1930

The Utility of Mathematics

Jack R. Dunn

THE UTILITY OF MATHEMATICS

JACK R. DUNN

This Thesis

is

submitted

in

partial fulfillment

of the

requirements

for the

Baccalaurate Degree

in

Science.

Xavier College

New Orleans, La.

::::::::::::::::::

CONTENTS

-)(--

INTRODUCTION

The following work is not a series of speculations. It has been done to show the utilities of mathematics which can be scarcely over estimated in any practical field.

Mathematics, the sciencs of quantity which treats of the measures of quantites and their relations to each other, can rightly lay claim to this pre-eminence. Mathematics, imparts knowledge to the mind that is deep, profound and abiding.

Pure mathematics embraces the principles of science, and all explanations of the processes by which those principles are derived from the laws of the abstract quantities, number and space. Mixed mathematics, embraces the appliacation of those principles to all investigations and to the solution of all questions of a practical nature, whether they relate to abstract or concrete quantity.

The laws of mathematical science are generalized truths derived from the consideration of number and space. All the processes of inquiry and investigation are conducted accord- to fixed laws, and form a science, and every new thought, higher impression and practical application, forms an additional link and lengthens the chain of the practical uses of mathematics.

The educator regards mathematecal science as one great means of accomplishing his work. He knows that the trains of reasoning are combinations according to logical rules of what has been previously fully comprehended, and that the individual develops through mathematics, so that the thread of the science and the warp of the intellect entwine themselves and become inseparable.

The philosopher regards mathematical science as the mere tool to his higher education and vocation. Bythe high impulses towards nature and the great laws governing all things he neglects that thorough preparation in mathemati- science necessary to success, and is often obliged like Antaeus to renew his strength.

The mere practical man regards with favor only the results of science, deeming the reasonings through which these results are arrived at, quite unnecessary. Such should remember that the mind requires instruments as well as the hands, and that it should be equally trained in their combinations and uses.

Suchis, indeed, now the complication of human affairs, that to do one thing well, it is necessary to know the properties and relations of many things. Everything, whether an element of knowledge or a rule of art, has its connections and its law. To understand these connections and that law is to know the thing. When the principle is clearly apprehended, the practise is easy.

With these general views, and under a firm conviction that mathematical science is emmencely practical, and should be the great bases of education, I have spent much time and labor to show the practicalness of mathematics. I have endeavored, in a broad sense, to present every practical field separately, to point out and note the great and necessary uses of mathematics.

THE UTILITY OF MATHEMATICS

PART ONE

MATHEMATICS OF FINANCE

This part of my treatise on The Utility of Mathematics is to present the application of mathematics to a broad class of financial problems.

It is very obvious that individuals and business organizations are familiar with the term, "Interest." Interest means that a consideration is being given for the use of capital during an interval time. The capital is called the principal and is ordinarily expressed in money, but may be expressed in any units of value. It is my purpose. however, to show the use of mathematics in solving the interest problems.

By the processes of mathematics the rate of interest is solved. It can only be solved by the principles of mathematics, which readily shows the object of a bank, the inside purpose of its existence, is calculated, simply, by the principles of mathematics. In calculation of interest at certain rates we find, that we need a good knowledge of algebra and percentage. Percentage is very important in building up an interest table, and algebraic graphing is essential for graphical representation of interest.

In working for the exact calculation of interest the formulae $I = Pni$, and $S = P \quad I$, from mathematics, are seen to be constantly in use. When the question arises as to the time required for money to double itself at a given rate of compound interest, deeper fields of mathematics are absolutely necessary, e.g. formulae such as $n \; \frac{\log 2P - \log P}{\log (1+i)}$ equals $\frac{\log 2}{\log (1+i)}$ enters and has tobe used to calculate the required time.

Example: Find the time required for a sum of money to double itself at 6% interest, convertible annually.

Solution: $n = \frac{\log 2}{\log(1 \quad i)} = \frac{0.30103}{0.02531} = 11.898$ years.

Banks have, continuously, many calculations in discount, force of discount, and continuously convertible interest. In all the calculations compound formulae from pre-eminent mathematics spring into use. Annuities, a succession of periodical payments, are worked out by progressions, both geometric and arithmetic. When questions arise as to the amount of an annuity payable p times a year, formulae from calculus come into use, e.g. $S \frac{P}{N} = \frac{1}{P} S_{\overline{NP}}$, where $S_{\overline{NP}}$ is to be computed at a given rate. The meaning of these letters are explained andthe process of finding their origin is also shown in the field of mathematics called calculus. If annuities are deferred or if they are due, and on them a certain rate of annuities for n years is deferred m years, the solving factor is mathematics which introduces another of its forecalculated formulae to offset the difficulty.

Example: Find the present value of an annuity of $1200 per annum in monthly instalments if money is worth 5% effective.

Solution: $A = \$1200.00 \times a_{\overline{7}|} \times \frac{i}{j(n)}$

From the table of annuities,

$a_{\overline{7}|} = 5.7863734$, and $\frac{i}{j\ (12)} = 1.0227147.$

$\therefore A = 1200.00 \times 5.7863734 = \$7101.37.$

In stocks and bonds questions arise as to the price of a bond to yield a given rate of interest for a certain time. The question is thus answered, let c = the price at which the bond is redeemed, n = the number of years to redemption date, g = rate of interest or dividents paid to the redemption price C, i = the effective investment rate. Then the present values of the redemption price and the interest payments are added, and we have then the purchase price of the bond to yield an effective rate i. $A = C\ i^n + g\ C\ a\ \frac{P}{N/}$ at rate i. The formula is easly substituted in thus giving the particularly desired case. This could not have been accomplished without the necessary mathematical procedure.

Example: Find the purchase price per $100.00 face value of a $4\frac{1}{2}\%$ bond interest payable semi-annually to be redeemed at par in 18 years, if it is to be purchased at an investment rate of 6% per annum, payable annually.

Solution: In this case, $n = 18$, $g = 0.0425$, $j = 0.06$, $m = 1$, and $p = 2$. Substituting in the formula $A = \frac{C}{(1+i)^n} + Cg\ \frac{i}{j(p)} \cdot a_{\overline{n}|}$ at the rate i, we have

$$A = \frac{100}{(1.06)^{18}} + \frac{4.25}{2} \cdot \frac{1 - (1.06)}{(1.06)^{\frac{1}{2}} - 1}$$

$$= \$35.034 + (4.25) \times \frac{0.06}{j(2)} \times a_{\overline{18}|} \text{ @6\%.}$$

$$= \$35.034 + (4.25)\ (1.01478)\ (10.8276)$$

$$= \$81.73.$$

Bond tables have also been established to give the purchase prices to maturity for rates named in the bond, and for the investment rate. These rates are standardly adjusted for a particular number of years. These are calculated algebraically by formulae preebtained. Again in the determination of the investment rate of a given bond from the purchase price, the three different cases follow a definite mathematical formula. Serial bonds involve the most mathematics in their process of solving.

