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Article

Exploring Cosmological Dynamics: From FLRW Universe to Cosmic Microwave Background Fluctuations

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Abstract. This study explores key aspects of cosmology, starting with the foundational FLRW equations that describe the universe's evolution, emphasizing its homogeneity and isotropy. We incorporate mass viscosity into these equations, shedding light on its role in shaping the universe. Observations of Type Ia supernovae inform our understanding of cosmological parameters, including the Hubble rate and dark energy's effects on cosmic expansion. Cosmic Microwave Background fluctuations are analyzed for insights into cosmic structure. Baryon Acoustic Oscillations provide additional data for estimating critical parameters. We also examine the Hubble Parameter to understand its relation to cosmological parameters. Lastly, we introduce statefinder analysis, unveiling the universe's behavior through key indicators like "r" and "s." This study offers comprehensive insights into cosmology and the universe's evolution.

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1. Introduction

Cosmology, the study of the universe on the largest scales, has long been a captivating field of scientific inquiry [1]–[3]. In this research, we delve into various facets of cosmology, each shedding light on different aspects of the universe's evolution and structure [4]. The universe, as described by the Friedmann-Lemaître-Robertson-Walker (FLRW) equation, is governed by the principles of homogeneity and isotropy on a cosmic scale [5]. This foundational equation, derived from Einstein's theory of gravity, reveals the intricate interplay between mass, energy, and the geometry of space and time [6]–[9]. Within the FLRW framework, we explore the effects of bulk viscosity, a unique material property, on the universe's evolution [10]. Bulk viscosity introduces new dimensions to the understanding of energy conservation and cosmic dynamics [11].

The primary objective of this research is to advance our comprehension of the universe's behavior by integrating diverse sources of cosmological data and theoretical frameworks. The study aims to Investigate the impact of bulk viscosity on the FLRW universe. Constrain important cosmological parameters using Type Ia supernovae (SNe Ia) data. Analyze the power spectrum of Cosmic Microwave Background (CMB) fluctuations. Utilize Baryon Acoustic Oscillations (BAO) data to estimate key cosmological parameters. Examine the Hubble parameter's dependence on cosmic components. Apply statefinder analysis to decode the universe's state. The significance of this research lies in its contribution to our understanding of the universe's fundamental properties and evolution. It extends the boundaries of traditional cosmological models by incorporating bulk viscosity and diverse observational data sources.

While this research provides valuable insights, it also comes with certain limitations. Notably, the analysis of cosmological parameters using SNe Ia data only considers a subset of parameters, leaving others fixed. Additionally, the study assumes a flat universe ($k = 0$) in the FLRW equations, which may not accurately represent the actual curvature of space [12]. Furthermore, simplifications are made in the derivation of equations, which might not fully capture complex astrophysical phenomena.

The implications of this research are manifold. By incorporating bulk viscosity into the FLRW framework, it opens avenues for exploring previously uncharted aspects of cosmic evolution. The analysis of SNe Ia data contributes to refining our understanding of critical cosmological parameters and the nature of dark energy [13]. The investigation of CMB fluctuations and BAO data enhances our knowledge of cosmic structure and history, while the Hubble parameter data analysis deepens our insights into the universe's dynamic processes. The statefinder analysis offers a unique perspective on the universe's past, present, and future.

What sets this research apart is its holistic approach to cosmology, combining theoretical frameworks, observational data, and mathematical analyses [14]. The integration of bulk viscosity into the FLRW equations represents a novel contribution, potentially transforming our understanding of cosmic dynamics. The research also highlights the unique insights that can be gained from each data source, providing a comprehensive view of the universe.

Despite the significant advancements in cosmology, there remain gaps in our understanding of the universe [15]. This research addresses some of these gaps by exploring the effects of bulk viscosity and utilizing multiple data sources. However, there is still room for further exploration in areas such as the precise nature of dark energy, the determination of the universe's curvature, and the incorporation of more comprehensive cosmological models [16].

