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1 The Free-Linking Task: A graph-inspired method for generating non-disjoint similarity 2 data with food products

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7 8 **Keywords**

9 sensometrics, free sorting, free linking, DISTATIS, graph theory, network theory, rapid methods

10 11 **Abstract**

12 “Free sorting”, in which subjects are asked to sort a set of items into groups of “most similar”
13 items, is increasingly popular as a technique for profiling sets of foods. However, free sorting
14 implies an unrealistic model of sample similarity: that similarity is purely binary (is/is not
15 similar) and that similarity is fully transitive (similarities {A, B} and {B, C} imply {A, C}).
16 This paper proposes a new method of rapid similarity testing—the “free-linking” task—that
17 solves both problems: in free linking, subjects draw a *similarity graph* in which they connect
18 pairs of samples with a line if they are similar, according to the subject’s individual criteria. This
19 simple task provides a more realistic model of similarity which allows degrees of similarity
20 through the *graph distance* metric and does not require transitive similarity. In two pilot studies
21 with spice blends (10 samples, 58 subjects) and chocolate bars (10 samples, 63 subjects), free
22 linking and free sorting are evaluated and compared using DISTATIS, *RVb*, and the graph
23 parameters *degree*, *transitivity*, and *connectivity*; subjects also indicated their preferences and
24 ease-of-use for the tasks. In both studies, the first two dimensions of the DISTATIS consensus
25 were highly comparable across tasks; however, free linking provided more discrimination in
26 dimensions three and four. *RVb* stability was equivalent for the two methods. Graph statistics
27 indicated that free linking had greater discrimination power: on average subjects made similarity
28 groupings with lower degree, lower transitivity, and higher connectivity for free linking in both
29 studies. However, subjects did overall find free sorting easier and liked it more, indicating a
30 higher cognitive difficulty of free linking. The free-linking task, therefore, provides more robust,
31 realistic similarity maps at the cost of higher panelist effort, and should prove a valuable
32 alternative for rapid sensory assessment of product sets.

33 34 **1. Introduction**

35 Methods for rapidly identifying similarities and differences in sets of food products have become
36 increasingly popular in sensory evaluation (Delarue, 2015; Valentin et al., 2012; Varela & Ares,
37 2014). In particular, “free sorting”, in which subjects are asked to sort a set of items (in this
38 case, foods or beverages) into groups of “most similar” items is increasingly popular as a
39 technique for profiling sets of foods (e.g., Lahne et al., 2018). Free sorting presents several
40 advantages: it does not require that subjects be trained, it is sensitive and stable with relatively
41 low numbers of subjects (usually as low as 25 subjects), it can accommodate relatively high
42 numbers of samples (as many as 20), and it has been shown to give product “maps” or
43 “configurations” (through multivariate analyses) that bear a close resemblance to those from
44 traditional and more work-intensive methods like Descriptive Analysis. Furthermore, unlike
45 other rapid methods like Projective Mapping or Flash Profiling (Dehlholm et al., 2012), free

46 sorting only requires that subjects make simple, holistic decisions of similarity or difference,
47 rather than requiring a scaled degree of difference that may induce a higher cognitive load.
48

49 However, a key disadvantage of free sorting is that the task of sorting samples makes some
50 strong assumptions about the underlying similarities between the products that are being
51 modeled. Groups in free sorting are *disjoint*, meaning that no element can belong to two groups.
52 Given samples A, B, and C there is no way that the same subject can create two similarity sets
53 such as {A, B} and {B, C} without creating a superset {A, B, C}. This simplifies the task for the
54 subjects and reduces the time and amount of samples required (because retasting is minimized),
55 but this restriction has two potentially undesirable consequences. The first is that the same
56 subject cannot represent different *types* or *dimensions* of similarity in the same sort: it is easily
57 conceivable that A and B are similar in terms of one attribute, say, “sweetness”, while A and
58 C are similar in terms of another, say “appearance”. It is quite easy to imagine real-world
59 situations in which this occurs. The second consequence is that similarity is necessarily modeled
60 as fully transitive: if A is similar to B, and B is similar to C, then A must be similar to C, and
61 furthermore the data can only indicate that all three samples *are equally similar*. This is also
62 clearly contrary to easily imagined real circumstances: perhaps A, B, and C are all “sweet”, but
63 while A and B are equally sweet, C is only half as sweet. Should a single subject be required to
64 group these together?
65

66 Two closely related alternatives have been suggested for the simple free-sorting task that address
67 these issues: free *multiple*-sorts (Blanchard & Banerji, 2016; Dehlholm, 2015; Dehlholm et al.,
68 2012) and *hierarchical* free-sorts (Koenig et al., 2020, 2021). The former modification asks
69 subjects, after they have completed a simple free-sorting task, to repeat the task until they feel
70 they have exhausted all possible grouping configurations (Dehlholm, 2015); the latter asks
71 subjects, once they completed a simple free-sorting task, to continue making groups *of groups*
72 until they cannot proceed further (Koenig et al., 2021). Thus, free multiple-sorting solves the
73 first problem highlighted above, and hierarchical free-sorting solves the second problem.
74 However, neither approach solves *both* problems, and they both introduce problems of panelist
75 motivation, in that they require a much more extensive data-collection procedure that will be
76 discouraging for some subjects. This is a more major problem when a large number of samples
77 is used, as in Koenig et al. (2020), but difficulty and motivation problems are reported with as
78 few as 18 complex samples sorted by taste (Kessinger et al., 2020). In addition, the data
79 collection for both methods is much more complicated and more poorly supported in practical
80 data-management programs (based on the authors’ personal communications with major sensory
81 and survey software providers in pursuit of these methods), which appears to have limited the
82 adoption of either approach in academia and industry in favor of the simple free-sorting task.
83 For example, Spencer et al. (2016) had to write custom software to support hierarchical free-
84 sorting, and authors as recent as Koenig et al. (2020, 2021) have used paper ballots because of
85 the lack of software supporting hierarchical free-sorting, requiring extensive transcription of
86 results.
87

88 Therefore, in this manuscript we propose an alternative task to the free-sorting task, inspired by
89 graph theory (Gross et al., 2014), which we term the “free-linking” task. In the free-linking task,
90 subjects are given a set of samples just as in free sorting, but rather than forming disjoint groups,
91 subjects are asked to indicate, for each pair of samples, whether the samples are similar. This

92 connect-the-dots interface was implemented in the SensoGraph system (Orden et al., 2019,
93 Alcalá, ES) in order to support this task, in which subjects are asked to draw “links” between
94 samples if they are similar (Figure 1). However, a paper-based system for free linking would be
95 no harder to implement than a paper-based simple free-sorting task.

96
97 FIGURE 1 GOES HERE
98

99 While the free-linking task solicits binary similarity data on a pair-wise basis for samples—two
100 samples are either similar or they are not—it does not impose the disjoint, restrictive model of
101 similarity implied by free sorting. Given 3 samples A, B, and C it is possible for a subject to
102 indicate, pairwise, that there are similar pairs {A, B} and {B, C} without indicating that A and C
103 are directly similar. Put another way, the free-linking task asks each subject to draw their own
104 similarity graph for the samples (Lahne, 2020; Orden et al., 2019, 2021). Unlike previous graph-
105 based approaches to similarity in food products, where just the presence or absence of a
106 connection was considered, in free linking we make use of the *graph distance* between samples
107 as a basis for a dissimilarity matrix for further analysis (Chartrand & Zhang, 2014). In the
108 example above, $distance(A, B) = distance(B, C) = 1$, while $distance(A, C) = 2$. This allows the
109 analyst to *infer* from a single subject’s data that, for the example above, there might be some
110 shared similarity between A and C without the link {A, C} actually being drawn. This same
111 change also addresses the second problem with simple free-sorting: subjects can now indicate
112 pairwise whether samples are similar, but because there are not larger similarity groups (e.g., {A,
113 B, C} in free sorting) it is not required that all samples that are connected be similar *in the same*
114 *way*. This allows more flexibility for a subject’s holistic similarity judgments (Figure 2; see also
115 Figure 3 for details on the dissimilarity).

116
117 FIGURE 2 GOES HERE
118

119 The free-linking task can be analyzed by the same tools that exist for the free-sorting task:
120 dimensionality reduction (through MDS, DISTATIS, and other approaches) and graph-based
121 approaches like Sorting Backbone Analysis. This allows analysts used to free sorting to easily
122 employ free linking, and for direct comparison of results.

123
124 Therefore, it is reasonable to hope that the free-linking task will provide results that are
125 comparable to free-sorting in terms of ease of deployment and data collection, but might allow
126 for more realistic and detailed results. In particular, the lack of forced memberships to a group
127 should allow for easier distinction among similar but not identical samples—that is, a more
128 multidimensional structure of similarity and difference. In order to investigate the utility of the
129 free-linking task, we report the results of two pilot studies in which subjects used both free
130 sorting and free linking to report their perceptions of different food products. In both pilot
131 studies subjects completed both free-sorting and free-linking tasks for the same samples in a
132 counterbalanced order. In the first study, subjects evaluated 10 blends of 4 dried spices
133 (cinnamon, turmeric, pepper, and cardamom) for similarity by aroma. In the second study,
134 subjects evaluated 10 commercial chocolate samples for similarity by taste. We hypothesized
135 that the overall similarity configuration should be similar between the two methods, and that the
136 results of the two methods should be equally stable, but that the free-linking results would
137 provide more realistic, multidimensional models of similarity, which should be evident in

138 parameters for the graphs derived from the similarity measurements as well as in visualizations
139 from DISTATIS.

140

141 **2. Materials and Methods**

142 The two studies reported were very similar in most details besides sample type, and so the basic
143 information distinguishing the studies is given below, followed by details on methodology and
144 analysis that were the same for both studies.

145

146 *2.1. Study 1–Spice sorting*

147 Study 1 was conducted in November and December of 2019, and used spices and spice blends as
148 stimuli. Sample details are given in Table 1. All spices were purchased at Kroger (Blacksburg,
149 VA, see Table 1). Samples were presented to subjects in foil-wrapped glass vials in order to
150 avoid visual discrimination, and evaluation was entirely orthonasal.

151

152 A total of N = 58 subjects (38 female, 20 male, average age 29 years old) participated in Study 1.
153 Subjects were recruited from the Virginia Tech/Blacksburg community. Subjects were not
154 trained sensory panelists (e.g., for Descriptive Analysis), but some had participated in previous
155 untrained sensory tests at Virginia Tech. Subjects received no compensation, but were given
156 snacks after completing Study 1.