Example: What is the purchase price of $5000.00 of serial bonds issued January 1, 1920, with 5% cupons payable annually, and maturing in ten equal annual instalments, to net the purchaser an effective rate of 6%?

Solution: The pre-calculated formula, $k = \frac{a^{F}}{r}$ at the investment rate r, $k = \left(1 - \frac{a_{\overline{10|}}}{10}\right) \frac{0.05 - 0.06}{0.06}$, then the formula, $k = \left(1 - \frac{r^{a}}{p}\right) \frac{g - i}{i}$, has been substituted in where $a_{\overline{10|}}$ at the rate $0.06 = 7.360087$.

In the gross operations of a business enterprise there are losses that arise out of the physical and functional deterioration of property consisting of buildings, machinery, and equipment. Part of the deterioration is handled by current repairs, but part of it is such that it can not be satisfactorily provided for out of a repair fund. Thus, parts of a plant must be renewed and such a renewal is likely to be an unfair charge on the current years operations. The losses thus cared for by current revenue are called depreciation, and are handled in the accounts of the business by means of formal charges to profit and loss, known as depreciation charges. The honor goes to mathematics for solving the depreciation charges. Five methods are considered in this process, the straight line method, constant percentage of book value method, the sinking fund method, the annuity method, and the unit cost method. The last method is very important due to the complicated formulae involved. Thus deep mathematics is illustrated in a very practical way.

For example, a machine with a remaining service life of 5 years turns out 20 units of work in a year. Its operation costs $250.00 per year, and repairs cost $200 per year. A new machine that turns out 25 units costs $1200. It has a probable life of 12 years, will cost $300 per year for operation, and $200 a year for repairs. Find the value of the old machine on the basis of interest at 5%. Again the pre-calculated formula is used,

$$c = ya_{\overline{n|}} \left(\frac{O+R+XC+iC}{Y} - \frac{O \quad R}{y} \right) \quad \text{where } x = \frac{1}{s_{\overline{n|}}} .$$

Solution: $C = \$1200$, $O = \$300$, $R = \$200$, $Y = 25$, $n = 5$, $a = 250$, $r = 200$, $y = 20$, $X = \frac{1}{s_{\overline{12|}}} = 0.062825$. $a_{\overline{N|}} = a_{\overline{5|}} = 4.32948$.

Then, $c = 20(4.32948)\ \frac{300+200+1200(0.06285)+1200(0.05)}{25} - \frac{250+20}{20} = 86.5896(25.4156 - 22.50) = \$252.46$. With such difficulties the giant of solution, mathematics, can not be diregarded. Mathematics can not be said unuseful to any degree in any financial affair.

In the operations of funds in building and loan associations, of which the primary purpose is to provide funds to be invested upon sufficient security to those members or share holders who need funds to build homes, much mathematics is involved. The first instance to be taken into consideration is the calculation of withdrawal values.

A man has paid $40 a month on 40 shares of stock in a building and loan association for 66 payments. At the end of 66 months, just before making payment number 67, he withdraws his money at its withdrawal value, which is the sum of his payments plus simple interest at 6% per annum on his payments. If the value of the stock has been accumulating at 7% per annum convertible monthly, what is the difference between book value and withdrawal value?

Solution: Book value $= 40 \cdot s_{\overline{66}|}$ @ 7/12 %

$$\text{Book value} = \$40.00 \; \frac{(1.005833) - 1}{0.005833333} \; (1.00583)$$

$= \$3227.62.$

To find the simple interest we note that the first payment was invested 66 months, the second 65 months, and the last 1 month. This is equivalent to $40 invested for 66 + 65 + 64 ....1 = 2211 months, or 184.25 years at 6% = $422.20. The total withdrawal value equals (66x40) + $422.20 = $3082.20. Book value - withdrawal value = difference $3227.62 - $3082.20 = $145.42.

Another type of problem presenting itself in building and loan isthe rate ofinterest from the stand point of the borrower, in which a problem arises involving deep mathematics. If the borrower should consider the interest on his loan and the dues for the purchase of stock to be entirely separate transactions, the rate of interest the borrower pays is that specified in the loan. But these two features are not separate because the borrower is an investor in stock. In order to extinguish his debt, the borrower has the advantage of creatinga sinking fund at the rate ofinterest earned on stock of the association. The mathematical problem arises then to calculate the rate of interest a borrower can afford to pay in a building andlloan association in place of a smaller rate when money to pay off the principsl has to be invested in a ordinary savings account or in some form of sinking fund, instead of earning the rates on stock in a building and loan association. A practical example follows: A man building a house can borrow from a building and loan association on a 7% nominal interest basis in which the stock that he pays $2.00 per month on would mature to $100 in 78 months. Just after making payment number 79, he can borrow from another source at 6% interest payable monthly in advance, and invest the balance of what he would put into the building and loan association into a sinking fund at 4% interest payable monthly. How much would he have saved at the end of the 78th month by choosing the building and loan proposition?

Solution: On each $100 borrowed he would pay each month to the building and loan association $1.5833. Under the second proposition, his interest payment per month would be $0.50. Hence, with the payment of $1.5833 per month, he would under the second plan have available each month $1.0833 to put into the sinking fund at 4% convertible monthly. This would, at the end of 78 months just after making payment 79, amount to $s = 1.0833 \frac{(1.00\frac{1}{3})^{79} - 1}{0.00\frac{1}{3}}$ equal $97.72. He would still owe $100.00 - $97.72 = $2.28, on each hundred dollars borrowed when his debt would have been discharged by taking the building and loan proposition. With such appliacations of a science its utility can not and will not be over estimated

In insurance a particular field of mathematics is advantageously used. Probability is stressed and is the business base of issuing policies of values according to the age of the applicant. By probability a table of mortality was arranged showing the probability of persons to live to be a certain age, a great factor in insurance.

Questions arise in problems of life annuities, in which a base formula is worked out to solve definite problems and cases. The value of an n-year pure endowment of $1.00 is equal to the present value of $1.00 to be received at the end of n years, multiplied by the probability $_nP_x$ that a person now age x, we have $nE_x = V^n {}_nP_x$. Then $l_x n$ represents the number of these persons who survive the period of n years. The total payments to them would be a sum $l_x n$. The present value of the sum $l_x n$ is $V^n l_x n$. The present value of $l_x n$ for each person in the group in $_nE_x$ $\frac{V^n l_x n}{l_x} = V^n {}_nP_x$.

Since the source of the formula is summed up, a practical problem illustrates it. A father's will provides that a son now aged 20 is to receive $10,000.00 upon attaining the age of 25. Find the present value of his inheritance, assuming interest at 3½% and the Amaercan Experience Table of Mortality.

