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Modeling Emotions and Reason in Agent-Based Systems

Fernando S. Oliveira, ESSEC Business School, Oliveira@essec.edu

Abstract: We analyze how to develop an agent-based system in which agents evolve co-evolutionary endogenous rules of behavior by using best response and emotions. We show that best response is not sufficient to define complete and consistent rules of behavior and we prove that the use of emotions, which complement reason, is necessary to learn rules of behavior. We model four different emotions (apathy, patience, anger and confidence) which enable the agent to deal with the rewards and with others. We propose an algorithm to model automata-based systems incorporating rationality and emotions.

Keywords: Agent-based systems; Automata; Emotions; Evolution; Rationality.

1. Introduction

Since the foundation of social sciences with Adam Smith it has been argued that emotions are an important determinant of human behavior (Ashrat et al., 2005), even though the development of economics has been mainly focused on the rational side of behavior. Furthermore, it has been shown by neuroscience that emotions and sentiments play an important role in the way people relate to each other and in the processes defining rational behavior (Damasio, 1994; Fiedler and Bless, 2000; Capra 2004; Rosati et al., 2007). Moreover, the converse is also true, as the ability to explain rationally an event has an impact on emotions, e.g., Lazarus (1991).

For this reason, researchers in artificial intelligence are studying: the interaction between emotions and decision making (e.g., Frijda and Swagerman, 1987; Botelho, and Coelho, 2001; Breazeal, 2002); the development of agents that provide the illusion of life (e.g., Bates, 1994; Pozanski and Thagard, 2005), that can interact with humans (e.g., Nicholson et al., 1998; Picard, 2000; Murphy et al., 2002; Gmytrasiewicz and Lisetti, 2002; Marinier and Laird, 2004), that can understand the sentiments and mental processes of others (e.g., Carmel and Markovitch, 1998; Weld, 1999), and that have "moral sentiments" (e.g., Bazzan et al., 2002).

In this article we study situations in which the agents have incomplete information and, in this context, analyze how rationality and emotions are used to learn rules of behavior. We show that reason, by itself, may fail to produce rules of behavior for three motives: a) it cannot be used to chose an action when there is no information on how the opponents or the environment respond; b) it is not useful when the agent has no impact on the system through unilateral actions; c) it cannot be used to decide when to change behavior.

We test four different emotions as a way to fill the gap left open by rationality. These emotions are *anger*, e.g., Zizzo (2003) (the agent punishes or threats to punish his opponents when the outcome of the system is not desirable for him), *apathy* (the agent does not respond to the rewards received from the system), *confidence,* e.g., Rothovius (2007) (the agent does not follow the policy recommended by best response as he believes that, instead, others will adapt to his behavior), and *patience, e.g.,* Rosati et al. (2007) (the agent attempts to reward his opponents if they change their behavior in order to benefit the community). We chose these four emotions as they directly relate to the way the agents act, deal with rewards (apathy and patience) and with others (anger and confidence).

Section 2 gives a general background on automata systems. Section 3 shows how emotions are necessary, together with reason, to define rules of behavior. Section 4 presents the algorithm used to model the agent-based system. Section 5 presents the computational results and Section 6 concludes the paper.

2. Background on Agent-Based Automata Systems

A finite automaton (Definition 2.1) is a decision rule consisting of a finite set of states, a transition function (which defines the transition between states) and a behavioral function (defining how the agent behaves in each state). Each agent i holds a product automaton P^i (Definition 2.2) which specifies how the agent models his opponents behavior. An automata system (Definition 2.3) describes how agents interact, e.g. Hopcroft and Ullman (1979).

Definition 2.1: *A finite automaton* $A^i = \left(Q^i, q_0^i, \Sigma^i, Z^i, \delta^i, \lambda^i\right)$ *is a 6-tuple in which* Q^i *stands for the finite non-empty set of internal states of i,* q_0^i *is the initial internal state,* Σ^i *is a non-empty set of possible actions of agent i,* Z^i *represents a finite non-empty set of possible* *outcomes of the system,* δ^i *is a transition function* $\left(\delta^i : Q^i \times Z^i \rightarrow Q^i\right)$ *and* λ^i *is a behavioral function* $\left(\lambda^i : Q^i \rightarrow \Sigma^i\right)$ *associating one action with each possible internal state.*

Definition 2.2: *The product automaton* $P^i = \left(Q^{pi}, q_0^{pi}, \Sigma^{pi}, Z^{pi}, \delta^{pi}, \lambda^{pi}\right)$ *represents the model the agent i holds of the system.*