157

158 *2.2. Study 2–Chocolate sorting*

159 Study 2 was conducted in November of 2020, and used commercial chocolate bars as stimuli.
160 Sample details are given in Table 1. All chocolate bars were purchased at Kroger (Blacksburg,
161 VA, see Table 1). Samples were presented in souffle cups with the bars' identifying details (e.g.,
162 logos) effaced, in natural light, and evaluation was by taste and retronasal flavor.

163

164 A total of N = 63 subjects (49 female, 14 male, average age 34 years old) participated in Study 2.
165 Subjects were recruited from the Virginia Tech/Blacksburg community. Subjects were not
166 trained sensory panelists (e.g., for Descriptive Analysis), but some had participated in previous
167 untrained sensory tests at Virginia Tech, including some who had participated in Study 1.
168 Subjects received no compensation, but were given snacks after completing Study 2.

169

170

TABLE 1 GOES HERE

171

172 *2.3. Overall study design*

173 Both studies used the same overall design. Subjects were recruited to participate in free-linking
174 and free-sorting of the same samples. In order to obtain within-subjects data, subjects were
175 randomly assigned one of the two tasks first, then took a short break, then completed the other of
176 the two tasks, then completed a short survey that asked them about their perceptions of the tasks
177 and some basic demographic details. In both free-sorting and free-linking studies, subjects were
178 seated at tables with a 36" x 36" workspace available and allowed to organize their samples
179 spatially prior to entering their judgments into the data-collection software.

180

181 *2.4. Free-sorting task*

182 In the free-sorting task, subjects received all 10 samples at the same time in a randomized order.
183 Sorting data was collected using the Compusense Cloud (Guelph, ON) system. Subjects were

184 prompted to “sort into groups based on similarities”. They were informed that there was no right
185 answer, and told that they could make any number of groups between two (2) and nine (9), with
186 as many samples as they chose in each group.

187 188 2.5. Free-linking task

189 In the free-linking task, subjects received all 10 samples at the same time in a randomized order,
190 positioned as the vertices of a regular polygon (see Figures 1 and 2). Linking data was collected
191 using the SensoGraph (Orden et al., 2019, Alcalá, ES) system. Subjects were prompted to “join
192 with a line those pairs of products you consider similar, dragging from one to the other with the
193 finger or the mouse” (see Figure 1). The codes presented on the screen for the SensoGraph
194 interface were given in random order for each subject. Subjects were able to remove lines they
195 had previously made (in case of mistakes or revisions in judgment) before submitting their
196 answers.

197 198 2.6. Data Analysis

199 Results from both free sorting and free linking were analyzed in parallel in order to compare the
200 results of the method. This parallelism is enabled by the data structure provided by both
201 methods: the dataset for each analysis is an $N \times K \times K$ array of (dis)similarity matrices, where N
202 is the number of subjects and K is the number of samples. In free sorting, each $K \times K$ slice is
203 composed by cell entries a_{ij} which are binary (either 0 or 1), representing whether, for the
204 current subject, samples i, j were sorted together. The raw data is a *similarity* measure in which a
205 1 indicates similarity through group membership, and the dissimilarity matrix, which is obtained
206 by subtracting every entry from 1, can be treated as binary distance and is analyzed via MDS or
207 DISTATIS (Abdi et al., 2007). In free linking, the graph drawn by the current subject provides a
208 graph distance between each pair of samples i and j , as an integer between 1 (if the connection
209 $\{i, j\}$ is present) and ∞ (if there is no path between i and j on the graph). The raw graph distance
210 is the number of edges comprising the shortest path between the two pairs of samples in the
211 graph (see Figures 2 and 3). For the dissimilarity matrix actually analyzed by DISTATIS we
212 adapt the cophenetic dissimilarity from Koenig et al. (2021, see Figures 2 and 3): the
213 corresponding cell entry a_{ij} of the $K \times K$ slice is defined as the subtraction from 1 of the inverse
214 of the graph distance between i and j (defining $1/\infty$ as 0, and setting a minimum of 0 for
215 dissimilarity of a sample with itself or with samples to which it is directly linked), so that the cell
216 entries a_{ij} are no longer binary but range in the interval $[0, 1]$, with larger values indicating
217 lower similarity and smaller values standing for higher similarity. The diagonal of the matrix is
218 set to 0, indicating that all samples are identical with themselves as would be expected for a
219 distance matrix.

220
221 FIGURE 3 GOES HERE

222
223 Data were first analyzed by DISTATIS in order to compare consensus similarity configurations
224 for samples across methods (Abdi et al., 2007). Confidence ellipses were generated through
225 bootstrapping (Beaton et al., 2013). A key property of any rapid sensory method is how well
226 samples and groups of samples are distinguished: this is clearly related to (but also not identical
227 to) discrimination ability for the method. Examination of product separation on the first four
228 DISTATIS axes for both methods via actual observations as well as bootstrapped confidence
229 intervals were considered as evidence. Choice of 4 axes for examination (out of a possible 10 for

230 each sample) were motivated by examination of scree plots for the DISTATIS \mathbf{S}_+ matrices (Abdi
231 et al., 2007, not shown) as well as by general practice in industry and the literature for
232 “significant dimensions” for interpretation.

233
234 The stability of results for a given number of subjects—that is, the required number of subjects—
235 for each method was evaluated through a bootstrapping approach to simulate panels of different
236 sizes and compare these simulated results to the actual, observed results. Specifically,
237 generalized stability, termed RVb by Blancher et al. (2012), was calculated for free sorting and
238 free linking: bootstrapped samples of subjects, of sizes 2 to N (where N is the number of subjects
239 in the particular study) were drawn (with $i = 100$ replicates at each sample size), and the average
240 RV between the DISTATIS \mathbf{F} (factor score) matrices from the bootstrap sample and the full
241 dataset was calculated at each sample size. Blancher et al. (2012) recommend that stability can
242 be considered achieved at the number of subjects for which the bootstrapped average RVb
243 exceeds 0.95.

244
245 Graph theory was also used to evaluate whether individual subjects’ free-sorting and free-linking
246 groupings were in fact different. For sorting, each individual’s $K \times K$ slice was treated as the
247 (symmetric) adjacency-matrix representation of an undirected graph (Gross et al., 2014). For
248 linking, the undirected graph drawn by each individual was used. In each subject’s graph, the
249 nodes represent the samples, and an edge between two nodes indicates that the subject sorted or
250 linked two samples as similar (Lahne, 2020; Orden et al., 2019). This graph representation
251 provides several simple parameters that give insight into the similarity structure.

252
253 The *degree* of each node indicates how many edges are incident to it (Gross et al., 2014); thus, in
254 sorting or linking higher degree for a node means the corresponding sample was considered
255 similar to more other samples. Comparison of average degree per subject and sample for each
256 method gives an indication of discrimination capacity: higher average degree indicates less
257 discrimination between samples, as subjects consider more samples similar.

258
259 The *transitivity on triads* (Arney & Horton, 2014) is the fraction indicating, for the total number
260 of node triads A, B, C with connections $\{A, B\}$, $\{B, C\}$, how many of them also contain the
261 connection $\{A, C\}$. In the literature this is also called the graph “clustering coefficient”
262 (Kolaczyk & Csárdi, 2014). In terms of the sorting and linking tasks, this is a measure of the
263 likelihood that similarities $\{A, B\}$ and $\{B, C\}$ imply that similarity $\{A, C\}$ also exists; when
264 transitivity is higher it may indicate a lower discrimination capability.

265
266 The *average connectivity* of a graph (Beineke et al., 2002) is a parameter that measures, in each
267 subject’s results, the average over all pairs of nodes A and B , how many independent paths
268 connect A and B . In the context of sorting and linking, lower average connectivity will be
269 associated with more disjoint groups, which is an indicator of less robust or realistic models of
270 similarity.

271
272 Subjects’ preferences for method were evaluated for each study using simple contingency-table
273 measures, and their opinions of the sorting and linking tasks’ ease of use and enjoyability were
274 evaluated using repeated measures ANOVA.

275

276 Data analyses were conducted in R (version 4.0.2). Code for analyses is available from the
277 corresponding author upon request.

278
279 *2.7. Ethics statement*

280 All research methods were reviewed and approved by the Virginia Tech Human Research
281 Protection Program (IRB # 19-1030).

282
283 **3. Results**

284 *3.1. Product configurations (via DISTATIS)*

285 The overall DISTATIS results for both the spice samples (Study 1) and the chocolate samples
286 (Study 2) are quite similar (Figures 4 and 5). In the first 2 dimensions of the DISTATIS
287 solutions the configurations of samples are almost identical, although it is worth noting that the
288 derived distances among samples in the chocolate study are larger (Figure 5). However, for both
289 studies it is apparent that the 3rd and 4th dimensions of the solution contain more valuable
290 discrimination information for free linking than for free sorting. In each case, more samples are
291 clearly discriminated (as can be seen from non-overlapping confidence ellipses) by subjects
292 using free linking than by subjects using free sorting.

293
294 FIGURE 4 GOES HERE

295
296 The same basic product differences are identified by both methods, but with better resolution
297 through free linking. For the spices, the first dimension separates cinnamon-containing mixes
298 from the rest of the samples, while the second dimension separates cardamom-containing mixes
299 (in both analyses the cinnamon+cardamom mixture falls in between these groups, with a stronger
300 attraction to the cardamom region on the second axis). The third dimension for both studies
301 separates pepper from the remaining samples, but with free linking it is also possible to infer that
302 pepper is being directly opposed to turmeric-containing samples (Figure 4). In the fourth
303 dimension, two samples that both contain turmeric are opposed: cardamom+turmeric and
304 cinnamon+cardamom, but again in the free-linking study several other samples
305 (cinnamon+pepper, cardamom) separate clearly on this dimension).

306
307 For the chocolate, the first dimension distinctly separates premium, dark chocolates from milk
308 chocolates, while the second axis separates mass-market dark chocolates (Hershey's and
309 Cadbury's) from the other samples. In the third dimension, the sole premium, milk chocolate
310 (Endangered Species) is separated from the remaining samples, but only in the free-linking study
311 is it clear that this dimension is capturing similarities between both chocolates from this producer
312 (Figure 5). Finally, the fourth dimension separates the dark chocolate from Endangered Species
313 from the remaining chocolates, but, again, in the free-linking study it is clear that there is more
314 separation on this axis, with a strong separation between the two dark chocolates from Green &
315 Black on this axis as well as separation among the other samples.

316
317 FIGURE 5 GOES HERE

318
319 *3.2. Stability (via RVb)*

320 In order to investigate stability of the solutions as a function of the number of panelists, *RVb* was
321 calculated as described in Blancher et al. (2012). Figure 6 shows the *RVb* results for free sorting

322 and free linking. As is apparent, the desired level of stability (the 0.95 level) is achieved with
323 essentially the same number of subjects for both sorting and linking—although an average of
324 about 1 subject less is required for stability in free sorting than in free linking. Given that this
325 level of stability is achieved at between 8-10 subjects in these studies, this difference of a single
326 subject is unlikely to be important in practical applications. In contrast to Blancher et al. (2012),
327 we used all 10 dimensions to calculate RVb , but results for *only* Dimensions 1 and 2 (as
328 calculated in Blancher et al. 2012) were almost identical to the full factor bootstraps (results not
329 shown).