Solution: The present value is given by 10,000 5E20 equals 10,000 $V^5 {}_5P_{20}$. The probability that the son will to receive the money is $_{15}P_{20} = \frac{l_{25}}{l_{20}} = \frac{89032}{92637} = 0.9610846$. Then from the mortality table $V^5 0.8419732$. The present value to the son is $10,000 x 0.8419732 x 0.9610846 = $8,092.07.

In calculation of premiums for simple forms of life insurance mathematics is looked forward to again. For such little problems as, "What is the net annual premium of an ordinary life insurance policy of $5000 of a life aged 25?" This is solved mathematically by the formula $P = \frac{M}{N}$. $P_{25} = \frac{M_{25}}{N_{25}}$ $\frac{11631.1}{770113} = 0.01510$. For a policy of $5000 we have for the net annual premium $5000 \times P_{25} = 5000 \times 0.01510$ equals \$75.50.

In endowment insurance calculations are necessary for the net single premium. It is found by substitution in a pre-worked mathematical formula for the special case, as in the following problem: Find the net annual premium on a twenty year endowment policy for \$10,000, purchased at the age of 25. The formula $P_{xn} \; \frac{M_x - M_{xn} + D_{xn}}{N_x - N_{xn}}$ is used thus $P_{\overline{25 \cdot 20}|}$ $\frac{M_{25} - M_{45} + D_{45}}{D_{25}}$. Values for M, D, and P are found in the annuity tables.

In the valuation of life insurance policies and terminal reserves still another formula is introduced and applied. This is the demonstration of a problem of this type: Find the terminal reserve of the tenth year of an ordinary life policy of \$5000 taken at the age of 25. The formula $nVx = A_{x\ n} - P_x(1 + A_{x\ n})$ is substituted in and the terminal reserve is found.

Solution: $_{10}V\,25 = A_{35} - P_{25}(1 + a_{35})$

By the table $A_{35} = 0.37055$, and $(1 + a_{35}) = 18.6138$. and $P_{25} = 0.0151031$. Substituting these values we have the required results. $_{10}V_{25} = 0.8942$, for \$1.00. $5000 \cdot 0.8942 =$ \$4471, which is the terminal reserve of the tenth policy on an ordinary life policy of \$5000.00 taken at the age of 25.

With these illustrations of the application of mathematics it is easily seen that in all financial problems, mathematics, both simple and complex, is deeply involved and cannot be avoided. It is readily seen that seven eighths of the calculations that arise in finance require pre-eminent mathematics for solution. Since finance, which is the basis and is of great importance to business, is practically overwhelmed with mathematics, it is reasonable to assume that nothing in more important in matters of business and finance than mathematics.

PART TWO

MATHEMATICS AS A PRODUCER OF THOUGHT

In thought, just as in finance, recreation, engineering, geography, electricity, business, etc., mathematics lays claim to a great part of the instigation. In bringing about the process of thinking, there is no greater instrument than mathematics. Mathematics can partly be defined as the science involving much thought.

The utility of mathematics as means of thought begins with the child in his first grade at school. This continues all through the grades through college and out in the daily process of life. Every problem that involves the process of mathematics, instigates thought, and therefore helps greatly to produce a strong thinking mind.

This part of my treatise, on the utility of mathematics as an instigator of thought, is specific, involving the more matured mind, rather than the younger and less developed mind. I therefore hope to show how necessary mathematics really is for thought, for it is a contemporary of philosophy, and I think deserves the edge over philosophy for its production of thought.

To begin I make the statement, "NUMBER is independent of the order of counting", and see what thought is involved and brought about. Immediately it is necessary for us to think. Take the word NUMBER which contains six letters. In order to find out that there are six letters we count them; (N) one, (U) two, (M) three, (B) four, (E) five, (R) six. In this process we have taken the letters one by one, and have put beside them six words which are the first six words out of a series of words that we always carry about us, the names of the numbers. After putting these six words, one to each of the letters of the word number, we found that the word was six; and accordingly we called that set of letters by the name six. But now the question of thought arises. Let us suppoes that the letters of the word number are written upon separate pieces of paper belonging to a box of letters, that we put these into a bag and shake them up and bring them out, putting them down in any order, and then count them again; we find that there are still six of them. For example, if they come out in the alphabetical order BEMNRU, and we put to each the names of one of the numbers that we have before used, we find that the last name will be six. In the assertion that any group of things consists of six things, it is implied that the word six will be the last of the ordinal words used, in whatever order we take up this group of things to count them. In other words, the number of any set of things is the same in whatever order we count them.

This primary example serves as an introduction, to examples that are more difficult and involve much more thought. It is hoped that it will serve to show that even the primary mathematical numbers, need thought to comprehend, and those who comprehend it are thinking due to the instigation of mathematics in one of its most primary forms. Then by the constant use of the thinking process, in the solution of mathematics, the power of thought is developed, thus making a clear mind that grasps other procedures readily and easy.

If a person is taught to reason a problem out, and to indulge deeply in the immediate and logical solution of a problem, he has a very good thinking power. For example: The area of a triangle is 96 sq. ft., and the base is 4 ft. longer than the altitude. Find the base and the altitude. The first thought involved is, "How do we find theearea of a triangle?" Since we are dealing withthe more matured mind it is fore known that the area of a triangle equals one half the product of the base times the altitude. It is therefore necessary for him to visualize on what is given and what he has to find. As fore said, he knows that the area of a triangle equals one half the base times the altitude. Therefore, $\frac{1}{2}$ b.a equals 96 sq. ft. If we let x equal the altitude, the base which is 4 ft. longer than the altitude equals $x + 4$. Then by the formula $A = \frac{1}{2}$ b.a, he substitutes his values, thus finding logically and thoughtfully, the wanted results.

x = altitude, $x + 4$ = base, $A = \frac{1}{2}x(X + 4)$, or $A = \frac{1}{2}x(x + 4) = 96$ or $x + 4x = 192$. The equotion to be solved involves more thought and fore knowledge. He must apply his forelearned principle for solving a quadratic equation to get the solution. His thinking power is taxed very much but after he obtains the end, he is ready, without much difficulty, to enter other fields involving thought or not. Continuing our problem we then have, $x + 4x = 192$. Completing the square:

$x + 4x + 4 = 196$. ____ Factoring: $(x + 2)^2 = 196$

Taking the square root. ____ $x + 2 = \pm 14$

Solving for x. ____ $x = -2 - 14$, $x = -16$; $x = 12$.

(Note). An altitude may not be negative, therefore a +12 is the real value. The altitude then equals 12. The base is x + 4 or 16.