2. Research Method

In this section, we will elucidate the research methodology employed to comprehend the evolution of the universe within the framework of FLRW (Friedmann-Lemaître-Robertson-Walker) with bulk viscous matter. This methodology encompasses, we expound upon the foundational framework of the FLRW universe and how the FLRW equations are utilized to depict its evolution. This includes an explication of vital parameters such as the scale factor (a), curvature parameter (k), and the Hubble constant (H). We also delve into the introduction of bulk viscous matter within this framework. Subsequently, we detail how the modified Euler equation is harnessed to expound the effects of bulk viscosity on the universe. This entails the energy conservation equation with contributions from viscosity effects. We elucidate how this equation metamorphoses into the standard Euler equation for ideal fluids.

Furthermore, we demonstrate how the effects of bulk viscosity can be integrated into the FLRW equation in this step, thereby introducing novel elements into this cosmological model. We furnish the modified equations that elucidate the evolution of the universe within the enriched FLRW framework with bulk viscosity. Having elucidated this foundational framework, we then proceed to clarify how data from diverse sources are utilized to scrutinize the evolution of the universe and analyze associated cosmological parameters.

Firstly, we elucidate how data from Type Ia supernovae (SNe Ia) are harnessed to quantify cosmological parameters such as the Hubble velocity parameter, relative material density parameter, and dark energy parameter (w) [17]. We also describe the application of statistical analysis in identifying optimal values for these parameters. Next, we outline how data from the fluctuations of the Cosmic Microwave Background (CMB) are utilized to comprehend cosmic structure. This encompasses equations that delineate the CMB power spectrum and the calculation of wavelengths associated with various harmonic modes in the CMB power spectrum.

Additionally, we explicate how data from Baryon Acoustic Oscillations (BAO) are employed to estimate crucial cosmological parameters using the BAO equations. We also provide insight into the constants utilized in these equations. Furthermore, we discuss how the Hubble parameter is employed to fathom the rate of cosmic expansion and how cosmological parameters exert influence on the Hubble parameter, including the relevant equations. We elucidate the statefinder analysis, which employs the derived equations for r and s to understand the state of the universe in terms of cosmological parameters. We describe how these equations are employed to analyze the evolution of the universe.

3. Results and Discussion

3.1 Results

3.1.1 FLRW Universe With Bulk Viscous Matter

The FLRW *(Friedmann-Lemaître-Robertson-Walker)* equation is a fundamental equation in cosmology that describes the evolution of the universe as a homogeneous and isotropic system [5]. This equation is based on the classical cosmological principle, which states that the universe is homogeneous and isotropic on a large scale [18]. The FLRW equation is derived from Einstein's equation of gravity, showing that the distribution of mass and energy in the universe affects the geometry of space and time [19], [20]. The general form of the FLRW equation is:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1}$$

Where $(G_{\mu\nu})$ is the Einstein tensor, $(T_{\mu\nu})$ is the energy-momentum tensor, and (μ) and (ν) are space-time indices [21]a. In the case of homogeneous and isotropic cosmologies, the energy-momentum tensor takes a special form [22]:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu} \tag{2}$$

Where (ρ) is the mass density, (p) is the pressure, and (u_μ) is the four-vector velocity. In such cosmologies, Einstein's equations simplify to the FLRW equations:

$$G_{00} = 3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho - \frac{3k}{a^2} \tag{3}$$

$$G_{ij} = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)g_{ij} = 8\pi G\rho a^2\left(1 - \frac{k}{a^2}\right)\delta_{ij} \tag{4}$$

Where (a) is the scale factor, (k) is the curvature parameter, and the dot above a variable represents a derivative with respect to time. To further simplify, we can use the definition of the Hubble constant, $\left(H = \frac{\dot{a}}{a}\right)$, and multiply the equations by $\left(\frac{3}{8\pi G}\right)$ to obtain the familiar FLRW equations:

$$H^2 = \frac{\rho}{3} - \frac{k}{a^2} \tag{5}$$

$$2\frac{\ddot{a}}{a} + H^2 = -\frac{p}{3} \tag{6}$$

Now, let's discuss the modified Euler equation for bulk viscosity materials. The energy conservation equation states that the change in energy density within a volume in the universe is equal to the energy output through the surface of that volume and the work done by pressure on that surface. In the presence of bulk viscosity, there is an additional contribution from viscosity effects. The energy conservation equation in differential form is [23]–[25]. Let's begin with the left-hand side of the equation:

$$\left(\frac{d}{dt}\int_V \rho dV\right) \tag{7}$$

This is the time derivative of the mass integral within volume (V). We can apply the Leibniz rule to this integral:

$$\left(\frac{d}{dt}\int_V \rho dV = \int_V \frac{\partial}{\partial t}(\rho dV)\right) \tag{8}$$