330
331 This is a quite low number of subjects when compared to those calculated by Blancher et al.
332 (2012)—it corresponds most closely to the results in that study for a similar dataset of chocolate
333 aromas (DS1, a free sort of 11 samples). While Blancher et al. (2012) do not give details on
334 sample-inclusion criteria, in the case of both Study 1 and Study 2 samples were chosen
335 specifically for their potential to be grouped by subjects (i.e., blends of the same spices and
336 chocolates from the same manufacturers, see Table 1), which may explain the high stability
337 observed here. It is also noticeable that the number of subjects required is slightly lower in
338 Study 2 (chocolate, solid line) than in Study 1 (spice, dashed line). This difference seems like it
339 may be attributed to the difference in modality—taste and flavor for Study 2, and only aroma for
340 Study 1; differences in the products themselves may also be in play. This difference is also
341 evident in the relative size and overlap of confidence ellipses for DISTATIS results (in which the
342 RV coefficient is a key statistic) seen in Figures 4 and 5. However, there is no evident difference
343 in the RVb patterns between sample type, modality, and methodology (sorting vs. linking). The
344 apparent stability of each method is equivalent.

345
346 FIGURE 6 GOES HERE

347
348 *3.3. Graph parameters*
349 Three key graph parameters were investigated for this study. In a graph, the degree of a node
350 represents the number of incident edges; for the sorting and linking studies, for each subject the
351 degree of each sample indicates the number of other samples to which it was judged similar.
352 Higher degree thus indicates a potentially lower discrimination ability among subjects, as fewer
353 distinctions are made. For both Study 1 and Study 2, the degree distribution for free linking is
354 clearly skewed more right than the degree distribution for free sorting (see Figure 7). Wilcoxon
355 rank-sum tests indicate that the free-sorting task produces significantly larger degrees per node
356 than the free-linking task for both the spice ($W = 143331, p < 0.05$) and the chocolate ($W =$
357 $167328, p < 0.05$) studies. This indicates that free linking better discriminates the samples than
358 free linking.

359
360 FIGURE 7 GOES HERE

361
362 The *transitivity* on triads of a graph indicates the likelihood, given three nodes A, B, and C and
363 edges $\{A, B\}$ and $\{B, C\}$, that there will also be an edge $\{A, C\}$. In terms of free sorting and
364 free linking, transitivity gives another indication of discrimination ability—it is a direct
365 measurement of the degree to which similarities among samples are forced by the method or are
366 allowed to be indicated by the subjects, and ranges from 0 to 1. In Figure 8, transitivity is plotted
367 on the Y-axis against degree (see above) on the X-axis. By the nature of the sorting task,

368 transitivity is always 0 or 1; it is only 0 in the degenerate case, when subjects made only pairs of
369 samples, which happened several times in the spice study. For free sorting there is a much
370 broader range of transitivity values in the [0, 1] range, indicating a higher likelihood of actual
371 discrimination by the subjects.

372
373 FIGURE 8 GOES HERE
374

375 Finally, the *connectivity* of a graph is a measure, for each subject, of the number of distinct,
376 connected paths between all pairs of nodes. Higher connectivity indicates a less disjoint (or
377 disconnected) graph; in terms of free sorting and free linking, lower connectivity would mean
378 more disjoint graphs, which are likely the result of a less realistic similarity model. In Figure 9,
379 connectivity is plotted on the Y-axis against degree on the X-axis. For both studies, free linking
380 tended to exhibit higher connectivity values than free sorting, as expected, but the differences
381 were in general rather smaller than the differences in connectivity or degree. Thus, while
382 subjects did produce more connected graphs using free linking than free sorting, they did not
383 always produce fully connected graphs.

384
385 FIGURE 9 GOES HERE
386

387 3.4. Subject preferences

388 Finally, it is important to consider subjects' experience of the two tasks. In a simple question of
389 overall preference ("Did you prefer the free-sorting or free-linking task?"), panelists preferred
390 free-sorting to free-linking narrowly but insignificantly in Study 1 ($\chi_1^2 = 1.10, ns$), and by a
391 broad and significant margin in Study 2 ($\chi_1^2 = 19.44, p < 0.05$; see Table 2). In neither task did
392 it matter which task the subjects completed first (Study 1: $\chi_1^2 = 0.69, ns$; Study 2: $\chi_1^2 =$
393 1.49, *ns*). This can potentially be explained by the difference in complexity of the relative tasks:
394 in Study 1, the test was by aroma only, whereas in Study 2 the subjects had to taste the chocolate.
395 Therefore, it is possible that Study 2 involved a more fatiguing sensory task and a more taxing
396 memory task, and in these circumstances it would make sense that subjects would prefer the
397 simpler free-sorting task, which involves fewer pairwise comparisons. Alternatively, it is
398 possible that the difference may be that the set of samples evaluated in Study 1 was "designed"
399 by blending spices, providing an "easier" similarity structure.

400
401 Subjects also answered questions about ease-of-use and rated liking for each task, both on
402 unstructured line scales converted to 10-pt values. Results were analyzed by mixed-effects
403 ANOVA, with the dependent variable (liking or ease) modeled as dependent on the random
404 effect of the particular subject, with the task (free sorting or free linking) as a within-subjects
405 variable and the order of task completion as a between-subjects variable. For all tests, there was
406 no effect of order of task, and no interaction between order and the task itself, so these results
407 will not be reported in detail. For Study 1, subjects indicated that they did not find any
408 difference in ease-of-use for the two tasks (effect of task on ease-of-use: $F_{1,56} = 0.016, ns$; free
409 sorting $M = 7.62, SD = 1.78$, free linking $M = 7.14, SD = 2.08$), but they did report a
410 significantly higher liking for the free-sorting task (effect of task on liking: $F_{1,56} = 5.14, p <$
411 0.05; free sorting $M = 7.57, SD = 1.70$, free linking $M = 6.85, SD = 2.08$). For Study 2,
412 subjects indicated significant differences in both ease-of-use (effect of task on ease-of-use:
413 $F_{1,61} = 24.76, p < 0.05$; free sorting $M = 8.36, SD = 1.48$, free linking $M = 6.96, SD = 2.38$)

414 and liking (effect of task on liking: $F_{1,61} = 19.40, p < 0.05$; free sorting $M = 7.54, SD = 1.61$,
415 free linking $M = 6.11, SD = 1.92$). These results can be explained in the same way as the
416 preference results: possibly a significantly higher memory and sensory-fatigue loads for tasting
417 would make free linking a more difficult and less pleasant task than free sorting, or possibly the
418 set of samples evaluated in Study 1 was slightly “easier” than the chocolates in Study 2. In both
419 cases, it is also possible that subjects are simply more familiar with free sorting than with free
420 linking, and familiarity has bred comfort with and preference for that method: while subjects
421 were not surveyed about previous experience, our lab frequently conducts free-sorting studies
422 and some subjects were definitely previous participants.

423

424 **4. Discussion**

425 Free sorting, as a rapid method for assessing similarities among a set of samples, has become an
426 extremely popular method in both industry and academia (Dehlholm, 2015; Koenig et al., 2020,
427 2021; Valentin et al., 2012). However, the basic instruction of free sorting—that subjects form
428 disjoint groups according to similarity—implies a model of similarity among the products that is
429 likely to be unrealistic. Specifically, sorting requires that similarities be fully transitive and
430 essentially unidimensional. In contrast, the method of pairwise free-linking, which we have
431 formalized and demonstrated in this paper, provides results that are comparable to free sorting,
432 while avoiding these restrictive assumptions.

433

434 In particular, on the same product sets, free linking results in significantly lower vertex degree
435 measurements for each product, indicating that subjects are making more discriminating
436 similarity judgments (Figure 7). In addition, the transitivity (or “clustering coefficient”
437 Kolaczyk & Csárdi, 2014) of the similarity graphs from free linking were significantly more
438 diverse than those from sorting, which are in general fully transitive (Figure 8); this explicitly
439 indicates that subjects in free-linking studies are not forced to “close the triangle” when they
440 want to indicate that A and B are similar, as are B and C. At the same time, the connectivity of
441 the free-linking graphs was also noticeably higher than that of the free-sorting graphs (Figure 9),
442 indicating that individual models of similarity generated through free linking were more robust,
443 with graph distance giving a non-binary similarity measure (Chartrand & Zhang, 2014), which
444 should capture a more multidimensional model of similarity.

445

446 This more “multidimensional” similarity is evident in DISTATIS biplots of results of free sorting
447 and free linking on the same samples. Although for both spices (Figure 4) and chocolate (Figure
448 5) gross similarities, represented by Dimensions 1 and 2 of the biplots, are almost identical, there
449 is much better discrimination of samples in Dimensions 3 and 4 for both sample sets. This
450 follows naturally from the two different models of similarity implied by free sorting and free
451 linking. Free sorting emphasizes rapidly finding gross similarities; free linking, while more
452 intensive because of the need for multiple pairwise judgments (Figure 1), focuses on
453 multidimensional similarity. Nevertheless, both methods provide stable results, as indicated
454 by RVB , at approximately similar numbers of subjects (Figure 6). However, it is important to
455 note that, on the whole, subjects found free sorting less taxing and more pleasant than free
456 linking. It will be important to take subject fatigue into account when designing future studies
457 that employ free linking. We might imagine that free linking would also be less fatiguing for
458 trained subjects, who are used to making frequent, analytical, sensory judgments.

459

460 4.1. Limitations and future work

461 A key limitation of this study was the artificial nature of the sample sets: for both the spices and
462 the chocolates, the samples were chosen to span a product category. In a real product-
463 development or other applied situation, it is unlikely that there would be such a structured set of
464 products. Arguably, free linking, which relies on pairwise comparisons, should perform better in
465 these real situations, but this could not be determined from these sample sets. It also remains to
466 be seen whether the lower preference and liking ratings for free linking by subjects will result in
467 lower compliance or lower quality data when the method is used in a non-comparative setting.