Algebra, a branch of mathematics, comes much in use in the instigation of thought. It deals with unknowns, letters, numbers, symbols, (e.g)---(.) is times, (+) is plus, (-) is minus, (=) is equal, etc. The origin of these symbols is discussed in the history of algebra. We are only interested presently in the origin of thought due to algebra. There will follow, a simple example, solved algebracially, of a problem that bring about thought and is also very practical.

"In his will a man bequeathss $7000 to his wife and daughter with the provision that his wife is to receive $1000 more than twice the amount received by his daughter. How much does each receive?"

Solution. Clear thinking being done we find that, by the conditions of the problem, the wife and daughter together receive $7000 and the wife receives $1000 more than twice that received by the daughter. The problem requires that the amount received by each be determined. If we consider the given conditions we see that the implied equation is: amount received by the wife plus the amount received by the daughter equals $7000. Our process begins.

Let d = the number of dollars received by the daughter.

2d plus 1000 = the number of dollars received by the wife.

Together - 2d plus 1000 plus d equals $7000, or 3d equals $6000, then d equals $2000, the amount received by the daughter. $7000 - $2000 = $5000, the amount received by the wife. The solution is then completed, the problem clearly understood, the process of thought made more accurate, logical and definite.

Many algebraic problems deepen the thought and instigate more careful and accurate study of the thought producing science. Another algebraic example follows:

"Four times a certain number increased by 7 equals 5 times the number, diminished by 7. Find the number."

Solution: Let J equal the number.

4 times J increased by 7 equals 5 times the number diminished by 7, or $4J + 7 = 5J - 7$.

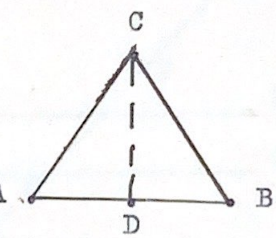
Collecting the knowns and the unknowns we have:

$4J - 5J = -14$, or $-J = -14$, where J equals 14.

Checking: $56 + 7 = 70 - 7$, or $63 = 63$.

In logical order we take the greatest instigator of thought, Geometry. Geometry can rightly be defined as that branch of mathematics that produces great thought and reasoning. This science is briefly composed of axioms, postulates, theorems, problems, propositions, corollaries, proofs, and thought. An axiom is a general statement admitted to be true without proof.(e.g) If equals are added to equals, their sums are equal. A postulate is a geometricl statement admitted to be true without proof. (e.g) All straight angles are equal. A theorem is a statement to be proven. (e.g) The square on th e hypotenuse of a right triangle equals the sum of the squares on the other two sides. A problem is a construction to be made so that it shall satisfy certain given conditions. (e.g) Required to construct a triangle all of whose sides shall be equal. A proposition is a statement of a theorem to be proven or a problem tobbe solved. A corollary is a truth that follows from another with little or no proof. (e.g) All right angles are equal. A proof is the demonstration of a theorem, problem, corollary, postulate, etc. Lastly but most important, thought, that which is required for the solution of geometry and the purpose for the production of the geometry.

The parts of geometry were given because of the gross amount of thought necessary to prove the perserved truths of geometry. I am, therefore, well convinced that geometry is a genuine instigator of thought, by everyone who begins to reason out a proposition. Some very good examples of solutions with the triangle, parallel lines and a circle follow: Theorem. "In an isosceles triangle the angles opposite the equal sides are equal."



Given the isosceles triangle ABC, with AC equal to BC.

To prove that $\angle A = \angle B$.

Proof. Suppose that CD is drawn so that it bisects $\angle$ ACB.

Then in the triangles ADC and BDC,

AC = BC, Given

CD = CD Iden.

(That is , CD issdrawn common to the two triangles.)

and $\angle$ACD = $\angle$DCB. Hyp.

(For CD bisects $\angle$ ACB.)

Therefore triangle ADC is congruent to triangle BDC.

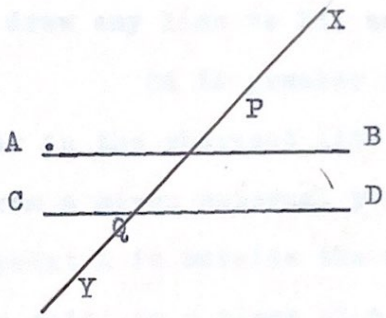
(Two triangles are congruent if two sides and the included angle of one equals to two sides and the included angle of the other.)

Therefore $\angle A = \angle B$.

(Corresponding parts of congruent figures are equal.)

In the procedure of the solution of the proposition, known as the "Pons asinorum", thought is necessary to carry out the logical procedure, and thought is well developed and worked out thus developing the brain. Likewise, in all geometrical procedures, thought is instigated cleverly and wisely and is firmly built up. Another example of a thought rending problem from geometry is displayed by parallel lines.

Theorem. "If two parallel lines are cut by a transversal, the exterior-interor angles are equal."



Given AB and CD, two parallel lines, cut by the transversal XY in the points P and Q respectively.

To prove that $\angle BPX = \angle DQX$.

Proof. $\angle BPX = \angle APQ$.

(If two // lines intersect the vertical $\angle$s are equal.)

$\angle APQ = \angle DQX$.

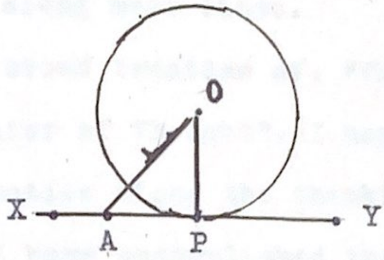
(If two // lines are cut by a transversal, the alternate-interior angles are equal.)

Therefore $\angle BPX = \angle DQX$.

(Quantities equal to the same quantity are equal to the each other.)

The circle problems involve as much thought. A good example of this type follows:

Theorem. "A line perpendicular to a radius at its extremity on the circle is tangent to circle."



Given a circle, with XY perpendicular to the radius OP at P.

To prove that XY is tangent to the circle.

Proof. From O draw any line to XY, as OA.

Then OA is greater than OP.

(A perpendicular is the shortest line that can be drawn to a given line from a given external point.)

Therefore the point A is outside the circle.

(The locus of a point in a plane at a given distance from a fixed point is a circle.)

Therefore XY is tangent to the circle at P.

(A straight line of unlimited length that has one point and only one, in common with a circle is called a tangent to the circle.)

It is readily seen, therefore, how mathematics is the big source of organized thought and one of the biggest instigators of it. By problems in analytic geometry, which is composed of algebra, geometry, and trigonometry, thought deepens, for upon approaching this phase of mathematics a systematized way of thinking would have been built up and the person is more prepared for the deep thought. Then going into solid geometry and solid analytic geometry and calculus, a process is completely developed and thought is clear and accurate. The person who has had such cources, as a rule, is a very good thinker along most lines.

