A didactic proposal for the construction of the ellipse

Rosana da Costa Ferreira
Private School Teacher Oficina da Criança - Cametá – Pará - Brazil
https://orcid.org/0000-0002-0114-2540
rosanaoficinadacrianca@gmail.com

Manoel Batalha Silvino de Araújo
Federal University of Pará – Belém – Pará - Brazil
https://orcid.org/0000-0002-6825-5216
silvino@ufpa.br

Geraldo Mendes de Araújo
Federal University of Pará – Belém - Pará
https://orcid.org/0000-0002-0884-1432
gera@ufpa.br

Gustavo Nogueira Dias
Federal College Ten. Rêgo Barros, Belém, Pará, Brazil
https://orcid.org/0000-0003-1315-9443
gustavonogueiradias@gmail.com

José Carlos Barros de Souza Júnior
Federal College Ten. Rêgo Barros, Belém, Pará, Brazil
https://orcid.org/0000-0003-4465-8237
barrosctrb@gmail.com

Fabricio da Silva Lobato
State University of Pará, Belém, Pará, Brazil
https://orcid.org/0000-0002-8240-8039
fabriciolobatomat15@hotmail.com

Gilberto Emanoel Reis Vogado
State University of Pará, Belém, Pará, Brazil
https://orcid.org/0000-0003-4763-4767
gvogado@globo.com

Vanessa Mayara Souza Pamplona
Federal Rural University of the Amazon, Paragominas, Pará, Brazil
https://orcid.org/0000-0002-2461-2103
vanessamayara2@gmail.com

Alessandra Epifanio Rodrigues
Federal Rural University of the Amazon, Paragominas, Pará, Brazil
https://orcid.org/0000-0002-8375-2923
alessandra.epifanio@ufra.edu.br

Cássio Pinho dos Reis
Federal University of Mato Grosso of South, Brazil;
https://orcid.org/0000-0002-2211-2295
cassio.reis@ufms.br

Washington Luiz Pedrosa da Silva Junior
Federal College Ten. Rêgo Barros, Belém, Pará, Brazil
https://orcid.org/0000-0002-1413-0047
jwl_pedrosa@hotmail.com

Eldilene da Silva Barbosa
Federal Rural University of the Amazon, Belém, Pará, Brazil.
https://orcid.org/0000-0002-9980-2286
eldilene.barbosa@gmail.com

Antônio Thiago Madeira Beirão
Federal Rural University of the Amazon, Belém, Pará, Brazil.
https://orcid.org/0000-0003-1366-5995
thiago.madeira@ufra.edu.br

Katiane Pereira da Silva
Federal Rural University of the Amazon, Belém, Pará, Brazil.
https://orcid.org/0000-0001-7864-6467
katiane.silva@ufra.edu.br

Abstract

The purpose of the present is to propose the construction of the ellipse in a didactic way in which the work deals with the exposure of a specific methodology for the 3rd year of high school when dealing with ellipse content. Followed by the results of the realization of a workshop aimed at the construction of the ellipse *in a didactic way, constructed with low cost material, thereby proposing the teaching of mathematics in a more pleasant and effective way, where the ellipse equations were defined with the theoretical concept and afterwards, the gardener's method was used to construct this curve in a terrain, using ropes and civil construction fasteners, so that the student could concretize and apply this content in practice. After that, it was used for construction in the classroom using thread threads or thin strings to make the tracing and to finish it used the Geogebra software to finish the work showing the perfection of the curve explained in the classroom. The didactic process carried out showed a real interest of the students mainly with the use of the field practice. In conclusion, teaching mathematics in a didactic and interactive way provides a favorable environment for learning, stimulating students with a critical sense and an investigative spirit.*

Key words: Ellipse; Mathematics teaching; Gardener's method; Geogebra.

1.Introduction

The search for new discoveries brought science to the curiosity of several mathematicians, which two scientists stood out in the study of Analytical Geometry: Pierre de Fermat and René Descartes. Both graduated in law. The first exercised an advisory position with parliament and the second in the military career.

The analytical geometry proposed by Fermat, contributed brilliantly to the study of geometrical places. This study was published in 1679, where it stated an equation with two variables, being a description of a line and a curve.