Definition 2.3: *An automata system is a 5-tuple* $G = \left(\mathrm{N}, \left\{\mathrm{Z}^{\mathrm{i}}\right\}_{i=1}^{N}, \left\{u^i\right\}_{i=1}^{N}, \left\{Q^i\right\}_{i=1}^{N}, \left\{\Sigma^i\right\}_{i=1}^{N}\right)$ *in which N denotes the number of agents,* $z^i \in Z^i$ *is dependent on the actions of each agent,* $z^i = z\left(a^1,...,a^i,...,a^N\right)$, $u^i = u\left(z^i\right)$ *represents the utility function of agent i, i.e., it is the payoff an agent i receives from his action,* $a^i \in \Sigma^i$ *represents the agent i's actions and, for all* $j \neq i$*, the action* $a^j \in \sum^j$ *represents his opponents' actions.*

From Definition 2.3 it follows that, in a situation with incomplete information, the outcome received by an agent is a function of his actions and it is different for each one of them, as each agent receives his own outcome. In summary, the interaction between the different automata in the system can be described as follows. At stage 1 each agent *i* plays $\lambda^i\left(q_0^i\right)$. At a stage $t \geq 1$, after receiving an outcome $z^i = z\left(a^1,...,a^i,...,a^N\right)$, the state of automaton A^i changes from the state q_t^i to the state $\delta^i\left(q_t^i, z_t^i\right)$. Then, each agent *i* chooses a new move, $\lambda^i\left(q_{t+1}^i\right)$.

In this article we define rationality as the ability to compute the optimal policy by best response, Definition 2.4. In automata systems this is achieved by the computation of the *best response automaton*, i.e., a finite automaton that maximizes the total of discounted rewards, e.g. Gilboa (1988) and Banks and Sundaram (1990). In this optimization problem the decision variables are agent *i's* actions in each state of the product automaton.

Definition 2.4: *Let* ρ_i^j *represent agent's i discount factor at time j. In an evolutionary N-agent automata system each agent aims to maximize the present value of his expected utility, i.e.,* $V_t^i = \sum_{j=t}^{+\infty} \rho_i^j E_t\left(u_j^i\left(z_j^i\right) \backslash A_t^i\right)$*, by choosing his automaton,* $A^i = \left(Q^i, q_0^i, \Sigma^i, Z^i, \delta^i, \lambda^i\right)$.

3. Rationality and Emotions (Anger, Apathy, Confidence, and Patience)

Next, we define closed automaton (Definition 3.1) which, as shown in Proposition 3.1, does not follow from rational choice, i.e., best response.

Definition 3.1: *A finite automaton* $A^i = \left(Q^i, q_0^i, \Sigma^i, Z^i, \delta^i, \lambda^i\right)$ *is closed if for every state in* Q^i *and every possible reward* $z_t^i \in Z^i$ *there is a transition function* $\delta\left(q_t^i, z_t^i\right)$ *defining the subsequent state of the automaton.*

Proposition 3.1: *In an automata system with incomplete information best response builds an automaton that is not closed.*

Proof. Let $P^i = \left(Q^{pi}, q_0^{pi}, \Sigma^{pi}, Z^{pi}, \delta^{pi}, \lambda^{pi}\right)$ represent agent *i*'s product automaton. From best response, Definition 2.4, it follows that for each state $q_t^{pi} \in Q^{pi}$ we choose an action a_t^i such that $S^i = \arg\max_{a_t^i}\left\{u_t^i\left(\lambda^{pi}\left(q_t^{pi}\right)\right) + \rho_i V_{t+1}^i\left(\delta^{pi}\left(q_t^{pi}, a_t^i\right)\right)\right\}$. Hence, for each $q_t^{pi} \in Q^{pi}$ only the optimal action a_t^i is defined, and therefore the automaton is incomplete.■

There are two main reasons why we need to use emotions to produce rules of behavior: we need them to close the automaton (as proved in Proposition 3.1) and to improve coordination among agents when best-response leads to undesirable equilibria that cannot be avoided by unilateral actions (i.e., the avoidance of this equilibrium requires behavior coordination among agents).

Let us start by analyzing three different emotions (*apathy*, *patience* and *anger*) that are sufficient to close an automaton (we call this the closing function) and can be used when best response fails to avoid undesirable equilibria (we call this the adaptation function). It is assumed that the adaptation function is applied after best response and the closing function have been used.

