \bar{\mu}_T^-(p(xy))e^{i\bar{\alpha}_T^-(p(xy))}$$

using definition of complement

$$\bar{\mu}_T^+(xy)e^{i\bar{\beta}_T^+(xy)} = \mu_S^+(x)e^{i\beta_S^+(x)} \wedge \mu_S^+(y)e^{i\beta_S^+(y)} - \mu_T^+(xy)e^{i\beta_T^+(xy)}$$

$$\bar{\mu}_T^-(xy)e^{i\bar{\alpha}_T^-(x)} = \mu_S^-(x)e^{i\alpha_S^-(x)} \wedge \mu_S^-(y)e^{i\alpha_S^-(y)} - \mu_T^-(xy)e^{i\alpha_T^-(xy)}$$

using the above equations

$$\mu_T^+(xy)e^{i\beta_T^+(xy)} = \mu_T^+(p(xy))e^{i\beta_T^+(p(xy))}, \mu_T^-(xy)e^{i\alpha_T^-(xy)} = \mu_T^-(p(xy))e^{i\alpha_T^-(p(xy))}$$

Hence $G_1 \cong G_2$. $\square$

Theorem 3.4. *Let G_1 and G_2 be two highly edge improper ivcf graphs. If G_1 is weak isomorphism with G_2, then $\bar{G}_1$ is weak isomorphic with $\bar{G}_2$.*

Proof. If p is weak isomorphism between G_1 and G_2 then $p\colon F_1 \rightarrow F_2$ is a bijective function such that, $\mu_S^+(x)e^{i\beta_S^+(x)} = \mu_S^+(p(x))e^{i\beta_S^+(p(x))}, \mu_S^-(x)e^{i\alpha_S^-(x)} = \mu_S^-(p(x))e^{i\alpha_S^-(p(x))}$

$\mu_T^+(xy)e^{i\beta_T^+(xy)} \leq \mu_T^+(p(xy))e^{i\beta_T^+(p(xy))}, \mu_T^-(xy)e^{i\alpha_T^-(xy)} \leq \mu_T^-(p(xy))e^{i\alpha_T^-(p(xy))}$

As, $p^{-1}\colon F_2 \rightarrow F_1$ is also bijective, for every $w \in F_2$, there exists $x \in F_1$ such that $p^{-1}(w) = m_i$

$\mu_S^+(x)e^{i\beta_S^+(x)} = \mu_S^+(p^{-1}(w))e^{i\beta_S^+(p^{-1}(w))}, \mu_S^-(x)e^{i\alpha_S^-(x)} = \mu_S^-(p^{-1}(w))e^{i\alpha_S^-(p^{-1}(w))}$

$$\bar{\mu}_T^+(xy)e^{i\bar{\beta}_T^+(xy)} = \mu_S^+(x)e^{i\alpha_S^-(x)} \wedge \mu_S^+(y)e^{i\alpha_S^-(y)} - \mu_T^+(xy)e^{i\alpha_T^-(y)}$$

$$\bar{\mu}_T^+(p^{-1}(wz)))e^{i\bar{\beta}_T^+(p^{-1}(wz))} \geq \mu_S^+(p(x)))e^{i\beta_S^+(p(x))} \wedge \mu_S^+(p(y)))e^{i\beta_S^+(p(y))} - \mu_T^+(p(xy))e^{i\beta T^{+(p(xy))}}$$

$$= \mu_S^+(w)e^{i\beta_S^+(w)} \wedge \mu_S^+(z)e^{i\beta_S^+(z)} - \mu_S^+(wz)e^{i\beta_S^+(wz)} = \bar{\mu}_T^+(wz)e^{i\beta_T^+(wz)}$$

$$\bar{\mu}_T^+(wz)e^{i\beta_T^+(wz)} \leq \bar{\mu}_T^+(p^{-1}(wz))e^{i\bar{\beta}_T^+(p^{-1}(wz))}$$

$$\bar{\mu}_T^-(xy)e^{i\bar{\alpha}_T^-(xy)} = \mu_S^-(x)e^{i\alpha_S^-(x)} \wedge \mu_S^-(y)e^{i\alpha_S^-(y)} - \mu_S^-(xy)e^{i\alpha_S^-(xy)}$$