Analytical geometry, on the other hand, is now known as the main characteristic of placing two orthogonal axes, the x-axis, the abscissa and the y-axis of the ordinates, where I can locate geometric figures on these axes as well as any points.

With regard to ellipse, it is part of a context belonging to the conics that was a subject dealt with at the time of Euclid (325 - 265 BC). The ellipse has numerous applications where we find in physics, such as the trajectory of the planets; in health as the treatment of kidney stones, where the shock waves created resemble an ellipse, called lithotripsy; in the field of acoustics used in large rooms, churches, theaters, where we place the application of the properties of the ellipse with a person located in one of the foci, what happens to the sound, when the auditorium behaves like an ellipsoid.

The Conical Section, in the view of Winterle (2000), is still considered by many to be the most accentuated point in Greek mathematics. Apollonius was the author of this grandiose work composed of eight volumes, of which one was lost and only three are written in Greek. He starts his discovery by showing that, with a single cone, the three conical sections can be obtained, for that it is only necessary to vary the inclination of the intersection plane; the perfection of his work is due to the fact of the completeness of his studies, since, until the 19th century, no new properties were discovered regarding the conics.

In the National Curriculum Parameters (PCN) for the teaching of mathematics, the objectives for the classroom discipline are “to identify characteristics of geometric figures, perceiving similarities and differences between them, through composition and decomposition, symmetries, enlargements and reductions ”(BRASIL, 1998, p. 56). In this context, it is important to highlight the importance of the construction of the ellipse, as it provides the student with the notion of the difference between circumference and ellipse, as well as other curves with characteristics of equality, laterality, among others. According to Kaleff (2004, p. 02),

> one of the main objectives of teaching geometry, both in elementary and high school, is the development of visual perception, which can be encouraged through the exploration of visual effects obtained from concrete models of representations of geometric transformations (Kaleff, 2004, p. 02).

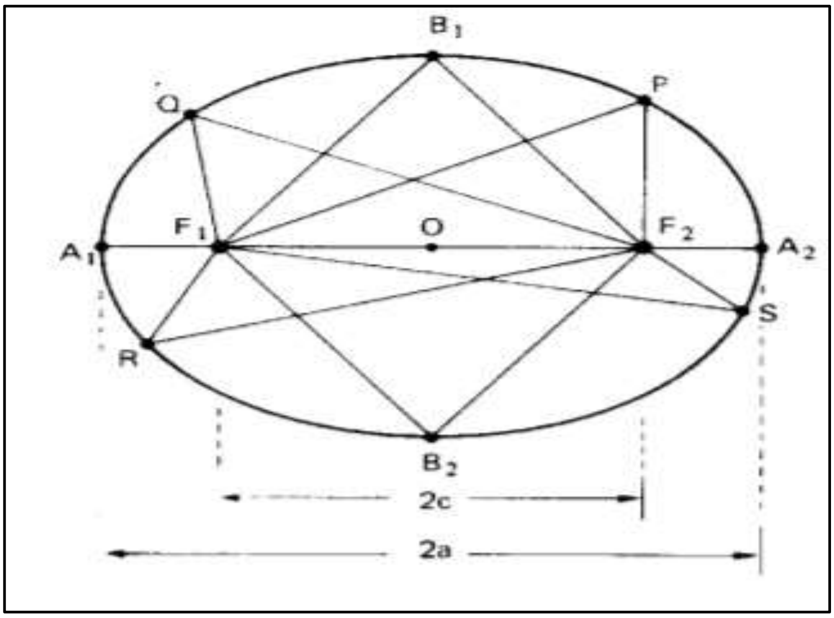
According to this perspective, it can also be articulated that this concept serves as a basis for the issue of interdisciplinarity, as Fortes (2011) says, interdisciplinarity is characterized by the intensity between specialists and the degree of real interaction between disciplines within the same research project. Brazilian education has undergone countless changes in the last 20 years, in general there is a school in which our parents attended and another totally different one in which our children attend. Information and extremely easy access to technology have put certain old educational concepts in disuse, for example, it is practically impossible to supervise the application of tests at school, as there are thousands of ways used by students to circumvent the inspection against glues and / or cell phones , however rigid the students are, they always find a way to achieve these goals.

