Optimization of filament antennas using the Gauss-Newton method

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Abstract

The project of the Yagi-Uda antenna was optimized using the Gauss-Newton method. The optimization consisted of specifying value interval for directivity, front-to-back ratio and beamwidth and, starting from a pre-defined initial model, the best values for the length and spacing of the elements were determined. For the direct modeling, the method of moments on the integral Pocklington equation was used, which consisted of obtaining the values of directivity, front-to-back ratio and beamwidth from the length and spacing between known elements. The procedure was applied to the synthesis of Yagi-Uda antennas with five and six elements and the results were found to be as good as those obtained in the literature using other optimization methods.

Keywords: Yagi-Uda antenna, gauss-Newton method, optimization.

1. INTRODUCTION

The Yagi-Uda antenna was introduced in 1920 by S. Uda [1]. A conventional Yagi-Uda antenna consists of parallel linear dipoles of which only one, usually the second element, is powered by a source, the rest are parasitic elements. The first element acts as a reflector, which is larger in size than the energized element. From the third to the n-th are guiding elements and are smaller than the source element. Figure 1 shows a Yagi-Uda antenna with six elements.

Despite the simple appearance of the Yagi-Uda antenna, the project of this device is not an easy task, mainly because there are many interrelationships between the variables involved in the project, for example, the elements are electromagnetically coupled and a small variation in length and / or in spacing between antenna elements can change the current distribution over all components.

The difficulties of the project caused the attention of some researchers to focus on the optimization of the Yagi-Uda antenna [2-4].

Cheng [2] used the gradient method to optimize the gain and input impedance of the Yagi-Uda antenna, his results increased the gain of an initial project of a Yagi-Uda device by 80%.

Jones and Joines [3] and Ramos et al. [4] used a genetic algorithm for the project of the Yagi-Uda antenna, their results were as good as those presented by [2].

For optimization problems [5-10] with few parameters, local search methods, such as Newton, Quasi-Newton and Gauss-Newton have a good performance. In addition, these techniques are computationally, in these cases, as fast as the global search methods, such as the genetic algorithm. Thus, it becomes attractive to develop the optimization of the Yagi-Uda antenna by applying a local search technique.

This work applies the Gauss-Newton method to optimize directivity, beamwidth and front-to-back ratio of the Yagi-Uda antenna by adjusting the values of the lengths and spacing between elements of this radiation device. For these parameters to remain within a range of pre-established values, a transformation technique will be applied.

2. GAUSS-NEWTON METHOD

The objective of the project is to develop a Yagi-Uda antenna that brings together some characteristics required for a good performance of this device. The characteristics that will be specified in this work are: number of elements, being a reflector and an energized element; directivity; front-to-back ratio; and half angle. The values of the sizes and spacing between the elements will be adjusted by the Gauss-Newton optimization process.

An initial project of the antenna will be established and, based on this data, the Gauss-Newton method will be applied until the value ranges of the pre-established characteristics are reached.

The quality of the optimization process is related, mathematically, to a cost function.

The cost function used in the Gauss-Newton optimization process, in this work, was the same developed by [5,6,8], and is given by the expression (1).

$$C(m) = \frac{1}{2}\left\{ \mu \left[\left\| \overline{\overline{W}}_d . (f(m) - d^{obs}) \right\|^2 - \chi^2 \right] + \left\| \overline{\overline{W}}_m (m - m_r) \right\|^2 \right\} \tag{1}$$

Where μ ($0<\mu<\infty$) is the regularization parameter (Lagrange multiplier); χ^2 is a pre-established value for adjusting the data; m is the vector containing the values to be adjusted (model parameters), which in this work were the sizes and spacing between the elements; $f(m)$ is the direct modeling operator; d^{obs} is the vector containing the values required for the antenna design (directivity, front-to-back ratio and half power angle); W_m is the inverse of the covariance matrix of the data to be adjusted; and W_d is the inverse of the covariance matrix of the pre-established data for the project.

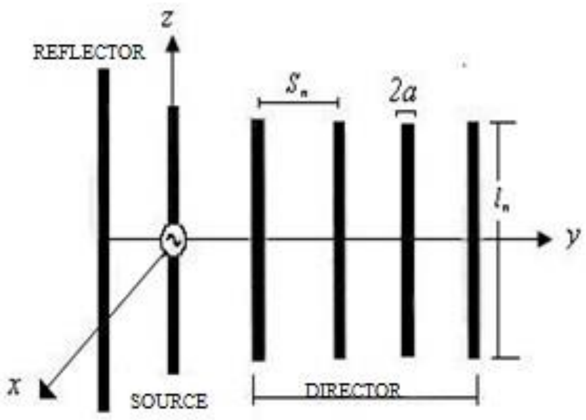

Figure 1: Yagi-Uda antenna with six elements.

