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Keyword: Fuzzy number; VaR; Portfolio selection; Risk; Fuzzy Laplace distribution; Fuzzy Normal distribution.

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Abstract

The propose of this work is applied the fuzzy Laplace distribution on a possibilistic mean-variance model presented by Li et al which appliehe fuzzy normal distribution. The theorem necessary to introduce the Laplace distribution in the model was demonstrated. It was made an analysis of the behavior of the fuzzy normal and fuzzy Laplace distributions on the portfolio selection with VaR constraint and risk-free investment considering real data. The results showns that were not difference in assets selection and in return rate, however, There was a change in the risk rate, which was higher in the Laplace distribution than in the normal distribution.

Keywords: Fuzzy number; VaR; Portfolio selection; Risk; Fuzzy Laplace distribution; Fuzzy Normal distribution.

1. Introduction

A financial portfolio is a distribution of financial resources among the various investment assets such as stocks, bonds and derivatives. With countless possible combinations of assets, the aim is to select the optimal portfolio, where the optimization depends on the purpose of the investor. The two most commonly searched objectives are: maximization return, to give acceptable level of risk, and risk minimization, to achieve a predefined level of return. The Nobel Prize winner in Economic Sciences, Harry Markowitz M. demonstrated that it is impossible to increase returns without increase risk; thus, it is common that higher expected return is associated with higher risk. Therefore, a profit-oriented investor, in order to guarantee an increase on his capital, would diversify by investing in several assets, instead of investing in only one asset, (HC Investment).

The least complex and most natural way to represent the problem of optimal portfolio selection is a constrained optimization problem. The aim is to maximize or minimize an objective function (usually maximize returns or risk minimization) subject to constraints. However, the objective function and constraints are usually not simple functions. They often rely on more than one characteristic of each asset, and these characteristics are usually combinations of functions that are much more complex than a linear or quadratic function. So, finding a solution to this optimization problem requires more complex techniques.

For these and other reasons, many researchers seek models that can measure all of these variables. In this sense, Markowitz [9],[10] proposed a model for the mean-variance portfolio selection and probability theory associated to optimization techniques to model the performance of investment under uncertainty.

In particular, [1],[17] and [18] analyzed the economic implications of using a mean-VaR model for portfolio selection and the portfolio selection implications arising from imposing a value of risk constraint on the mean-variance model.

However, due to the complexity of financial systems, there are several situations where the input data are not precise but only fuzzy. Therefore, the decision makers should not consider parameters (goals and constraints) using numbers or unique distribuition functions, but instead they should use fuzzy numbers or fuzzy probability distribution functions (see, for example [19]). Recently, researchers investigated many fuzzy portfolio selection problems (see, e.g., [15], [11], [7], [13], [12] and [20]). Knowledge of methods to rank fuzzy numbers is extremely important for this purpose ([2], [3], [5], [6], [14], [16]). Carlsson and Fuller [4] introduced the notations of upper and lower possibilistic mean values, and introduced the notation of crisp possibilistic mean values and crisp possibilistic variance of continuous distributions. Zhang and Nie [21] extended the concepts of possibilistic mean and possibilistic variance proposed by [4], and introduced the concepts of upper and lower possibilistic variances and covariances of fuzzy numbers.

Li et al [8] proposed a model portfolio of possibilistic investment restrictions under the VaR and risk-free. This model shows that risk-averse investors want to not only achieve the expected return rate on their current investment, but also they would prefer to ensure that the maximum of their potential future risk is lower than the VaR. With the assumption that returns of assets are fuzzy variables with normal distribution, with VaR restriction and risk-free.

The propose is to make a comparison using their model, but instead it will be applied fuzzy Laplace distribution and fuzzy normal distribution. We also demonstrate the theorem which is necessary for the inclusion of this distribution to the model proposed by [8]. So, we evaluate the behavior of this model with these two distributions functions.

This paper is organized as follows. In section 2, it is proposed a possibilistic portfolio model under constraints of VaR and risk-free investment. In section 3, it is presented a fuzzy normal distribution demonstrated in [8]. In section 4, it is presented fuzzy Laplace distribution. In Section 5, numerical examples are given to illustrate our effective proposed approaches. And finally, Section 6 presents our conclusions.

