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Full length article

CONSTITUTIVE MODELLING OF SANDS UNDER MONOTONIC LOADING

M. Safdar[1], S.H. Shah[2], H.A. Qureshi[3*], M. Waseem[4], and K. Mahmood[5]

1. Earthquake Engineering Center, University of Engineering & Technology Peshawar University Road, Peshawar, Pakistan
2. Department of Civil Engineering, University of Engineering & Technology Peshawar University Road, Peshawar, Pakistan
3. Department of Civil Engineering, Cecos University of IT and Emerging Sciences, Peshawar, Pakistan
4. Civil Engineering Department, University of Engineering & Technology Peshawar University Road, Peshawar, Pakistan
5. Civil Engineering Department, Sarhad University of Science and Information Technology (SUIT), Pakistan

ABSTRACT

This paper presents the drained and undrained behavior of soils using a modified version of the original cam clay constitutive model. The strain hardening behavior of soils is one of the major challenges in geotechnical engineering. The constitutive equations are numerically integrated over fixed time steps to apply effective stress to the derived elastoplastic soil model. Convergence of solution is controlled by a constitutive relation, namely the associated flow rule. This study provides step by step Python and octave programs to solve for q''-'' p by solving the associated non-linear system. The problem is formulated by assuming small strains in the elastic region and large strains in the plastic region. The transition from over-consolidated to normally consolidated states is predicted to be smooth by this elastoplastic model. The model is recognized and solved as a boundary value problem with only two effective stress variables namely q''-'' p which is an approximation of three-dimensional invariants.

KEYWORDS: Constitutive Modelling, Cam Clay Model, Triaxial Test, Flow Rule, Python

*Corresponding author: (Email: hamzaahmad@cecos.edu.pk/18pwciv4995@uetpeshawar.edu.pk)

1. INTRODUCTION

Constitutive modelling is the practice of using mathematical models to describe the mechanical properties of materials in mechanics. Cam clay model is a constitutive formulation to predict the elastoplastic behavior of the soil continuum [1]. The building blocks of a structured cam clay model include a link between consolidation, elasticity and plasticity. Cam clay model is a family of elastoplastic strain hardening models that are formulated based on critical state theory. These models attempt to explain the strength, volume changes and critical state aspects of soil behavior [2]. At critical states, large displacements are produced in soil elements without any changes in stress. The soil is treated as a porous elastic continuum [3]. The original cam clay model was introduced by Schofield et al. [4] as an extension of the Granta-gravel model which derives an expression of yield surface under triaxial loading for lightly consolidated clays. The nature of the yield surface at the far end (pc) makes it harder to use incremental stress models as normal to the surface is not defined at that point. An elliptical yield surface is introduced in Roscoe et al. [5] and Abdelmalek [6] to allow for incremental volume changes. The problem still does not have an analytical solution for arbitrary stress paths [7]. Unlike its predecessors, cam clay and modified cam clay describes soil accurately for arbitrarily large stress ratios when the stress path is above the critical state line. The use of these models in practical applications such as the general boundary value problem is not recommended [8]. Several renowned researchers have worked on constitutive modelling of sands [9-11] and clays [12-13].

We have programmed the modified cam clay model described in Wood [14] with the architecture of the structured cam clay model. The model is based on critical state soil mechanics. The yield surface also serves as plastic potential. The results are compared with PLAXIS 3D analysis and found to be accurate matching. An Abaqus plugin inside the Visualization workbench is created for quick analysis of drained/undrained behavior of modified cam clay. The plugin does not use finite element formulation and thus cannot be used as a UMAT/VMAT material model. It can only be used to run triaxial tests on solved soil elements. The previous statement is redundant after mentioning that the plugin works in the visualization workbench.

Triaxial testing is an entry step to analyzing complex soil properties under various loading conditions. A large amount of data is generated at every loading step of such tests [15]. This data cannot be directly used with FEM or DEM program as it has to be adjusted for each node [16]. A fluent way to integrate this data with our FEM programs is via the construction of a constitutive model with various parameters based on experimental data such as cam clay. Parameters controlling the behavior of the sample are extracted from this data. This is termed as model calibration. Once a model is calibrated it can predict other soil properties in diversely different FEM simulations. These predictions are as good as the accuracy of the soil model. The benefit of a hierarchical cam clay model is that it can describe a wide range of non-linear soil behavior due to its modular nature. By modular we imply the ease of introducing new non-linear behaviors. This study describes the constitutive formulation of the cam clay model and modified cam clay. Programs and visual aids for interpretation of the behavior of triaxial tests under cam clay models have been explored. The results from the modified cam clay model are compared with those from PLAXIS-3D and found to be accurate. Furthermore, a post-processing plugin for modified cam clay was developed in Abaqus.

