Evaluation of Equity-linked structured products and pricing

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Abstract

Based on a unique data set, this research paper examines the pricing of equity-linked structured products in the market. The following section of this paper look at examining a few popular products available in the market, describing their key characteristics, and identifying one such product which will be examined closely for the purposes of determining if the issuing institution has priced the same fairly. The daily closing prices of a large variety of structured products are compared to theoretical values derived from the prices of options traded on the Eurex (European Exchange). This research paper also provides a brief background on the pricing of equity-linked structured products ('products') and issues around valuation of these products and look in detail fair pricing of the zero-coupon bond and the basket option. Comparing this with the market price of the instrument I could draw conclusions based on how close the real market price of the instrument is with the recomputed price.

Keywords – Structured products, Pricing, Option, Implied Volatility

1. Introduction

Equity-linked structured products are derivative instruments wherein the underlying asset is an equity security. Although every product is different in several ways all of them promise tailor made risk return profiles. Even though equity derivatives are amongst the oldest form of derivatives, structured equity products gained popularity in Europe only in the mid- 1990's during a period of low interest rates. The market today for these products has become extremely complex and has evolved significantly from traditional single stock equity derivatives to index-based products. Traditional forms of equity derivatives were generally used to raise equity capital for companies; today these products are increasingly being used in the process of debt fund raising through new issue arbitrage techniques. Most forms of equity linked structured products have their performance linked to an underlying benchmark such as an equity index, real estate index, commodities, interest rates etc. For instance, a FTSE 100 linked note will have returns linked to the FTSE 100 index and consequently the better the FTSE 100 performs the better will be the returns of the note. Another common feature of many structured products is capital protection. A capital protected structured product protects a portion (up to 100%) of the capital invested, making the product very desirable for retail investors with a low-risk appetite. Some of the major reasons for the rapid evolution of these products include changes in the pattern of equity investments including the increased importance of indexed strategies and increase in cross border investments, demand for customized equity investments and structured equity risk profiles, tax factors, including the optimization of after-tax returns from equity instruments.

The Structured Products Association (SPA) estimates $45 to $50 billion worth of products were deposited within the U.S. in 2005 which there will be a 20% to 25% growth in 2006.In terms of listed registered products, the American Stock Exchange has reported an 18 percent increase in the number of structured product issues listed in 2005 on the exchange over 2004, bringing the notional number of structured products on the AMEX to over 13 billion. Despite the large size and rapid growth of the market for these products, very little empirical research has been conducted on pricing.

The following paragraphs list out a few studies conducted on re-pricing of structured products:

Chen and Kensinger (1990) for a period of two months in 1988 and 1989 have conducted an analysis of 'Market-Index-Certificate of Deposit' (MICD) within the US market, which guaranteed a minimum rate and a variable rate pegged to the performance of S&P 500.A comparison of the implied volatility of the S&P 500 option with the implied volatility of the (MICD)'s option components revealed significant positive and negative differences between theoretical and market values.

Chen and Sears (1990) investigated the exchange traded 'S&P 500 Index Note' (SPIN) issued by the Salomon Brothers. Computing the differences between market and model prices for the period 1986 to 1987 they diagnose overpricing in the first sub-period and under-pricing in the second and third sub-periods.

Baubonis et al (1993) analyze the cost structure of equity-linked certificates of deposits and demonstrate, using Citicorp product as an example that the bank can earn 2.5%-4% of the selling price in the primary market. Wasserfallen and Schenk (1996) examine the pricing of capital protected products issued in 1991-1992 in the Swiss market. The comparison of the products option components from those derived from historical and implied volatility of country Market Index shows that the securities are sold slightly above the theoretical value. In the secondary market, model vales exceeded observed prices. In all the above-mentioned studies the authors have found overpricing of structured equity products in the favor of the issuing institution. The purpose of this study is to undertake a firsthand investigation of a sample of equity linked structured products (hereinafter referred to as 'products') and analyze product from the selected sample.

2. An Overview of Types of Products Available

There is no formal definition of a structured product however for the purpose of this study the focus is maintained on the market for equity linked products, i.e., instruments with stocks or stock indices as underlying. At a macro level 'products' can be classified as ones with plain-vanilla option components and ones with Exotic option components. Another way of classifying these 'products' are by their objectives, i.e., growth products: which may provide some or 100 percent capital protection and income products: which promise high fixed income but carry risk on capital return (no capital protection) For this study, the discussion will remain focused around 'Capital-Guaranteed Products'. In the following sections different variant of 'Capital-Guaranteed Products' are described. Most Capital Protected Products are redeemable with a minimum guaranteed percentage of the face value (often 100%), have low or no interest rates and promise participation in the performance of an underlying asset. Investor participation in the performance of the underlying asset can take a wide variety of forms which allows creation of a variety of Capital Protected Products.

The most basic form of this type of an instrument is where the investor is guaranteed a percentage of the principal invested and participation in the percentage of growth of the underlying asset.
This can also be expressed as the following formula:

$$R = N \cdot \left(1 + max\left(0; \frac{S_T - S_0}{S_0}\right)\right) = \\ = N + \frac{N}{S_0} \cdot max(0; S_T - S_0) \quad (1)$$

where,
R redemption amount
N face value
S0 original price of underlying asset
ST price of underlying asset at maturity

Such products will have several European call options on the underlying asset embedded in them. The number of call options is equal to the face value of the instrument divided by the initial price. In summary the instrument can be interpreted as a combination of zero-coupon bonds with European options.
Most of the other types of Capital Protected Products are essentially permutations and combinations of zero-coupon bonds with all conceivable types of options depending on what the product sets out to achieve.