468
469 The free-linking task also provides some new possibilities for the design of sensory studies. For
470 example, to this point it has not been feasible to conduct free-sorting tests (or indeed projective-
471 mapping tests) in an incomplete-block design, because the sorting space depends simultaneously
472 on all samples. This has restricted the number of samples that can practically be analyzed in a
473 free-sorting study to around 25 actual samples (the number is much higher for visual or text
474 samples). This restriction should not apply to the free-linking task, which is based on a
475 similarity graph of *pairwise* comparisons, but provides results that are similar or arguably
476 superior to free sorting. Therefore, a logical future study is the investigation by free-linking of
477 similarities in a set of samples large enough to present with an incomplete block design, but
478 small enough to also investigate in full with free sorting in order to determine the comparability
479 of this approach. Incomplete blocks for similarity would be a significant boon to food-sensory
480 researchers in both industry and academia. In addition, given that free sorting appears to become
481 exponentially more fatiguing as the number and sensory complexity of samples increases (see for
482 example Kessinger et al., 2020), it may be hoped that free linking, which requires a larger
483 number of simpler judgments, may perform better with large sample sets, especially when
484 implemented in incomplete blocks as described above.

485 4.2. Conclusions

486 In this paper, we present a new, rapid method for assessing similarities among a set of samples:
487 the “free-linking task”. In the free-linking task, subjects are given a set of samples and asked to
488 indicate pairwise similarity according to their own criteria; in effect, as we have demonstrated,
489 subjects are drawing their own individual similarity graph for the samples. The data from free
490 linking can be treated using existing tools for analyzing similarity data, such as DISTATIS,
491 MFA, or even MDS.

492
493
494 The free-linking task explicitly solves two issues with the currently popular free-sorting task: in
495 free sorting, subjects can only indicate one degree of similarity (is/is not similar) and are forced
496 to make fully transitive similarity groups. While previously proposed modifications of sorting
497 like the *hierarchical* and *multiple* free-sorting tasks can solve these respective tasks with
498 replicated or multiple passes of sorting for each sorting, free linking solves both problems at
499 once with only a single task. As we have demonstrated, therefore, the results of free linking
500 provide a more realistic representation of similarity and allow finer and more powerful
501 interpretations than free sorting. However, while the results of free linking are more realistic and
502 robust, the cost is that free linking, because it involves more pairwise comparisons, is also more
503 demanding for the participants. The multidimensionality of free-linking data is also greater, which
504 can be considered either a cost or a benefit, depending on the sensory analyst’s goals. Therefore,
505 we believe that the free-linking task will be a significant addition to the sensory analyst’s arsenal

506 of tools for rapidly assessing similarities, and we expect to see improvements and new uses cases
507 for the tool in the near future.

508

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515 coordinate data collection.

516

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581
582
583

Table 1. Sample information for Study 1 and Study 2.

<i>Study 1 – Spices*</i>		
Sample Name	Recipe	
<i>Cinnamon</i>	<i>1 g ground cinnamon</i>	
<i>Cardamom</i>	<i>1 g ground cardamom</i>	
<i>Pepper</i>	<i>1 g ground black pepper</i>	
<i>Turmeric</i>	<i>1 g ground turmeric</i>	
<i>Cinnamon + cardamom</i>	<i>0.5 g ground cinnamon + 0.5 g ground cardamom</i>	
<i>Cinnamon + pepper</i>	<i>0.5 g ground cinnamon + 0.5 g ground black pepper</i>	
<i>Cinnamon + turmeric</i>	<i>0.5 g ground cinnamon + 0.5 g ground turmeric</i>	
<i>Cardamom + pepper</i>	<i>0.5 g ground cardamom + 0.5 g ground black pepper</i>	
<i>Cardamom + turmeric</i>	<i>0.5 g ground cardamom + 0.5 g ground turmeric</i>	
<i>Pepper + turmeric</i>	<i>0.5 g ground black pepper + 0.5 g ground turmeric</i>	
<i>Study 2 - Chocolate</i>		
Manufacturer	Chocolate type	Cocoa content
<i>Cadbury</i>	<i>Dark</i>	<i>35%[?]</i>
<i>Hershey's</i>	<i>Dark</i>	<i>45%[?]</i>
<i>Green & Black's</i>	<i>Dark</i>	<i>70%</i>
<i>Endangered Species</i>	<i>Dark</i>	<i>72%</i>
<i>Green & Black's</i>	<i>Dark</i>	<i>85%</i>
<i>Pascha</i>	<i>Dark</i>	<i>85%</i>
<i>Cadbury</i>	<i>Milk</i>	<i>26%[?]</i>
<i>Hershey's</i>	<i>Milk</i>	<i>30%[?]</i>
<i>Green & Black's</i>	<i>Milk</i>	<i>34%</i>
<i>Endangered Species</i>	<i>Milk</i>	<i>48%</i>

*All spices are McCormick Gourmet Organic line ground spices (no whole spices were used for the purpose of blending the recipes).

[?]information gathered indirectly from manufacturer's website rather than packaging.

Table 2. Counts of preference for free-sorting or free-linking task for each study, counted by which test was completed first.

Task Completed First	Prefer Free-Sorting	Prefer Free-Linking
<i>Study 1: Spices (by smell)</i>		
<i>Free-Sorting</i>	15	15
<i>Free-Linking</i>	10	18
<i>Study 2: Chocolate (by taste)</i>		
<i>Free-Sorting</i>	24	10
<i>Free-Linking</i>	25	4

589 **Figures**

590

591 **Figure 1.** Interface for individual subjects' free-linking task, as rendered in SensoGraph (Orden
592 et al., 2019). Note that sample order is randomized between subjects.

593

594 **Figure 2.** Schematic representation of free sorting (top) and free linking (bottom). From the
595 same samples (presented in random order to each subject) in (1), the methods diverge. For free
596 sorting, subjects group samples (2) and their groupings are transformed directly to binary
597 dissimilarities (3). For free linking, subjects indicate pairwise similarity (2), which is
598 transformed into graph distances (3), and then to [0,1]-range dissimilarity (4, with details given
599 in Figure 3). At this point, the same analyses can be conducted on the each of the dissimilarity
600 matrices.

601

602 **Figure 3.** Schematic for deriving dissimilarity from graph distance, based on Koenig et al.
603 (2021).

604

605 **Figure 4.** DISTATIS biplots for free sorting (top, in purple) and free linking (bottom, in orange)
606 of spice-study results. The left-hand column gives Dimensions 1 and 2, while the right-hand
607 column gives Dimensions 3 and 4 of the respective spaces.

608

609 **Figure 5.** DISTATIS biplots for free sorting (top, in purple) and free linking (bottom, in orange)
610 of chocolate-study results. The left-hand column gives Dimensions 1 and 2, while the right-hand
611 column gives Dimensions 3 and 4 of the respective spaces.

612

613 **Figure 6.** Stability of consensus solutions as assessed by *RVb* for free linking (purple) and free
614 sorting (orange) in spice (dashed) and chocolate (solid) studies.

615

616 **Figure 7.** Degree distributions for spice (left) and chocolate (right) studies for free linking
617 (purple) and free sorting (orange). In these studies, higher degree indicates less power to
618 discriminate among samples.