2. Portfolio Model under constraints of VaR and risk free investment

In order to define the model, it is necessary to make the following considerations. First, there are n risk assets and one risk-free asset for investment and the asset return rate $\tilde{\varphi}_i$ is a fuzzy number, $i = 1,2, \ldots , n$. x_i represents the proportion invested in assets i, and r_f is the risk-free asset return. From this, and taking into account the definitions of Carlsson and Fuller [4] for upper and lower possibilistic means and upper and lower possibilistic variances and covariances of a fuzzy number $\tilde{A}$ with γ

Theorem 3.1. Assume that the return rates of assets are fuzzy variables with fuzzy normal distribution expressed as $_i$ $FN($ $_i,$ $_i), i = 1,2, \dots, n,$ then

$$\sum_{i=1}^{n} x_i \widetilde{\varphi_i} \sim FN\left[\sum_{i=1}^{n} x_i \mu_i, \left(\frac{4-\pi}{8}\right)\left(\sum_{i=1}^{n} x_i \sigma_i\right)^2\right], \tag{4}$$

where x_i $i = 1,2, \dots, n.$

Proof. Refer to [8].

Moreover, Li et al [8] defines the possibilistic portfolio model under constraints of VaR and risk-free investments, whereas the variables are fuzzy with fuzzy Normal distribution. Thus:

$$\text{s.t}\quad \begin{cases} \min \quad \overline{\sigma}^{2} = (\frac{1}{2} \quad \frac{\pi}{8})\ (\ {}_{i=1}^{n} x_i^2 \sigma_2^2 + 2 \quad {}_{1>i=1}^{n} x_i x_j \sigma_i \sigma_j) \\ \sum_{i=1}^{n} x_i(\mu_i - r_f) + r_f \geq \tilde{r} \\ \left(VaR - \sum_{i=1}^{n} x_i \mu_i\right)^2 \leq ln(1-\beta)\left(\frac{1}{2} - \frac{\pi}{8}\right)\left(\sum_{i=1}^{n} x_i \sigma_i\right)^2 \\ \sum_{i=1}^{n} x_i \leq 1 \\ l_i \quad x_i \quad u_i, i = 1,2, \dots, n. \end{cases} \tag{5}$$

4. Fuzzy Laplace Distribution

Suppose that the return rate of asset i is a Laplace distribution fuzzy variable expressed as $_i$ $FL($ $_i,$ $_i)$, and its membership function is

$$A_i(t/\ _i, b) = 21b\ \overline{exp} \quad |\underline{\qquad} t \quad b \quad _i|)$$

$$= \frac{1}{2b}\begin{cases} exp\left(\frac{\ _i+t}{b}\right) if\ t \quad _i \\ exp\left(\frac{t \quad _i}{b}\right) if\ t \quad _i, \end{cases}$$

$\sqrt{2}$

where $^2{}_i = 2b^2$ $b = {}_2\,\sigma_i,$

The level set of $\tilde{\varphi}_i$ is defined as

$$[\widetilde{\varphi_i}]^{\gamma} = \left[\mu_i - \frac{\sqrt{2}}{2}\sigma_i \ln\sqrt{2}\,\sigma_i\gamma, \mu_i + \frac{\sqrt{2}}{2}\sigma_i \ln\sqrt{2}\,\sigma_i\gamma\right] \tag{6}$$

Theorem 4.1. Assume that the rates of return of the assets are Laplace fuzzy distributions variables expressed as $_i$ $FL(_i, _i), i = 1,2, \dots, n$. Then

$$\sum_{i=1}^{n} x_i \widetilde{\varphi_i} \sim FL\left(\sum_{i=1}^{n} x_i \mu_i, \frac{1}{8}\left(\sum_{i=1}^{n} x_i \sigma_i\right)^2\right) \tag{7}$$

where x_i $i = 1,2, \dots, n$.