2 LITERATURE REVIEW

Cam clay model is a constitutive formulation to predict of elastoplastic behavior of the soil continuum. The building blocks of a structured cam clay model include a link between consolidation, elasticity and plasticity. The Cam clay model is a family of elastoplastic strain hardening models that are formulated based on critical state theory. These models attempt to explain the strength, volume changes and critical state aspects of soil behavior. At critical states, large displacements are produced in soil elements without any changes in stress. The soil is treated as a porous elastic continuum. The original cam clay model was introduced by Schofield et al. [4] as an extension of the Granta-gravel model which derives an expression of yield surface under triaxial loading for lightly consolidated clays. The nature of the yield surface at the far end (pc) makes it harder to use incremental stress models as normal to the surface is not defined at that point. An elliptical yield surface is introduced in Roscoe et al. [5] and Abdelmalek [6] to allow for incremental volume changes. The problem still does not have an analytical solution for arbitrary stress paths [7]. Unlike its predecessors cam clay and modified cam clay describes soil accurately for arbitrarily large stress ratios when the stress path is above the critical state line. The use of these models in practical applications such as the general boundary value problem is not recommended.

This paragraph summarizes some of the drawbacks of the cam clay model. It is clear from the literature review that the deformations within the yield surface are not purely elastic as assumed by the cam clay models. The undrained stress path in p^' "-" q space is not necessarily vertical in real soils. In the associated flow rule it is assumed that the plastic potentials follow the yield surface. This is not true for real soils. The plastic potential is a rotated version of the yield surface [14]. Due to the non-homogeneous nature of soil, there is a possibility for the development of slip planes in over-consolidated samples. Such behavior cannot be predicted by clay models. The rotation of the plastic potential surface depends on Lode's angle [17]. There is no standardized way of performing extension [17]. However, it is assumed by Wood [14] that extension behavior is the same as compression with different values of state variables. The modified cam clay model cannot accurately predict post-failure non-linear behavior. This problem was solved in Grammatikopoulou [18]. If applied to settlement problems, the value of settlement obtained is reliable at a depth of 4 m or more [14].

We have implemented the modified cam clay model described in Wood [14]. The model is based on critical state soil mechanics. The yield surface also serves as plastic potential. The results are compared with PLAXIS 3D analysis and found to be accurate matching. An Abaqus plugin inside the Visualization workbench is created for quick analysis of drained/undrained behavior of modified cam clay. The plugin does not use finite element formulation and thus cannot be used as a UMAT/VMAT (User Material) material model. It can only be used to run triaxial tests on solved soil elements. The previous statement is redundant after mentioning that the plugin works in the visualization workbench.

2.1 Applications of Cam Clay Model

- It packages strength and stress-strain behavior within a single framework [19].
- It combines the theories of critical state, and state boundary with plasticity (yielding, plastic flow and hardening) [14].
- The cam clay model can be easily extended to expansive soils under cyclic loading as seen in Abdelmalek [6] and Carter et al [20].
- It can be applied to dense granular soil [21] as well as sand [14].
- It is good at predicting behavior of material in a triaxial test.
- It accurately predicts the behavior of reconstituted soil [22].
- By using anisotropic elasticity formulation, the model adapts to the anisotropic nature of soil as mentioned in [23] and Wood [14].

2.2 Scope of the Work

The study covers only the constitutive formulation of the modified cam clay model. The calibration procedure for obtaining the cam clay model is not included. The constitutive formulation of the cam clay model and modified cam clay is described in this study. Programs and visual aids for interpreting triaxial test behavior under cam clay models have been investigated. When the results of the modified cam clay model are compared to those of PLAXIS-3D, they are determined to be accurate. In addition, Abaqus was used to create a post-processing plugin for modified cam clay.

Up to this point, we have distinctly separated soil shear strength from consolidation behavior and evaluated them as two different phenomena. However, the truth is that soil shear strength and consolidation behavior are connected and can be used together to better understand the overall behavior of the soil and predict its response to load. The framework involves concepts of one-dimensional behavior (represented by e-log(p') curves) and deviator stress vs. mean effective stress behavior (represented by p' -q curves or stress paths).