2.1 Capital Protected Products Using European Call Options

Background:

These instruments as described above are the simplest of Capital Protected Products. They guarantee a percentage of the principal sum invested (often up to 100%) plus a return which depends on the performance of the underlying asset and the agreed participation in the performance of the underlying asset. If the price of the underlying asset increases the investor stands to gain while on the other hand if the price drops the investor enjoys capital protection.

Example: ATX guarantee certificate 1998–2000

Maturity	November 24, 1998, to July 25, 2000 (1 year, 8 months)
Redemption rate	Face value * (1 + b * ((ATX_T – ATX_0) / ATX_0)), at least a% of face value
Participation	b = 50%
Capital guarantee	a = 95%
ATX_0	value of the ATX on November 24, 1998
ATX_T	value of the ATX on July 25, 2000
Issue price	100%
Denomination	EUR 1,000

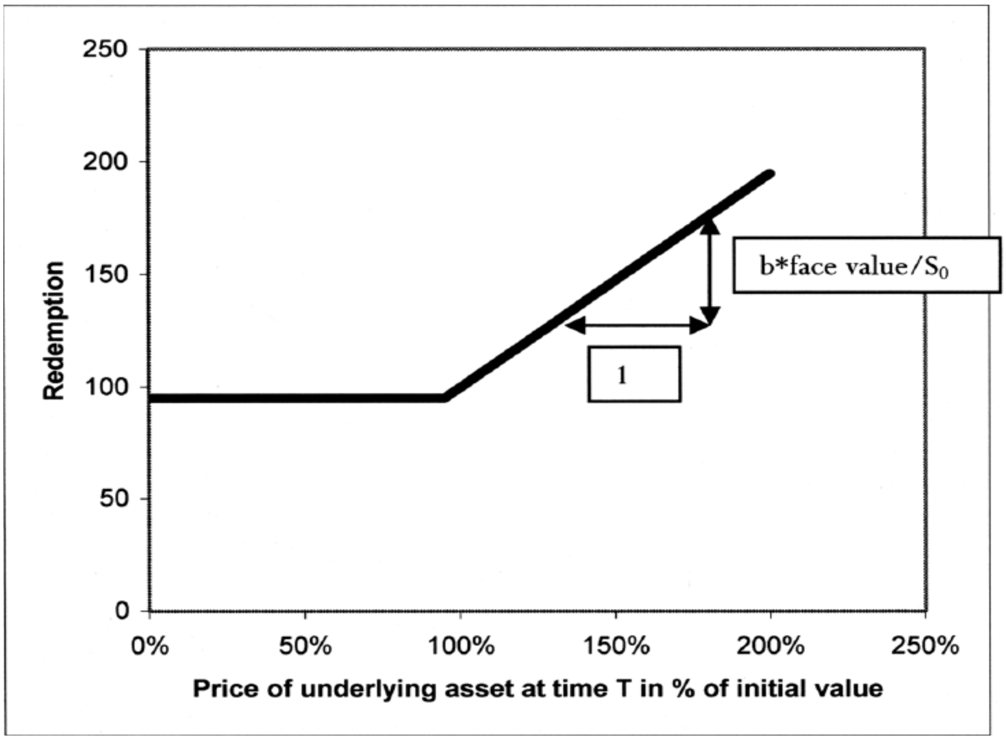

Figure 1. (Example of the Capital Protected Product and diagram showing payoff profile taken from 'Financial Instruments –The Structured Products Handbook' Oesterreichische Nationalbank)

Replication

This product can be broken down into a zero-coupon bond and a European Call Option.

Redemption in this case can also be described mathematically as:

$$R = max\left(N \cdot a; N \cdot \left(1 + b \cdot \frac{S_T - S_0}{S_0}\right)\right) =$$
$$= N \cdot a + \frac{N}{S_0} \cdot max(0; (1-a) \cdot S_0 + b \cdot (S_T - S_0)) =$$
$$= N \cdot a + \frac{N \cdot b}{S_0} max\left(0; S_T - S_0\left(1 - \frac{1-a}{b}\right)\right) \quad (2)$$

where,
R redemption amount
N face value
S_0 original price of underlying asset
S_T price of underlying asset at maturity
a guaranteed redemption amounts
b participation rate

The product can be replicated as follows:

+ Capital-guaranteed bonds with embedded European call option =	+ zero coupon bonds + $\frac{face\ value^{*} b}{S_0}$ European call options

(3)

The strike price of the option is:

$$S_0\left(1 - \frac{1-a}{b}\right)$$

(4)

Assuming a principal of 1,000, we can replicate this instrument with:

- The purchase of a zero-coupon bond which reaches maturity on July 25, 2000 and has a face value of EUR 950
- The purchase of 1,000 * 0.5/ATXnov 24, 98 European call options on the ATX with a strike price of 0.9 (¼ 1 _ (1-a) / b) * ATX_{nov} 24, 98 and expiring on July 25, 2000

Valuation

Zero coupon bonds are valued using the relevant spot rates. Black-Scholes model gives a closed formula for calculating option premium. Assuming that the currency of the bond and that of the underlying asset are the same:

$$c = Se^{-qT}N(d_1) - Xe^{-rT}N(d_2)$$

where

$$d_1 = \frac{ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

(5)

c	premium of a European call option on one share with price S at $t = 0$. X is the exercise price, and the option expires in T years.
r	risk-free interest rate (constant) over the period of the option
q	dividend yield
σ	volatility of the stock
$N(d)$	cumulative standard normal distribution at d

2.2 Capital Protected Products using European Put Options

Background:

Capital Protected Products with embedded European put options are generally characterized by 100% capital protection plus a percentage return based on the difference in underlying asset's price between the issue and maturity date.

The percentage return is paid only if the price of the underlie drops, while on the other hand the investor enjoys capital protection if the price rises.