The problem presented here is one of nonlinear optimization for which Newton's iterative method is applied. This technique is based on a quadratic representation of the cost function. The quadratic model is obtained by taking the first three terms of the Taylor series expansion of the cost function (1) around the k-th iteration (m_k), like this:

$$C(m_k+\Delta m_k)=C(m_k)+g^T(m_k).\Delta m_k++\frac{1}{2}\Delta m_k{}^T.G(m_k).\Delta m_k \quad (2)$$

where T denotes transposed matrix and $\Delta m_k = m_{k+1}-m_k$ is the increment of the vector of the parameters towards the stationary point of the cost function $C(m_k)$; $\bar{g}(m)=\nabla C(m)$; is the gradient vector of the cost function, that is:

$$g(m)=\nabla C(m)=\left[g_n\equiv\frac{\partial C}{\partial m_n},\quad n=1,2,3,\cdots,N\right]=\mu J^T(m).W_d{}^T\cdot W_d\cdot e(m)+W_x{}^T\cdot W_x\cdot(m-m_r)$$

m_n is the n-th component of the vector of the parameters of model m; $e(m)=f(m)-d^{obs}$ is the residual error vector; and $J(m)$ is a matrix whose dimension is $M\times N$ called Jacobian (or sensitivity) and is given by:

$$J(m)=\left[\frac{\partial e_l(m)}{\partial m_n},\quad l=1,2,3,\cdots,M;\quad n=1,2,3,\cdots,N\right]=$$

$$=\begin{bmatrix}\frac{\partial f_1}{\partial m_1} & \cdots & \frac{\partial f_1}{\partial m_j} & \cdots & \frac{\partial f_1}{\partial m_N}\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ \frac{\partial f_i}{\partial m_1} & \cdots & \frac{\partial f_i}{\partial m_j} & \cdots & \frac{\partial f_i}{\partial m_N}\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ \frac{\partial f_M}{\partial m_1} & \cdots & \frac{\partial f_M}{\partial m_j} & \cdots & \frac{\partial f_M}{\partial m_N}\end{bmatrix}$$

$G(m)=\nabla\nabla C(m)$ is the Hessian of the cost function which is a symmetric matrix of order N×N given by:

$$G(m) = \nabla\nabla C(m) = \left[G_{nl} = \frac{\partial^2 C}{\partial m_n \partial m_l},\ n,l = 1,2,3,\cdots,N \right] =$$
$$= W_m \cdot W_m^{\ T} + \mu \left[J^T(m) \cdot W_d^{\ T} \cdot W_d \cdot J(m) + Q(m) \right]$$

with $Q(m) = \sum_l^M f_l(m) F_l^T(m)$ being $f_l(m)$ the l-th element of the vector $f_l(m) = W_d . e(m)$, and

$$F_l(m) = \nabla\nabla f_l(m) = \left[\frac{\partial^2 f_l}{\partial m_i \partial m_j} \quad i,j = 1,2,3,\cdots,N \right]$$

The minimum of (2) is obtained when Δm_k is a minimum of the quadratic function

$$\phi(\Delta m) = g^T(m_k) \cdot \Delta m + \frac{1}{2} \Delta m^T \cdot G(m_k) \cdot \Delta m \tag{3}$$

The function $\phi(\Delta m)$ has a stationary point (critical point) in Δm_k only if the gradient of $\phi(\Delta m)$ is to zero in Δm_k, that is:

$$\nabla\phi(\Delta m_k) = g^T(m_k) + G(m_k) \cdot \Delta m_k = 0 \tag{4}$$

Thus, the stationary point Δm_k of the function $\phi(\Delta m)$ will be the solution of the system of linear equations:

$$G(m_k) \cdot \Delta m_k = -g(m_k) \tag{5}$$

Depending on the definition of the Hessian matrix, the stationary point given by (5) can be a minimum, maximum or saddle point.

According to [5], the condition on the Hessian matrix $G(m)$ in being singular or non-singular and its definition (positive, negative defined or undefined) can be adjusted by an appropriate choice of the regularization parameter μ.