Key features of critical state theory are highlighted in this section. These are important in modelling behavior of cam clay under a particular stress path. As the regime at which elastic deformation switches to plastic deformation is described by a critical state, it is therefore important to understand the concept. Most papers and books dealing with cam clay use different notations for state parameters p,q,v. The notation used in this study is the same as Wood [14]. Nowadays the elliptical modified cam clay model is often used in books such as Wood [14], [24] and literature such as [5], [25], as a cam clay model whereas Schofield [4] is called the original cam clay model. A similar naming convention is followed here.

3. METHODOLOGY

Our approach to solving these problems comprises a proper review of critical state soil mechanics and an in-depth literature review. We will also write programs for variants of cam clay models while doing a literature review. These models will either be implemented in Octave, Python or JavaScript. These models will be validated against popular finite element programs such as PLAXIS 3D and Abaqus. Fig. 1 shows the methodology adopted in this research article. Fig. 12 shows the Working of the Abaqus Plugin.

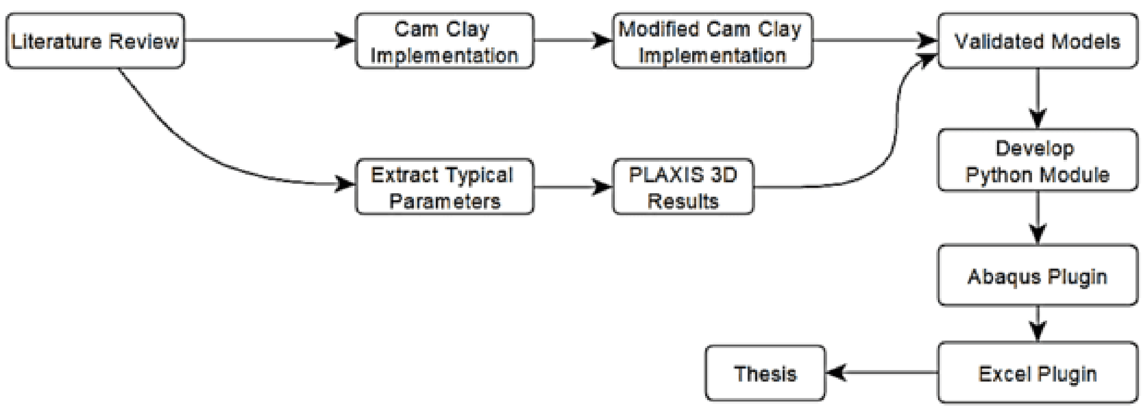

Fig. 1. Proposed Methodology

4. FORMULATION

4.1 Stress Stages

In the consolidation stage, there is no shear stress (q=0) and the mean effective stress is equal to the initial loading ($p' = \sigma_3$) [14]. In the shearing stage, the mean effective stress and shear stress is given by Wood [14];

$$p' = \frac{\sigma_1' + 2\sigma_3'}{3} \quad (1)$$

$$q = \sigma_1' - \sigma_3' \quad (2)$$

4.2 Stress Changes

The change in mean effective stress under axisymmetric loading in a triaxial test is given by:

$$\Delta p' = \frac{\Delta\sigma_3'}{3} \quad (3)$$

Similarly, the change in shear stress is as;

$$\Delta q = \Delta\sigma_1' \quad (4)$$

4.3 Yielding

Yielding starts as soon as the stress path touches the yield surface after passing through the critical state line. At yielding the soil element can behave in three ways Ortigao [26].

1. Linear Strain
2. Strain Softening
3. Strain Hardening

4.4 Key Parameters of Modified Cam Clay Model

A detailed discussion of the parameters of cam clay models is done in this section. Starting with consolidation parameters. The slope of the isotropic normal consolidation line and over-consolidation line is represented by λ and κ respectively. It is to be noted that λ is always greater than κ. This can be used to validate input data to our model. μ' represents the effective Poisson's ratio. The critical state friction angle is given by φ. It is used in the calculation of M value (M_c or M_e). M represents the slope of the critical state line or shape factor for ellipse.

