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Strategic Procurement in Spot and Forward Markets Considering Regulation and Capacity Constraints

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Abstract: With the generalization of business-to-business electronic exchanges, online spot markets have become an important component of suppliers' procurement strategies in their aim to increase flexibility and reduce transaction costs. In this article we analyze, both analytically and computationally, how these online spot markets interact with forward contracts as strategic procurement tools. We consider non-storable commodity markets in which the suppliers have market power. We derive the equations describing the equilibrium of this game considering capacity constraints and regulation. We show that price caps increase forward trading and we analyze the conditions under which, in the capacitated model, some suppliers can *buy* forward to *sell* spot. Furthermore, we prove that inefficient producers continue to operate in the market as arbitrageurs, selling forward and buying spot. We model the game with asymmetric suppliers, identifying the situations in which it is well defined, and describing how these asymmetries are important for market equilibrium. Finally, we analyze a game with multiple sequential forward contracts: we prove that, when suppliers readjust their forward positions until the start of the spot market, the number of time periods (i.e., market liquidity) has neither effect on the suppliers' strategic procurement nor on market efficiency.

Keywords: Supply Chain Management, Forward Contracts, Oligopoly, Procurement, Regulation, Spot markets.

1. Introduction

The procurement problem is gaining importance in the management of the supply chain, as the development of new markets, the design of complex types of contract, and real-time information, have contributed to an increase in the procurement strategies available to firms. For example, with the development of electronic exchanges in which commodities are traded real time (e.g., Teich et al. 2006), business-to-business electronic procurement (e.g., De Boer et al., 2002; Chen and Liu, 2007) has become an essential instrument to reduce transaction costs and increase managerial flexibility (e.g., Seifert et al., 2004).

Such electronic markets have become central to the supply chain operations: for example, these days we have online markets for agriculture products (cereals, coffee, cotton, livestock, soybeans, etc.), metals (copper, gold, and silver), energy (electricity, gas and petroleum), carbon emissions trading, and freight services. These markets allow the trade to occur from a few months (even years), in the case of the futures markets, to ½ hour ahead delivery (such as in wholesale electricity markets). In general, the availability of new markets and trading opportunities give more flexibility to the firms involved in the supply chain, as they can choose to trade via bilateral contracts, future contracts or on spot markets. A question raised by the development of these markets is how firms should modify their procurement strategies to account for the interactions between the different trading opportunities. From the regulators' perspective, we need to understand how the interaction between forward contracts and spot markets affects spot prices, production, and the impact of the regulation instruments: for example, it is crucial to understand if continuous trading in forward contracts increase market efficiency, i.e., helps in delivering lower prices, or not. Greenstone (1981) explains how some coffee producing countries, in the 1970s, used futures markets to control prices, buying their own production, taking delivery, as one of the instruments in their attempt to control coffee prices. A different view on this issue was taken by Allaz and Vila (1993) who explain how the introduction of a futures and forward markets could actually increase overall efficiency leading to lower prices.

In order to address these issues we need to explain how spot markets (in which commodities are traded over the internet or using specialized exchanges very close to delivery time) interact with bilateral contracts (e.g., Dyer and Ouchi, 1996; Cohen and Agrawal, 1999, Chen and Paulraj, 2004) to improve coordination in supply chains (e.g., Lariviere and Porteus, 2001; Cachon, 2003, Cachon and Lariviere, 2005; Oliveira et al., 2013) in the strategic procurement of commodities (e.g., Antelo and Bru, 2002; Seifert et al., 2004; Haksoz and Seshadri, 2007). This issue has several facets which include the relationships between: spot trading and long-term contracts, where the problem of contracting with a long-term supplier vs. buying in the spot market is analyzed (e.g., Akella et al., 2001; Serel et al., 2001; Bonser and Wu, 2001; Chen and Liu, 2007); spot and forward contracting (e.g., Antelo and Bru, 2002; Wu, Kleindorfer, and Zhang, 2002; Seifert et al., 2004; Gulpinar and Oliveira, 2012, 2014; and Oliveira et al., 2013) that aims to explain the conditions under which a firm should contract in the spot market, trade in a futures market (an organized exchange for trading contracts on the future delivery of commodities in specific spot markets) or forward contract directly with a specific supplier or buyer using bilateral contracts.