With this broad treatise of, "The Utility of Mathematics as an Instigator of Thought", I hope the necessity and great use of mathematics along the thinking line is brought out. Hoping that I have accomplished the point on this field of my treatise, I will proceed to tell of the utility of mathematics in the field that follows.

PART THREE

MATHEMATICS IN GEOGRAPHY

It is seldom, if ever, realized that in the study of geography, mathematics enters. Little is taught of the use of mathematics in this field, but as the object of this treatise is to show the utilities of mathematics, I shall, in a broad way, devote a part to its uses in geography.

In the field of gergraphy comes the term "centrifugal force." the literal meaning of the word suggests its current meaning. It comes from the latin, centrum, center; and fugere, to flee. Therefore centrifugal force is one, directed away from a center. Mathematics enters in the calculation of centrifugal force on the earth. The rotating earth imparts to every portion of it, save along the axis, a centrifugal force which varies according to the foregoing formula, r being the distance to the axis, or the radius of the parallel. It is obvious that on the surface of the earth the centrifugal force, due to its rotation, is greatest at the equator and zero at the poles.

At the equator centrifugal force (c) amounts to about $\frac{1}{289}$ that of the earths attraction (g), and thus, an object there whichs weighs 288 lbs. is lightened just one pound by centrifugal force, that is, it would weigh 289 lbs. were the earth at rest. At latitude 30°, c equals $\frac{g}{385}$ (that is, centrifugal force is $\frac{1}{385}$ the force of the earths attraction); at 45°, c = $\frac{g}{579}$; at 60°, c = $\frac{g}{1156}$. For any latitude the lightering effect of centrifugal force due to the earths rotation equals $\frac{g}{289}$ times the square of the cosine of the latitude ($c = \frac{g}{289}$ times $\cos^2 \emptyset$). For example, say the latitude of the observer is 40°. $\text{cosine}^2 40° = .7660^2$, the equation is then, $\frac{g}{289}$ times $.7660^2 = \frac{g}{492}$. Thus the earthsaattraction for an object on its surface at latitude 40° is 492 times as great as centrifugal force there, and an object weighing 491 lbs. at that latitude would weigh one pound more were the earth at rest. This is the first simple example of geographical mathematics.

Again in 1851, a French physicist, M. Leon Foucoult, suspended, from the dome of the Pantheon, in Paris, a heavy iron ball by a wire 200 ft. long. A pin was fastened to the lowest side of the ball so that when swinging, it traced a slight mark in a layer of sand placed beneath it. Carefully the long pendulum was set to swing. It was found that the path gradually moved around towards the right. Now either the pendulum changed its plane or the building was gradually turned around. By experimenting with a ball suspended from a ruler, one can readily see that gradually turning the ruler will not change the plane of the swinging pendulum. If the pendulum swings back and forth in north and south direction, the ruler can be entirely turned around without changing the direction of the pendulum's swing. If at the north pole a pendulum was set swinging towards a fixed star, say Ariturus, it would continue swinging towards the same star and the earth and thus br seen to turn around in the length of a day. The earth would not seem to turn but the pendulum would seem to deviate toward the right or, in other words, clockwise.

To calculate the amount of deviation the mathematics comes forward. At first thought it might seem as though the floor would turn completely around under the pendulum in a day, regardless of the latitude. It will be readily seen, however, that it is only at the pole that the earth makes one complete rotation under the pendulum in one day, or show a deviation of 15° in an hour. At the equator the pendulum will show no deviation, and at intermediate latitudes the rate of deviation varies. Now the rate of variation from the pole considered as one and the equator as zero, is shown in the tables of natural sines. It can be shown, that the number of degrees the plane of the pendulum will deviate in 1 hr. at any latitude, is found by multiplying 15 by the sine of the latitude. In other words, $d = \sin \phi$ times 15°. For example, suppose the latitude is 40°. $\sin 40^{\circ} = .6428$. The hourly deviation of the latitude, then is .6428 times 15° or 9.64°. Since the pendulum deviates 9.64° in 1 hr., for the entire circuit it will take as much time as that number of degrees is contained in 360° or $37\frac{1}{3}$ hours.

It is obvious that the involved mathematics in calculations of this sort, is not difficult, but very necessary for the solution. By the great necessity and ues of mathematics, geographical tables of variations, velocity of rotation and the table of uniform rate of rotation was calculated. These tables are very necessary to the geologist.

In calculating the actual measurement in canal digging a nd laying water mains, allowance is necessary for the curvature of the earth; also in surveying. For such difficulties, a mathematical rule has been brought about for finding the amount of curvature for any given distance. The rule: "Square the number of miles representing the distance, and two thirds of this number represents, in feet, the departure from a straight line." For example: Suppose the distance is 1 mile, that the number squared is one and two thirds of that number of feet is 8 inches. Thus, an allowance of 8 inches must be made for 1 mile. If the distance is 2 miles, that number squared is 4, and two thirds of 4 ft. is 2ft. 8in. An object, then, 1 mile away sinks 8 in. below the level, and at 2 miles it is below 2 ft. 8 in. To find the distance, the height being given from the level, we have the converse of the foregoing rule: "Multiply the number representing the height in feet by 1½, and the square root of this product represents the number of miles distant the object is situated". For example let us say an object is 10 ft. from the level line. 10 times 1½ equals 15 and the square root of this is 3.8730. The number of miles distant the object is situated is then 3.8730. Then such situations can only be brought controllable by pre-eminent mathematics.

Mathematics is used to determine the longitude, the correctness of a ship's choronometer and to adjust one of lifes important occurrences, time. By the usefulness of mathematics, equations of time have been worked out to serve man in its own useful way. The equation of time is indicated in various ways. The ususl method is to indicate the time by which the apparent sun is faster than the average, by a (-) sign and the time by which it is slower by a (+) sign. The apparent time and the equation time is thus indicated, when combined, will give the mean time. Thus, if the sun indicates noon (apparent time), aand we know the equation time to be 7m (sun fast, 7m), we know that it is 11h. 53m., A.M. by mean solar time. Any almanac shows the equation of the time for any day of the year. There is also, calculated by the principles of mathematics, "The Analemma", that is used to ascertain the longitude, to set watches, etc.

It is very evident that mathematics cannot be denied use in any field. In bringing about the calendar, the authors, figured mathematically, the number of days to the year and the division of their calendars.

In finding the width of the zones, it is only determined by the distance the vertical ray travels on the earth, and with the moving of the vertical ray, the shifting of the day circle. This distance is in turn determined trigonometrically by the angle which the equator, andthe earths orbit forms. The planes of the equator, and of the orbit form an angle of 23½; the vertical ray travels that many degrees each side of the equator, and the torrid zone is 47° wide. The circle of illumination never extends more than 23½° beyond each pole and the frigid zones are thus 23½° wide. The remaining temperate zones between the torrid and the frigid zones must each be 43° wide. It is therefore, by this simple mathematical method that we know the width of the zones and the non-difficult manner in which to solve them.