When performing the closing function, the emotions are used as follows: a) *Apathy*, $\delta\left(q_j^i, z_j^i\right) = q_j^i$, in this state, the automaton always plays the same action independently of the rewards received. b) *Patience*, $\delta\left(q_j^i, z_j^i\right) = q_m^i$, in which q_m^i is such that $\lambda\left(q_m^i\right)$ maximizes his opponents' rewards. c) *Anger*, $\delta\left(q_j^i, z_j^i\right) = q_M^i$, where q_M^i is a state such that $\lambda\left(q_M^i\right)$ aims to minimize the opponents' rewards.

The emotions' adaptation function can be summarized in the following way: a) *Apathy*, the agent keeps the same behavior $\lambda_{t+1}^i\left(q_j^i\right) = \lambda_t^i\left(q_j^i\right)$ independently of the rewards received. b) *Patience*, the agent chooses $\lambda_{t+1}^i\left(q_j^i\right) = \lambda\left(q_m^i\right)$, increasing the possible rewards of his opponents. c) *Anger*, the agent punishes his opponent's deviation by choosing actions that minimize their rewards, $\lambda_{t+1}^i\left(q_j^i\right) = \lambda\left(q_M^i\right)$.

Another issue that cannot be solved by best response alone arises when an agent chooses between adapting to others and waiting for others to adapt to his behavior: each agent decides between following best response (optimizing his behavior to his perceptions P^i of the automata held by his opponents) or waiting for the other agents to adapt to his behavior. We propose that a fourth emotion (*Confidence*, Definition 3.2) is important in this case to guide the agent's actions, allowing him to gain credibility (as in Crandall and Goodrich, 2004), and to impose the agent's behavior on others, as best response behavior (if leading to frequent automaton switching) decreases the agent's credibility.

Definition 3.2: *Confidence*: *an agent i keeps the same automaton, i.e.,* $A_t^i = A_{t-1}^i$, *not adapting by best response, i.e.,* $A_{t,BR}^i = BR\left(P_t^i\right)$.

Let $z_j^i \setminus BR(P_t^i)$ and $\zeta_j^i \setminus A_t^i$ represent the outcomes received by agent *i* at iteration *j*, respectively by following best-response (i.e., rationality) and by keeping the same automaton (i.e., by being *confident* about his current strategy). Proposition 3.2 shows that, under certain circumstances, confidence is necessary for maximizing the present value of the utilities.

Proposition 3.2: *Confidence is a necessary condition for an agent i to maximize* $V_t^i = \sum_{j=t}^{+\infty} \rho_i^j E_t\left(u_j^i \setminus A_t^i\right)$ *if the expected payoff of playing best-response is not higher than the expected payoff of keeping the same automaton, i.e.,* $\sum_{j=t}^{+\infty} \rho_i^{j-t} E_t\left(u_j^i\left(z_j^i \setminus BR(P_t^i)\right)\right) \leq \sum_{j=t}^{+\infty} \rho_i^{j-t} E_t\left(u_j^i\left(\zeta_j^i \setminus A_t^i\right)\right)$.

Proof. Let $A_{t,BR}^i$ stand for the best-response strategy against the product automaton P_t^i, $A_{t,BR}^i \equiv BR\left(P_t^i\right)$. Assume that $A_{t,BR}^i \neq A_t^i$, then agent *i* expects $\sum_{j=t}^{+\infty} \rho_i^{j-t} E_t\left(u_j^i\left(z_j^i \setminus A_{t,BR}^i\right)\right) > \sum_{j=t}^{+\infty} \rho_i^{j-t} E_t\left(u_j^i\left(z_j^i \setminus A_t^i\right)\right)$, i.e., he expects best response to lead to increased utility. By keeping A_t^i he enhances the reputation of his behavior, as for any agent $j \neq i$ the product automaton P_t^j depends on the automata used by *i*. If these automata are stable then P_t^j is stable as well. Then, agent *j* chooses $A_{t,BR}^j = BR\left(P_t^j\right)$ adapting his behavior to agent *i*'s automaton, A_{t-1}^i, changing the outcomes received by agent *i* from z_t^i to ζ_t^i, and increasing *i*'s present value of utility as $\sum_{j=t}^{+\infty} \rho_i^{j-t} E_t\left(u_j^i\left(z_j^i \setminus BR(P_t^i)\right)\right) \leq \sum_{j=t}^{+\infty} \rho_i^{j-t} E_t\left(u_j^i\left(\zeta_j^i \setminus A_t^i\right)\right)$. ■

4. The Evolutionary Automata System and its Properties

We now present the evolutionary automata system summarized in Table 4.1.