Proof. According to Li et al [8], the possibilistic mean value of $\sum_{i=1}^{n} x_i \widetilde{\varphi_i}$ can be calculated by

$$M \quad \left(\sum_{i=1}^{n} x_i \widetilde{\varphi_i}\right) = \sum_{i=1}^{n} x_{i\overline{M}}(\widetilde{\varphi_i}) = \sum_{i=1}^{n} x_i \mu_i.$$

From definitions of upper M_U and lower M_L possibilistic means and the equation (4), it can be deducted that

$$M_U(\widetilde{\varphi_i}) = 2\int_0^1 \gamma\left(\mu_i + \frac{\sqrt{2}}{2}\sigma_i \ln\sqrt{2\sigma_i\gamma}\right) dy$$

$$= \mu_i + \sqrt{2}\,\sigma_i \int_0^1 \gamma \ln\sqrt{2}\,\sigma_{i}\gamma\, dy \tag{8}$$

$$= \mu_i + \frac{\sqrt{2}}{2}\sigma_i\left(\ln\sqrt{2}\,\sigma_i - \frac{1}{2}\right).$$ and

$$M_L(\widetilde{\varphi_i}) = 2\int_0^1 \gamma\left(\mu_i - \frac{\sqrt{2}}{2}\sigma_i \ln\sqrt{2\sigma_i\gamma}\right) dy$$

$$= \mu_i - \frac{\sqrt{2}}{2}\sigma_i\left(\ln\sqrt{2}\,\sigma_i - \frac{1}{2}\right). \tag{9}$$

In the same way, the following results can be obtained:

$$\sigma^2{}_U = 2\int_0^1 \gamma\,[M_U(\widetilde{\varphi_i}) - a_2(\gamma)]^2\, dy$$

$$= 2\int_0^1 \gamma\left[\mu_i - \frac{\sqrt{2}}{2}\sigma_i\left(\ln\sqrt{2\sigma_i} - \frac{1}{2}\right) - \left(\mu_i + \frac{\sqrt{2}}{2}\sigma_i \ln\sqrt{2\sigma_i\gamma}\right)\right]^2 dy$$

$$= \frac{1}{8}\,{}^2\sigma_i,$$

and

$$\sigma^2{}_L = 2\int_0^1 \gamma\,[M_U(\widetilde{\varphi_i}) - a_2(\gamma)]^2\, dy$$

$$= \frac{1}{8}\,{}^2\sigma_i,$$

8

Thus, the possibilistic variance can be written as

$$\overline{\sigma^2_{\widetilde{\varphi_i}}} = \frac{\sigma_U^2 + \sigma_L^2}{2} = \frac{1}{8}\sigma_i^2 \cdot \qquad (10)$$

Furthermore, the upper and lower possibilistic covariances are given by

Cov_U

$(\widetilde{\varphi_i}, \widetilde{\varphi_j}) = 2\int_0^1 \gamma\, [M_U(\widetilde{\varphi_i}) - a_2(\gamma)][M_U(\widetilde{\varphi_i}) - b(\gamma)]_{dy},$

$= \frac{1}{8}\ \sigma_i\sigma_j,$

The possibilistic covariance is

$$\overline{Cov}(\ _i,\ _j) = \frac{Cov_U(\ _i,\ _j) + Cov_L(\ _i,\ _j)}{2} \ \frac{1}{8} = \ _i\ _j. \qquad (11)$$

According to [8] and considering $(x_i$, it can be computed the possibilistic variance of $\sum_{i=1}^n x_i \widetilde{\varphi_i}$

$$_{ni=1\, xi\ \ i} = \sum^n x^2\overline{\sigma^2} + 2\sum_{i=1}\ \ i\ \ \tilde{\varphi}i\ \ in=1\, x_i x_j \overline{Cov}(\ _i,\ _j)$$

$$= \sum_{i=1}^n \frac{1}{8} x_i^2\, \sigma_i^2 + 2\sum_{i=1}^n \frac{1}{8}\ x_i x_j\ \ _i$$

$$= \frac{1}{8}\left(\sum_{i=1}^n x_i\, \sigma_i\right)^2 \cdot$$

So the proof of the theorem is complete.