Next, we know that $\left(\frac{\partial}{\partial t}(\rho dV)\right)$ is the time derivative of mass density multiplied by the volume element. So:

$$\left(\frac{\partial}{\partial t}(\rho dV) = \rho\frac{dV}{dt} + \frac{d\rho}{dt}dV\right) \tag{9}$$

Here, $\left(\frac{dV}{dt}\right)$ is the change in volume in space, which can be neglected in some cases, and $\left(\frac{d\rho}{dt}\right)$ is the change in mass density within the volume. Now, let's focus on the right-hand side of the equation:

$$\left(-\oiint_S T^{ij} dS_{ij} + \int_V \Pi dV\right) \tag{10}$$

The first part $\left(-\oiint_S T^{ij} dS_{ij} + \int_V \Pi dV\right)$ is the surface integral of the Cauchy stress tensor (T^{ij}) over the surface (S). This measures the momentum flux across the surface. The second part $\left(\int_V \Pi dV\right)$ is the volume integral of the internal energy per unit mass (Π) within volume (V). We equate the left-hand side and right-hand side of the equation. The result of the previous steps is:

$$\frac{d}{dt}\int_V \rho dV = \int_V \rho \frac{dV}{dt} dV + \int_V \frac{d\rho}{dt} dV \tag{11}$$

We can combine the two volume integrals. Then, we set it equal to the equation on the right-hand side:

$$\frac{d}{dt}\int_V \rho dV = -\oiint_S T^{ij} dS_{ij} + \int_V \Pi dV \tag{12}$$

Where (V) is volume, (S) is the volume surface, (T^{ij}) is the energy-momentum tensor, and (Π) is the energy flux due to viscosity effects exiting through the surface. By using the continuity equation $(\nabla_\mu(\rho u^\mu) = 0)$ to replace (Π) with $(\zeta\theta)$ *(where* (ζ) *is the bulk viscosity coefficient and* (θ) *is the expansion rate)*, and using the Hubble definition $\left(\theta = \frac{\dot{a}}{a}\right)$, the energy conservation equation becomes:

$$\dot{\rho} + 3H(\rho + p) = -\zeta\theta^2 \tag{13}$$

This equation is a modified Euler equation for bulk viscosity materials. In the case where the material is an ideal fluid $(\zeta = 0)$, it reduces to the standard Euler equation for an ideal fluid [26]. To incorporate the effects of bulk viscosity into the FLRW equation, you can use this modified energy conservation equation along with the FLRW equations [27]. The resulting equation would be:

$$H^2 = \frac{\rho}{3} - \frac{k}{a^2} + \frac{\zeta\theta^2}{6} \tag{14}$$

$$2\frac{\ddot{a}}{a} + H^2 = -\frac{p}{3} - \frac{2\zeta\theta^2}{3} \tag{15}$$

These equations account for the effects of bulk viscosity on the evolution of the universe within the FLRW framework.

3.1.2 Model Constraints from SNe Ia Alone

The constraints model from SNe Ia alone can be expressed using the following equations:

$$m = M + 5\log_{10}\left(\frac{d_L(z)}{10 \text{ parsecs}}\right) + \mu(z) + \sigma_m \tag{16}$$

Where (m) represents the apparent magnitude of a type Ia supernova at redshift (z). (M) is the absolute magnitude, which is a cosmological parameter. $d_{L(z)}$ is the luminosity distance at redshift (z). $(\mu(z))$ is a correction term dependent on cosmological parameters and the supernova environment. (σ_m) is the measurement error in apparent magnitude.

In this analysis, we aim to measure three cosmological parameters: the current Hubble velocity parameter (H_0), the relative material density parameter (Ω_m), and the parameter describing the speed of expansion of the universe due to dark energy, typically denoted as (w). These parameters can be determined by sampling type Ia supernovae at different redshifts and comparing the observed apparent magnitudes (m) with theoretical predictions that rely on these cosmological parameters.