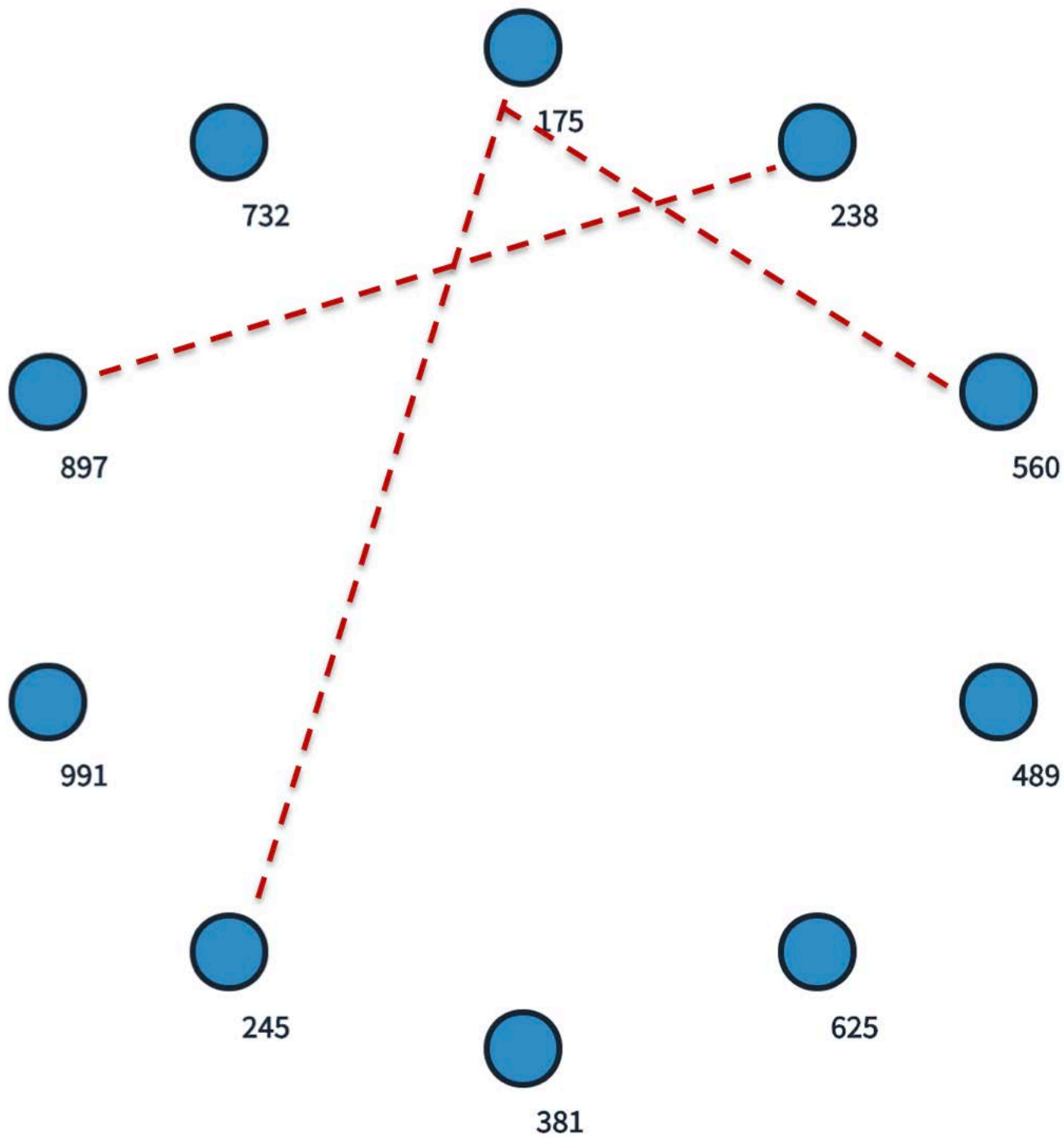
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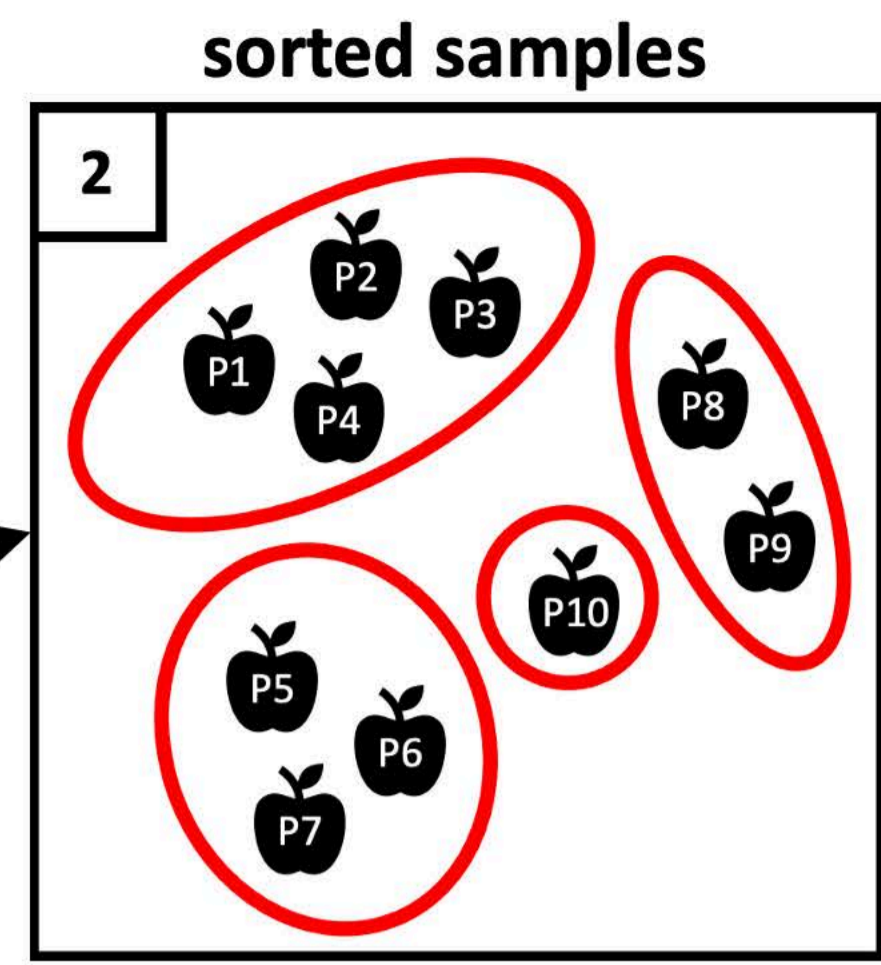
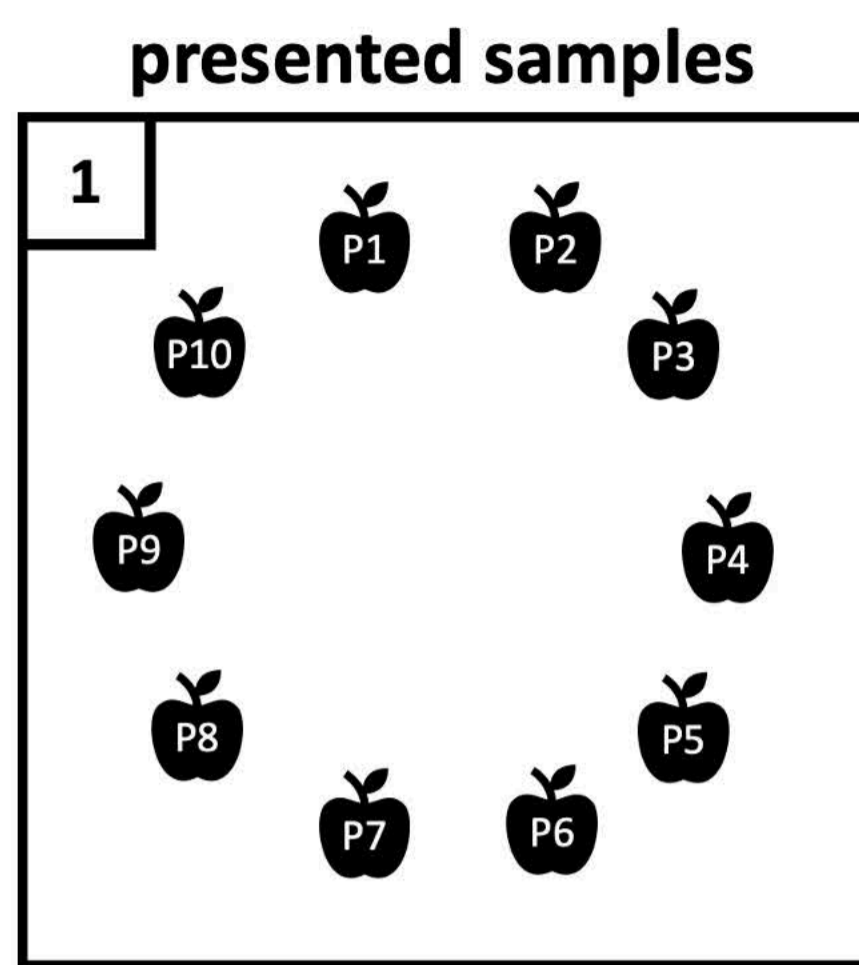
620 **Figure 8.** Scatter plots of individual subjects' degree (with lower degree indicating higher
621 discrimination power) against transitivity (clustering coefficient, with higher values indicating
622 forced grouping/similarity) for free linking (purple) and free sorting (orange). Note that for free
623 sorting, transitivity is *always* equal to 1 except in the rare degenerate case in which subjects only
624 make groups of 2 or fewer samples (bottom left).

625

626 **Figure 9.** Scatter plots of individual subjects' degree (with lower degree indicating higher
627 discrimination power) against connectivity (with higher values indicating ability to detect
628 multiple levels of similarity). Note that for free sorting, only high values of degree guarantee
629 higher connectivity, whereas in free linking higher connectivity is achieved at lower degree (with
630 higher discrimination power).

631



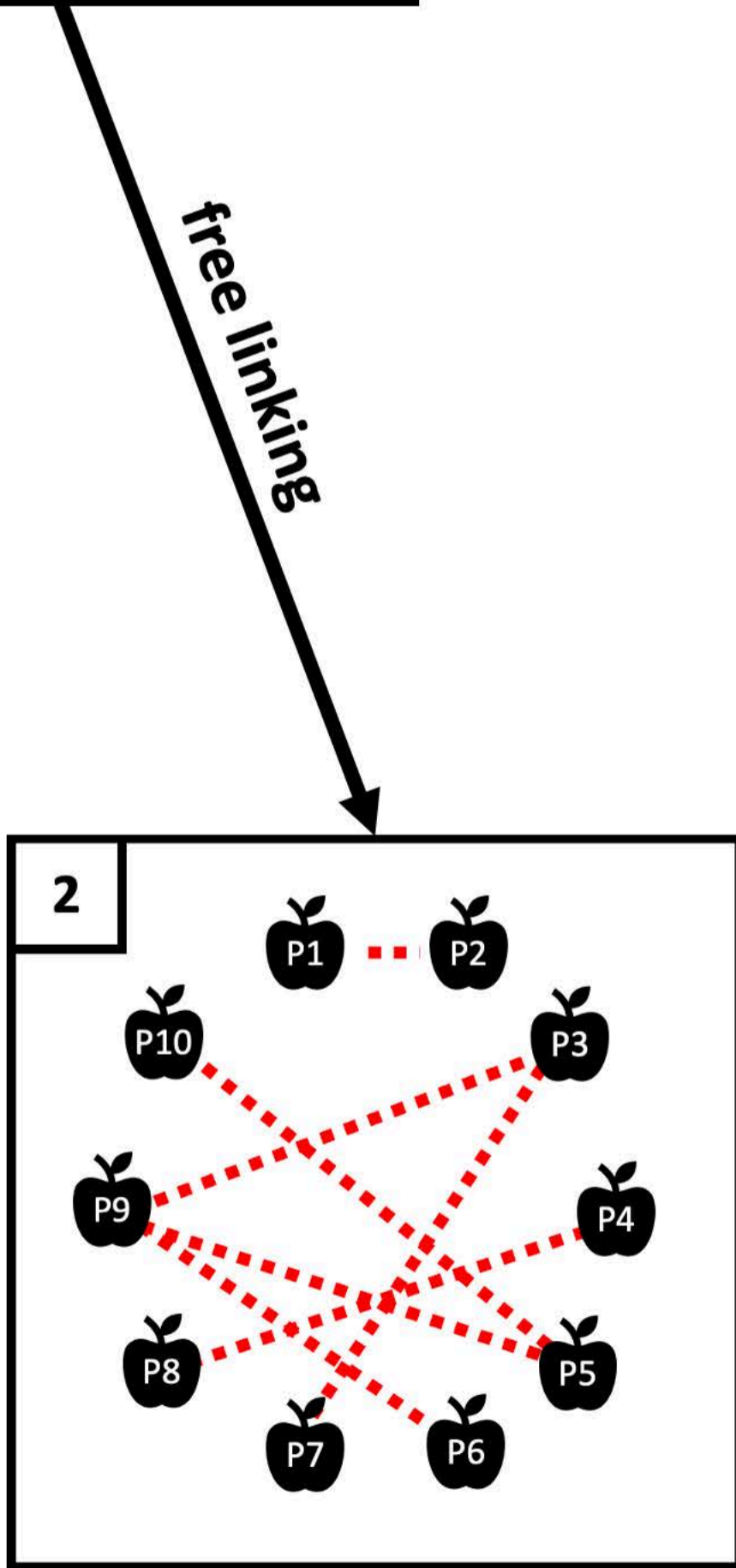


dissimilarity matrices

3

P#	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	1	1	1	1	1	1
2	0	0	0	0	1	1	1	1	1	1
3	0	0	0	0	1	1	1	1	1	1
4	0	0	0	0	1	1	1	1	1	1
5	1	1	1	1	0	0	0	1	1	1
6	1	1	1	1	0	0	0	1	1	1
7	1	1	1	1	0	0	0	1	1	1
8	1	1	1	1	1	1	1	0	0	1
9	1	1	1	1	1	1	1	0	0	1
10	1	1	1	1	1	1	1	1	1	0