In offsetting difficulties of the refraction of light the effects of refraction on celestial altitudes, the length of twilight, etc., mathematical procedure is introduced and eliminates the difficulties. Calculations are condensed to simple rules for the determination of specified things. For example, to determine the latitude of any place proceed thus: By ascertaining the noon altitude of the sun, and referring to the analemma table the altitude is easy to compute. First determine when the sun will be on your meridian and its shadow strike a north-south line. Secondly, by a trigonometrically foremade device, measure the altitude of the sun at apparent noon; that is, when the shadow is north. Then angle A, (Fig.1) the shadow on the quadrant, is the altitude of the sun. This is apparant since XY is the line to the sun and angle B equals angle A. (Fig.2)

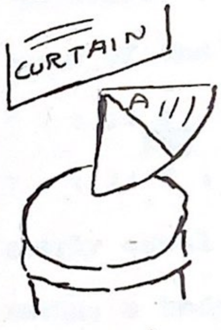


FIG. 1

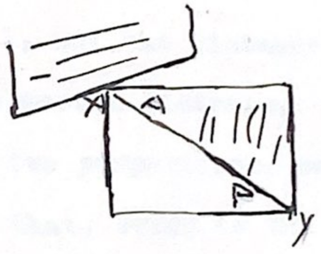


FIG. 2.

Consult the analemma and ascertain the declination of the sun. Add this to the suns altitude if south declination, and subtract it if north declination. If, however, you are south of the equator you must subtract declination south and add declination north. Subtract the result from 90° and the remainder is your latitude. A practical example follows: Suppose you are at San Francisco, October 23, and wish to ascertain your latitude. Assume that you have a north-south line. (The sun's shadow will cross it on that date at 11h 54m., 33s.,A.M., Pacific time.) The altitude of the sun when the shadow is northhis found to be 41°. The declination of the celestial equator is 11° s. Adding we get 52°, the altitude of the celestial equator. 90° - 52° equals 38°, the latitude of the place of the observer.

In geography, as well as in physics, the question arises that the tide producing force of a body varies inversely as the cube of its distance and directly to its mass. This is applied geographically to the moon and the sun.

Let T = the suns tide producing power.

Let t = " moons " " ".

The sun's mass is 26,500,000 times the moon's mass.

Therefore, T : t :: 26,500,000 : 1, but the distance from the earth of the sun is 390 times the moon's diatance.

T : t :: $\frac{1}{390}$: 1 . Combining the two porportions, we get:

T : t :: 2 : 5. It is foreknown that, owing to the very nearly equal attraction of the sun for different parts of the earth, a body's weight is decreased when the sun is overhead, as compared with the weight six hours from then, by $\frac{1}{20,000,000}$; that is, an object weighing a ton, varies in weight ¾ of a grain from sunrise to noon. In case of the moon, thediff-erence is about 2.5 times as great, or nearly 2 grains.

In planning geographical maps it is necessary to approach the exact science for correctness and accuracy. For example: Suppose we wish a map about 20 in. wide to include the 70th parallel. We find that, in a pre-calculated table, 5944.3, is the distance to the equator. Then since the map is to extend 10 in. on each side of the equator, $\frac{10}{5944.3}$ is the mathematical scale to be used in making the map; that is, 1 inch on the map will be represented by 10 in. divided by 5944.3. Then if we wish to lay off the parallels 10° apart, the first parallel to be drawn north of the equator has, according to the table, 599.1 for its meridional distance. This times $\frac{10}{5944.3}$ equals slightly more than one, hence the parallel 10 should be laid off 1 in. from the equator. The 20th parallel has for its meridional distance 1217.3. This times the scale, gives 3.15 inches. In like manner all the other parallels are laid off. The meridians will be $\frac{10}{5944.3}$ times 60 or 600 in. divided by 5944.3 for every degree, or 10 , 6000 in . divided by 5944.3 = 1.01 in. This makes the map 36.36 inches long, (1.01 times 36). We then see that the same scale of miles cannot be used for different parts of the map, though within 30° of the equator representations of areas will be in very nearly true porportions. The parallels in a map not wider than this, say for Africa, may be drawn equdistant and the same distant apart as the meridians, the inaccuracy not being very much.

Trigonometry is a branch of mathematics that is very important in measuring the heights of geographical objects. By porportion, trigonometry and other parts of mathematics, establish facts about the universe. By geometry, astronomers measure the dimensions of the planets, the moon and the sun. It is upon these measurements and facts that the study of geography gets a firm bases. Mathematics ranks again as one of the necessary principles, in the field of geography.

PART FOUR

MATHEMATICS IN THE SHOP AND DRAFTING ROOM

It is known by everyone that in shop work, mathematical problems arise innumberable times. More specifically, in about every undertaking in the shop some mathematical problem arises. The problems may be simple or complex, but, nevertheless, they are all practical problems and go to futher the utility of mathematics.

Shop work requires a knowledge of the measures of length. The unit measure for length is the yard. The standard yard was obtained from Great Britain and its distance between the centers of the two cylindrical bars of gold set in a bar of Bailey's metal whem the metal has a temperature of 62 F. The yard is seldom used in shop work, the foot, inch and parts of an inch being commonly used.

Shop problems are often found in board measure. A board foot means a piece of lumber having an area of 1 sq. ft. on its flat surface, and a thichness of 1 inch or less. To find the number of board feet in a piece of lumber a rule is summed up to simplify problems that may arise. The rule is : To find the number of board feet in a piece of lumber, multiply the number of sq. ft. in its flat surface by the number of inches in thickness, counting a thickness less than one inch as an inch. To illustrate, a practical example follows: Find the number of board feet necessary, in a piece of lumber $1\frac{1}{2}$ inches thick, 9 inches wide, and 14 ft. long.

$\frac{9 \cdot 14 \cdot 1\frac{1}{2}}{12} = 15\frac{3}{4}$ ft. which equals the number of board feet necessary.

Mathematics is used for calculations of house building, general construction, heights of trees and other measurements. In laying floors the amount of lumber needed is necessarily calculated. For example: If a floor to be laid is 12 ft. square, the amount of flooring required is 144 ft. plus $\frac{1}{4}$ of 144 or 36 ft., making it all 180 ft. The $\frac{1}{4}$ is for matching and fitting corners for there are usually a few pieces of wood unusable. In getting stair measurements, calculations are always necessary. For example: The rise of a flight of stairs is 9 ft. 8 in. and it is desired to have the stairs as near 7 inches high as possible. 9 ft. 8in. reduced to inches and divided by 7 equals $16\frac{4}{7}$; now we, in turn, divide the 9 ft. 8 in. by 16, we have $7\frac{1}{4}$ in., we have the width of each riser, or the height of each step. In the shingling of the roof, mathematical calculations are necessary. For example: Find the number of squares (100 sq. ft.) in the roof; then divide this number by $1\frac{1}{4}$ and multiply by 1000. Problem: Find the number of shingles required, if a roof is 25 feet long and 20 feet wide. Solution. $\frac{20 \cdot 25}{100} = 5$ squares. $\frac{5 \cdot 1000}{1.25} = 4000$, the number of shingles required for the roof.