The modified cam clay model consists of

1. Isotropic Elasticity
2. Yield Surface
3. Plastic Potentials
4. Hardening Rule

4.5 Elasto-Plastic Response

$$\delta\varepsilon^e = D \ \delta\sigma' \quad (5)$$

$$\delta\varepsilon^p = D^p \delta\sigma' \quad (6)$$

Where,

$$D = \begin{bmatrix} k\frac{\nu'}{p} & 0 \\ 0 & \frac{1}{3G'} \end{bmatrix} \quad (7)$$

and,

$$D^p = \frac{\lambda - \kappa}{\nu \ p'(M^2 + \eta^2)} \begin{bmatrix} M^2 - \eta^2 & 2 \ \eta \\ 2\eta & \frac{4\eta^2}{M^2 - \eta^2} \end{bmatrix} \quad (8)$$

4.6 State Boundary of Modified Cam Clay Model

The parametric equation for state boundary is given in Wood [14] as:

$$\begin{bmatrix} \nu \\ p \\ q \end{bmatrix} = \begin{bmatrix} N - \lambda \ln(p_0') + \ln\left(\frac{p_0'}{p}\right) \\ p \\ Mp\sqrt{\frac{p_0}{p} - 1} \end{bmatrix} \quad (9)$$

This equation defines a logarithmic line in ν-p-q space where the yield surface contracts or expands. Code for plotting this yield surface is provided. The state boundary of modified cam clay has a logarithmic shape in ν-p space. This is due to the consideration of actual consolidation in a sample due to applied load. Fig. 2 shows the state boundary of the Modified Cam Clay Model

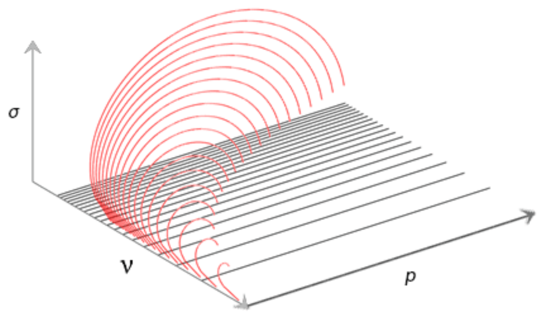

Fig. 2. State Boundary of Modified Cam Clay Model

5. Results and Discussions

The results for modified cam clay models are plotted in this section. The results of model output from modified cam clay can be seen in Figs. 3-5. These results are compared with those from PLAXIS 3D simulation in Figs. 4-5. The model lies agrees perfectly with simulation results from PLAXIS 3D for similar values of input parameters.

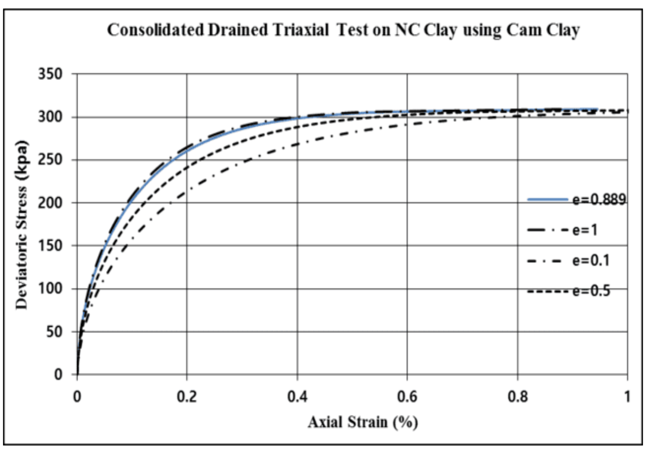

Fig. 3. Variation of $q-\varepsilon_1$ behavior with e

With variation of e from 1 to 0.1 it is inferred from Fig. 3 that the stiffness degradation is more subtle at lower values of void ratio, e. The physical explanation of this phenomenon is that, as the void ratio reduces, the p_c is large enough to fail the sample therefore, a smooth transition between linear and elastoplastic behavior is observed in Figs. 4-5. A comparison of q-ε_1 plot using our modified cam clay and PLAXIS 3D is shown in Fig. 4. The results are statistically the same for the same values of input parameters.