In the Gauss-Newton method, second order derivatives of the cost function are disregarded in relation to the vector components of the model parameters (m), that is, the term Q is not considered. Thus, the Hessian, in the Gauss method -Newton, will be given by:

$$G(m) = W_m \cdot W_m^{\ T} + \mu J^T(m) \cdot W_d^{\ T} \cdot W_d \cdot J(m) \tag{6}$$

The method is reduced to solving a system of linear equations presented in (7).

$$\begin{aligned} &[J^H.W_d^{\ T}.W_d J^H + W_m^{\ T}.W_m].\Delta m_j = \\ &\qquad \mathrm{J}^{\mathrm{H}}.W_d^{\ T}.W_d.[d^{obs} - f(m_j)] - W_m^{\ T}.W_m.m_j \end{aligned} \tag{7}$$

The system of linear equations (7) was solved using the Gaussian elimination method.

The direct modeling, that is, the design of the Yagi-Uda antenna from the length and spacing entries between the antenna elements and as the output, directivity, beamwidth and front-to-back ratio, is based on the method of moments presented. in 1].

In order that the optimized parameters are always within a range of pre-established values, the following transformation technique was applied [5]:

$$m_{k+1} = m_{\min} + \frac{m_{\max} - m_{\min}}{\alpha_k^2 + (m_k - m_{\min})(m_{\max} - m_k)^3}\alpha_k^2$$

with

$$\alpha_k = (m_k - m_{\min})(m_{\max} - m_k) + \frac{1}{2}(m_{\max} - m_{\min})\nu_k \Delta m_k$$

where ν_k is the search direction (see [5]).

3. RESULTS

To apply the procedure described in this work in the optimization of the directivity, the beamwidth and the front-to-back ratio of the Yagi-Uda antenna, we will use the examples of antennas with six elements, also used in the studies of [2] and [4], and with five elements.

3.1 Antenna With Six Elements

The antenna optimized by [4] will serve as the initial project for this work. The initial data, in wavelength (λ), are shown in Table 1.

Table 1. Initial data of the project of the six-element antenna to be optimized.

Element	Length	Element	Spacing
1º director	0.448 λ	Source and 1º director	0.152 λ
2º director	0.434 λ	1º and 2º director	0.229 λ
3º director	0.422 λ	2º and 3º director	0.435 λ
4º director	0.440 λ	3º and 4º director	0.272 λ
Reflector	0.478 λ	Reflector and Source	0.182 λ
Source	0.450 λ	-	-

The initial design of the antenna provides as output data the values shown in Table 2.

Table 2. Output data of the initial antenna project.

Beamwidth(plan E)	40,06 degrees
Beamwidth(plan H)	43,76 degrees
Front-to-back ratio (plan E)	16,0552 dB
Front-to-back ratio (plan H)	16,0457 dB
Directivity	11,758 dB

The radiation diagram of this antenna for the plan H and for is shown in Figure 2. Applying the Gauss-Newton optimization process described above, the values presented in Table 3 were obtained for length and spacing between the antenna elements.

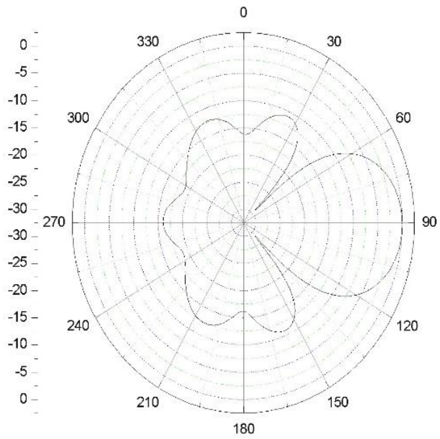

Figure 2: Radiation diagram (plan H) of the six-element Yagi-Uda antenna with the initial data.

Tabela 3. Data optimized by the Gauss-Newton method of the six-element antenna.

Element	Length	Element	Spacing
1º director	0.44900λ	Source and 1º director	0.15292 λ
2º director	0.42301 λ	1º and 2º director	0.25761 λ
3º director	0.42028 λ	2º and 3º director	0.42740 λ
4º director	0.41125 λ	3º and 4º director	0.24082 λ
Reflector	0.50197 λ	Reflector and Source	0.14675 λ
Source	0.44900 λ	-	-

These optimized data provide the values shown in Table 4 as an output.

Table 4. Project output data optimized by the Gauss-Newton method of the six-element antenna.