**analysis with
DISTATIS,
MFA, SBA,
etc**



graph-distance matrices

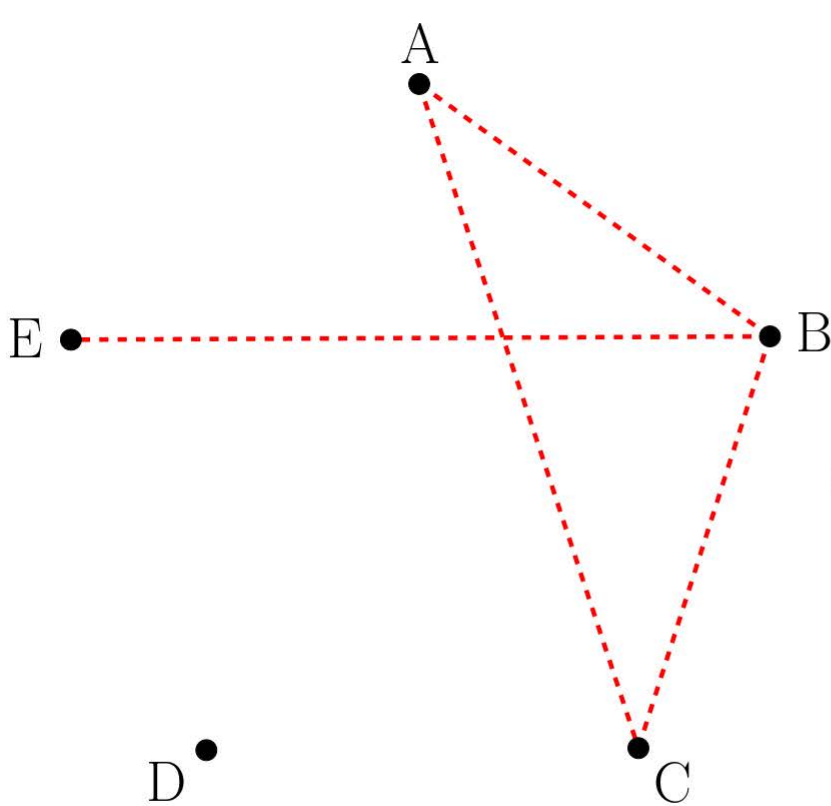
3

P#	1	2	3	4	5	6	7	8	9	10
1	0	1	∞	∞	∞	∞	∞	∞	∞	∞
2	1	0	∞	∞	∞	∞	∞	∞	∞	∞
3	∞	∞	0	∞	2	2	1	∞	1	3
4	∞	∞	∞	0	∞	∞	∞	1	∞	∞
5	∞	∞	2	∞	0	2	3	∞	1	1
6	∞	∞	2	∞	2	0	3	∞	1	3
7	∞	∞	1	∞	3	3	0	∞	2	4
8	∞	∞	∞	1	∞	∞	∞	0	∞	∞
9	∞	∞	1	∞	1	1	2	∞	0	2
10	∞	∞	3	∞	1	3	4	∞	2	0

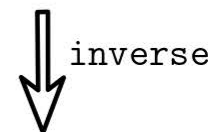
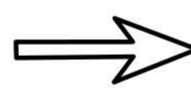
dissimilarity matrices

4

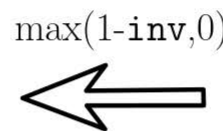
P#	1	2	3	4	5	6	7	8	9	10
1	0	0	1	1	1	1	1	1	1	1
2	0	0	1	1	1	1	1	1	1	1
3	1	1	0	1	1/2	1/2	0	1	0	2/3
4	1	1	1	0	1	1	1	0	1	1
5	1	1	1/2	1	0	1/2	2/3	1	0	0
6	1	1	1/2	1	1/2	0	2/3	1	0	2/3
7	1	1	0	1	2/3	2/3	0	1	1/2	3/4
8	1	1	1	0	1	1	1	0	1	1
9	1	1	0	1	0	0	1/2	1	0	1/2
10	1	1	2/3	1	0	2/3	3/4	1	1/2	0



- distance(A, A)=0
- distance(A, B)=1
- distance(A, C)=1
- distance(A, D)=∞
- distance(A, E)=2
- distance(B, C)=1
- distance(B, D)=∞
- distance(B, E)=1
- distance(C, D)=∞
- distance(C, E)=2
- distance(D, E)=∞



- inv(A, A)=1/0
- inv(A, B)=1/1
- inv(A, C)=1/1
- inv(A, D)=1/∞
- inv(A, E)=1/2
- inv(B, C)=1/1
- inv(B, D)=1/∞
- inv(B, E)=1/1
- inv(C, D)=1/∞
- inv(C, E)=1/2
- inv(D, E)=1/∞

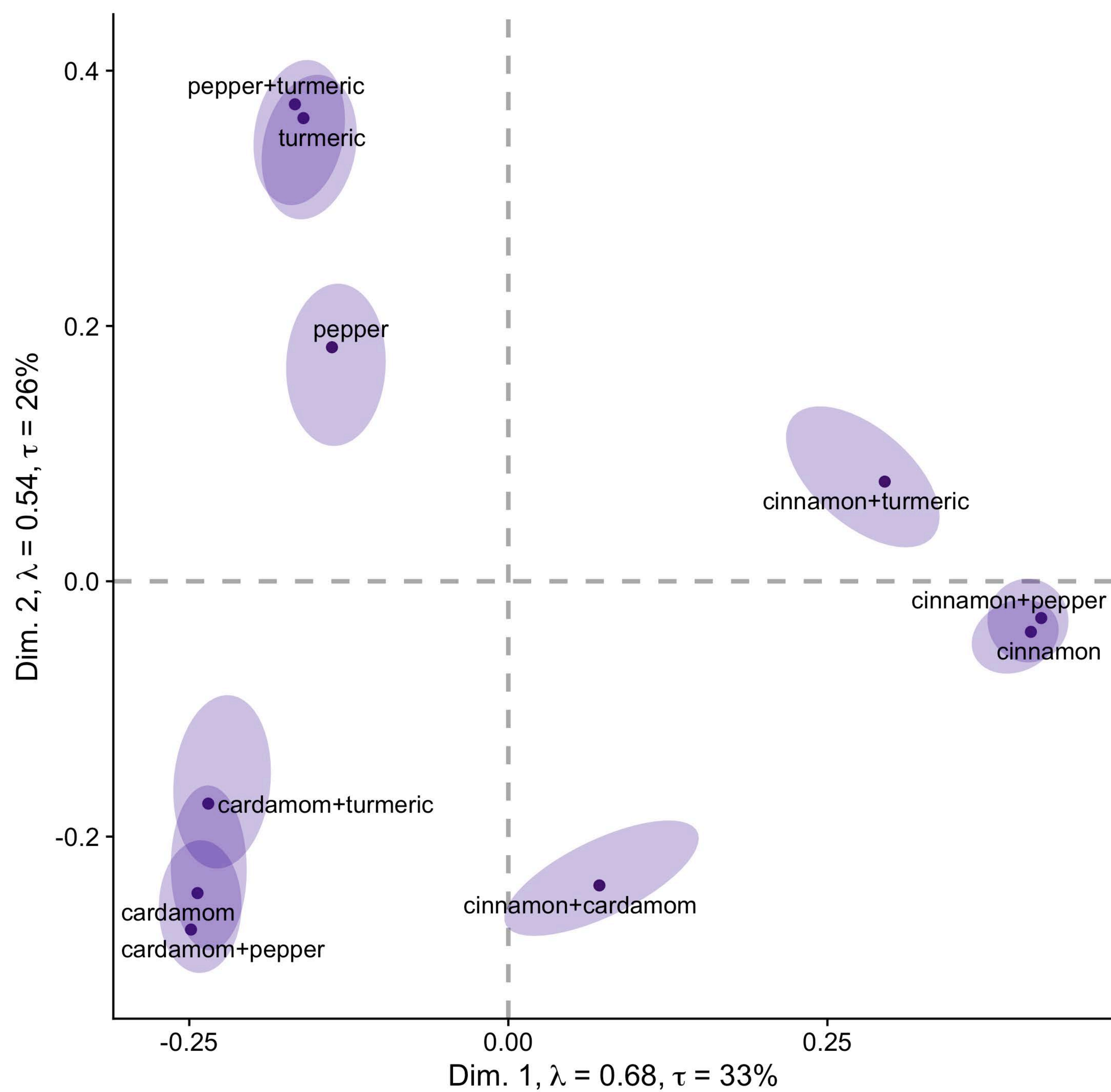


	A	B	C	D	E
A	0	0	0	1	$\frac{1}{2}$
B	0	0	0	1	0
C	0	0	0	1	$\frac{1}{2}$
D	1	1	1	0	1
E	$\frac{1}{2}$	0	$\frac{1}{2}$	1	0