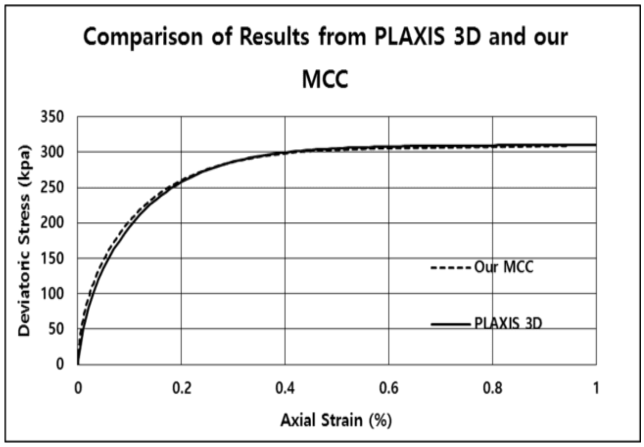

Fig. 4. Comparison of PLAXIS 3D MCC and MCC of this Study

The effect of drained and undrained loading conditions can be seen in Figs. 5-8. With drained loading, a significant effect of pore water pressure on the result is assured to be expected. This is confirmed by comparing Fig. 6 with Fig. 7, representing undrained deviator stress vs axial strain plot.

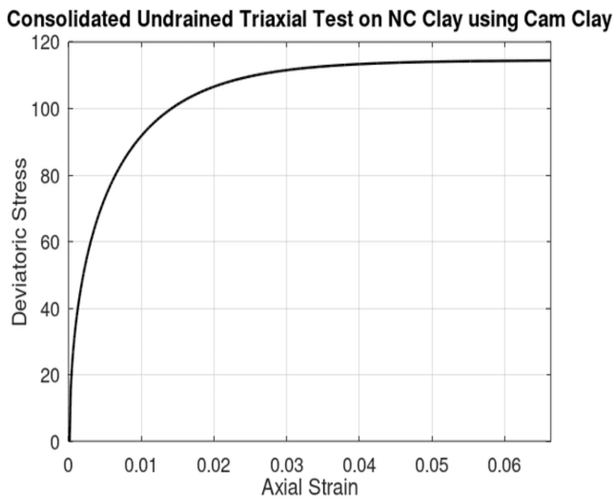

Fig. 5. Deviator Stress- Axial Strain behavior of NCC using Modified Cam Clay

Fig. 5-9 appear identical to the figures presented in Enyue (2017). These figures are as; Figs. 10-11. It is because modified cam clay was initially developed to comprehend the physics of such soils.

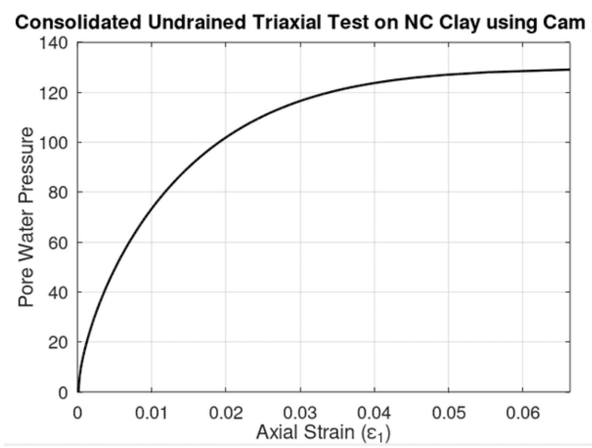

Fig. 6. Pore Water Pressure using Modified Cam Clay

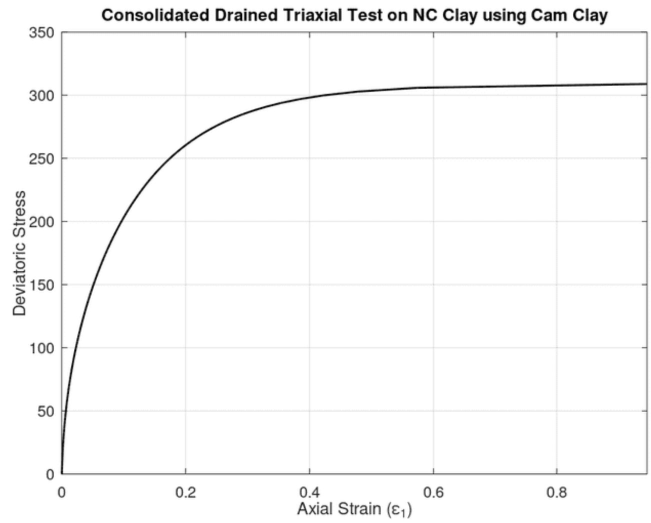

Fig. 7. Deviator Stress vs Axial Strain behavior of NCC using Modified Cam Clay

A separate comparison of PLAXIS 3D results with screenshots from the software is shown for brevity and validation of the aforementioned agenda. A plot of pore water pressure versus axial strain in consolidated undrained triaxial test simulation of normally consolidated clay in PLAXIS3D.

