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CONSTRAINTS ON $f(R, \phi)$ (SANDERS-LIKE) GRAVITY POTENTIAL FROM ORBIT OF S2 STAR

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Abstract. We investigate the possibility to explain theoretically the S2 star orbital precession around the massive object at Galactic Centre using Extended Theories of Gravity, specifically $f(R, \phi)$ a Sanders-like gravitational potential in total absence of dark matter. To this aim an analytic fourth-order theory of gravity, non-minimally coupled with a massive scalar field is considered. The interaction term is given by an analytic functions $f(R, \phi)$ where R is the Ricci scalar and ϕ is the scalar field. We simulated orbit of S2 star around Galactic Centre in Sanders-like gravity potentials and compared it with NTT/VLT observations. We presented maps of reduced χ^2 over the $\{\alpha - m_\phi\}$ parameter space in the case of NTT/VLT observations. The approach we are proposing seems reliable to constrain modified gravity models at astronomical level.

1. INTRODUCTION

Extended Theories of Gravity (see e.g. Capozziello & De Laurentis 2011) are alternative theories of gravitational interaction. These theories are developed from the exact starting points like General Relativity. They aimed from one side to extend the positive results of General Relativity and, on the other hand, to cure its shortcomings. These theories have been proposed like alternative approaches to Newtonian gravity in order to explain galactic and extragalactic dynamics without introducing dark matter (see e.g. Capozziello & De Laurentis 2012, Nojiri & Odintsov 2011, Capozziello

2002, Capozziello et al. 2003, Capozziello & Faraoni 2010).

S-stars closely orbit the massive compact object at the center of Milky Way, named Sgr A* (see e.g. Ghez et al. 2000, Gillessen et al. 2009a, Gillessen et al. 2009b, Genzel et al. 2010). These stars, together with recently discovered dense gas cloud falling towards the Galactic Centre (see e.g. Gillessen et al. 2012), indicate that the massive central object is probably a black hole. There are some observational indications, for at least S2, that its orbit may deviate from the Keplerian case (see e.g. Gillessen et al. 2009a, Meyer et al. 2012).

2. THEORY

2. 1. $f(R,\phi)$ THEORIES OF GRAVITY

We can consider a generic function of Ricci scalar and scalar field. Then the action becomes (see e.g. Stabile & Capozziello 2013):

$$\mathcal{A} = \int d^4x\sqrt{-g}\Big[f(R,\phi) + \omega(\phi)\,\phi_{;\alpha}\,\phi^{;\alpha} + \mathcal{X}\mathcal{L}_m\Big]. \tag{1}$$

We get $\phi = \phi^{(0)} + \phi^{(1)} + \phi^{(2)} + \ldots$ and the function $f(R,\phi)$ with its partial derivatives (f_R, f_{RR}, f_ϕ, $f_{\phi\phi}$ and $f_{\phi R}$) and $\omega(\phi)$ can be substituted by their corresponding Taylor series. In the case of $f(R,\phi)$, we have:

$$f(R,\phi) \sim f(0,\phi^{(0)}) + f_R(0,\phi^{(0)})R^{(1)} + f_\phi(0,\phi^{(0)})\phi^{(1)}... \tag{2}$$

In the $f(R,\phi)$-gravity the gravitational potential is found by setting the gravitational constant as

$$G \;=\; \left(\frac{2\,\omega(\phi^{(0)})\,\phi^{(0)} - 4}{2\,\omega(\phi^{(0)})\,\phi^{(0)} - 3}\right)\frac{G_\infty}{\phi^{(0)}} \tag{3}$$

where G_∞ is the gravitational constant as measured at infinity and by imposing $\alpha^{-1} = 3 - 2\,\omega(\phi^{(0)})\,\phi^{(0)}$, the gravity potential is (see e.g. Stabile & Capozziello 2013):

$$\Phi_{ST}(\mathbf{x}) \;=\; -\frac{G_\infty M}{|\mathbf{x}|}\Big\{1 + \alpha\, e^{-\sqrt{1-3\alpha}\,m_\phi|\mathbf{x}|}\Big\} \tag{4}$$

and then a Sanders-like potential is obtained (see e.g. Sanders 1990, Sanders 1984).