$$\bar{\mu}_T^-(p^{-1}(wz))e^{i\bar{\alpha}_T^-(p^{-1}(wz))} \leq \mu_S^-(p(x))e^{i\alpha_S^-(p^{-1}(x))} \wedge \mu_S^-(p(y))e^{i\alpha_S^-(p^{-1}(y))} - \mu_S^-(p(xy))e^{i\alpha_S^-(p(xy))}$$

$$= \mu_S^-(w)e^{i\alpha_S^-(w)} \wedge \mu_S^-(z)e^{i\alpha_S^-(z)} - \mu_S^-(wz)e^{i\alpha_S^-(wz)} = \bar{\mu}_T^-(wz)e^{i\alpha_T^-(wz)}$$

$\bar{\mu}_T^-(wz)e^{i\alpha_T^-(wz)} \geq \bar{\mu}_T^-(p^{-1}(wz))e^{i\bar{\alpha}_T^-(p^{-1}(wz))}$.

So, $p^{-1}\colon F_1 \rightarrow F_2$ is weak isomorphism between $\bar{G}_1$ and $\bar{G}_2$. □

4 μ-Complement of Highly Edge improper Interval-valued complex Fuzzy Graphs

Definition 4.1. Let $G : (S, T)$ be a ivcf graph. The μ-complement of G is defined as $G^\mu : (S, T^\mu)$ where $T^\mu = ({\mu_T^+}^\mu, {\mu_T^-}^\mu)$

$${\mu_T^+}^\mu(xy)e^{i{\beta_T^+}^\mu(xy)} = \begin{cases} \mu_S^+(x)e^{i\beta_S^+(x)} \wedge \mu_S^+(y)e^{i\beta_S^+(y)} & \\ -\mu_T^+(xy)e^{i\beta_T^+(xy)} & \mu_T^+(xy)e^{i\beta_T^+(xy)} > 0 \\ 0 & \mu_T^+(xy)e^{i\beta_T^+(xy)} = 0 \end{cases}$$

$${\mu_T^-}^\mu(xy)e^{i{\alpha_T^-}^\mu(xy)} = \begin{cases} \mu_S^-(x)e^{i\alpha_S^-(x)} \wedge \mu_S^-(y)e^{i\alpha_S^-(y)} & \\ -\mu_T^-(xy)e^{i\alpha_T^-(xy)} & \mu_T^-(xy)e^{i\alpha_T^-(xy)} > 0 \\ 0 & \mu_T^-(xy)e^{i\alpha_T^-(xy)} = 0. \end{cases}$$

Theorem 4.1. *Let G_1 and G_2 be two highly edge improper isomorphic ivcf graphs. Then μ-complement of G_1 and G_2 are also isomorphic. But the μ complements need not be highly edge improper ivcf graphs.*

Proof. Assume that G_1 and G_2 are isomorphic. There exists bijective map $p\colon F_1 \to F_2$ satisfying

$\mu_S^+(x)e^{i\beta_S^+(x)} = \mu_S^+(p(x))e^{i\beta_S^+(p(x))}$, $\mu_S^-(x)e^{i\alpha_S^-(x)} = \mu_S^-(p(x))e^{i\alpha_S^-(p(x))}$

$\mu_T^+(xy)e^{i\beta_T^+(xy)} = \mu_T^+(p(xy))e^{i\beta_T^+(p(xy))}$, $\mu_T^-(xy)e^{i\alpha_T^-(xy)} = \mu_T^-(p(xy))e^{i\alpha_T^-(p(xy))}$