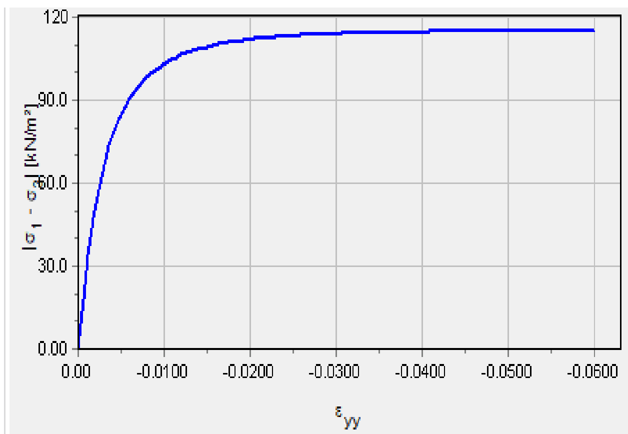

Fig. 8. Deviator Stress in CU Triaxial Test on NC using PLAXIS 3D

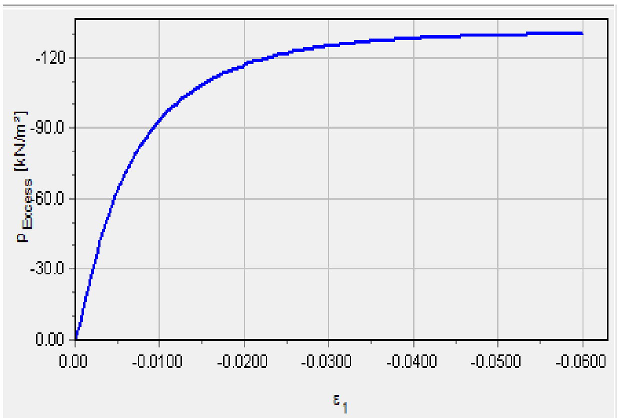

Fig. 9. Pore Water Pressure in CU Triaxial Test on NC Clay using PLAXIS 3D

5.1 Abaqus Plugin

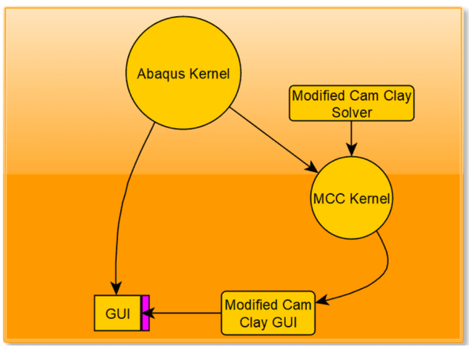

Fig 10. Interface of Abaqus Plugin

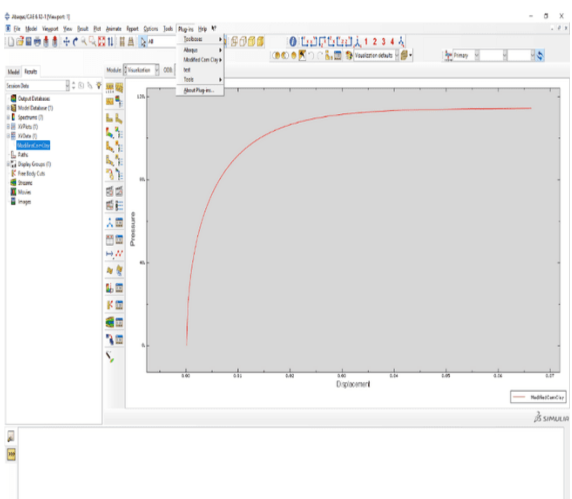

Fig. 11. Result of Abaqus Plugin Plotted in Visualization Workbench

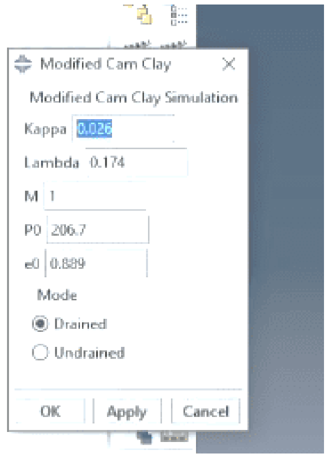

Fig. 12. Working of Abaqus Plugin

The solvers and increment update procedures in these programs are inspired by Borja [27] and Borja [28]. The rounding off if any is done as per the "Standard Practice for Using Significant Digits in Geotechnical Data" [29].