According to Theorem 4.1 the membership $= \frac{\sqrt{2}}{2}\sigma \quad = \frac{\sqrt{2}}{2}\left(\frac{1}{8}\left(\sum_{i=1}^n x_i\,\sigma_i\right)^2\right)$ function of $\sum_{i=1}^n x_i \widetilde{\varphi_i}$ is defined by:

— $1\ |t\ \ _{ni=1\, xi\ \ i)}|\}$, where b b. $(\sum_{i=1}^n \varphi_i\, x_i \leq$

So pos

$A(t) = exp$

$2b$ b

$VaR) = sup_t\ _{VaR}\ \{\frac{1}{2b}\, exp\ \{\ \ |t\ \ _{i=1} b1nxi\ \ i)|\}\},$

$= \ _1\, exp\ \ |\ \ _{VaR}\ \ ^n{}_{i=1\, xi\ \ i)}|\},$

$2b \qquad b$

$$= \frac{1}{2b} exp \frac{\; in=1 xi \; i \; VaR}{b} \; if, VaR < \sum_{i=1}^{n} x_i \mu_i$$

$$= \{_n \begin{array}{l} \frac{1}{} exp \frac{in=1 xi \; i \; VaR)}{} \; if \; VaR < \sum_{i=1}^{n} x_i \mu_i \\ 22\overline{1b}b exp \frac{VaR \quad bbi=1 xi}{} \; i) \; if \; VaR \geq \sum_{i=1}^{n} x_i \mu_i \end{array} \quad (12)$$

According to equations (2) and (12), it is obtained

$$\begin{array}{ll} (VaR - \sum_{i=1}^{n} x_i \mu_i) \leq b \ln[2b(1-\beta)] & if \; VaR \quad i=1 \, x_i \quad i \\ (\sum_{i=1}^{n} x_i \mu_i - VaR \quad b \ln[2b & if \; VaR \quad i=1 \, x_i \quad i. \end{array} \quad (13)$$

It is known from equations (1), (8) and (9), that when the return rate of assets are Laplace distribution fuzzy variables, the upper and lower possibilistic means of $\tilde{r}_p$ are given by:

$$M_U(\tilde{r}_p) = \sum_{i=1}^{n} \left(\mu + \frac{\sqrt{2}}{2}\sigma\left(\ln\sqrt{2}\,\sigma - \frac{1}{2}\right) - r_f\right) x_i + r_f$$

$$M_L(\tilde{r}_p) = \sum_{i=1}^{n} \left(\mu - \frac{\sqrt{2}}{2}\sigma\left(\ln\sqrt{2}\,\sigma - \frac{1}{2}\right) - r_f\right) x_i + r_f$$

Thus, the possibilistic mean of $\tilde{r}_p$ is written as

$$\bar{M}(\tilde{r}_p) = \frac{M_U(\tilde{r}_p) + M_L(\tilde{r}_p)}{2} = \sum_{i=1}^{n} (\mu_i - r_f) x_i + r_f.$$

Moreover, for a Laplace fuzzy variable distribution, the portfolio model of VaR restrictions and risk free investment can be formulated as:

$$\begin{array}{ll} \min & \overline{\sigma^2} = \frac{1}{8}\left(\sum_{i=1}^{n} x_i^2 \sigma_i^2 + 2\sum_{n_i > j=1} x_i x_j \; i \; j\right) \\ s.t & \left\{ \begin{array}{l} \sum_{i=1}^{n} x_i(\mu_i - r_f) + r_f \geq \tilde{r} \\ (VaR - \sum_{i=1}^{n} x_i \mu_i) \leq b \, ln[2b(1-\beta)] \quad p/VaR < \sum_{i=1}^{n} x_i \mu_i \\ (\sum_{i=1}^{n} x_i \mu_i - VaR) \leq b \, ln[2b(1-\beta)] \quad p/VaR \geq \sum_{i=1}^{n} x_i \mu_i \\ w \; ere \quad b = \frac{\sqrt{2}}{16}(\sum_{i=1} 1^n x_i \sigma_i)^2 \\ \sum_{i=1}^{n} x_i \leq 1 \\ 0 \leq l_i \leq x_i \leq u_i, i = 1,2,\ldots,n, \end{array} \right. \end{array} \quad (14)$$

5. Numerical Example

We may believe that in order to have a more profitable investment, we need only increase the risk. So in an ideal world, you would choose the desired risk and receive the expected return. In the real world, the history is different. Risky investments do not imply or guarantee higher returns. If it were in this way, the idea of risk would not make sense. For this reason, we have created a rating for the investor as a result of desired risk and from there select the best expected portfolio allocation for this profile (see Table 1).

For $P_i(i = 1, \ldots, 100)$ denoting the set of the i% of the portfolio with least risks. The investor profile is defined in the Table 1, with Su
Little Aggressive (Aggressive (and Very Aggressive (in term of P_i's.