Table 4.1. The Evolutionary Automata System

Consider the following notation: ρ_i: Discount factor for agent *i; D^i*: Data collected by agent *i* during a cycle of interactions with others; a_t^i: An action of agent *i* at stage *t*; u_t^i: The reward of agent *i* at stage *t*; z_t^i: The outcome of the system at stage *t.*

While the last iteration is not reached

Step 1. Simulate a cycle of interactions

Each agent makes a move given his current automaton

$$a_t^i = \lambda^i\left(q_t^i\right)$$

Compute the new state and the reward of each agent

$$q_{t+1}^i = \delta^i\left(q_t^i, z_t^i\right)$$

$$u_t^i = u^i\left(z_t^i\right)$$

Step 2. For each agent *i* infer the opponents' behavior, $P_t^i = P\left(D_t^i\right)$

Step 3. For each agent *i* compute the new automaton using RE(A^i, P^i, V^i, ρ_i), Table 4.2.

In step 1 we simulate the interaction between the different automata used by the agents. In step 2, each agent models the current behavior of his opponents: we assume the agents have the ability to learn the correct model; Oliveira (2009) compares three possible algorithms that can be used in this task. In step 3, each agent, using the Rationality & Emotions (RE) algorithm presented in Table 4.2, computes the strategy to play against his opponents.

Table 4.2. Rationality & Emotions (RE) Algorithm, RE(A^i, P^i, V^i, ρ_i)

Consider the notation: A^i: The current agent i's automaton, $A^i = \left(Q^i, q_0^i, \Sigma^i, Z^i, \delta^i, \lambda^i\right)$;
P^i: The product automaton; ρ_i: Discount factor for agent i, $0 \le \rho_i \le 1$; S^i: Optimal policies generated by the automaton A^i; $V_{i,t}^*$: Optimal value of the automaton used by an agent i at time t.

Repeat for each agent:

Step 1. If the agent has *confidence* in his current automaton

Return $A_t^i = A_{t-1}^i$ and terminate

Otherwise: go to Step 2

Step 2. Compute the Algorithm Best-Response $BRA_t^i = BR\left(P_t^i\right)$:

2.1 Compute S^i the optimal policy play against P_t^i:

$$S^i = \arg\max_{a_t^i}\left[u_t^i\left(\lambda^{pi}\left(q_t^{pi}\right)\right) + \rho_i V_{t+1}^i\left(\delta^{pi}\left(q_t^{pi}, a_t^i\right)\right)\right]$$

$$s.t.$$

$$q_{t+1}^{pi} = \delta^{pi}\left(q_t^{pi}, a_t^i\right)$$

$$q_1^{pi} = q_0^{pi}$$

2.2 Compute A^i from the optimal policy S^i

Let g represent the optimal policy such that

$g: Q^{pi} \to S^i$ and $S^i = g\left(Q^{pi}\right)$

The automaton A^i has the same number of states as p_t^i, $Q^i = Q^{pi} \Leftrightarrow q_t^i = q_t^{pi}$

Compute the initial state and action of A^i, $q_0^i = q_0^{pi}$

Compute the behavior function: assign an action to each state of A_t^i,

$$\lambda^i\left(q_t^i\right) = g\left(q_t^{pi}\right)$$

Compute the transition function, $\delta^i\left(q_t^i, \lambda^{pi}\left(q_t^{pi}\right)\right) = \delta^{pi}\left(q_t^{pi}, \lambda^i\left(q_t^{pi}\right)\right)$

Step 3. Complete the rule using the closing function of the emotions anger, apathy, and patience

$$CA_t^i = Closing\left(BRA_t^i\right)$$

Step 4. Apply the adaptation function of the emotions anger, apathy, and patience

$$A_t^i = AO\left(CA_t^i\right)$$

The RE algorithm starts in Step 1 by applying the confidence emotion, Definition 3.2. Let θ^i represent the *confidence decay* such that $0 < \theta^i < 1$, and I_t^i represent the probability of using confidence, $I_t^i = \begin{cases} 1 & \Leftarrow A_t^i \neq A_{t-1}^i \\ \theta^i I_{t-1}^i & \Leftarrow A_t^i = A_{t-1}^i \end{cases}$. This means that an agent, after changing his behavior, waits for the other agents to adapt to his new automaton, changing strategy with a probability (1-I_t^i). Step 2: in step 2.1, having decided to change strategy, an agent computes his best response to the product automaton P_t^i. In step 2.2 the agent builds his new automaton by making a one-to-one correspondence between his new automaton, A_t^i, and the product automaton P_t^i. This correspondence defines the transition function and the behavioral function for A_t^i that implement the optimal policy to play against P_t^i. Step 3: the algorithm closes the automaton using the emotions: *anger*, *apathy*, and *patience*, getting $CA_t^i = Closing(BRA_t^i)$. Step 4: if the agent perceives that best response is not enough to avoid an undesirable state, he can change the closed automaton by using the emotions *anger*, *apathy*, and *patience*, and deriving $A_t^i = AO(CA_t^i)$.