6. CONCLUSIONS

A modified version of the original Cam clay constitutive model is used to illustrate the drained and undrained behaviour of soils in this study. One of the most difficult problems in geotechnical engineering is the behaviour of soils under strain hardening. The constitutive equations are numerically integrated over fixed time steps to apply effective stress to the derived elastoplastic soil model. The study shows the application of a new version of a modified cam clay model to real-world problems. The modification provides a realistic transition from normally consolidated behavior to over-consolidated clays. The proposed elastoplastic stiffness matrix is consistent with the current implementation of a modified cam clay model in PLAXIS 3D for different values of the input parameters.

DECLARATIONS

Data availability statement:

Some or all data, models, or codes that support the findings of this study are available from the corresponding author upon reasonable request.

Authors Contribution

Muhammad Safdar proposed topic, Muhammad Waseem worked on basic study Plan. Literature review, manuscript writing, and coding were done by Syed Haseeb Shah and Hamza Ahmad Qureshi. Khalid Mahmood worked as quality insurer by removing SI and Template formatting.

Conflict of interest

The authors declare no conflict of interest.

Appendix

Code for programs

The updated version of the code presented in this section can be found on my GitHub repository [30].

Original Cam Clay Model

```
def yield_surface(p, M, p_c):
    return M*p*(-log(p/p_c))
def diff_eq(p, q, M):
    return -M + q/p

f= integrate.solve_bvp(lambda q,p: diff_eq(p, q, M), bc, p, y_a)

plt.plot(p, f.sol(p)[0], 'r-')
plt.plot(p, -f.sol(p)[0], 'r-')
```

Modified Cam Clay

The solvers and increment update procedures in these programs draw inspiration from Borja [27-28]. The rounding off if any is done as per "Standard Practice for Using Significant Digits in Geotechnical Data".

Python code form consolidated drained triaxial test on normally consolidated clays.

```
from numpy import *
import matplotlib.pyplot as plt

# Calculations for a Cons. Dr. Triaxial Test on NC Clays
# sigma_ 3
p_0 = 206.7 # kpa
dp = 1
dq = 3*dp

M = 1;
kappa = 0.026;
lmbda = 0.174;
e0 = 0.889;
e = e0;

pf = 3*p_0/(3-M);
qf = 3*M*p_0/(3-M);

p = append([0], arange(p_0,pf,dp))
q = append([0], arange(0,qf,dq))

eta = append([0], q[1:]/p[1:])
eta_n = roll(eta, 1);
eta_n[0] = 0;
deta = eta - eta_n;
dev = lmbda/(1+e) * (dp/p + (1-kappa/lmbda) * 2*eta *deta/(M**2 + eta**2));
dev[0] = 0;
evk = cumsum(dev);
de = (1+e)*dev;
e = append([e], zeros((0, len(de)-1)))
for i in range(1, len(e)):
  e[i] = e[i-1]-de[i];
des = (lmbda - kappa)/(1+e) * (dp/p + 2 * eta *deta/(M**2 + eta**2)) * (2*eta/(M**2 - eta**2));
```

```
des[0] = 0;
es = cumsum(des);
e1 = evk/3 + es;
plt.plot(e1, q);
plt.show()
```