Example: ATX guarantee certificate 1998–2000

Maturity	November 24, 1998, to July 25, 2000 (1 year, 8 months)
Redemption rate	Face value * (1 + b * ((ATX_0 – ATX_T) / ATX_0)), at least a% of face value
Participation	b = 50%
Capital guarantee	a = 95%
ATX_0	value of the ATX on November 24, 1998
ATX_T	value of the ATX on July 25, 2000
Issue price	100%
Denomination	EUR 1,000

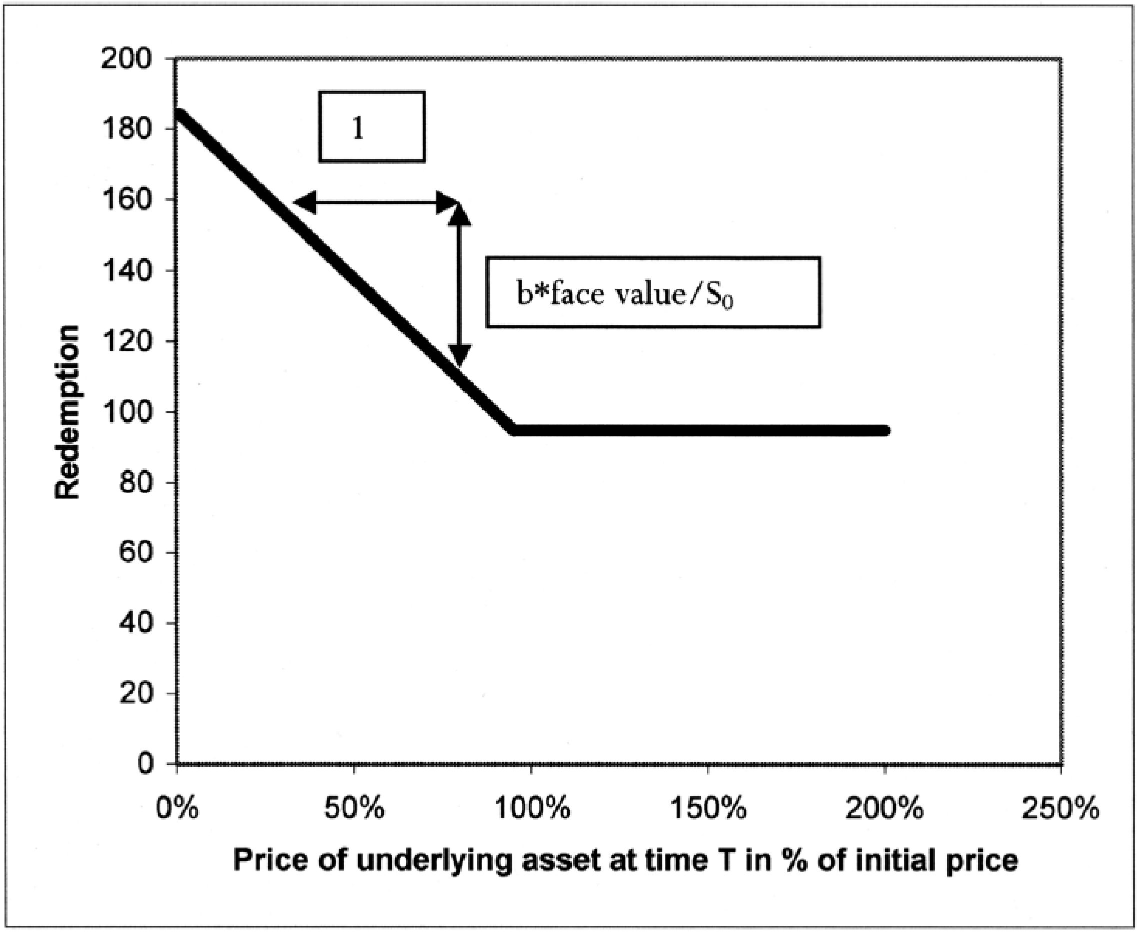

Figure 2. (Example of the Capital Protected Product and diagram showing payoff profile taken from 'Financial Instruments –The Structured Products Handbook' Oesterreichische Nationalbank)

Replication

As explained in section 3.1, the Capital Protected Product can be broken into components of a zero-coupon bond and European Put Option.

Redemption in this case can also be described mathematically as:

$$R = max\left(N \cdot a; N \cdot \left(1 + b \cdot \frac{S_0 - S_T}{S_0}\right)\right) =$$
$$= N \cdot a + \frac{N}{S_0} \cdot max(0; (1-a) \cdot S_0 + b \cdot (S_0 - S_T)) =$$
$$= N \cdot a + \frac{N \cdot b}{S_0} max\left(0; \left(1 + \frac{1-a}{b}\right) \cdot S_0 - S_T\right) \qquad (6)$$

where,

R redemption amount

N face value

S_0 original price of underlying asset

S_T price of underlying asset at maturity

a guaranteed redemption amounts
b participation rate

The product can be replicated as follows:

+ Capital-guaranteed bonds with embedded European put option	= + zero coupon bonds + $\frac{face\ value^{*}b}{S_0}$ European put options

(7)

The strike price of the option is:

$$S_0\left(1+\frac{1-a}{b}\right) \quad (8)$$

Assuming a principal of 1,000, we can repeat this instrument with:

- The purchase of a zero-coupon bond which reaches maturity on July 25, 2000, and has a face value of EUR 950
- The purchase of 1,000 * 0.5/ATXnov 24, 98 European put options on the ATX with a strike price of 1.1 (= 1 - (1-a) / b) * ATXnov 24, 98 and expiring in July 25, 2000

Valuation

Zero coupon bonds are valued using the relevant spot rates. Black-Scholes model gives a closed formula for calculating option premium. Assuming that the currency of the bond and that of the underlying asset are the same:

$$p = Xe^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$$

where

$$d_1 = \frac{ln(S/X)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (9)$$

p	premium of a European put option on one share with price S at $t=0$. X is the exercise price, and the option expires in T years.
r	risk-free interest rate (constant) over the period of the option
q	dividend yield
σ	volatility of the stock
$N(d)$	cumulative standard normal distribution at d

2.3 Capital Protected Products using asian options (average rate options)

Background:

Where a Capital Protected Product has an embedded Asian option, the amount paid out at maturity depends on the average value of the underlying asset. This average is calculated using prices of the underlying at regular intervals.