Type of Investor		Rating Interval
Super Conservative	0.10	Maximum return of risks lesser than P10
Conservative	0.25	Maximum return of the risks between P10 and P25
Moderate	0.50	Maximum return of the risks between P26 and P50
Little Aggressive	0.75	Maximum return of the risks between P51 and P75
Aggressive	0.90	Maximum return of the risks between P76 and P90
Very Aggressive	1.00	Maximum return of the risks greater than P91

Table 1. Investor Profile

In order to compare the results using the two models of fuzzy distributions, namely, Normal and Laplace, whenever they are in the context of possibilistic mean-variance model described by [8], it was selected three assets of the Bank of Brazil, correspond to three period of the application (12, 14 and 36 months), conform shows the Table 2.

In a next step, a database has been generated x_i, $i = 1,2,\ldots,n$, representing the proportion invested in assets i, namely, l_i x_i u_i. For this work, will be considered $n = 3$ assets and return rate for each asset $_i$ $FN(\ _i,\ _i)$ and $_i$ $FL(\ _i,\ _i)$ is calculated from the frequency distribution of monthly returns of the tree assets in each period of the application, see Table 2.

	Investment Portfolio								
	STPF – Short-term portfolio funds			LTPF – Long-term portfolio funds			Stock Market		
	Rate Return (%)			Rate Return (%)			Rate Return (%)		
Months	12	24	36	12	24	36	12	24	36
Accumulated	5,026667	15,10417	26,50833	10,43517	16,22708	31,75975	24,5475	48,646	40,64295
µ	0,418889	0,41956	0,736343	0,869597	0,676128	0,882215	2,045625	2,068583	1,128971
	0,186846	0,29036	0,403642	1,412328	1,680308	1,310872	6,024559	7,289093	5,449996

Table 2. The possibilistic distributions of returns of tree assets, data from 2018, the Bank of Brazil

The level set of, is given by $_i(i = 1, \ldots, 4)$ from the $FN(\ _i,\ _i)$, *is given by*

$$[\varphi_1]^\gamma = \left[0.0161 - 0{,}0194\sqrt{ln\gamma^{-1}}, 0.0161 + 0.0194\sqrt{ln\gamma^{-1}}\right]$$

$$[\varphi_2]^\gamma = \left[0.0247 - 0.0228\sqrt{ln\gamma^{-1}}, 0.0247 + 0.0228\sqrt{ln\gamma^{-1}}\right]$$

$$[\varphi_3]^\gamma = \left[0.0282 - 0.0171\sqrt{ln\gamma^{-1}}, 0.0282 + 0.0171\sqrt{ln\gamma^{-1}}\right] \quad (15)$$

$$[\varphi_4]^\gamma = \left[0.0392 - 0.0104\sqrt{ln\gamma^{-1}}, 0.0392 + 0.0114\sqrt{ln\gamma^{-1}}\right]$$

Suppose \% and $VaR = 0.4\%$ and considering each investor profile defined in Table 1, the results obtained of the selected portfolios to 12, 24 and 36 months are presented in Tables 3, 4 and 5, respectively.

Portfolio						
	0.1 (1)	0.25 (2)	0.5 (3)	0.75 (4)	0.9 (5)	1 (6)
x_1 (%)	47.75	28.75	9.75	5.00	5.00	5.00
x_2 (%)	0.00	60.00	65.00	55.00	30.00	20.00
x_3 (%)	10.00	10.00	23.50	37.00	59.50	73.00
Risk-Free (%)	42.25	1.25	1.75	3.00	5.50	2.00
Return (%)	5.1610	10.1741	13.0583	15.0986	18.0304	20.2754
Risk (%)	0.6177	2.9124	7.1280	11.7204	20.8184	28.3697

Table 3. Normal - the Bank of Brazil year 2018/12

Portfolio						
	0.1 (1)	0.25 (2)	0.5 (3)	0.75 (4)	0.9 (5)	1 (6)
x_1 (%)	5.00	33.50	71.50	62.00	38.25	14.50
x_2 (%)	0.00	0.00	0.00	0.00	0.00	10.00
x_3 (%)	10.00	10.00	19.00	32.50	59.50	73.00
Risk-Free (%)	85.00	56.50	9.50	5.50	02.25	2.50
Return (%)	6.2675	10.4327	20.3036	25.5408	35.3316	40.0681
Risk (%)	1.4225	1.7563	6.5253	16.7199	50.9219	78.7441