Python code form consolidated undrained triaxial test on normally consolidated clays

```
import matplotlib.pyplot as plt
from numpy import *
# Calculation for a Cons. Und. Triaxial Test on NC Clays
N = 14;
p0 = 100;
dp=1;
dq = 3*dp;
e0 = 0.889;
nu = 0.3;
kappa = 0.026;
lmbda = 0.1740;
M = 1;
e_lmbda = e0 + (lmbda - kappa) * log(p0/2) + kappa * log(p0)
print(e_lmbda)
pf = exp((e_lmbda - e0)/lmbda);
p = arange(p0, pf-1, -dp)
pc = append(
    [p0],
    zeros((1, len(p)-1))
    )
for i in range(1,len(pc)):
    pc[i] = pc[i-1] * (p[i-1]/p[i])**(kappa/(lmbda-kappa));
q = M * p * sqrt(pc/p - 1);
eta = q/p;
deve = kappa/(1+e0) * - dp/p;
deve[0] = 0;
print(pc)
devp = -deve;
desp = devp * 2 *eta /(M**2 - eta**2);
G = 3 * (1-2*nu) * (1+e0) * p/(2*(1+nu)*kappa);
dese = dq/(3*G);
des = dese + desp;
es = cumsum(des);
e1 = es;
pu = p0+q/3;
du = pu-p;
plt.plot(e1, q)
plt.show()
```

Octave Programs for the same simulations

```
clc, clear
% Cons. Dr. Triaxial Test on NC Clays
% sigma_ 3
p_0 = 100; % kpa
dp = 1;
dq = 3*dp;
M = 1;
kappa = 0.026;
lmbda = 0.174;
e0 = 0.889;
e = e0;
pf = 3*p_0/(3-M);
qf = 3*M*p_0/(3-M);
p = [0 p_0:dp:pf];
q = [0 0:dq:qf];
eta = [0 q(2:end)./p(2:end)];
eta_n = circshift(eta, 1);
eta_n(1) = 0;
deta = eta - eta_n;
%[eta;eta_n;deta]'
dev = lambda/(1+e) * (dp./p + (1-kappa/lmbda) * 2*eta .*deta./(M.^2 + eta.^2));
dev(1) = 0;
evk = cumsum(dev);
de = (1+e)*dev;
e = [e zeros(1,length(de)-1)];
for i=2:length(e)
  e(i) = e(i-1)-de(i);
```

```
end
des = (lmbda - kappa)./(1+e) .* (dp./p + 2 .* eta .*deta./(M.^2 + eta.^2)) .* (2*eta./(M.^2 - eta.^2));
des(1) = 0;
es = cumsum(des);
e1 = evk/3 + es;
[p;q;eta;deta;dev;evk;de;e;des;es;e1]'
%plot(e1, q);
plot(e1,q, 'color','k','Linewidth',2);
xlim([0 max(e1)])
xlabel('Axial Strain (\epsilon_1)', 'Fontsize',18)
ylabel('Deviatoric Stress', 'Fontsize',18)
grid on;
title('Consolidated Drained Triaxial Test on NC Clay using Cam Clay', 'Fontsize',18)
set(gca,'FontSize',20)
%%%%% SECOND PROGRAM %%%%%%
clc, clear
% Cons. Dr. Triaxial Test on NC Clays
% sigma_ 3
p_0 = 100; % kpa
dp = 1;
dq = 3*dp;
M = 1;
kappa = 0.026;
lmbda = 0.174;
e0 = 0.889;
e = e0;
pf = 3*p_0/(3-M);
qf = 3*M*p_0/(3-M);
p = [0 p_0:dp:pf];
q = [0 0:dq:qf];
eta = [0 q(2:end)./p(2:end)];
eta_n = circshift(eta, 1);
eta_n(1) = 0;
deta = eta - eta_n;
%[eta;eta_n;deta]'
dev = lmbda/(1+e) * (dp./p + (1-kappa/lmbda) * 2*eta .*deta./(M.^2 + eta.^2));
dev(1) = 0;
evk = cumsum(dev);
de = (1+e)*dev;
e = [e zeros(1,length(de)-1)];
for i=2:length(e)
  e(i) = e(i-1)-de(i);
end
des = (lmbda - kappa)./(1+e) .* (dp./p + 2 .* eta .*deta./(M.^2 + eta.^2)) .* (2*eta./(M.^2 - eta.^2));
des(1) = 0;
es = cumsum(des);
e1 = evk/3 + es;
[p;q;eta;deta;dev;evk;de;e;des;es;e1]'
%plot(e1, q);
plot(e1,q, 'color','k','Linewidth',2);
xlim([0 max(e1)])
xlabel('Axial Strain (\epsilon_1)', 'Fontsize',18)
ylabel('Deviatoric Stress', 'Fontsize',18)
grid on;
title('Consolidated Drained Triaxial Test on NC Clay using Cam Clay', 'Fontsize',18)
set(gca,'FontSize',20)
.
```

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