Example: Equity-linked bond

Maturity	November 30, 1998, to November 29, 2003 (5 years)
Redemption rate	face value (N)+ bonus return Bonus return $= N \times max\left(0; 60\%^* \frac{\varnothing ATX_T - ATX_0}{ATX_0}\right)$
Participation	$b = 60\%$
Capital guarantee	100%
ATX_0	closing value on November 30, 1998
$\varnothing ATX_T$	arithmetic mean of 20 quarterly prices, starting on November 30, 1998, plus the price on November 17, 2003 (21 observation dates).
Issue price	100%
Denomination	EUR 1,000

Table 1. Example of the Capital Protected Product and diagram showing payoff profile taken from 'Financial Instruments –The Structured Products Handbook' Oesterreichische Nationalbank

Depending on whether an Asian call or put option is embedded, the redemption amount is calculated using one of the following formulas:

(1) Call option (see example):

$$R = N + N \cdot max\left(0; b \cdot \left(\frac{\varnothing S_T - S_0}{S_0}\right)\right)$$

or

$$R = N + \frac{N \cdot b}{S_0} \cdot max(0; \varnothing S_T - S_0)$$

(2) Put option:

$$R = N + N \cdot max\left(0; b \cdot \left(\frac{S_0 - \varnothing S_T}{S_0}\right)\right)$$

or

$$R = N + \frac{N \cdot b}{S_0} \cdot max(0; S_0 - \varnothing S_T) \quad (10)$$

where

R redemption amount

N face value

S_0 initial value of underlying asset

$øS_T$ average value at maturity

b participation rate

An Asian option (also called an average option) is an option whose payoff is linked to the average value of the underlier on a specific set of dates during the life of the option. There are two basic forms:

An **average rate** option (or average price option) is a cash-settled option whose payoff is based on the difference between the average value of the underlier during the life of the option and a fixed strike.

An **average strike** option is a cash settled or physically settled option. It is structured like a vanilla option except that its strike is set equal to the average value of the underlier over the life of the option (http://www.riskglossary.com/link/asian_option.htm)

Replication:

The product can be replicated as follows:

Assuming a principal of 1,000, we can replicate this instrument with:

-The purchase of a zero-coupon bond which reaches maturity on November 29, 2003, and has a face value of EUR 1,000

-The purchase of 1,000 * 0.6/ATXnov 30, 98 Asian call options on the ATX with a strike price of ATXnov 30, 98 and expiring on November 29, 2003

Valuation

The zero-coupon bonds are valuated using the relevant spot interest rates. Asian options for which payments are based on a geometric average are relatively easy to valuate. A closed-form valuation formula exists, the zero-coupon bonds are valuated using the relevant spot interest rates. Asian options for which payments are based on a geometric average are relatively easy to valuate. A closed-form valuation formula exists.

However, because the interest on bonds with Asian options generally depends on the arithmetic average value of the underlying asset, this straightforward formula can't be used to determine the precise value. As the arithmetic average of a log-normally distributed value is itself not log-normally distributed, these options can only be valuated using numerical procedures or with the help of analytical approximation.

Approximations have been developed by Turnbull and Wakeman (1991), Levy (1992) and Curran (1992), for example. *E. G. Haug, "The Complete Guide to Option Pricing Formulas", 1997, McGraw-Hill,(p. 96,97)* In Curran's model, the value of an Asian option can be approximated using the following formula:

$$c \approx e^{-rT}\left[\frac{1}{n}\sum_{i=1}^{n} e^{\mu_i+\sigma_i^2/2} N\left(\frac{\mu - ln(\hat{X})}{\sigma_x}+\frac{\sigma_{xi}}{\sigma_x}\right) - XN\left(\frac{\mu - ln(\hat{X})}{\sigma_x}\right)\right]$$

$$\mu_i = ln(S) + (r - q - \sigma^2/2)t_i$$

$$\sigma_i = \sqrt{\sigma^2[t_1 + (i-1)\triangle t]}$$

$$\sigma_{xi} = \sigma^2\{t_1 + \triangle t[(i-1) - i(i-1)/(2n)]\}$$

$$\mu = ln(S) + (r - q - \sigma^2/2)[t_1 + (n-1)\triangle t/2]$$

$$\sigma_x = \sqrt{\sigma^2[t_1 + \triangle t(n-1)(2n-1)/6n]}$$

$$\hat{X} = 2X - \frac{1}{n}\sum_{i=1}^{n} exp\left\{\mu_i + \frac{\sigma_{xi}[ln(X)-\mu]}{\sigma_x^2} + \frac{\sigma_i^2 - \sigma_{xi}^2/\sigma_x^2}{2}\right\} \quad (11)$$

Where,

c	premium of an Asian call option
S	current value of underlying asset
X	strike price
r	risk-free interest rate (constant) over the period of the option
q	dividend yield
T	term in years
t_1	first observation point
t	time between observation points
n	number of values sampled
σ	volatility of the underlying asset
$N(d)$	cumulative standard normal distribution at d

Curran's approximation for Asian Call options

2.4 Capital Protected Products using capped call options

Background:

Theoretically the redemption amount for a capital protected product with an embedded call option can be infinitely high. The product described in this section places a cap, expressed as a percentage of the instrument's face value, on the redemption amount. In this way the bearer only participates in the relative performance of the underlying asset up to a certain maximum value.