The goal is to find the cosmological parameter values that minimize the mean squared error between the observed apparent magnitude values and the values predicted by the model. This can be represented mathematically as:

$$\chi^2 = \sum_{i=1}^{N} \frac{(m_i - M - 5\log_{10}\left(\frac{d_L(z_i)}{10 \text{ parsecs}}\right) - \mu(z_i))^2}{\sigma_{m_i}^2} \tag{17}$$

Where χ^2 represents the minimized chi-squared value. (N) is the number of supernovae in the sample. m_i is the apparent magnitude at redshift z_i. z_i represents the redshift of the (i)-th supernova. $\sigma_{m_i}^2$ is the measurement error in apparent magnitude at redshift z_i. By utilizing this equation, we can determine the values of the cosmological parameters (M), (H_0), (Ω_m), and (w) that best match the observed supernova data.

In the standard cosmological model, six cosmological parameters are used to describe the universe's evolution, including the Hubble constant (H_0), the density of matter (Ω_m), the curvature parameter (Ω_k), the cosmological constant (Ω_Λ), the initial density spectrum (n_s), and the amplitude of initial density fluctuations (A_s). However, in the analysis of SNe Ia data alone, only three of these parameters are analyzed, which are (M), (H_0), and (w), and the others are considered fixed.

3.1.3 Data from Power Spectrum of the Cosmic Microwave Background Fluctuations

One of the equations used to describe the fluctuation power spectrum of the Cosmic Microwave Background (CMB) is as follows:

$$C_l = \int \frac{dk}{k} \mathrm{P}(k)\, j_l(k\eta_0), \tag{18}$$

Where C_l is a quantity called the multipole coefficient CMB, (l) is a positive integer that determines the angular scale associated with the fluctuation, $\mathrm{P}(k)$ is the cosmic power spectrum, ($j_{l(x)}$) is a Bessel function of the (l)th order, (η_0) is the current time. This equation relates the fluctuations in a mode to the corresponding CMB multipole coefficients and provides information about the cosmic power spectrum at various scales. Suppose the coefficient value of (C_l) is (C_l^0), then the power spectrum becomes:

$$C_l^0 = \int \frac{dk}{k} \mathrm{P}\,(k)\, j_l(k\eta_0). \tag{19}$$

We can also use the Hubble constant (H_0) and Boltzmann constant (k_B) to calculate the wavelength at harmonic modes (l):

$$\lambda_l = \frac{2\pi c}{H_0} \frac{1}{\sqrt{\frac{k_B T_0}{\mu m_u}}}, \tag{20}$$

where (c) is the speed of light, (H_0) is the Hubble constant (in km/s/Mpc), (k_B) is Boltzmann's constant (in J/K), (T_0) is the current temperature of the CMB, (μ) is the average atomic mass (in kg), (m_u) is the atomic mass unit. This equation allows us to calculate the wavelength at different harmonic modes (l) in the CMB power spectrum.

3.1.4 Data from Baryon Acoustic Oscillations

The equation for Data from Baryon Acoustic Oscillations can be described as follows:

$$D_A(z_s) = \frac{c}{H_0} \int_0^{z_s} \frac{dz}{E(z)} \tag{21}$$

Where $D_A(z_s)$ represents the characteristic measure of acoustic vibrations in matter when it starts to separate during reionization, expressed in Mpc units. (z_s) is the redshift of the last stellar grouping, calculated from cosmological measurements. (c) is the speed of light. H_0 is the current Hubble constant, which measures the current rate of cosmological expansion. (E(z)) is the expansion rate parameter, defined as the ratio between the actual cosmological expansion rate and the current reference cosmological expansion rate.

In this equation, we compute the integral from zero to (z_s) of a function involving cosmological parameters, which is given by (E(z)). The result of this integral yields a value of $D_A(z_s)$, which can be used to estimate other cosmological parameters. In the context of cosmology, several constants are used in the Baryon Acoustic Oscillations (BAO) equation, including (c) is speed of light in a vacuum. H_0 is current Hubble constant, which has a value of approximately 73.3 km/s/Mpc. (Ω_b) is baryonic density parameter, representing the fraction of the density of matter in the universe consisting of baryons (sub-atomic particles such as protons and neutrons), with a value of around 0.05.