In finding a square that has an area equal to the areas of two given squares, the geometricial theorem is brought into use. It is simplified to the formula: $S^2 = s^2 + s^2$.

Example. Two grain bins of the same depth have bottoms 9 ft. and 11 ft. square respectively. What numbers must be the size of the square bottom of a third bin of the same depth that will hold as much as the two given bins?

Solution. $S^2 = s^2 + s^2$, where s = 9 and s = 11. With these two values we substitute in the given formula to find the third square. $S^2 = 81 + 121 = 202$, where S equals 14.21 ft. or 14 feet, 3 inches. Likewise in finding a circle equivalent to two circles, the process is sifted and simplified to the form: $D^2 = d^2 + d^2$. For example. Two branches of an iron pipe are respectively, 2 inches and 3 inches in diameter. What must the diameter of the pipe into which they empty be, in order that the water may be carried off, if the velocity of the water in all three pipes is the same? $D^2 = d^2 + d^2$, where d equals 2 and d equals 3. Substituting in the formula; $D^2 = 4 + 9 = 13$, D, therefore, equals 3.6 inches.

In finding the heiggts of trees or of any other objects, the fundamentils of trigonometry are,like geometry, simplified. There are six simplifications, begun to find the height of trees without trigonometry. These simplifications were made for shop men who have not had higher mathematics. In shop dealings with pulleys, belts etc., as in other fields, mathematics is involved. In difficulties that may arise for cutting speed, a mathematical rule has been simplified to off set the difficulty. The rule: "Multiply the number of feet in the circumference of the work being done by the number of revolutions per. minute, and the result will be the cutting speed in ft. per minute." For example. If the circumference of a piece of work that is being done, is 20 in., and the work makes 400 revolutions per minute, the cutting speed is 400. $\frac{20}{12}$ equals $666\frac{2}{3}$ ft. per minute.

In mold pressure problems more mathematics is exemplified. These problems follow the laws of pressure and a very simplified rule is stated for finding such pressure. The shop mathematics is almost identical with problems of physics, for involved is the micrometer, Venier, shafs, gears, pulleys a nd tapers. The methods of calculating tapers has two cases. (1) When the tailstock is to be offset, and (2) when the taper attachment is to be used. In this particular field of shop mathematics, slide tests are used. This brings about the graduation of angles and mathematics involving angles, their complements and graduation, is necessary for the solution.

The device called the screw, by itself, has mathematics attached. To find the depth of a thread of a screw, the followimg rule applies: "The depth of a V-thread is equal to the altitude of an equilateral triangle whose sides are equal to the pitch of the thread." the altitude is easly found by geometry. Considering 1" as a standard basis of measurement, we may find a value, C (really the depth of a thread 1" pitch) such that the following rule may be derived: "For a screw of any pitch, the depth of the thread is the same fractional part of C as the given pitch is the base pitch 1"."

Example-: Find the depth of a thread of $\frac{1}{8}$" pitch whose angle is 55°. By trigonometry, the altitude a, of the triangle A whose side P is equal to 1", is found thus:

Tan. $\frac{55}{2} = \frac{.5}{a} = 5.206$. The tan. $27\frac{1}{2}^\circ = .5206$, and a equals $\frac{.5}{;5206} = .96$" which is the value of C" for a 55 sharp thread.

Then the depth of a thread of $\frac{1}{8}$" is 12", or $\frac{1}{8}$"of 96.

An acme thread, but has the following porportions:

D equals the outside diameter, and N equals the number of threads per inch. The root diameter of the screw equals

$D_a\left(\frac{1}{N} \cdot .02\right)$, the outside diameter of the top equals $D - .02$, and the depth of a thread equals $\frac{1}{2n} - .01$, the width of a point of tool for the screw, equals $\frac{.3707}{N} - .0052$, and lastly the width of the screw thread at the top equals $\frac{.3707}{N}$.

It is therefore seen that shop problems may be solved by trigonometry, geometry, algebra, arithemetic or briefly, by mathematics in general. It is then absurd to think that mathematics, and its infinite uses can be over estimated.

In the drafting room, a knowledge of trigonometry, logarithms, geometry and algebra, is most necessary. These phases of the great scisncs come about in problems about the sides of buildings, the determination of the radius of an arc and the laying out of a brick arch. Calculations which involve not only the characteristic considerations of the usual setback problems, but which also require the solving of an unusual algebraic expression, where part of a problem which was presented to an architect when he was designing a building in which the upper floors were to used for dormitory purposes. In the design of a building of this type the upper floors were to be planned first, and the plan which provides the dark corners at the intersection of corridors, is the one which will influence the plans of all the other doors. By the correct draftsman procedure simple engineering problems can be solved.

It is then seen, in a very broad way, the abundance of mathematics necessary for the successful shop or draftsman. The utility of mathematics again scores in an essential way, and again we name mathematics as a most necessary science.

PART FIVE

MATHEMATICS IN RECREATION

In the preceeding parts of my broad treatise on the utility of mathematics, I have tried to discuss some of the many uses of mathematics, in practical fields. It is now, in this part of my treatise that I shall try to show some recreations that can be had by mathematics.

From the beginning to the present day, man has always had some desire for recreation. This desire has been carried all through the ages and will be until the end of man. The satisfaction of this desire for recreation, is gotten from many sources, but the source of mathematics is the only one that will briefly be discussed.

The early men of thought sought and obtained much recreation in the solution of mathematical problems. Today, the deep indulgers of mathematics also find much recreation in mathematics. The average man, however, does not dislike mathematics to any degree nor does he like mathematics to any great degree, seldom, in his quest for recreation, turns to mathematics. He sees little or no recreation that can be had with numbers, symbols, letters, etc. He defines his recreation as some refreshment after toil, or as some amusement and sport. This definition is all right, but by using mathematics as a recreation he can be amused and entertained and at the same time develop the faculties of his mind.

Today, thoughtful mathematical recreations are practised. A very interesting example is numbergrams. The main purpose of numbergrams is to provide a means of relaxation whereby the reader may, by the use of logical thinking, solve problems that at the outset appear very difficult of solution.