2. 2. SIMULATED ORBITS OF S2 STAR

In order to constrain the parameters of $f(R,\phi)$ model, we simulate orbits of S2 star in Sanders-like gravity potentials. We fit orbits to the astrometric observations obtained by New Technology Telescope/Very Large Telescope (NTT/VLT) (see e.g. Gilessen et al. 2009a) for different combinations of α and m_ϕ. Each simulated orbit is defined by four initial conditions: two components of initial position and two components of initial velocity in orbital plane at the epoch of the first observation. For each combination of α and m_ϕ, we obtain the best fit initial conditions corresponding to a simulated orbit. The fitting procedure is performed using LMDIF1 routine from MINPACK-1 Fortran 77 library which solves the nonlinear least squares problems by a modification of Marquardt-Levenberg algorithm (see e.g. Moré et al. 1980). Detailed descriptions are given in the papers of Borka et al. (2012, 2013).

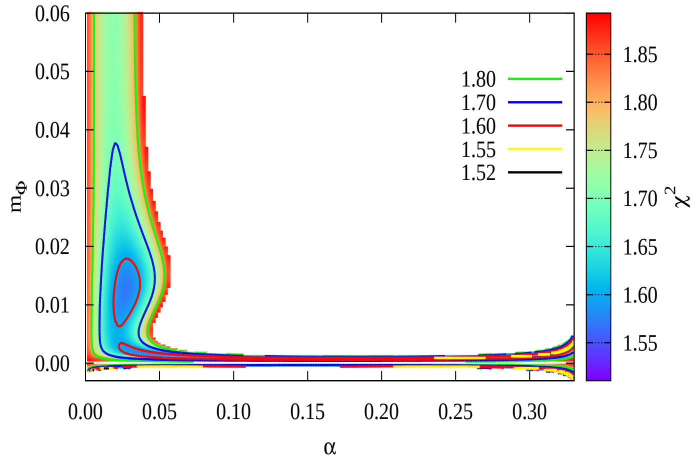

Figure 1: The map of reduced χ^2 over the $\{\alpha - m_\phi\}$ parameter space of $f(R,\phi)$ gravity in case of NTT/VLT observations of S2 star which give at least the same ($\chi^2 = 1.89$) or better fits ($\chi^2 < 1.89$) than the Keplerian orbits. The map corresponds to m_ϕ in $[0, 0.06]$ and α in $[0, 0.33]$. A few contours are presented for specific values of reduced χ^2 given in the legend.

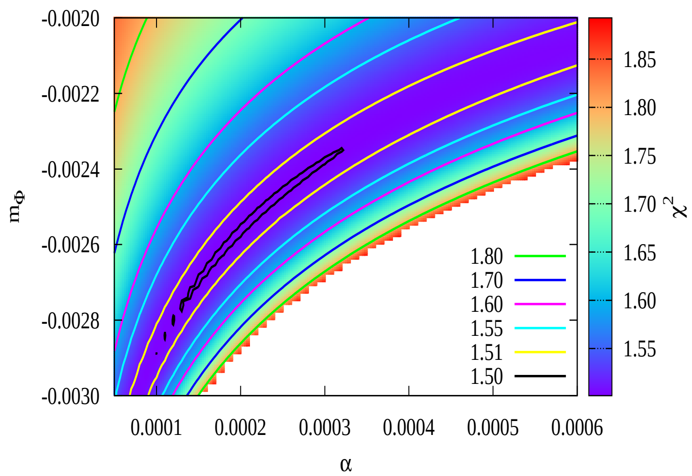

Figure 2: The same as in Figure 1, but for a narrow region in the $\{\alpha - m_\phi\}$ parameter space around the absolute minimum of the reduced χ^2. A few contours are presented for specific values of reduced χ^2 given in the legend.