Table 4. Normal - the Bank of Brazil year 2018/24

Portfolio						
	0.1 (1)	0.25 (2)	0.5 (3)	0.75 (4)	0.9 (5)	1 (6)
x_1 (%)	5.00	28.75	66.75	66.75	38.25	14.50
x_2 (%)	0.00	0.00	20.00	0.00	5.00	15.00
x_3 (%)	10.00	10.00	10.00	28.00	55.00	68.50
Risk-Free (%)	85.00	61.25	3.25	5.25	1.75	2.00
Return (%)	6.5015	10.2516	20.0598	25.2374	35.1677	40.0607
Risk (%)	1.2338	1.6875	4.4742	12.4493	39.9836	61.4422

Table 5. Normal - the Bank of Brazil year 2018/36

Now it will be considered the Laplace distribution to selected the three assets presented in Table 2. The level set of $\;_i(i = 1, \ldots, 4)$ from the $FN(\;_i, \;_i)$ (Figure 1), is given by

$$[\widetilde{\varphi_1}]^\gamma = \left[0.0161 - \frac{\sqrt{2}}{2}0.0194\ln\left(\sqrt{2}0.0194\gamma\right), 0.0161 + \frac{\sqrt{2}}{2}0.0194\ln\left(\sqrt{2}0.0194\gamma\right)\right]$$

$$[\widetilde{\varphi_2}]^\gamma = \left[0.0247 - \frac{\sqrt{2}}{2}0.0228\ln\left(\sqrt{2}0.0228\gamma\right), 0.0247 + \frac{\sqrt{2}}{2}0.0228\ln\left(\sqrt{2}0.0228\gamma\right)\right]$$

$$[\widetilde{\varphi_3}]^\gamma = \left[0.0282 - \frac{\sqrt{2}}{2}0.0171\ln\left(\sqrt{2}0.0228\gamma\right), 0.0282 + \frac{\sqrt{2}}{2}0.0171\ln\left(\sqrt{2}0.0171\gamma\right)\right] \quad (16)$$

$$[\widetilde{\varphi_4}]^\gamma = \left[0.0392 - \frac{\sqrt{2}}{2}0.0104\ln\left(\sqrt{2}0.0228\gamma\right), 0.0392 + \frac{\sqrt{2}}{2}0.0104\ln\left(\sqrt{2}0.0104\gamma\right)\right]$$

Again, suppose and $VaR = 0.4\%$ and considering each investor profile defined in Table 1, the results obtained of the selected portfolios for 12, 24 and 36 months are presented in Tables 6, 7 and 8, respectively.

Portfolio						
	0.1 (1)	0.25 (2)	0.5 (3)	0.75 (4)	0.9 (5)	1 (6)
x_1 (%)	47.75	28.75	9.75	5.00	5.00	5.00
x_2 (%)	0.00	60.00	65.00	55.00	30.00	20.00
x_3 (%)	10.00	10.00	23.50	37.00	59.50	73.00
Risk-Free (%)	42.25	1.25	1.75	3.00	5.50	2.00
Return (%)	5.1610	10.1741	13.0583	15.0986	18.0304	20.2754
Risk (%)	0.7196	3.3928	8.3049	13.6537	24.2523	33.0493

Table 6. Laplace - the Bank of Brazil year 2018/12

Portfolio						
	0.1 (1)	0.25 (2)	0.5 (3)	0.75 (4)	0.9 (5)	1 (6)
x_1 (%)	5.00	33.50	71.50	62.00	38.25	14.50
x_2 (%)	0.00	0.00	0.00	0.00	0.00	10.00
x_3 (%)	10.00	10.00	19.00	32.50	59.50	73.00
Risk-Free (%)	85.00	56.50	9.50	5.50	02.25	2.50
Return (%)	6.2675	10.4327	20.3036	25.5408	35.3316	40.0681
Risk (%)	1.6571	2.0460	7.6017	19.4778	59.3214	91.7328