In analyzing the relationship between forward contracts and spot markets, in the context of oligopolies, Vila (1992), and Allaz and Vila (1993) have modeled endogenous prices, showing that forward contracts increase competition between producers, reducing prices and profitability. Su (2007) has extended Allaz and Vila (1993) by analyzing the equilibrium of the game when considering asymmetrical linear cost functions and the non-negative constraint for production and futures sales. A modification to the Allaz-Vila (1993) model has been introduced by Thille (2003) to include inventories, showing that, in this case, prices tend to be higher than in the Allaz-Villa model, but lower than in the Cournot model without forward trading. The relationship between forward contracts and spot markets has been tested empirically by Herguera (2000) who found evidence supporting the hypothesis that, given a concentrated market structure, the introduction of forward contracts leads to lower spot prices. This thesis has also been corroborated by Le Coq and Orzen (2006) in laboratory experiments. Nonetheless, this relationship is still controversial: Mahenc and Salanie (2004) have shown that, in a Bertrand duopoly, suppliers buy their own production forward, increasing equilibrium prices, when compared to the scenario without forward trading; and Antelo ana Bru (2002), have proved in a Cournot game with a dominant supplier competing with a price taking fringe, that the dominant supplier buys in the forward markets; both of which explaining the behavior of the coffee producing countries described in Greenstone (1981). Gulpinar and Oliveira (2012, 2014) have analyzed the impact of risk aversion on the quantity traded forward and on the equilibrium prices an oligopoly, based on an electricity market, and considering demand and cost uncertainty. Oliveira et al. (2013), have analyzed the relationship between forward and spot prices in the context of the Spanish electricity market, considering the interaction between wholesale and retail markets, under pool based and bilateral contracts, proving the existence of multiple equilibria.

In this article we analyze the strategic procurement problem, focusing on the relationship between forward contracts and spot markets in oligopolistic industries of non-storable commodities (e.g., electricity and freight markets), taking only into account the strategic component of this interaction, as in Allaz and Vila (1993), Antelo and Bru (2002), Su (2007), Erhun et al. (2008), Oliveira et al. (2013) and Gulpinar and Oliveira (2012, 2014). The analysis of the strategic component aims to explain why firms trade in forward contracts even in the absence of uncertainty.

In a similar context to ours, strategic procurement has been analyzed by Erhun et al. (2008) and Oliveira et al. (2013), who both concluded, as in Allaz and Vila (1993), that the end consumers benefit from multiple trading opportunities, and by Antelo and Bru (2002) who have shown that the presence of a dominant supplier competing with a price taking fringe can lead to a reduction in total production, an increase in prices, as the dominant firm *buys* in forward markets.

The specific questions we address in this article are the following: In the absence of risk considerations, why would suppliers engage forward trading when there are spot markets? How much would they trade in each market? How would regulation, capacity constraints and the liquidity of forward markets influence suppliers' behavior? These questions are important as they are at the core of the explanation for the strategic component of procurement (which is complementary to its hedging role), as in Su (2007), Erhun et al. (2008) and Oliveira et al. (2013).

This article extends the two-period model in Allaz and Vila (1993) by deriving the equilibrium solution of the two-stage model with N asymmetric players, capacity constraints and regulatory instruments, such as price caps and production controls. We show that technological asymmetries influence the number of firms that can enter the industry, their production, and the proportion traded forward (in the uncapacitated case). We prove that, under capacity constraints, a possible optimal procurement strategy for some suppliers is to *buy* in the forward market and to sell in the spot market (as described in Greenstone, 1981); additionally, we show that inefficient players can still act as arbitrageurs, selling forward and buying in the spot market, and that the presence of price caps increases forward trading.