In the perspective of Stenhouse, (1987) apud Contreras (2002), each class, each student, each teaching situation reflects unique and singular characteristics. It is not possible to know what a teaching situation is or will be until it takes place. In this sense, Dias (2011) states that it is impossible to have a knowledge that provides us with the methods that should be followed in teaching, when the important thing in education is to attend to the circumstances that each case presents and not to intend to standardize the educational processes.

In the conception of Iezzi (2005), figure 01, given two distinct points F_1 and F_2, belonging to a plane α, be it 2c the distance between them Ellipse, is the set of points of α whose sum of distances F_1 and F_2 is constant 2a (2a> 2c).

Ellipse = $\{P \in \alpha \mid PF_1 + PF_2 = 2a\}$

Figure 01: Ellipse's relative image referring to its definition.



Source: IEZZI, 2005

In general, the goal of most teachers is to try to pass on the contents in a very easy and accessible way to the student, using several didactic transpositions that are not exposed in textbooks or in books recommended for entrance exams. In this sense, Chevallard contribution (1991, p.39) is important:

> A content of knowledge that has been defined as knowing how to teach, undergoes, from then on, a set of adaptive transformations that will make it able to occupy a place among the objects of teaching. The 'work' that makes an object of knowledge to be taught, an object of teaching, is called didactic transposition (Chevallard 1991, p.39).

According to Vogado et al. (2020), the student has to leave the posture of a mere spectator and meet a posture in which he will produce his own knowledge, starting to reflect on his ideas, this being the first step towards the constitution of meanings by students. Cabral (2017, p. 10) describes the need that the student needs to leave the posture strengthened by the traditional model of teaching, definition, example and exercise, that is, leaving the passive posture for an active posture and the teacher adopts a conduct of provocateur and organizer of ideas.

The interaction between teacher / student and student / student is interesting in order to enable innovations and improvement of teaching methods and techniques, sometimes forgotten and useless, and for a moment it becomes crucial for the development of the central idea of the problem. The teacher cannot consider himself the whole source of knowledge and existence in the classroom; often an absurd and nonsensical idea becomes plausible from another point of view of knowledge, importing at that moment the intellectual growth and the development of new ideas and applied methods.

Given the above, the present study aims to present a didactic teaching methodology, specifically for the construction of the ellipse.

2. Material and methods

In the analytical geometry, the Ellipse content designated by the curriculum proposal of the Ministry of Education (MEC) is present, with the objective of being worked on in the 3rd year of High School, this proposal can be considered innovative, as this content was taught in higher education area of exact sciences, and is now being taught in high school, which generated the need to adapt the curriculum to the proposed content.

Pereira et al (2018). It was used as a research source used in this article as formatting and indication of the means and ways of correction.

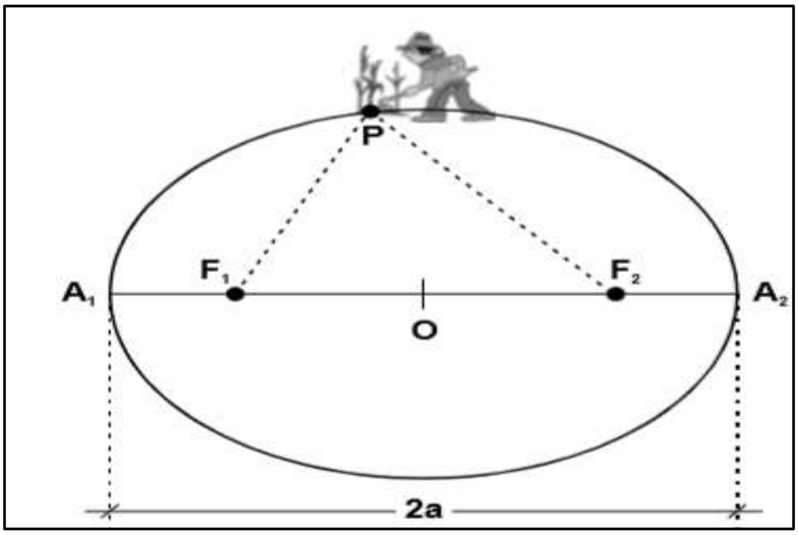
The study was carried out at the State School Osvaldina Muniz, located in the municipality of Cametá, state of Pará, with 18 students from the 3rd year of high school, class A, from August to September 2019. This is a diagnostic study with the quali-quantitative approach.