Beamwidth(plan E)	42,44 degrees
Beamwidth(plan H)	46,95 degrees
Front-to-back ratio (plan E)	19,119 dB
Front-to-back ratio (plan H)	19,099 dB
Directivity	12,859 dB

It can be seen that the Gauss-Newton process, applied in the optimization of the six element Yagi-Uda antenna design, obtained very significant results for directivity, increasing from 11,758 to 12,859 db and for the front-to-back ratio, both in the plan E and plan H. However, there was a small loss in the beamwidth parameter. Figure 3 shows the radiation diagram for the plan H of the optimized antenna in this work.

3.2 Antenna With Five Elements

For the antenna with five elements, the initial data shown in Table 5 were considered.

Table 5. Initial project data for the five-element antenna to be optimized.

Element	Length	Element	Spacing
1º director	0.419 λ	Source and 1º director	0.271 λ
2º director	0.437 λ	1º and 2º director	0.362 λ
3º director	0.407 λ	2º and 3º director	0.390 λ

Reflector	0.483 λ	Reflector and Source	0.224 λ
Source	0.434 λ	-	-

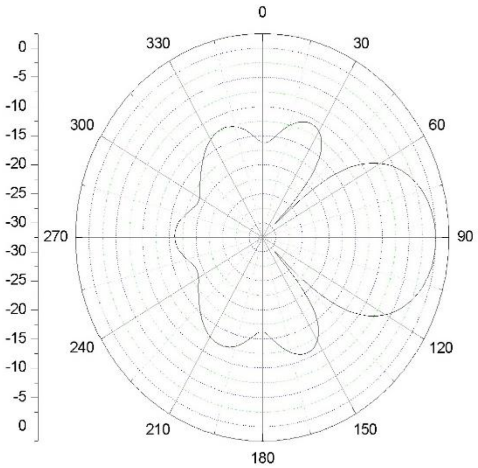

Figure 3: Radiation diagram (plan H) of the six element Yagi-Uda antenna with data optimized by the Gauss-Newton method.

Where the radiation diagram of this antenna for the plan H is shown in Figure 4.

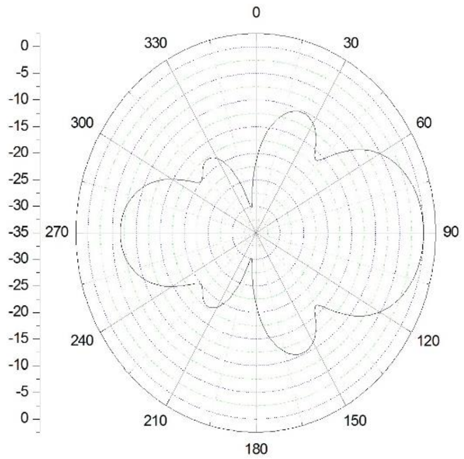

Figure 4: Radiation diagram (plan H) of the five-element Yagi-Uda antenna with the initial data.

Table 6. Output data of the initial project of the five element antenna.

Beamwidth(plan E)	43,08 degrees
Beamwidth(plan H)	47,81 degrees
Front-to-back ratio (plan E)	6,6602 dB
Front-to-back ratio (plan H)	6,6516 dB
Directivity	11,444 dB

After applying the Gauss-Newton method, the values shown in Table 7 for length and spacing between the elements of the five-element antenna were obtained.

Table 7. Data optimized by the Gauss-Newton method of the five element antenna.

Element	Length	Element	Spacing
1º director	0.4308273 λ	Source and 1º director	0.2308273 λ
2º director	0.4308273 λ	1º and 2º director	0.323309 λ
3º director	0.4308273 λ	2º and 3º director	0.323309 λ
Reflector	0.4884964 λ	Reflector and Source	0.1924818 λ
Source	0.4746618 λ	-	-

These optimized data provide the parameters of directivity, front-to-back ratio and half-power angle shown in Table 8.

Table 8. Project output data optimized by the Gauss-Newton method of the five-element antenna.

Beamwidth(plan E)	40,86 degrees
Beamwidth(plan H)	44,89 degrees
Front-to-back ratio (plan E)	10,2482 dB
Front-to-back ratio (plan H)	10,2372 dB
Directivity	12,170 dB

Note that the optimization process applied in this work in the synthesis of the five element Yagi-Uda antenna, obtained significant results for directivity, for the front-to-back ratio, both in the E and H plans, and for the angle of half power. Figure 5 shows the radiation diagram for the plan H of the optimized antenna.