By definition of μ-complement,

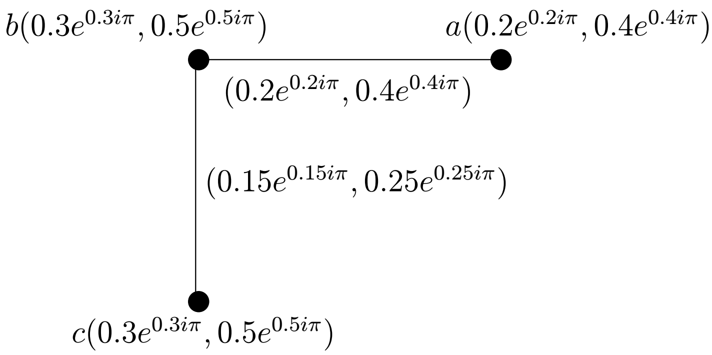
$$\begin{aligned}\mu_T^{+\mu}(xy)e^{i\beta_T^{+\mu}(xy)} &= \mu_S^+(x)e^{i\beta_S^+(x)} \wedge \mu_S^+(y)e^{i\beta_S^+(y)} - \mu_T^+(xy)e^{i\beta_T^+(xy)}\\ &= \mu_T^+(p(x))e^{i\beta_T^+(p(x))} \wedge \mu_T^+(p(y))e^{i\beta_T^+(p(y))} - \mu_T^+(p(xy))e^{i\beta_T^+(p(xy))}\end{aligned}$$

$$\begin{aligned}\mu_T^{-\mu}(xy)e^{i\alpha_T^{-\mu}(xy)} &= \mu_S^-(x)e^{i\alpha_S^-(x)} \wedge \mu_S^-(y)e^{i\alpha_S^-(y)} - \mu_T^-(xy)e^{i\alpha_T^-(xy)}\\ &= \mu_T^-(p(x))e^{i\alpha_T^-(p(x))} \wedge \mu_T^-(p(y))e^{i\alpha_T^-(p(y))} - \mu_T^-(p(xy))e^{i\alpha_T^-(p(xy))}\end{aligned}$$

Hence $\bar{G}_1 \sim \bar{G}_2$. □

Definition 4.2. An ivcf graph G is said to be self μ-complementary if $G \sim G^\mu$.

Example 4.1. *An ivcf graph is considered on $G^*(F, I)$.*



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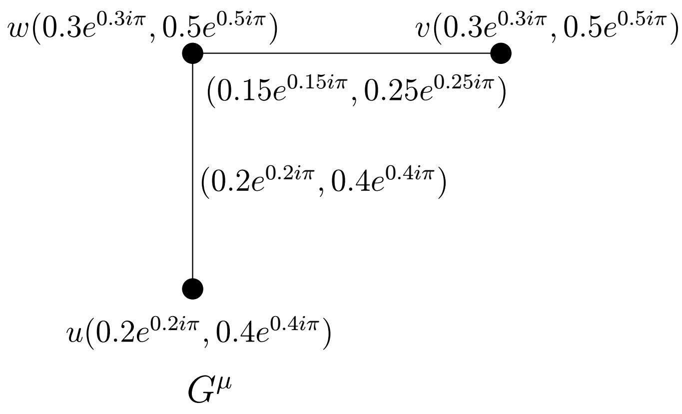

Figure 4: Self μ-complementary

In Figure 4, the two graphs are isomorphic. Hence G is Self μ-complementary ivcf graph.

Theorem 4.2. *Let G be self μ-complementary highly edge improper ivcf graph, then*
$\sum \mu_T^+(uv)e^{i\beta_T^+(uv)} = \frac{1}{2}\sum \mu_S^+(u)e^{i\beta_S^+(u)} \wedge \mu_S^+(v)e^{i\beta_S^+(v)}$ *and* $\sum \mu_T^-(uv)e^{i\alpha_T^-(uv)} = \frac{1}{2}\sum \mu_S^-(u)e^{i\alpha_S^-(u)} \wedge \mu_S^-(v)e^{i\alpha_S^-(v)}$.