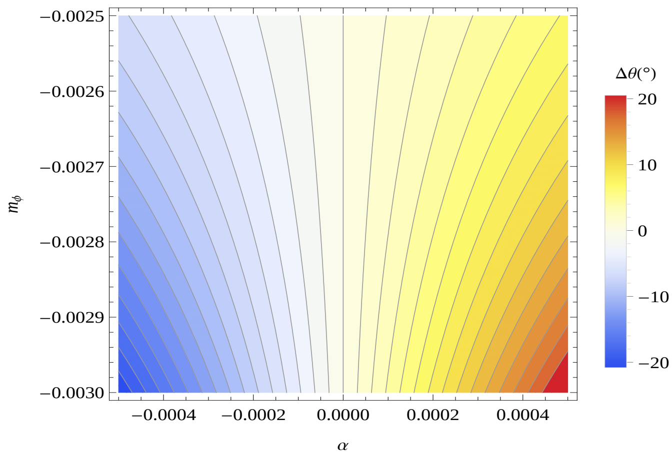

Figure 3: Numerically calculated angle of precession per orbital period as function of parameters α in the range $[-0.0005, 0.0005]$ and m_ϕ in the range $[-0.003, -0.0025]$ in case of Sanders-like potential. The pericenter advance (like in GR) is obtained for positive α, and retrograde precession for negative α.

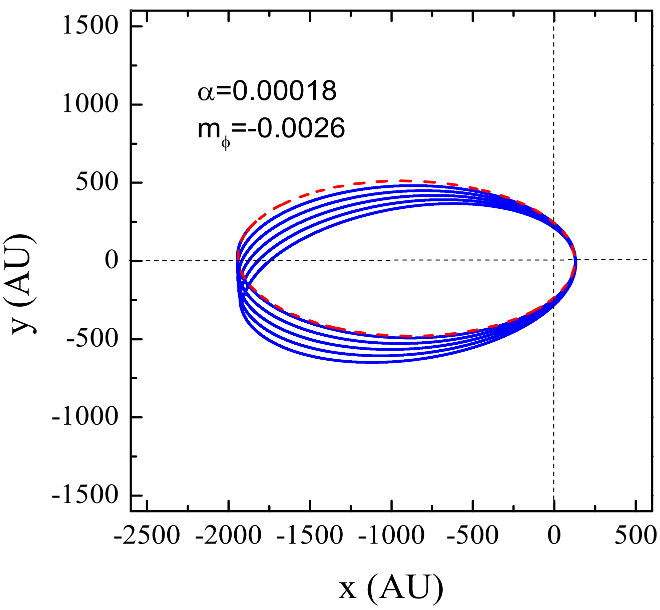

Figure 4: Comparison between the orbit of S2 star in Newtonian potential (red dashed line) and Sanders-like potential for the best fit parameters α = 0.00018 and m_ϕ = -0.0026 during 5 orbital periods (blue solid line).