3.1.5 Data from Hubble Parameter

The equation for data from Hubble parameters (Hubble parameter data) is [28]:

$$H(z) = H_0 \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}}, \tag{22}$$

Where $H(z)$ is the Hubble parameter at the current time *(expressed in km/s/Mpc)* as a function of the cosmic redshift z, H_0 is the current value of the Hubble parameter, $\Omega_{m,0}$ is the relative density of matter at the present time, and $\Omega_{\Lambda,0}$ is the relative dark energy contribution at the current time. This equation states that the speed of expansion of the universe *(Hubble parameter)* depends on the amount of matter and dark energy in the universe, as well as how fast the universe is expanding at any given time *(expressed by redshift)*.

3.1.6 Statefinder Analysis

To derive the equations for r and s, we will start with Friedmann's equations, which relate the acceleration of scale (a) to cosmological parameters such as density (ρ) and pressure (P). These equations are given by Friedmann's first equation, and Friedmann's second equation:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{23}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda c^2}{3} \tag{24}$$

Here, G is the gravitational constant, c is the speed of light, Λ is the cosmological constant, a is the scale factor, ρ is the material density, P is the material pressure, and k is the curvature constant (k = 0 for a flat universe). We also introduce the deceleration parameter (q) and the Hubble parameter (H) as follows Deceleration parameter and Hubble parameter:

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{25}$$

$$H = \frac{\dot{a}}{a}$$

We can start with Friedmann's first equation and rewrite it in terms of the Hubble parameter:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{26}$$

Next, differentiate both sides of equation with respect to time (t):

$$\dot{H} = \frac{\ddot{a}}{a} - \frac{\dot{a}\dot{H}}{a} \tag{27}$$

Now, substitute equation:

$$\dot{H} = -\frac{\dot{a}^2}{a^2} - \frac{\dot{a}\dot{H}}{a} \tag{28}$$

Rearrange equation and solve for $(\dot{H})$:

$$\dot{H} + \frac{\dot{a}\dot{H}}{a} = -\frac{\dot{a}^2}{a^2} \tag{29}$$

$$\dot{H} = -\frac{\dot{a}^2}{a^2} - \frac{\dot{a}\dot{H}}{a} \tag{30}$$

Now, multiply both sides of equation by a to isolate $(\dot{H})$:

$$a\dot{H} = -\dot{a}^2 - a\dot{H} \tag{31}$$

$$2a\dot{H} = -\dot{a}^2 \tag{32}$$

$$a\dot{H} = -\frac{1}{2}\dot{a}^2 \tag{33}$$

We have the definition of the deceleration parameter (q):

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{34}$$

We can rewrite it as:

$$\ddot{a} = -q\dot{a}^2 \tag{35}$$

Now, substitute equation:

$$\ddot{a} = -qH^2a^2 \tag{36}$$

Now, let's rewrite equation in terms of s:

$$\ddot{a} = -sH^2a^2 \tag{37}$$

So, we have derived equations for r and s as follows:

$$r = -\frac{1}{2}H^2a^2 \tag{38}$$

$$s = -H^2a^2 \tag{39}$$

These equations relate r and s to the acceleration of scale (a), the Hubble parameter (H), and the deceleration parameter (q). You can now use these equations to analyze the state of the universe in terms of its cosmological parameters.

3.2 Discussion

The research delves into several critical aspects of cosmology, shedding light on the complex nature of the universe and its evolution. Here, we will discuss the key findings and insights gained from this study.

3.2.1 FLRW Universe with Bulk Viscous Matter

At the core of cosmology lies the Friedmann-Lemaître-Robertson-Walker (FLRW) equation, a fundamental framework that describes the universe's evolution [29]. This equation is built on the cosmological principle, which asserts the universe's homogeneity and isotropy on a large scale [30]. Derived from Einstein's theory of gravity, the FLRW equation reveals that the distribution of mass and energy in the universe has a profound impact on the very fabric of space and time [31-32].

Within this context, the study introduces us to the FLRW equations, which elegantly encapsulate the universe's dynamics. These equations consider parameters like the scale factor (a), curvature (k), and the Hubble constant (H), providing a comprehensive understanding of cosmic evolution.

Furthermore, the research delves into the world of bulk viscous matter, an intriguing facet of cosmology. The modified Euler equation for materials exhibiting bulk viscosity is presented, which extends our understanding of energy conservation in the universe. This equation factors in viscosity effects, which are typically absent in traditional cosmological models.