Moreover, we analyze the implications, in the multiple-period model, of removing the assumption of binding pre-commitment in forward contracts. We show that, in this case, the number of time periods has no effect on equilibrium prices and, therefore, has no impact on the suppliers' profitability. This result complements the conclusions in Allaz and Vila (1993) and Erhun et al. (2008), which were derived for forward contracts with binding pre-commitments. Our result is representative of the workings of real forward and futures markets where firms can continuously revise their positions, and it is coherent with the behavior of real forward markets in which continuous trading has failed to deliver perfect competition, as in electricity markets (e.g., Wolfram, 1999; Bunn and Oliveira, 2016; Nalan and Oliveira, 2012, 2014; Oliveira et al. 2013), telecommunications (e.g., Antelo and Bru, 2002), and freight services (e.g., Fox, 1994; Sys, 2009), contrary to the predictions in Allaz and Vila (1993), as suppliers of these services revise their forward positions up to the time of delivery. In this article, as we prove that the T period problem, in the absence of uncertainty, is equivalent to the 2-period problem, we use the latter as the working framework in all other issues. Finally, this article also extends Su (2007), who has centered on the two-stage model with non-negative constraints for production and futures trading, by considering both capacity constraints on production and by analyzing the introduction of price caps in the spot market.

This article proceeds as follows. Section 2 presents the basic model analyzed in this article. Section 3 analyzes the strategic procurement problem with asymmetric players. Section 4 extends the model to include regulation and capacity constraints. Section 5 illustrates some of these findings using computer simulations. Section 6 concludes the article.

2. Modeling the Relationship between Spot Markets and Forward Contracts

We use the following notation: T, the length of the planning horizon; t, any given period in the planning horizon; $Q_{0,i}$ total production of supplier i; $f_{t,i}$, total forward trade at period t by supplier i; $F_{t,i}$, total forward trade from period T to period t by supplier i, $F_{t,i} = \sum_{k=t}^{T} f_{k,i}$; P_t, equilibrium forward price at period t, $t > 0$, P_0, equilibrium spot price; C_i, supplier i's marginal cost; Π_i, supplier i's total profit during the planning horizon.

The spot trading, and production, occurs at time zero and the forward trading occurs at time 1 (for the two stage model), and at times t= 1, …, T for the T stage model (occurring t periods ahead of the production period). The game starts in period T when firms decide how much to contract forward for the production to be delivered at time zero. Then, it moves on to period T-1, when firms re-assess their positions in the forward contracts, given their expectations on how the market will behave at time zero, and so on. The last opportunity to revise the forward contracts is at time 1. Then, at time zero, all the spot trading takes place and production occurs. From a mathematical perspective, the process of finding a solution of such game starts at time zero, when we compute the equilibrium production and spot trading for each firm. Then, given these equilibrium outcomes, we compute the equilibrium forward trading at time 1, then at time 2, and so on, and finally, the forward trading at time T. In the absence of arbitrage opportunities, the transaction price in each period is equal to the spot price.

In our model, as there is no binding pre-commitment in forward contracts, the suppliers are allowed to sell and buy in the forward and, for this reason, both $f_{t,i}$ and $F_{t,i}$ can be positive or negative: this is not the case of the Allaz and Vila (1993) model in which the binding pre-commitment in the forward contracts imposes positive $f_{t,i}$, for all i and t. The total profit earned during the planning horizon is represented by (2.1), in which $C_i Q_{0,i}$ is the variable cost, and $Q_{0,i} - F_{1,i}$ stands for the quantities traded in the spot market.

$$\Pi_i = P_0\left(Q_{0,i} - F_{1,i}\right) - C_i Q_{0,i} + \sum_{t=1}^{T} P_t f_{t,i} \tag{2.1}$$

Moreover, let $u_{0,i}$ represent the spot profit, (2.2), $u_{t,i}$ represent the forward profit at time *t*, (2.3), for supplier *i*, and (2.4) represents the inverse demand function at period zero, where $Q_0 = \sum_{i=1}^{N} Q_{0,i}$, and *N* stands for the number of suppliers.