Dissimilarity matrix

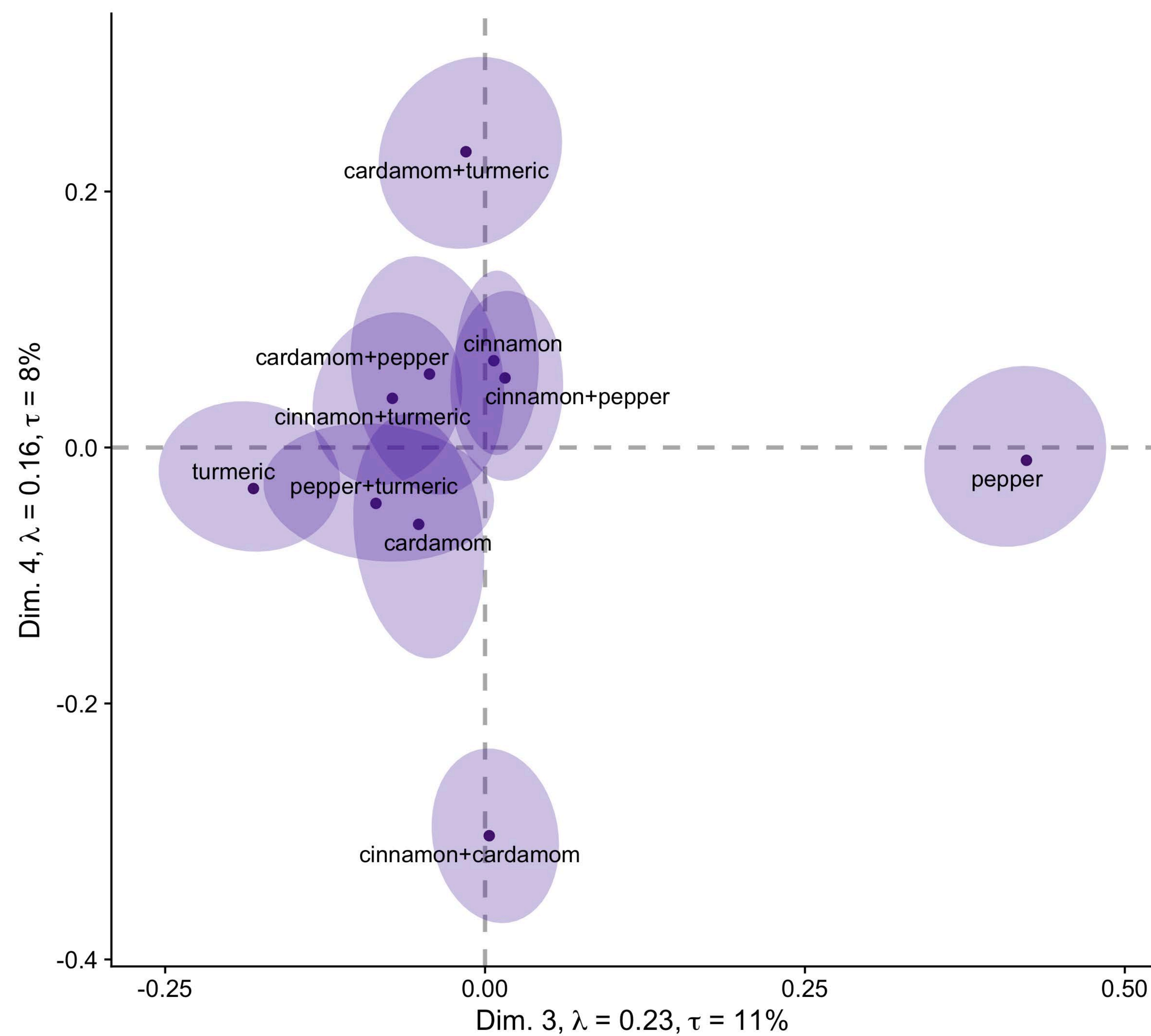
DISTATIS for Spice Sorting Data

Dimensions 1 & 2



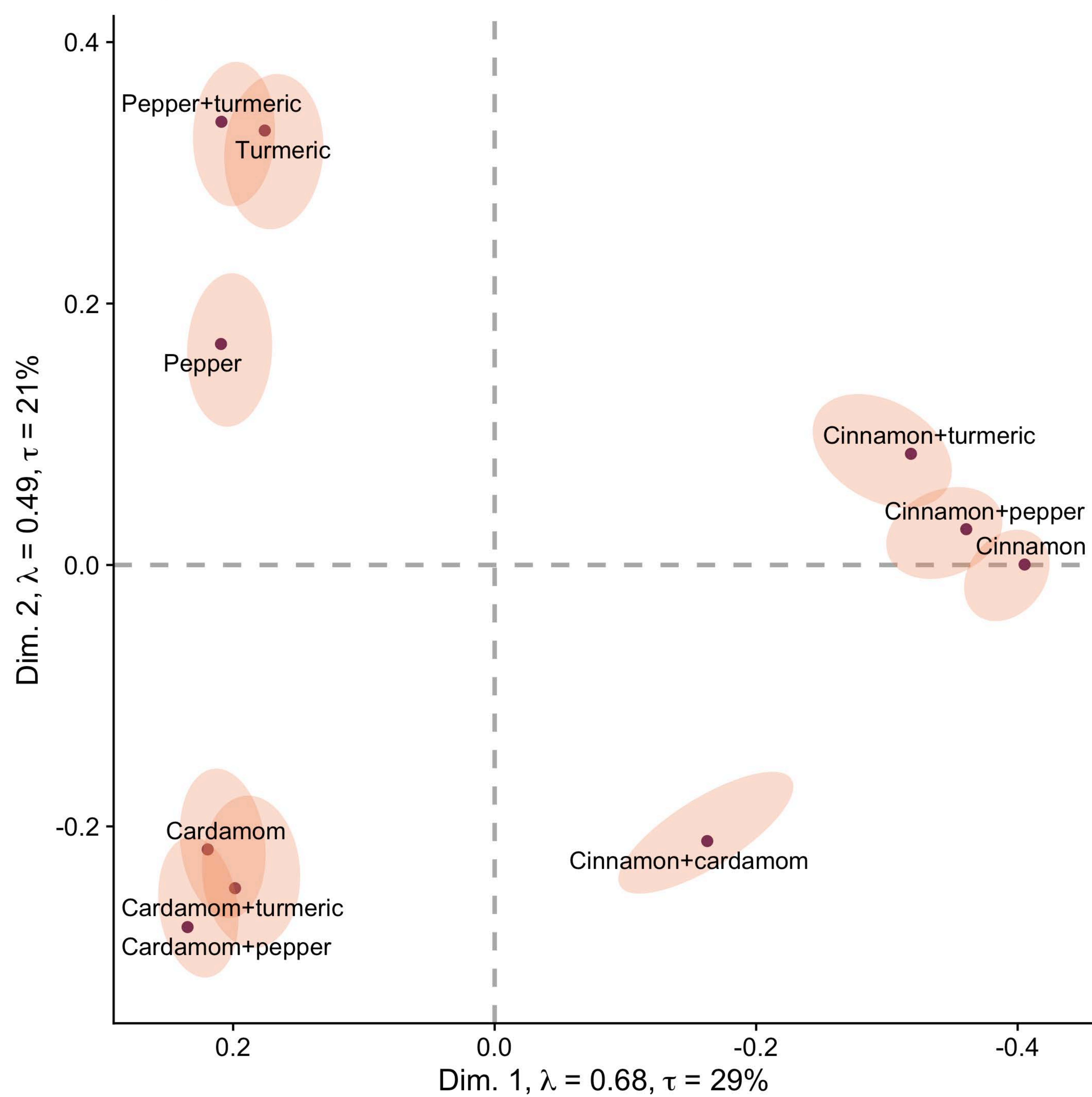
DISTATIS for Spice Sorting Data

Dimensions 3 & 4



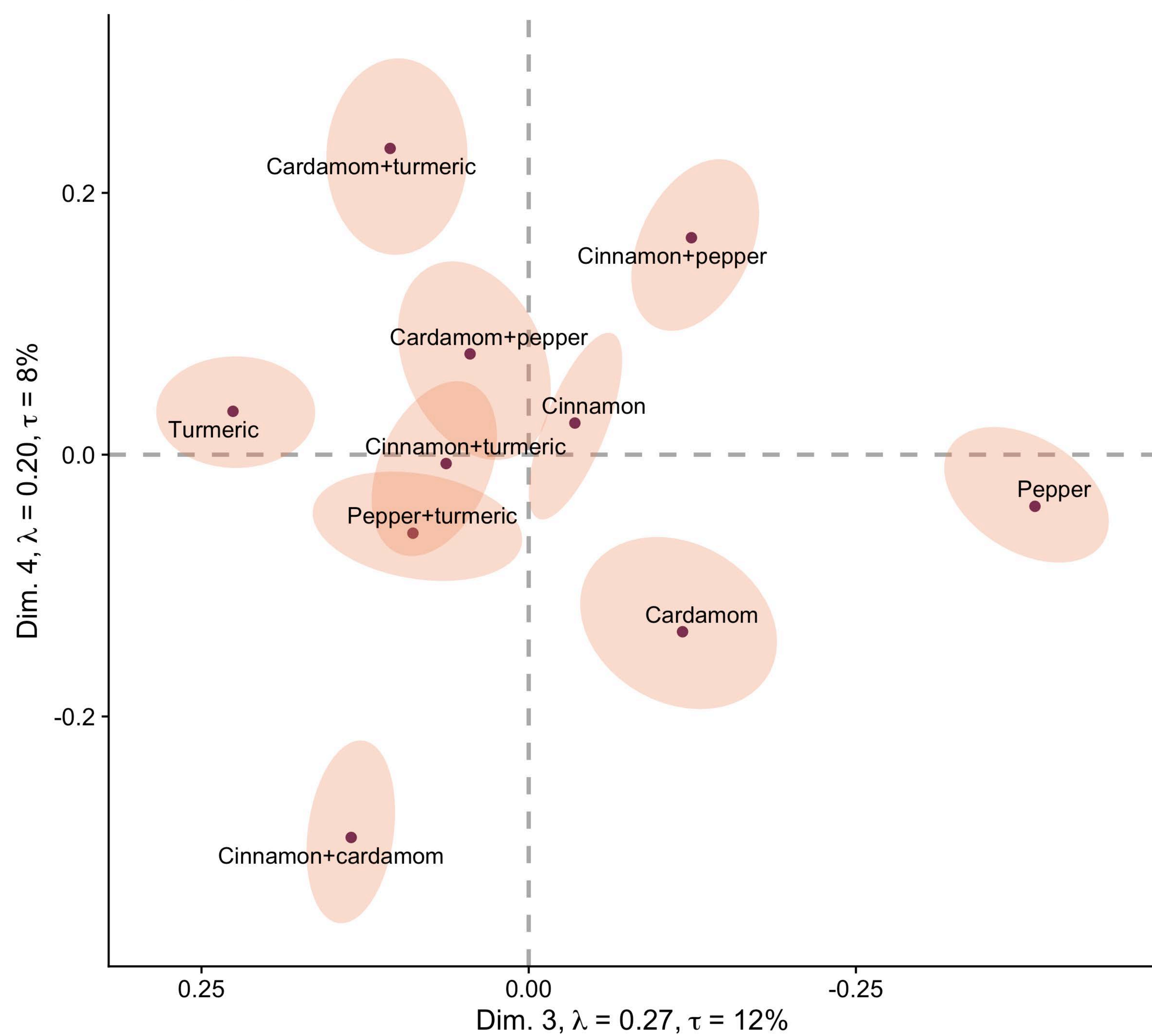
DISTATIS for Spice Linking Data (based on graph distances)