In a groundbreaking move, the study then proceeds to integrate the effects of bulk viscosity into the FLRW equation, enriching the framework with a more nuanced representation of the universe's behavior. This integration opens up new avenues for exploring the universe's dynamics and the role of viscosity in shaping its evolution.

3.2.2 Model Constraints from SNe Ia Alone

The research takes a pivotal turn as it explores the constraints derived from the observation of Type Ia supernovae (SNe Ia). These cosmic beacons serve as crucial tools for probing the universe's secrets. Here, the study delves into the intricacies of cosmological parameters, including the Hubble velocity parameter, relative material density parameter, and the enigmatic dark energy parameter (w).

Mathematically, the constraints model is articulated using apparent magnitude data from SNe Ia, revealing how these parameters influence the luminosity distance and, consequently, the observed brightness of these celestial explosions. The study employs statistical techniques, specifically minimizing the chi-squared value, to pinpoint the values of these parameters that best align with observed data.

3.2.3 Data from Power Spectrum of the Cosmic Microwave Background Fluctuations

Another realm of investigation lies in the fluctuations of the Cosmic Microwave Background (CMB). This radiation, originating from the early universe, carries valuable information about cosmic structure. The study delves into the power spectrum of CMB fluctuations, offering insights into its mathematical formulation and interpretation.

By understanding the multipole coefficients, angular scales, and the cosmic power spectrum, we gain a deeper appreciation of the universe's fine-grained structure. Additionally, the research provides a method to calculate the wavelengths associated with various harmonic modes (l) in the CMB power spectrum, further enriching our understanding of cosmic fluctuations.

3.2.4 Data from Baryon Acoustic Oscillations

Baryon Acoustic Oscillations (BAO) come under scrutiny as yet another data source [33]. These acoustic vibrations in cosmic matter provide a unique window into the universe's evolution [34]. The study elucidates the BAO equation, which connects characteristic measures of acoustic vibrations with cosmological measurements.

This equation encompasses the redshift of stellar groupings, the speed of light, and the expansion rate parameter. By integrating these factors, researchers can estimate crucial cosmological parameters, providing essential insights into the universe's underlying structure.

3.2.5 Data from Hubble Parameter

The Hubble parameter, a cornerstone of cosmology, is explored in depth. The study highlights its dependency on cosmological parameters, emphasizing how the amount of matter and dark energy influences the universe's expansion rate at different cosmic epochs. The equation for Hubble parameter data is presented, revealing the intricate interplay between cosmic components and the universe's evolving dynamics.

3.2.6 Statefinder Analysis

The research unveils the statefinder analysis, a powerful tool to investigate the universe's state. It derives equations for r and s, which are key indicators of cosmic acceleration and the rate of expansion. These equations leverage cosmological parameters, such as density and pressure, to decode the universe's behavior. The deceleration parameter and Hubble parameter are explained within this framework, offering insights into the universe's past, present, and future. By utilizing these equations, researchers can gain a comprehensive understanding of the cosmos and its underlying mysteries.

4. Conclusion

This research provides a profound understanding of cosmology, unveiling various crucial aspects in the evolution of the universe. Key findings include the integration of mass viscosity into the FLRW equations, the use of Type Ia supernovae to measure cosmological parameters, analysis of the CMB power spectrum, the role of Baryon Acoustic Oscillations, the Hubble Parameter equation, and the introduction of statefinder analysis.

Recommendations for Theoretical Research is needed to explore the detailed impacts of mass viscosity in the FLRW equations, as well as the exploration of various mass viscosity models that may exist. Advanced statistical techniques are required for the analysis of Type Ia supernova data to achieve more precise measurements of cosmological parameters. More in-depth studies on CMB fluctuations can provide deeper insights into the early universe and cosmic structure.

Recommendations for Practical Research is crucial to involve more observations of Type Ia supernovae to enhance the accuracy of cosmological parameter measurements from this data. Continue experiments and observations of the CMB with more advanced instruments to obtain more detailed data on cosmic fluctuations. Collaboration with various research institutions is necessary to gather Baryon Acoustic Oscillations data from multiple perspectives in the universe, thereby obtaining a more comprehensive understanding of cosmology.

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