Anyone may devise numbergrams with a minimum effort; the difficulty arising in making a problem that is neither too easy nor too hard, yet one which may be solved by gradual developement along common sense lines of reasoning. The following problem to be solved, is taken from "Numbergrams" by Sparhawk.

```
          N X X
A N V   K N A V K
        K G V
          A K V
          A N V
            Y G K
            A N V
            X K A
```

(1) At first glance we see that A N V multiplied by X equals A N V: therefore X must be 1. In the units column where A N V is subtracted from A K V --- V minus V equals G: therefore, G must be zero. (This holds good for all similar cases in the units column, but might not in the tens or any larger column since the value of the minuend may be depressed one unit because of borrowing by the number in the column to the right of it.) Filling in the X's and G's in the following skeleton we get:

```
          - 1 1
- - -   - - - - -
        - 0 -
          - - -
          - - -
            - 0 -
            - - -
            1 - -
```

(2). Referring back to the original problem: ANV multiplied by N equals KGN, in short, NV mulitplied by N equals a number ending in GV and we know that G equals zero. We need then two numbers filling the condition of NV and N which we will give a quotient ending in zero V. 67 times 6 giving 402 fulfills most of these, but fails to give a quotient ending in 7; which is essential if these two numbers are correct. Again, 72 times 7 gives us 504, but fails in not producing a quotient ending in 2. By a very few trials we find that 35 times 3 gives us 105 which fulfills all of the conditions. We neglect the 1 of the 105 that the value we know of A in the divisor has not been used thus far in our problem and will change the value of the 1 to some number much higher. Filling 3 for N, and 5 for V we get in addition to the numbers previously supplied, the two numbers we are now sure of:

```
              3 1 1
          ___________
- 3 5     - 3 - 5 -
          - 0 5
          ______
            - - 5
            - 3 5
          ________
              - 0 -
              - 3 5
            ________
              1 - -
```

(3). Continuing with our analysis of the divisor: A must be multiplied by 3 and gives K05. A must be smaller than 4; since 4, or any higher number gives 12 when multiplied by 3, which would make our quotient more than a thousand. In our problem it cannot be larger than 905. This leaves to be considered the numbers 1, 2, and 3. Since 1 and 3 are already in use, A must be 2 and gives us 705 for the quotient. Filling in once more:

```
            3 1 1
235   ) 7 3 2 5 7
        7 0 5
        -----
          2 7 5
          2 3 5
          -----
            4 0 7
            2 3 5
            -----
            1 7 2
```

The subtraction of 3 from 7 supplies the missing number, 4. Our problem is therefore solved.

We have, in the process of solution, amused ourself and promoted logical thinking. This is a means of recreation that is had by intelligent men and great thinkers.

There are other forms of good intelligent recreation made possible by mathematics, such as, arranging magic squares with numbers. The oldest and the simplest magic square consists of the quadratic arrangement of the nine numbers from 1 to 9 in such a manner that the sum of each horizontal, vertical or diagonal combination always remains the same, namely 15. The square is the adjoined.

2	7	6
9	5	1
4	5	8

It is also a very good recreation to arrange hexagons with numbers that give perscribed sums. Take the numbers from 1 to 73 they admit of being arranged about a centre, in which the number 37 is written, into three hexagons which contain respectively 3, 5, and 7 numbers in each side has the following properties: Each hexagon always gives the same sum, not only when the summation is made along its six sides, but also when it is made along the six diameters that join its corners and along the six that are constructed at right angles to its sides; this sum for the hexagon from within, is 111, for the second, 185, and for the third 259.

```
        1   5   6  70 60 59 58
       63                    8
      62   19 53 46 22 45     9
     61   20             24    64
    2    48   31 42 38   49    57
   3    47   39         40   44   56
  67    51   41   37    33   23     7
   66    50   34         35   54   11
    65    25   36 32 43   26    12
     10   30              27   13
      17   29 21 28 52 55    71
       18                   72
        16 69 68 4  14 15 73
```

Musing on such problems as the magic squares is facinating to thinkers of a mathematical turn of mind. We take delight in discovering a harmony that abides as an intrinsic quality in the forms of our thought. The problems of magic squares, and the like, are playful puzzles, invented as it seems for mere pastime and sport. It is thus so, but there is also a deeper problem underlying all significance. It is the philosophical problem of world order.

The formal sciences are creations of the mind. We build the science of mathematics, geometry, and algerba with our conception of pure forms which are abstract idears. And the same order that prevails in these mental constructions premeats the universe, so that an old philosopher, overwhelmed with the grandeur of law, imagined he had heard its rhythm in a cosmic harmony of the sphere.

Such is the predominance of such a great pre-eminent science, mathematics.

CONCLUSION

The preceeding treatise on the utility of mathematics, in a broad sense, was intended to be a link to bring back the interest of my fellow students in mathematics. The attitude toward one of the most logical sciences has become dull,and I hope thatby these examples of its importance, although broadly stated, the interestwill be reborn in mathematics.

Little is realized by the majority of students the necessity and practicalness of mathematics. If we would but realize this necessity, we would go forward as exceptionally clear thinkers. While gathering yhese few instances of the utilities of mathematics, I thoroughly see how practical, and necessary mathematics is.

In all branches of work be it trade or a profession mathematics is very important. Philosophers need it as a true foundation for their doctrines, scientists need it to calculate their findings, poets need it to give their minds a clearness and a quick perception. It is shown briefly in a part of my treatise, how much business men need it, and it is pessimistic to think that teachers can spare this lack of knowledge.

Therefore my fellow students, since mathematics is so useful, so intellectual, so thought building and so immensely practical, let us get an optimistic view and see its great importance. It is not a dry mess of formulae, numbers etc., but has a composition of interest, thought, and practicalness and after all what is more necessary for success than interest,thought utility and practicalness.

BIBLIOGRAPHY

----)(----

"MATHEMATICAL GEOGRAPHY"--------------------------JOHNSON

"SHOP PROBLEMS IN MATHEMATICS"------------MERSERAN, MOORE

"DRAFTING ROOM MATHEMATICS"------------------------POND

"GENERAL SHOP MATHEMATICS"-----------------------HARPER

"HISTORY OF MATHEMATICS" -------------------------CAJORI

"MATHEMATICS OF FINANCE" ------RIETZ, CATHORNE, RIETZ

"MATHEMATICAL PHILOSOPHY" --------------------- KEYSER

"MATHEMATICAL ESSAYS AND RECREATIONS" -----------SCHMBERT

"NUMBERGRAMS"------------------------------- SPARHAWK

"MATHEMATICS OF ACCOUNTING"----------------CURTIS, COOPER

"INTERMEDIATE PHILOSOPHIC ARITHMETIC" -------------SOULE'

"TEACHING OF GEOMETRY" --------------------------------

------------THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

"PLANE GEOMETRY"------------------------WENTWORTH, SMITH