3. Results and discussion

For the construction of a didactic proposal for teaching the construction of the ellipse geometric figure, initially, a relatively large area consisting of grass or earth was sought around the school, in order to drill the ground with two pre-fixed hands with a distance between them of three or four meters.

After the pointers were securely fixed to the ground, with a rope or a more resistant string, the two ends of the pointer were tied and with a relatively large gap, around 8 or 9 meters of rope between one pointer and another. tracing the ellipse, holding a movable pointer, as shown in figure 2.

Figure 2: Relative image of the construction of Ellipse, gardener method



Source: Venturi, 2OO3

After carrying out the experiment, the students were able to perceive the curve and the proposed shape in the shape of an ellipse, admitting the consideration of two Cartesian axes. Through practical application it was possible to see that,

$$d(P, F_1) + d(P, F_2) = 2a$$

In this way, the main objective of the work was successfully completed.

Subsequently, the students returned to the classroom and the subject was proposed on the board in an enlightening way, using the Cartesian axes and presenting the ellipse equation, and demonstrating the formula by means of the distance between points.

The material made with cardboard was delivered to the students, so that they could build their own ellipse in practice. She put her material on the floor to teach in detail how to make the ellipse on paper, using again string and cardboard, now on a smaller scale, where everyone was interested in doing the activity.

Then, the researcher Rosana taught all the components of the ellipse: The foci, center at focal distance, the measurements of the major and minor axis and the eccentricity.

The students soon realized the existence of the curve, because according to them the practice helped a lot to recognize the layout of the new curve. After the demonstration was carried out, which required algebra, the teacher proposed current exercises that involve the contemporary context:

The teacher inserted two proposals: One with the major axis in x and the other with the major axis in y.

Then follow the curve below with the major axis on the x axis.

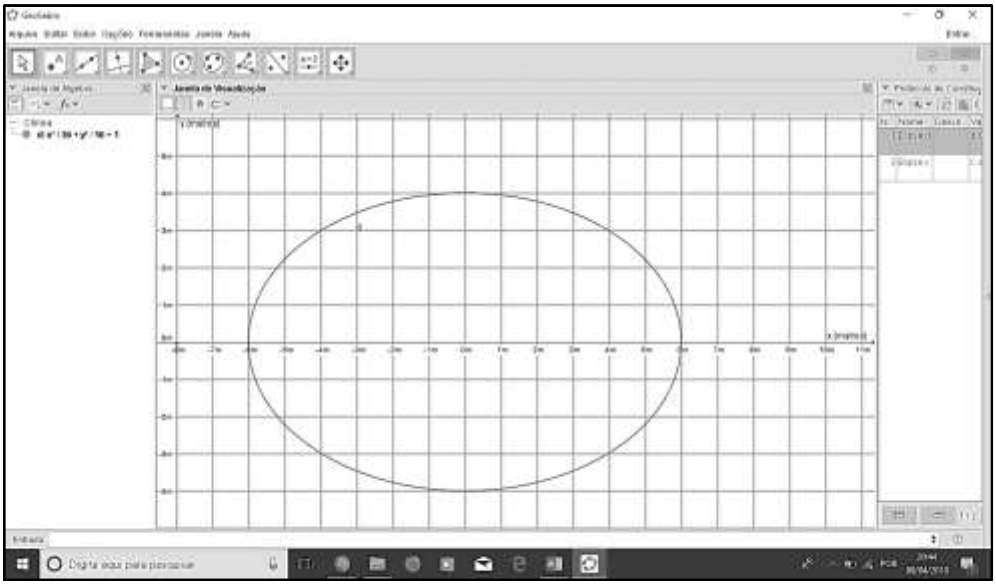
i) $\frac{x^2}{36} + \frac{y^2}{16} = 1$

He explained how to build, locating the axes and using the main relation $a^2 = b^2 + c^2$ and found that the construction by the gardener method is what worked the most for drawing the curve.

In addition to this process, he explained to students about a technological resource called geogebra, where you can download the program for free, as well as in the form of an application on your cell phone, which easily some students have already obtained this resource.