In cases where the price of the underlying asset decreases, the issuer guarantees a minimum redemption amount.

Example: "Europa" guarantee certificate

Maturity	December 15, 1998, to December 13, 2002 (4 years)
Redemption rate	The redemption rate (expressed as a percentage of the face value) is proportionate to the change in the underlying asset's price (S_T/S_0); minimum 100%, maximum 109%; or, expressed as a formula: $T = face\ value^{*}(100\% + min(9\%; max(0\%; (S_T - S_0)/S_0)))$
S_0	closing price of XY stock on December 15, 1998
S_T	closing price of XY stock on December 13, 2002
Issue price	100%
Denomination	EUR 1,000

Table 2. Example of the Capital Protected Product and diagram showing payoff profile taken from 'Financial Instruments –The Structured Products Handbook' Oesterreichische Nationalbank

If the price of the underlying asset increases between the issue date and the maturity date, then the investor will participate up to a rate of 9%.

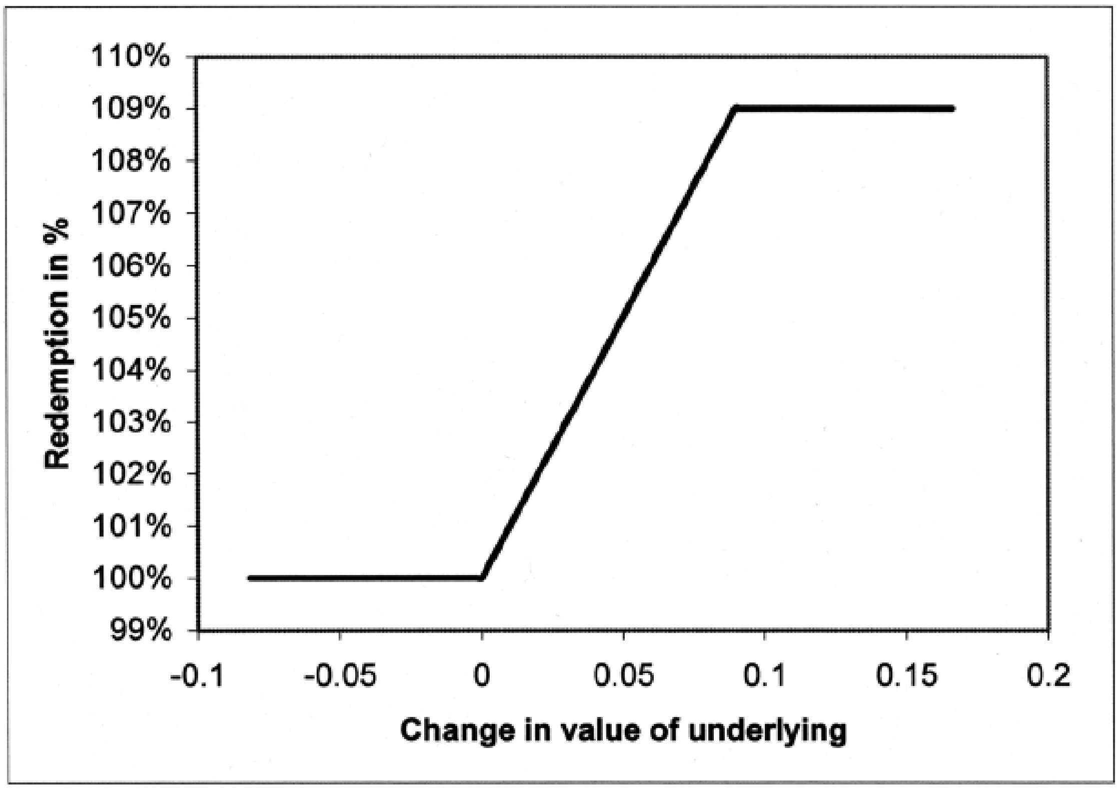

Figure 3. Guaranteed redemption at 100%, participation in positive changes in the underlying asset's price, maximum redemption rate 109%. (The payoff pattern in the illustration precisely reflects the redemption in the example.)

Replication:

Assuming a principal of 1,000, we can replicate this instrument with:

- The purchase of a zero-coupon bond which reaches maturity on December 13, 2002, and has a face value of EUR 1,000
- The purchase of 1; 000=S_0 European call options on the underlying asset with a strike price of S_0, expiring on December 13, 2002
- The sale of 1; 000=S_0 European call options on the underlying asset with a strike price of 1:09S_0, expiring on December 13, 2002

Valuation:

Valuation of this type of a product can be analyzed in the same way as described in section 3.3. i.e. by breaking down the product into zero coupon bonds and European call options.

3. Theoretical Aspects of Volatility

Volatility is a measure of price variability over some period. It typically illustrates the standard deviation of returns, in a particular context that depends on the definition used. Alternatively, we can saw that volatility is the standard deviation of the change in the logarithm of a price or a price index during a stated period. Volatility can be defined and interpreted in different ways.

Realized volatility

Realized volatility, also called historical volatility, is the standard deviation of a set of previous returns. This volatility can be stated in annual units as standard deviation (for *n* trading periods with *r* returns) times the square root of *N*, where *N* denotes the number of trading days in a year.

Conditional Volatility

Conditional volatility is that the standard deviation of future return that is conditional on known information like the history of previous returns. Unlike realized volatility, the expectation for the subsequent period is calculated using a time series model that has been selected and estimated using appropriate data.