Dimensions 1 & 2



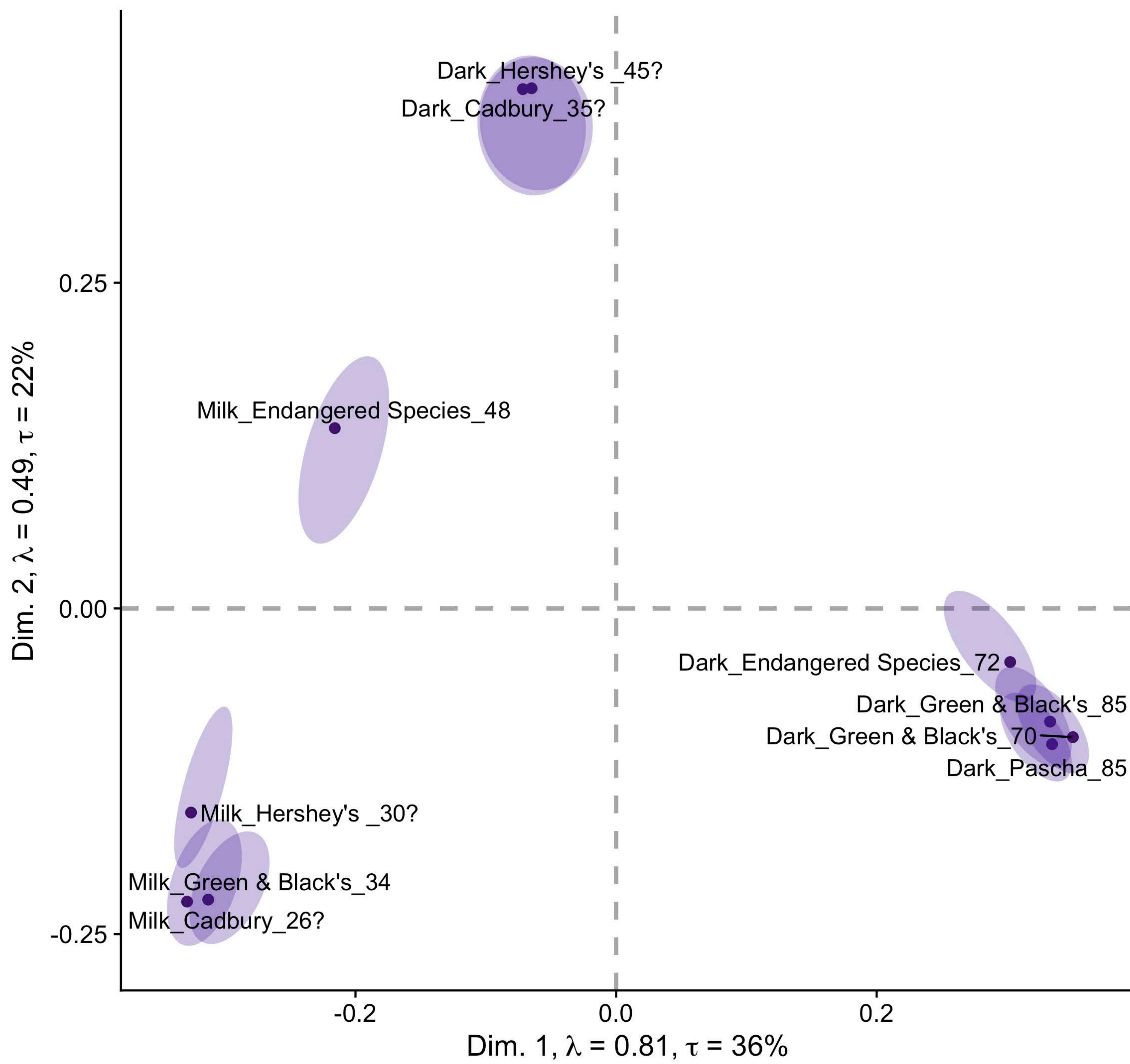
DISTATIS for Spice Linking Data (based on graph distances)

Dimensions 3 & 4



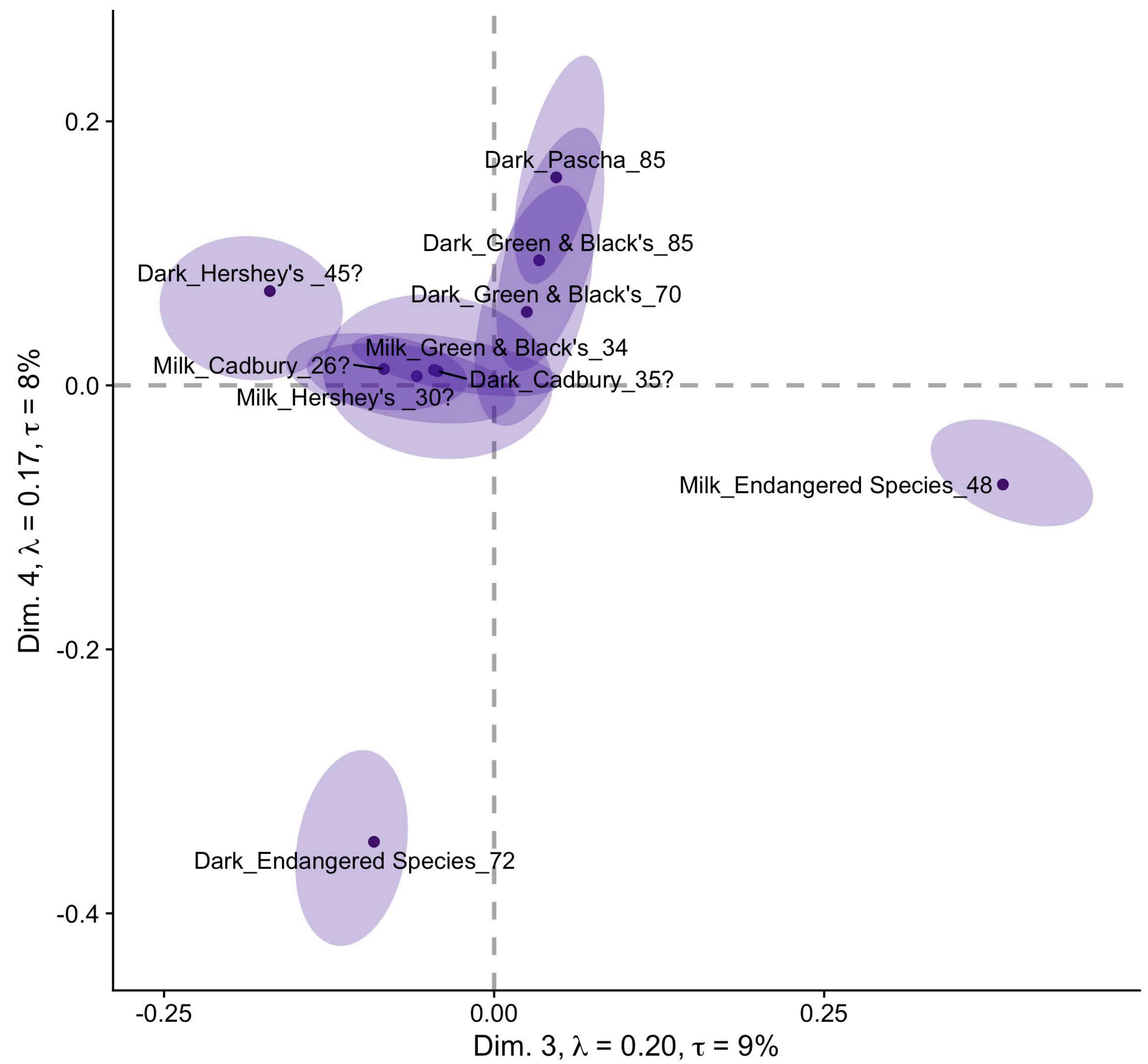
DISTATIS for Chocolate Sorting Data

Dimensions 1 & 2



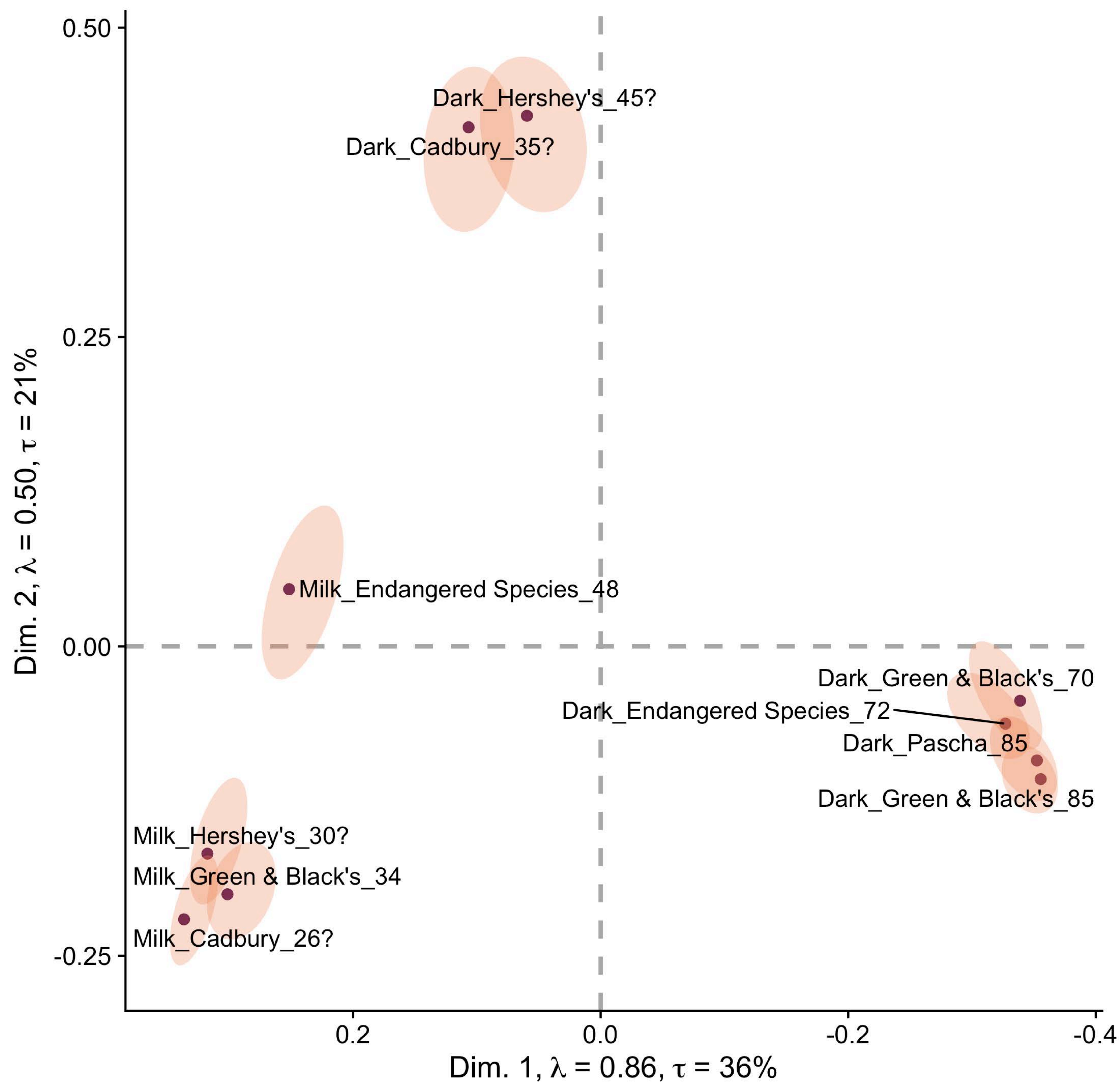
DISTATIS for Chocolate Sorting Data

Dimensions 3 & 4



DISTATIS for Chocolate Linking Data (based on geodesic distances)

Dimensions 1 & 2



DISTATIS for Chocolate Linking Data (based on geodesic distances)

Dimensions 3 & 4