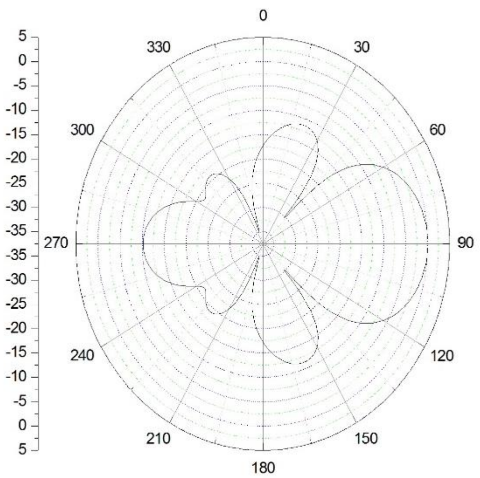

Figure 5: Radiation diagram (plan H) of the five-element Yagi-Uda antenna with data optimized by the Gauss-Newton method.

4. CONCLUSION

The project of a Yagi-Uda antenna bringing together some characteristics required for this device was developed using the Gauss-Newton optimization technique. For that, the number of elements was specified, being a reflector and an energized element, directivity, front-to-back ratio and half angle. The values of the sizes and the spacing between the elements were adjusted by the optimization process.

An initial project of the antenna was established and, based on these data, the Gauss-Newton method was applied until the value ranges of the pre-established characteristics were reached.

Direct modeling, that is, the project of the Yagi-Uda antenna from the length and spacing entries between the antenna elements and as an output, directivity, beamwidth and front-to-back ratio, was carried out from the application of the method of moments Pocklington integral equation.

This procedure was applied to antennas with six and five elements, the results showed that the Gauss-Newton optimization process presents itself as an efficient tool for synthesis of Yagi-Uda antennas. In the examples used in this work, results were obtained as good as other optimization methods presented in the literature for the project of the Yagi-Uda antenna, such as the genetic algorithm technique.

5. References

[1] C. Balanis, "Antenna Theory and Design". Jhon Wiley & Sons, 1997, ISBN 0-471-59268-4.

[2] D. K. Cheng, "Gain optimization for Yagi-Uda array", IEEE Antennas and Propagation Magazine, vol. 33, pp. 42 – 45, june 1991.

[3] E. A Jones and W. T. Joines, "Design of Yagi-Uda antennas using genetic algorithmis", IEEE Transactions and Propagation, vol. 45, pp. 1386 – 1392, setembro 1997.

[4] R. M. Ramos, R. R. Saldanha, R. H. C. Takahashi e F. J. S. Moreira,"Otimização multiobjeto aplicada ao projeto de antennas filamentares", Ciência e Engenharia, SBMag/CBMag 2002 – Edição Especial, pp. 67– 70, dezembro 2003.

[5] V. J. da C. Farias, "Interpretation of induced polarization data using the fractal model for complex resistivities and tomographic images", Postgraduate Program in Electrical Engineering, Technological Center, Federal University of Pará, PhD. Thesis, 151 p., 2004.

[6] J. J Xia, T. M. Habashy, and J. A Kong, "Profile inversion in a cylindrically stratified lossy medium". Radio Science, Vol. 29, pp. 1131 – 1141, 1994.

[7] V. J. da C. Farias, C. H. de M. Maranhão, B. R. P. da Rocha, N. P. O. de Andrade. Induced polarization forward modelling using finite element method and the fractal model. Applied Mathematical Modelling, v.34, p.1849 - 1860, 2010.

[8] V. J. da C. Farias, B.R.P. da Rocha, M. P. C. da Rocha, H. R. Tavares. The use of the fractal model to complex resistivity in the interpretation of induced polarization data. Applied Mathematical Modelling, v.37, p.1347 - 1361, 2013.

[9] M. P. C. ROCHA, L. L. COSTA, V. J. C. Farias; B. Bredegal, H. R Tavares. A comparison of the normal and laplace distributions in the models of fuzzy probability distribution for portfolio selection. International Journal for Innovation Education and Research, v.8, p.183 - 198, 2020.

[10] M. P. C. da Rocha, M. SOUZA, V. J. C. Farias, H. R. TAVARES. Optimization of soybean outflow routes from Mato Grosso, Brazil. International Journal for Innovation Education and Research, v.8, p.176 - 191, 2020.