Proof. Let G be self μ-complementary highly edge improper ivcf graph. Since $G \sim G^{\mu}$, there exists a bijective map $p\colon V \to V$ such that
$\mu_S^+(u)e^{i\beta_S^+(u)} = \mu_S^{+\mu}(p(u))e^{i\beta_S^{+\mu}(p(u))} = \mu_S^+(p(u))e^{i\beta_S^+(p(u))}$ and
$\mu_S^-(u)e^{i\alpha_S^-(u)} = \mu_S^{-\mu}(p(u))e^{i\alpha_S^{-\mu}(p(u))} = \mu_S^-(p(u))e^{i\alpha_S^-(p(u))}$

$\mu_T^+(uv)e^{i\beta_T^+(uv)} = \mu_T^{+\mu}(p(uv))e^{i\beta_T^{+\mu}(p(uv))}$
and $\mu_T^-(uv)e^{i\alpha_T^-(u)} = \mu_T^{-\mu}(p(uv))e^{i\alpha_T^{-\mu}(p(uv))}$

By definition of μ-complement, we have

$$\mu_T^{+\mu}(xy)e^{i\beta_T^{+\mu}(xy)} = \begin{cases} \mu_S^+(x)e^{i\beta_S^+(x)} \wedge \mu_S^+(y)e^{i\beta_S^+(y)} & \\ -\mu_T^+(xy)e^{i\beta_T^+(xy)} & \mu_T^+(xy)e^{i\beta_T^+(xy)} > 0 \\ 0 & \mu_T^+(xy)e^{i\beta_T^+(xy)} = 0 \end{cases}$$

$\mu_T^{+\mu}(p(uv))e^{i\beta_T^{+\mu}(p(uv))}$
$= \mu_S^+(p(u))e^{i\beta_S^+(p(u))} \wedge \mu_S^+(p(v))e^{i\beta_S^+(p(v))} - \mu_T^+(p(uv))e^{i\beta_T^+(p(uv))}$

Now, $\mu_T^+(uv)e^{i\beta_T^+(uv)}$
$= \mu_S^+(p(u))e^{i\beta_S^+(p(u))} \wedge \mu_S^+(p(v))e^{i\beta_S^+(p(v))} - \mu_T^+(p(uv))e^{i\beta_T^+(p(uv))}$

$$\mu_T^+(uv)e^{i\beta_T^+(uv)}+\mu_T^+(p(uv))e^{i\beta_T^+(p(uv))} = \mu_S^+(p(u))e^{i\beta_S^+(p(u))}\wedge\mu_S^+(p(v))e^{i\beta_S^+(p(v))}$$

$$\Rightarrow 2\mu_T^+(uv)e^{i\beta_T^+(uv)} = \mu_S^+(u)e^{i\beta_S^+(u)} \wedge \mu_S^+(v)e^{i\beta_S^+(v)}$$

$$\Rightarrow 2\sum \mu_T^+(uv)e^{i\beta_T^+(uv)} = \sum \mu_S^+(u)e^{i\beta_S^+(u)} \wedge \mu_S^+(v)e^{i\beta_S^+(v)}$$
$$\Rightarrow \sum \mu_T^+(uv)e^{i\beta_T^+(uv)} = \frac{1}{2}\sum \mu_S^+(u)e^{i\beta_S^+(v)} \wedge \mu_S^+(v)e^{i\beta_S^+(v)}$$

Similarly we can show that $\sum \mu_T^-(uv)e^{i\alpha_T^-(uv)} = \frac{1}{2}\sum \mu_S^-(u)e^{i\alpha_S^-(u)}\wedge\mu_S^-(v)e^{i\alpha_S^-(v)}$. $\square$

Remark 4.1. *If G is highly edge improper ivcf graph with $\sum \mu_T^+(uv)e^{i\beta_T^+(uv)} = \frac{1}{2}\sum \mu_S^+(u)e^{i\beta_S^+(u)} \wedge \mu_S^+(v)e^{i\beta_S^+(v)}$ and $\sum \mu_T^-(uv)e^{i\alpha_T^-(uv)} = \frac{1}{2}\sum \mu_S^-(u)e^{i\alpha_S^-(u)} \wedge \mu_S^-(v)e^{i\alpha_S^-(v)}$ then G need not be self μ-complementary.*

5 Conclusion

In this paper we have introduced the notions of isomorphic, weak isomorphic, co-weak isomorphic of neighbourly(highly) edge improper ivcf graphs. In future we study the properties for some standard graphs and have an idea to extend the concept of $\mu-$complement of neighbourly(highly) edge improper ivcf graphs. Though isomorphism exists on several fuzzy graphs isomorphic on interval-valued complex fuzzy graph is limited only on the case which involves time phenomena.

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