Stochastic volatility processes are motivated by noting that volatility is not constant and hence it is interesting to hunt to specify how volatility changes through time. Typical discreet-time models suppose that volatility is unobservable and so its stochastic properties could also be inferred from either absolute of squared returnsContinuous-time models are used to price options when the assumption of constant volatility is relaxed.

A square root process for volatility permits the rapid calculation of appropriate option prices.

Implied volatility is a value calculated from an option price. It equals the volatility parameter σ for which an option's market price equals its theoretical price according to a pricing formula. The Black-Scholes pricing formula provides theoretical prices for European Call options, say c(σ), and assumes that the asset price process is Geometric Brownian Motion with annual variance rate σ^2. As c(σ) is an increasing function of σ, for any market price c_M between the lower and upper bounds that exclude arbitrage profits there is a unique solution to the equation $c_M = c(\sigma)$ that defines the implied volatility.

These volatility measures depend on the time until expiry and the exercise price of an option.

At any time, the values of realized volatility, conditional volatility, unobservable stochastic volatility, and implied volatility will usually all be different, because different data and assumptions are employed when these values are calculated.

The Theta

The effect of a change in time to expiration on the theoretical values of calls and puts is called theta. All options, both calls and puts, loose value as expiration approaches. The *theta*, or time decay factor is the rate at which an option loses value as time passes. It is usually expressed in points lost per day when all other conditions remain the same. An option with theta of 0.5 will lose 0.5 in value for each day that passes with no change in other market conditions. If the option is worth 2.75 today, then tomorrow it will be worth 2.17. The day after that it will be worth 2.65. Time runs only in one direction and technically, theta is a positive number. However, theta sometimes is written as a negative number, consequently, a long option position will always have negative theta and short option position will always have positive theta. This is just the opposite of gamma, where a long option position will have positive gamma, and a short option position has a negative gamma.

Every option position is a trade off between market movements and time decay. If the price movement in the underlying contract will help a trader (positive gamma) the passage of time will hurt (negative theta) and vice versa.

The Vega

The vega of an option is usually given in point change in theoretical value for each 1%-point change in volatility. Since all options gain value with rising volatility the vega for both calls and puts is positive. If an option has a vega of 0.15, for each percentage point increase (decrease in volatility) the option will gain (lose) 0.15 in theoretical value and at-the-money option always has a greater vega than either an in-the-money or out-of-the-money option when all options are of the same type and have the same amount of time to expiration. This means that an at-the-money option is always the more sensitive in total points to a change in volatility.

3.1 Buying and Selling Volatility:

To make a profit, most individual investors and fund managers are forced to take a view on the direction of the price of something. The traditional or fundamental strategy is to study all aspects of the marketplace and decide on the value of the investment vehicle under study. However, whether following fundamental analysis or technical analysis or a combination of both, the ultimate investment decision is to buy or sell something. The traditional investor must take a view on the direction of the price.

If one is right a profit is enjoyed, if not loss is suffered. Between entering and exiting the trade many things can and often do happen. The price of the instrument may rise or fall. The price may begin to fluctuate violently or become moribund. Price may collapse 50%, stay at this depressed level for several months only then to rise in an orderly fashion and the settle back to the original entry level.
The value of the investment will vary from day to day to a greater or lesser extent looking on market conditions.

The view taker in a sense is looking at only one dimension of a price sequence – the direction. The view taker, once in, is just looking for an exit point. He needs to get the direction of this dimension correct. However, there is another dimension to investment – trading the volatility of a price and not the direction. The position of a volatility trader is such that on entering a position in each stock it does not matter whether the price goes up or down. (Connolly, 1999)

3.2 The Long Volatility Trade:

There are many different types of portfolios that will profit from volatility in each stock price but by far the most straight-forward is that of being long a call option and short shares.

To discuss the mechanics of buying volatility it is assumed that the price of the stock in question will fluctuate excessively. Assuming that it is known for sure that which ever way the price moves, it will be large and there will be many swings in price. Unfortunately, the investor does not know which direction the price will move and when the direction will change. The investor needs to get into a position which will profit whichever way the market moves, up or down. To achieve this the investor has to start with a position which is initially market neutral but gets long if the market rises and gets short if the market falls. The answer is to have a portfolio long of one option contract and to be simultaneously short the number of shares contained in one option contract. In this way the stock exposures of both components are identical in sign and must cancel out. This is referred to as a long volatility or hedged portfolio. In rising markets losses made on shorting stocks will have to be subtracted from option profits, conversely in falling markets profits made on shorting stocks will have to be added to option losses. If the stock price rise is small, the option profits are almost whacked out by short stock losses. If the stock price fall is small, the stock profits are almost completely whipped out by option losses. Hence for small price moves, up or down no profit or loss results. The position is perfectly hedged. The holder is such a position is said to be market neutral or delta neutral.If the stock price rises significantly, the option component always makes more than is lost by the stock component. If the stock price falls significantly, the profit on the short stock component always exceed the loss on the option component. The bigger the move up or down, the bigger the profit. Whichever way the stock price moves, the long volatility trader will always make a profit.