Inserting the curve in the geogebra program, we obtain the following image, figure 03:

Figure 03. Print of the construction of the ellipse used in the classroom using the Geogebra software.



Source: Authors.

When students installed the application on their cell phone, they found it to be a very important resource that consumes only 6 MB of the system, which compared to other applications is considered very low.

It was warned that this software builds all kinds of curves, including in 3D.

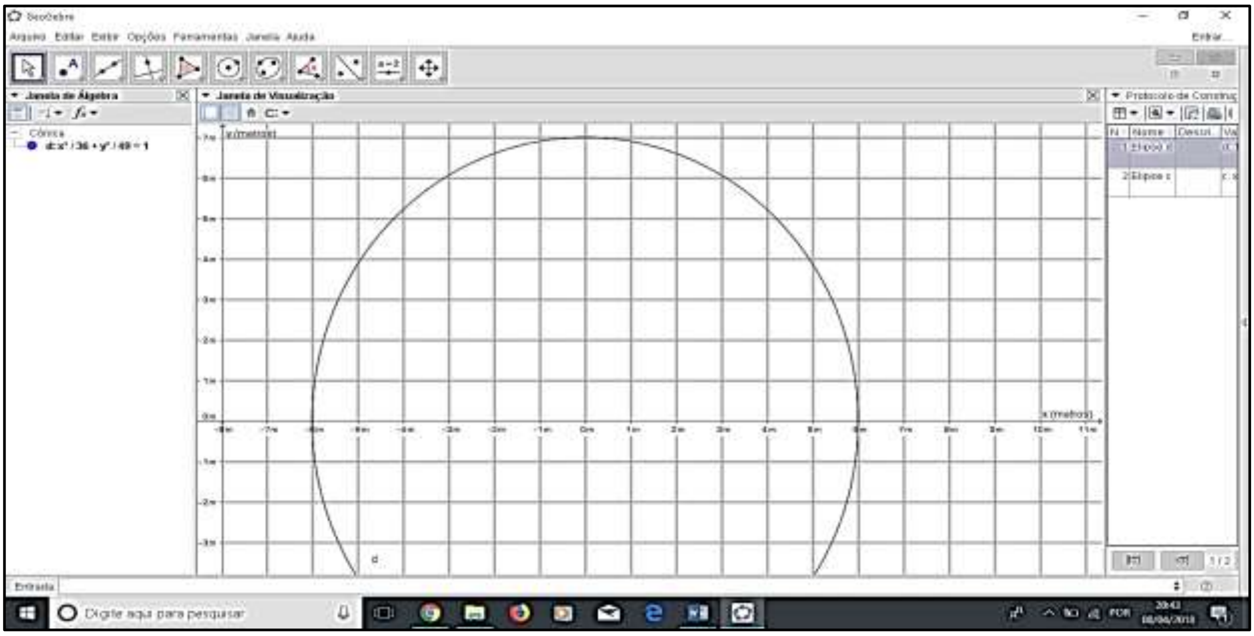
The teacher ratified the sense of applicability of the software in which it should be used for a better visualization and construction of the requested curve and that in tests and tests the student should try to solve and only check the construction in the application after the construction attempt.

Then follow the curve below with the major axis on the y axis.

ii) $\frac{x^2}{36} + \frac{y^2}{49} = 1$

Inserting the curve in Geogebra, we clearly perceive the change of axes, as well as the perfection of the created layout. To the side the curve appears in bold the typed curve, figure 4

Figure 04. Print of the construction of the ellipse used in the classroom using the geogebra software.



Source: Authors.

We observe that in the software the y-axis appears shifted to the right and not in the center (0,0) as indicated in the references. We realize that the curve comes out with complete geometric perfection.

With the students having a basic notion of what the ellipse is like, having already shown its elements and its reduced equations, the construction of the ellipse was proposed by the students, for this construction they used a square made of cardboard and covered with A4 paper, two thumbtacks , a piece of string and pencil.