Table 7. Laplace - the Bank of Brazil year 2018/24

Portfolio						
	0.1 (1)	0.25 (2)	0.5 (3)	0.75 (4)	0.9 (5)	1 (6)
x_1 (%)	5.00	28.75	66.75	66.75	38.25	14.50
x_2 (%)	0.00	0.00	20.00	0.00	5.00	15.00
x_3 (%)	10.00	10.00	10.00	28.00	55.00	68.50
Risk-Free (%)	85.00	61.25	3.25	5.25	1.75	2.00
Return (%)	6.5015	10.2516	20.0598	25.2374	35.1677	40.0607
Risk (%)	1.4374	1.9659	5.2122	14.5027	46.5789	71.5770

Table 8. Normal - the Bank of Brazil year 2018/36

To illustrate, assume that an investor wants to invest 1,000 in the Banco do Brasil portfolios presented in Table 1. Considering the results presented from the model for the two distributions studied and the investor profile (Table 1), Tables 9, 10, 11 shows a summary of the previous results, with the Normal and Laplace

present the best portfolio distribuition (Table 11).

Comparing the two distributions,it was observed that the returns for each portfolio (STPF, LTPF and Stock Market) are the same for the two fuzzy possibilistcs distributions considered (Normal and Laplace) with theories and equations (5) and (14) previously defined.However, the risks are different. Note that in all cases, the risk is higher when calculated using the Laplace distribution. From this, it possible in future work establish a gap between this difference in found risks in Normal and Laplace distribuitions, with the objective of defining the investor profile of the interval form.

Therefore when comparing the results for Normal and Laplace models of fuzzy distribution, whenever they are inserted into possibilistic mean-variance model described by [8], it can be found out they are very similar to each other, Differing only in the risk rate, which is higher in the Laplace distribution than in the

	0.1 (1)		0.25 (2)		0.5 (3)		0.75 (4)		0.9 (5)		1 (6)	
Portfolio	N	L	N	L	N	L	N	L	N	L	N	L
STPF	477.50	477.50	288.50	288.50	97.50	97.50	50.00	50.00	50.00	50.00	50.00	50.00
LTPF	0.00	0.00	600.00	600.00	650.0	650.0	550.00	550.00	300.00	300.00	200.00	200.00
Stock Market	100.00	100.00	125.00	125.00	235.0	235.0	370.00	370.00	595.00	595.00	730.00	730.00
Free – Risk	422.50	422.50	12.50	12.50	17.50	17.50	30.00	30.00	55.00	55.00	20.00	20.00
Return (%)	5.1610	5.1610	10.1741	10.1741	13.0583	13.0583	15.0986	15.0986	18.0304	18.0304	20.2754	20.2754
Risk (%)	0.6177	0.7196	2.9124	3.3928	7.1280	8.3049	11.7204	13.6537	20.8184	24.2523	28.3697	33.0493

Table 9. Result for an investment value of mil dollar – Normal (N) and Laplace (L) – Bank of Brazil years 2018/12

	0.1 (1)		0.25 (2)		0.5 (3)		0.75 (4)		0.9 (5)		1 (6)	
Portfolio	N	L	N	L	N	L	N	L	N	L	N	L
STPF	50.00	50.00	335.00	335.00	715.00	715.00	620.00	620.00	382.50	382.50	145.00	145.00
LTPF	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00