3.3 The Short Volatility Trade:

In the section above, a simple long volatility trade was discussed in which the strategy was to capitalise on future volatility in the stock price. If the option was cheap enough and/or the future volatility was high a profit would have resulted. However, to make a positive profit the circumstances would have to be right. Sometimes the investor might take a view that the options are very expensive and/or the stock volatility is going to fall. Paying up for expensive option premiums and witnessing little or no market movements may produce losses. If the options are expensive enough the investor may choose to take the opposite position and ***sell volatility***. The short volatility strategy is exactly the opposite of the long volatility strategy in every respect and can produce profits. The basic strategy involves selling call options and hedging with a long position in the stock. Selling expensive options can produce significant returns and the short volatility player sells options within the hope that the premium he receives over covers any future price movements. Simply selling naked call options can be very risky. As with the long volatility play it is assumed that the investor has no view on the direction of the underlying and so a simple short volatility portfolio will have short call options perfectly hedged with a long stock position. The following section discusses what happens to the portfolio under two scenarios. If the stock price rises, a loss is suffered on the short call and a profit is enjoyed on the long stock. If the stock price falls, a profit is enjoyed on the short call and a loss is suffered in the long stock. It is easy to see that the net profit and loss to the hedged portfolio will be the difference between the two value profiles. If the stock price rise is small, the option losses are completely cancelled by the profit on the long stock. If the stock price fall is small the option profits are completely cancelled by the losses on the long stock. The portfolio is perfectly hedged and remains so with small stock price moves. If the stock price rises significantly, the option component always loses more than is made by the stock component. If the stock price falls significantly, the losses on the stock component always exceed the profits on the option component. The bigger the move, up or down, the bigger the losses. After entering this position, the investor would hope for little or no stock price movement until expiry. And therefore, the strategy is called the short volatility trade. The reason the portfolio always produces losses whatever the direction of the underlying is again due to the curvature of the option price. In the short volatility portfolio, the curvature of the option is the opposite of that in the long volatility portfolio.

4. Managing Correlation Risk:

Correlation risk refers to the risk of a financial loss when correlation in the market changes. It plays a central role in risk management and the pricing of basket derivatives. In risk management, correlation risk refers to the danger of a loss in a very financial position occurring because of a difference between anticipated correlation and realized correlation. Particularly, this happens when the estimate of correlation was wrong or the correlation within the market changed. Sophisticated structured equity derivatives houses have shed up to 10% of their short correlation risk positions to hedge funds using a new instrument called a **correlation swap.** Dealers structuring equity products for retail investors have built up significant short equity correlation positions in the year 2004, estimated to be as large as e200 million per percentage point of correlation – or e10 billion at one-year Eurostoxx implied correlation levels. The sheer size of those one-way positions, also termed ‘book axe’, led to fears that dealers would be forced to scale down their activity in providing retail structured products

This potentially threatened the profitability of several investment banks, especially SG, which most dealers believe has the biggest short correlation positions – something that became particularly acute because the equity market rally within the year 2004 accentuated the short positions But the French bank has innovated its way out of the problem and inspired several its competitors to mimic its latest new trade – the correlation swap. These instruments typically dated between one-to-three years, and priced between e250,000 to e1 million per correlation point, require the dealers to structure a bespoke basket of 10 to 35 individual names where the dealer has significant correlation risk. It then sells the risk to hedge funds, and SG has laid off as much as 20% of its short correlation risk.

SG conducted its first deal in 2002, with an undisclosed large international reinsurer. While reinsurers have retreated from the market, hedge funds, pension funds and other real-money investors have stepped in to take the other side of correlation swaps. Unlike credit correlation trades, equity correlation is still relatively infrequently traded. And the correlation swaps are attractive to investors due to their exposure solely to correlation. For example, a one-year correlation swap on the Eurostoxx 50 index might have an implied correlation of 47%. If the trade is agreed at e1 million per percentage point, the dealer would pay the hedge funds e47 million on expiry. Should the realized correlation fall during that year to 43%, the hedge fund would pay the dealer e43 million. So, the trade would settle with the dealer paying the hedge fund e4 million. Correlation has become an increasingly familiar concept for hedge funds trading dispersion strategies using vanilla equity options. Here an investor trades an index versus its components or a subset of it components – for example, the Eurostoxx versus its 50 underlying or 20 of them. Dispersion trades are used by hedge funds most commonly as a vega-spread transaction with the correlation position forming an intrinsic part of the deal.

As index volatility has increased significantly relative to single-stock volatility, implied correlations have spiked sharply. The expectation is that the implied correlation level of 50% on the Eurostoxx 50 is rarely realized (see graph below).

Typically, vanilla option transactions have a listed expiry date, so there is a high level of transparency within the market and hedging is straightforward and comparatively cheap.

There are liquidity issues in some single-stock options and a desire to observe corporate actions. But perhaps the most important cost to hedge funds is that the investment required to risk-manage positions: As a result, many funds have used variance swaps to put dispersion trades. Variance swaps are useful to hedge funds as they do not need to develop the risk management and trading infrastructure required to delta hedge their positions. But the downside is that the use of variance swaps can prove more expensive – about 1.7% to 2% more than using vanilla instruments, according to industry participants. In addition, the whole skew of correlation is built into the pricing when using variance dispersion, whereas the implied correlation is different between strikes for vanilla options – for example, the 90% strike on Eurostoxx has a higher implied correlation than the at-the-money implied correlation.

But the purest way to take a correlation position is through correlation swaps. They are also the only way to gain direct longer-dated exposure – one-to-three years versus less-than-a-year for variance swaps. The main downside is that they are extremely illiquid products that are hard to mark-to-market

Unlike European options, where the gamma profile is linked to strikes, correlation swaps offer constant correlation sensitivity regardless of underlying equities moves – they are lever-independent

Figure 4

For example, when Finnish telecommunications company Nokia reported that first-quarter sales fell 2%, causing its share price to tumble 15% to e14.70 on April 6, with only a corresponding 0.9% fall in the Eurostoxx index – a relatively uncorrelated move – dealers' short correlation made money.

Correlation swaps are proving to be a vital risk management tool for structured product houses.