For construction, follow the steps below:

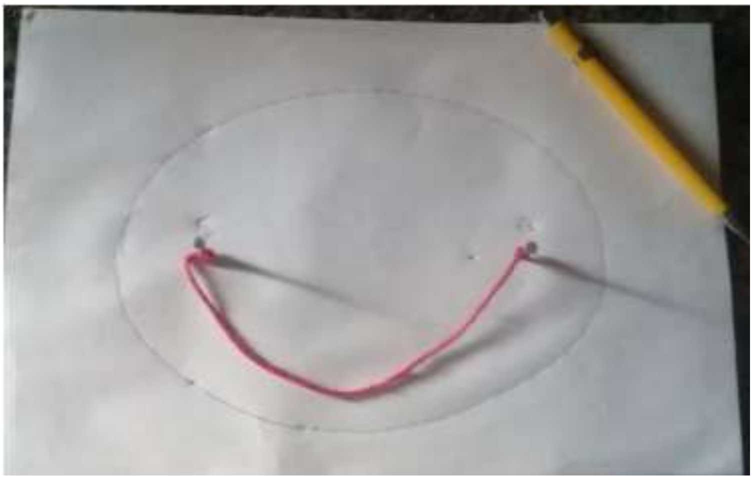
1. Determine in the material two fixed points that will be the foci and this point will be marked with a pencil.

2. Then take the two tacks and the string, tying each tack on the ends of the string; now fixing these

ends at the points marked with the pencil, which are the focal points; so that the string can form an arc, (this procedure can be adjusted depending on the location of the stitches).

3. Now with the pencil attached to the string and placed in the direction of, moving the pencil so that it traces on the paper an arc limited by the string, this trace will be made both above the foci and below, so that the two strokes meet and form the ellipse, figure 05.

Figure 05: Elipse photo built.



Source: Authors.

The students were very excited to do the construction. It was noticed that doing this procedure was new to them, and little by little they were able to draw the ellipse, it is worth mentioning that some felt motor difficulties to make a perfect layout, with two or three tentas they managed.

After the construction of the ellipse with the help of the teacher, the students were challenged to solve a mathematical problem with the ellipse, for that they would use the same material where the ellipse was built.

Activity:

Given the equation:

$$\frac{(x-1)^2}{25}+\frac{(y-2)^2}{9}=1$$

Determine.

a) The coordinates of the center of the ellipse. To determine such coordinates, we will make use of the given general equation, through which we observe that the ellipse in question is of the translated type; that is, the center is not at the origin. Therefore, its center is (h, k) $\neq$ (0, 0), so through the equation we see that C (1, 2).

b) The length of the major and minor axis. It is known that the length of the major axis is equal to 2a. So we have to know who it is.

Thus, we have to:

$$a^2 = 25$$
$$a = \sqrt{25}$$
$$a = 5$$

Therefore, 2.a = 2.5 = 10, is the length of the major axis. In a similar way we do to find the minor axis, which in turn is equal to 2b.

$$b^2 = 9$$
$$b = \sqrt{9}$$
$$b = 3$$

Therefore, 2.b = 2.3 = 6, it is the length of the minor axis.

c) The coordinates of the vertices and foci.

To find the coordinates of the vertices we need the value of " a ", as we already know that a = 5, starting from the center C we are going to move 5 spaces to the left and mark the point " v" and 5 spaces to the right and mark the point " v " which is the other vertex.

To find the other 2 vertices we need the value of " b ", as we know b = 3. Starting from the center we will walk 3 spaces upwards and mark the point " v " and 3 spaces downwards and mark the " v " point again.

With that, we will obtain the four vertices and their coordinates:

$V_1(1,5)$, V_2 (-4,2), $V_3(1, -1)$ e $V_4(6,2)$.