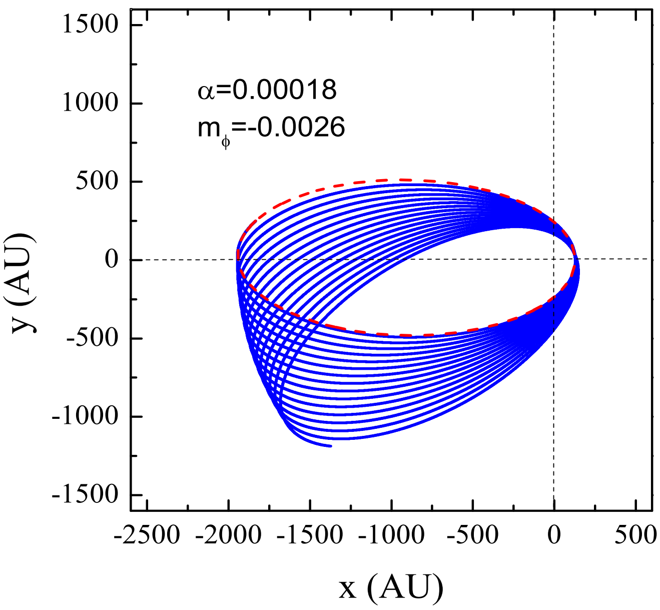

Figure 5: The same as in Figure 4, but for 15 orbital periods.

3. $f(R,\phi)$ RESULTS AND DISCUSSION

Our aim in this paper is to determine coefficients f_0, f_R, f_{RR}, f_ϕ, $f_{\phi\phi}$ and $f_{\phi R}$. For more details (see e.g. Stabile & Capozziello 2013, Capozziello et al. 2014). We obtained the following set of parameters $f_0 = 0$, $f_R = 3 - 1/\alpha$, $f_\phi = 0$, $f_{RR} = 0$, $f_{\phi R} = 1$ and $f_{\phi\phi} = -m_\phi^2$. These choices are physically reliable and mean that we can assume an asymptotic Minkowski background, i.e. $f_0 = 0$, that the General Relativity is recovered for $f_\phi = 0$, $f_{RR} = 0$, $f_{\phi R} = 1$, and effective massive modes (and then effective lengths) are related to $f_R = 3 - 1/\alpha$, and $f_{\phi\phi} = -m_\phi^2$. In particular, $f_0 = 0$ means that cosmological constant can be discarded at local scales.

Figures 1 and 2 presented the maps of the reduced χ^2 over the $\{\alpha - m_\phi\}$ parameter space in $f(R,\phi)$ gravity for all simulated orbits of S2 star which give at least the same or better fits than the Keplerian orbits ($\chi^2 = 1.89$). Figure 1 corresponds to m_ϕ in $[0, 0.06]$ and α in $[0, 0.33]$. Figure 2 corresponds to the zoomed range of parameters m_ϕ and α. For $\alpha < 0$, there is no region in the parameter space where $\chi^2 < 1.89$ (Keplerian case). For $0 < \alpha < 1/3$ there are two regions where $\chi^2 < 1.89$ (for $m_\phi < 0$ and $m_\phi > 0$), but the absolute minimum is for $m_\phi < 0$. We obtained absolute minimum of the reduced χ^2 for α in the interval $[0.0001, 0.0004]$, and m_ϕ in the interval $[-0.0029, -0.0023]$. The absolute minimum of the reduced χ^2 ($\chi^2 = 1.5011$) is obtained for $\alpha = 0.00018$ and m_ϕ = -0.0026, respectively.

Graphical presentation of precession per orbital period for α in the range $[-0.0005, 0.0005]$ and m_ϕ in $[-0.003, -0.0025]$ is given in Figure 3. As one can see pericenter advance (like in GR) is obtained for positive α, and retrograde precession for negative α. The fits better than Keplerian are obtained only for positive α, i.e. for the precession in the same direction as in GR.

The simulated orbits of S2 star around the Galactic Centre in Sanders gravity potential (blue solid line) and in Newtonian gravity potential (red dashed line) for α = 0.00018 and m_ϕ = -0.0026 during 5 and 15 periods, are presented in Figure 4 and Figure 5, respectively. We can see from Figures 4 and 5 that the best fit orbit in Sanders gravity potential precesses for about $3^\circ.1$ per orbital period. General Relativity predicts that pericenter of S2 star should advance by $0^\circ.08$ per orbital revolution (see e.g. Gillessen et al. 2009b) which is much smaller than the value of precession per orbital period in Sanders gravity potential, but the direction of the precession is the same.

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