5. Monte Carlo Simulation:

In this section the Monte Carlo method is described and two techniques for improving the efficiency of the method are discussed.' An excellent exposition of the Monte Carlo method is given by Hammersley and Handscomb (1964). Shreider (1966). Fishman (1973) and Meyer (1956) provide additional useful references. It is convenient to couch the discussion in terms of the evaluation of a definite integral although Monte Carlo methods have a much wider range of applicability. Consider the integral

$$\int_A g(y)f(y)\,\mathrm{d}y = \bar{g}, \tag{12}$$

where g(y) is an arbitrary function and f(y) is a probability density function with

$$\int_A f(y)\,\mathrm{d}y = 1. \tag{13}$$

A denotes the range of integration and is omitted for convenience in the sequel. To obtain an estimate of d a number (n) of sample values (vi) is picked (at random) from the probability density function f(y). The estimate of 3 is given by

$$\hat{g} = \frac{1}{n} \sum_{i=1}^{n} g(y_i). \quad (14)$$

The standard deviation of the estimate is given by S where

$$\hat{s}^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (g(y_i) - \hat{g})^2. \quad (15)$$

For large n the error involved in replacing (n- I) by II in this formula is of little consequence. The distribution

$$\frac{\hat{g} - \bar{g}}{\sqrt{\frac{\hat{s}^2}{n}}} \quad (16)$$

tends to a standardized normal distribution with increasing values of n. For the values of tr considered in this paper (n > 1000) the distribution can be regarded as normal and confidence limits on the estimate of 8 can be obtained on this basis. Since the standard deviation of 4 is equal to S/dn the confidence limits can be reduced by increasing n. To reduce the standard deviation by a factor of ten the number of simulations trials must be increased one hundredfold. An alternative approach is to concentrate on reducing the size of s^. Such techniques are known as variance reduction techniques and several them are described in Chapter 5 of Hammersley and Handscomb (1964). The thrust of these methods is to modify or distort the original problem in such a way as to improve the accuracy of the results obtained by crude Monte Carlo methods. One such approach is known as the control variate method.

The basic idea underlying this method is to replace the problem under consideration by a similar but simpler problem which has an analytic solution.

The solution of the simpler problem is used to increase the accuracy of the solution to the more complex problem. Suppose that the integral

$$\int g(y)h(y)\,\mathrm{d}y = G \quad (16)$$

can be evaluated analytically where h is a probability density function. From eqs. (I) and (5) it is clear that

$$\bar{g} = G + \int g(y)[f(y) - h(y)]\,\mathrm{d}y. \quad (17)$$

A revised estimate of g, g* can be obtained by evaluating the integral on the right-hand side of (6) by crude Monte Carlo methods. The function h is called the control variate. The gain in efficiency will be measured by the reduction in the variance of K* as compared to the variance of J'. The increase in efficiency will depend on the degree to which II mimics the behaviour off. Thus, in selecting an appropriate control variate there are two (usually conflicting) requirements.

First h must give rise to an integral that is easy to evaluate. Second h must model the behaviour off. In evaluating the integral in eq. (6) by crude Monte Carlo the same random number is used in the in the simulation trial to generate a value y, from s(y) and a value Y, from /z(y). For a given value of/t Ict.

9 = 1 s(u)f(v) du and 6 = j S(Y) 4~) dy be the estimates obtained by crude Monte Carlo under these conditions. Then
g* is given by
g* = G+(@C?). (7)

This is an unbiased estimate, and its variance is
var@) + var(6) - 2 COY@, 6).
This will be less than the variance of 8 as long as
or
This confirms the earlier observation that the efficiency gain is a function of the relationship between/and h. Furthermore eq. (8) furnishes a prescription for testing if a particular control variate will result in an efficiency gain. Whereas the control variate approach used a second estimate of the integral with a high positive correlation with the estimate of interest the antithetic variate approach exploits the existence of negative correlation between two estimates. In each problem there may be different methods of introducing an antithetic variate

6. Conclusion

Most of the products described above offer a downside protection and a capped return provided the investment is held to maturity. A large part of my research will be devoted to identifying the most suitable method to replicate the pricing of a given equity structured product. This research paper looks at the pricing of these structured products and find a fairly wide range across issuers, but nearly all involve a substantial premium over the cost of replication using exchange-traded options on the Eurex. The volume of the premium varies with moneyness, and there is a “life cycle” effect, as well, because the issuers make secondary markets in their securities and investors may sell them back prior to maturity.

By doing extensive research on equity linked structured product at this stage my thoughts on pricing such products could be done by buying a zero-coupon bond with the same maturity as that of the investment note. The balance amount of the remaining funds can be invested by identifying a ‘basket option’ which mirrors the basket of investments made by the Bank and hedging risks. Carrying the above idea further, this research to look at fair pricing of the zero-coupon bond and the basket option. Comparing this with the market price of the instrument I could draw conclusions based on how close the real market price of the instrument is with the recomputed price.

Bearing in mind the results presented in this paper, great care should be taken when judging quoted prices. Despite the very easy access to structured products, experienced investors should still consider buying or selling options at derivatives exchanges with liquid trade. Nevertheless, it must be acknowledged that a useful" packaging" of single components could justify the implicitly demanded margins as compensation for the issuers' structuring service for offering these products, including the commitment to provide liquid trading of the products. With the help of structured products, however, the issuers provide access to non-standard options positions, so that the observed premia can also be interpreted as commissions for this market extension. The market for structured products is still very attractive for issuers, due to the almost total absence of restrictions about underlying and contract conditions. Examples of recent developments in the German market for structured products are callable step-up bonds, bull, bear and condor bonds, kick start certificates, barrier, and two-asset products. Further research could concentrate on product classes apart from the ones considered in this paper, probably providing an even greater insight into the issuers' pricing policies. Broader market research could consider the pricing stability over time. In addition, taking transactions costs into account, an investigation of concrete arbitrage strategies between the market for structured products and derivatives exchanges seems promising.

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