5. Simulating the Agent-based Evolutionary Pie Sharing Problem

We illustrate the concepts presented in this article using the pie sharing problem: each agent can bid for a given number of pieces of pie, receiving the number of slices requested if the total number of bids by all the agents is less than or equal to the number of slices available (and obtaining an utility equal to the number of requested pieces). Otherwise, if the total number of pieces requested in total is greater than the total number of slices available then no agent gets any piece of pie (obtaining a utility equal to zero).

We exemplify this game with five agents and a number of pieces of the pie equal to eight and ten, in two different scenarios. The parameters are the number of possible slices an agent can ask for, $\Sigma^i = \{1,2,3\}$, the discount factor ρ^i = 0.9, and the confidence decay θ^i= 0.9. We simulate nine different scenarios for the possible combination of emotional states for closing and adaptation. In the experiments presented in Table 5.1 the closing and adaptation functions interact with the number of pieces available. In Table 5.1.a (with ten pieces) the null partition (the undesirable state) is not Nash equilibrium; however, in Table 5.1.b (with eight pieces) the null partition is Nash equilibrium.

Table 5.1: Average Number of Pieces of Pie Taken by the Agents

		Adaptation					
		ten pieces in total (5.1.a)			eight pieces in total (5.1.b)		
		apathy	anger	patience	apathy	anger	patience
Closing	apathy	9.8	4.4	9.8	3.9	2.7	7.8
	anger	9.8	0.1	9.6	3.9	0	7.8
	patience	9.1	4.9	5.6	0.2	0.1	2.3

In these simulations patience performed well in its adaptation function, while anger performed poorly in adaptation (anger only works if others are patient and accommodate to the agent's demands). However, anger worked well in its closing function, as the threat of punishment convinces others not to deviate from equilibrium. The best emotion for closing is apathy as it almost always dominates anger and patience (except in the case of ten pieces of pie when the best adaptation is achieved by anger).

Furthermore, we analyze the conditions under which the system converges on the Nash equilibrium. Figure 5.1 shows the bids towards which the system converged on, for each agent, in the experiment with eight slices of pie, and in the scenario in which each agent applies patience for adaptation and apathy for closing (one of the two best combinations).

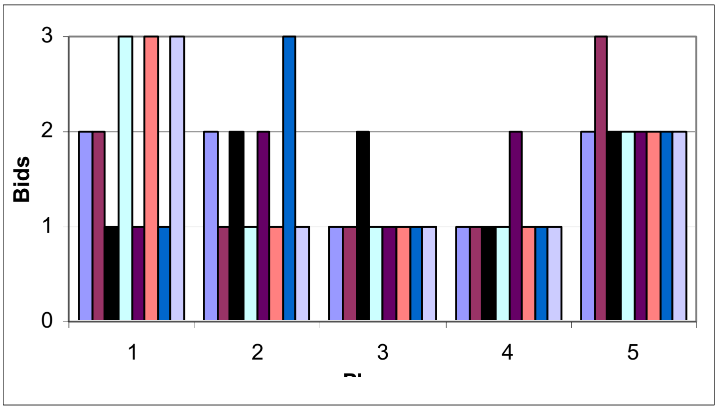

Figure 5.1: Automata Behavior: the bidding policy to which the system converged on for each agent (1 to 5).

As depicted in Figure 5.1, this system converged on a cycle of play of length eight, showing that the strategies emerging from these simulations are complex and allow all the agents to perform better than they would with a simple one state automaton (in this case some of the agents would receive one piece of the pie only, in equilibrium), see Figure 5.2.