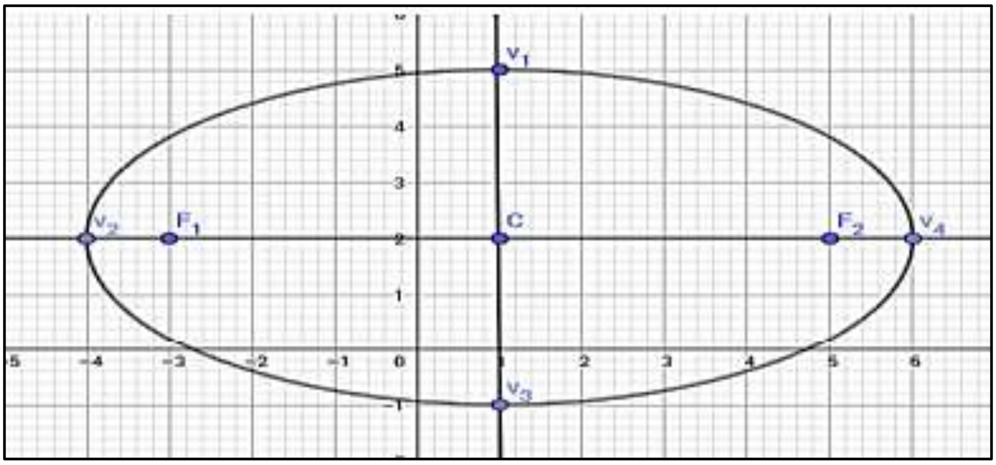
To find the coordinates of the focus we have to determine ‘’ c ”, it is known that the distance from center C to F_2 is equal to ‘’ c ”.

Using the relationship, we have:

$$a^2 = b^2 + c^2$$
$$5^2 = 3^2 + c^2$$
$$25 = 9 + c^2$$
$$25 - 9 = c^2$$
$$16 = c^2$$
$$\sqrt{16} = c$$
$$4 = c$$

So, starting from the center, we will go 4 spaces to the left and mark the point and in the same way mark. With that, we determine the coordinates requested and build the graph of the ellipse given in the Cartesian plane, figure 06.

Figure 06: Plot of results in Geogebra



Source: Authors.

d) The eccentricity of the ellipse. The eccentricity is given by the relation:

$$e = \frac{c}{a}$$

$$e = \frac{4}{5}$$

During the resolution of the activity, it was explained how to find the elements of the ellipse so that they could visualize and understand them, for this reason the placement “floor houses” was used.

The workshop was very enriching for both parties, both for the teacher (researcher) and for the class, they were very attentive, curious and participative during the workshop, it was a relaxed learning experience. At the moment when an overview of the conics and elements of the ellipse was being made with the use of slides, the students were very attentive, asked questions and showed doubts that were resolved.

The workshop enabled students to have a more interactive class, it was clear from their faces that the moment was being pleasurable, everyone worked hard to draw the ellipse, some experienced difficulties in the procedures, which were repeated more than once by some, but everyone succeeded build the ellipse.

To solve the proposed activity it was a little more complicated, some students felt very difficult to develop the calculations, and it was a little difficult to help everyone, so the doubts were resolved using the traditional method the table, it was shown how the calculations should be developed and how the exercise could be solved.

The students understood what the ellipse is and were able to identify its elements, they were also able, despite their doubts, to determine the center, the foci, the vertices. Students are used to not very didactic classes, pictures full of endless content, environments that are not conducive to learning. This is due to the lack of school infrastructure and numerous problems that are the responsibility of the state, remembering that the reality of the teacher is crowded classrooms and very short time.

Mathematics is challenging, which suggests that the teacher is constantly looking for new teaching didactics, making use of new tools, and this workshop, in turn, made this experience possible. This exchange with students, being in front of a classroom, allows to partially know the reality of the teacher, the reality of the classroom, knowing this plurality of behaviors and thoughts is challenging and enriching, given that knowledge is much more pleasurable it becomes a two-way street.

4. Final considerations

The construction of the ellipse was made using low-cost materials such as cardboard, A4 paper, thumbtacks, string and pencils. The intention was to make the students participate in the class and that they could build the conic themselves; the workshop was very constructive and enriching, and counted on the interaction between everyone, so it is believed that the objective had been achieved, since everyone managed to build the ellipse and the vast majority managed to identify its elements and carry out the proposed activity.

The workshop made it possible to perceive how productive an interactive class is, how something new captures the students' attention, giving them the opportunity to participate in the class in a concrete way allows them to learn more effectively and pleasurably. For a moment there, everyone seemed to like mathematics, even though some students had difficulty making a perfect layout, everyone seemed to be quite committed to the task. Interactive and enjoyable classes are essential to transform the classroom into an environment conducive to learning.

The world has changed, students have changed and there is no ready model of how to teach or how to learn, because of this the teacher must always seek to propose the new, use effective methodologies always with the objective of awakening in the student a critical sense and an investigative spirit , something that is extremely important in the acquisition of mathematical knowledge.

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