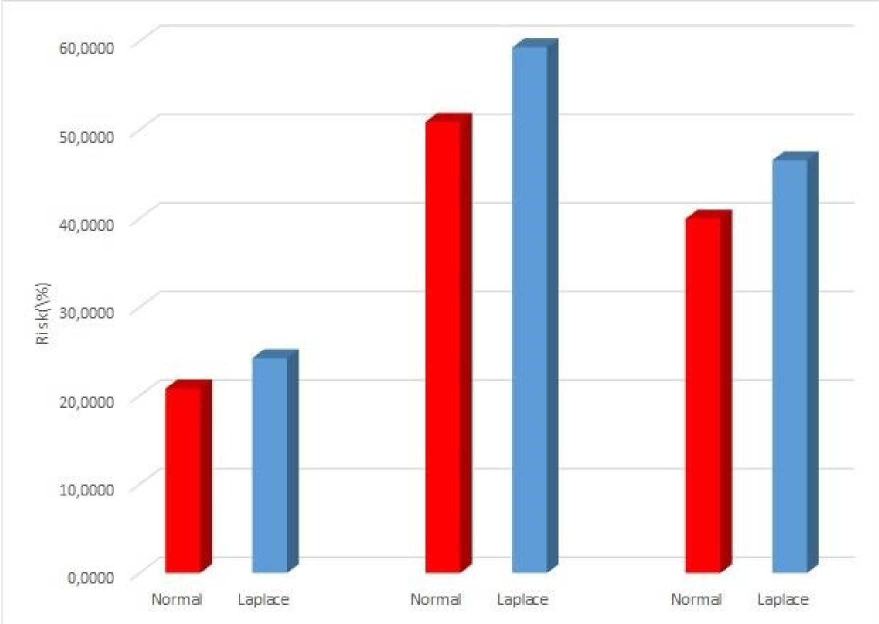
Stock Market	100.00	100.00	100.00	100.00	190.00	190.00	325.00	325.00	595.00	595.00	730.00	730.00
Free – Risk	850.00	850.00	565.00	565.00	95.00	95.00	55.00	55.00	22.50	22.50	25.00	25.00
Return (%)	6.2675	6.2675	10.4327	10.4327	20.3036	20.3036	25.5408	25.5408	35.3316	35.3316	40.0681	40.0681
Risk (%)	1.4225	1.6571	1.7563	2.0460	6.5253	7.6017	16.7199	19.4778	50.9219	59.3214	78.7441	91.7328

Table 10. Result for an investment value of mil dollar – Normal (N) and Laplace (L) – Bank of Brazil years 2018/24

	0.1 (1)		0.25 (2)		0.5 (3)		0.75 (4)		0.9 (5)		1 (6)	
Portfolio	N	L	N	L	N	L	N	L	N	L	N	L
STPF	50.00	50.00	287.50	287.50	667.50	667.50	667.50	667.50	382.50	382.50	145.00	145.00
LTPF	0.00	0.00	0.00	0.00	200.00	200.00	0.00	0.00	50.00	50.00	150.00	150.00
Stock Market	100.00	100.00	100.00	100.00	100.00	100.00	280.00	280.00	550.00	550.00	685.00	685.00
Free – Risk	850.00	850.00	612.50	612.50	32.50	32.50	52.50	52.50	17.50	17.50	20.00	20.00
Return (%)	6.0015	6.0015	10.2516	10.2516	20.0598	20.0598	25.2374	25.2374	35.1677	35.1677	40.0607	40.0607
Risk (%)	1.2338	1.4374	1.6875	1.9659	4.4742	5.2122	12.4493	14.5027	39.9836	46.5789	61.4422	71.5770

Table 10. Result for an investment value of mil dollar – Normal (N) and Laplace (L) – Bank of Brazil years 2018/36

	BB 2018/12		BB 2018/24		BB 2018/36	
Portfolio	N	L	N	L	N	L
STPF	50.00	50.00	382.50	382.50	382.50	382.50
LTPF	300.00	300.00	0.00	0.00	50.00	50.00
Stock Market	595.00	595.00	595.00	595.00	550.00	550.00
Free – Risk	55.00	55.00	22.50	22.50	17.50	17.50
Return (%)	18.0304	18.0304	35.3316	35.3316	35.1677	35.1677
Risk (%)	20.8184	24.2523	50.9219	59.3214	39.9836	46.5789



- Investor

Aggressive

5. Conclusion

This work compared models of fuzzy distribution, namely, Normal and Laplace, whenever they are inside the context of possibilistic mean-variance model described by in [8], where only fuzzy Normal distribution was used. The purpose was to make a comparison of model applying fuzzy Laplace distribution. The theorem which was necessary for the inclusion of the Laplace distribution to the model proposed by [8] was demonstrated. It is well known the importance of having other distributions as parameters for financial analysts, due the volatility of the stock market as well as the behavior of financial market.

It can also be emphasized the importance of working with several fuzzy probability distributions, as demonstrated by the significant variation in the return and risk rates. Besides, this work demonstrated that for the model proposed by [8] and that fuzzy Normal distribution and fuzzy Laplace distribution are the most appropriate ones.

For a comparison between the two fuzzy distributions, it was defined the investor profile with Super

Aggressive types. Next, we show the selected portfolio from your profile.

Therefore, risk-averse investors were given the possibility of a better evaluation, i.e, deciding the best way to distribute their funds in assets investment.

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