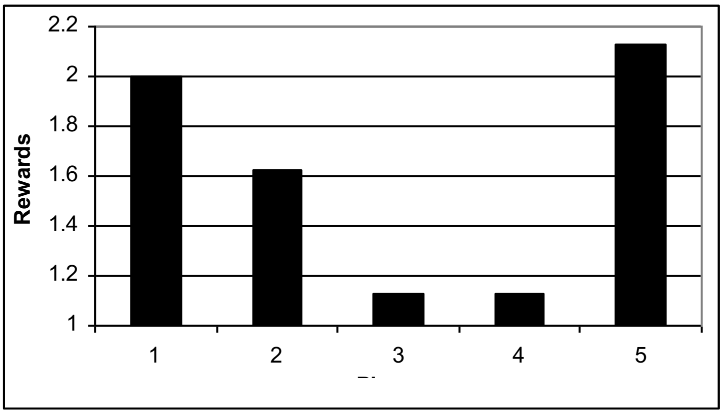

Figure 5.2: Average Reward per Agent: represents the average reward received by each agent (1 to 5) within the cycle toward which the system converged on.

As illustrated in Figure 5.2, the system converged on a solution where the minimum average reward was 1.12 pieces, and therefore all the agents received more than one piece each, on average. Moreover, this result is Nash equilibrium as the total number of bids is equal to eight pieces of pie available (Figure 5.1) and, therefore, no agent can increase his utility by unilaterally changing his bids.

Finally, Figure 5.3 reports the results for the experiment with ten pieces. It shows that the relationship between confidence decay, θ^i, and performance is non-linear. For low levels of confidence (low confidence decay) the agent tends to adapt too fast to his opponents' behavior, even when to wait would be a better option. Moreover, high confidence gives time for the agents to coordinate better their behavior, improving the overall performance of the agents. Excessive confidence (very high confidence decay) prevents the agent from attempting to optimize his behavior by adapting to others. This shows that there is an optimal level of confidence decay that is required to improve the performance of the agents.

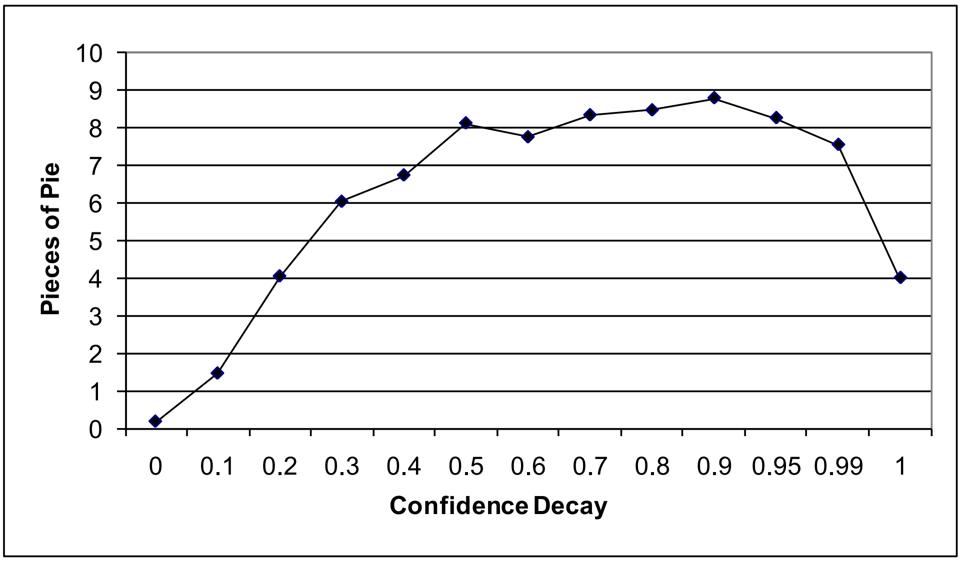

Figure 5.3: Total Number of Pieces Received by All the Agents, on Average.

6. Conclusion

In this paper we analyze how to develop an agent-based system in which the agents are endogenous automata that co-evolve by interacting with each other. At a behavioral level, we show that rationality is not sufficient for the maximization of the long-term discounted utilities and that emotions are required for the construction of behavioral rules and for the improvement of the long-term performance of the agents. We show that best response, by itself, cannot be used: a) To optimize the behavior of the agent when there is no information on the opponents' behavior; b) To choose actions when these do not have an impact on the state of the system; c) To decide between using the strategy that best fits the others' behavior and waiting for others to adapt to the agent's behavior.

Finally, through the use of simulation we illustrate how emotions, such as anger, apathy, and patience, can have a positive impact on the agents' performance when used correctly. The fourth emotion analyzed, confidence, is also important for improving the overall performance of the agents. The simulations suggest that there is an optimal level of confidence decay.

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