$$u_{0,i} = P_0 \left(Q_{0,i} - F_{1,i} \right) - C_i Q_{0,i} \tag{2.2}$$

$$u_{t,i} = P_t f_{t,i}, \; \forall t = 1,\ldots,T \tag{2.3}$$

$$P_0 = a - bQ_0 \tag{2.4}$$

Let M_i and K_i represent, respectively, the lower and the upper bounds on the production of supplier *i*, (2.5), as illustrated by Figure 2.1, and let $\underline{P}_0$ and $\overline{P}_0$ represent, respectively, the lower and the upper cap on the spot price, (2.6), as depicted in Figure 2.2.

$$Q_{0,i} \geq M_i \qquad \text{and} \qquad Q_{0,i} \leq K_i \tag{2.5}$$

$$P_0 \geq \underline{P}_0 \qquad \text{and} \qquad P_0 \leq \overline{P_0} \tag{2.6}$$

There are several factors that explain the minimum generation constraint, which can arise from technical specifications of the production plants, from legislation forbidding layoffs, etc. The maximum production constraint may arise from technical limits to production. The maximum price cap tends to be imposed by regulation, as it is the case of “essential foods” such milk and bread, or “strategic goods and services” such as electricity and combustibles, in several countries. The minimum price cap can arise when the firm does not want to damage the image of its product or service by lowering prices, such as in hotel chains, for example. The demand function is, for these reasons, non-linear.

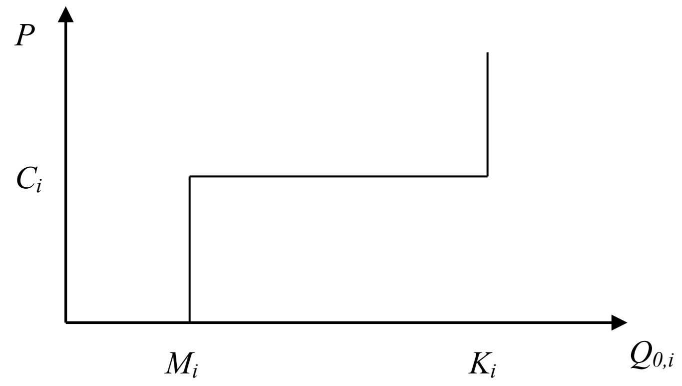

Figure 2.1: **Firm *i*'s production function.**

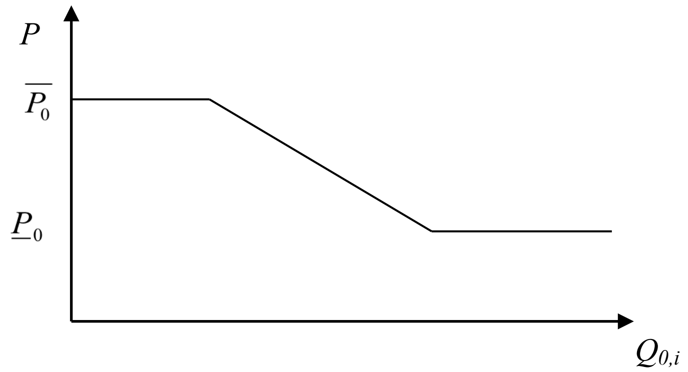

Figure 2.2: **Demand function.**

3. Strategic Procurement with Asymmetric Suppliers

In this section we analyze the asymmetric strategic procurement in oligopolistic commodity markets, which is essential to model industries in which there are different production technologies (such as electricity or freight services, for example), deriving the equilibrium solution of the game and analyzing its properties (a similar model, for the symmetric case, has been presented in Allaz and Vila, 1993).

In this section the procurement problem without considering the price and capacity constraints is analyzed. This is useful as the results are comparable with the literature on the topic and it allows the derivation of the main result that relates the two-stage with the multiple stages when there is no pre-commitment within forward transactions.

The solution of the two-stage asymmetric dynamic game proceeds by backward induction. First the suppliers choose the optimal policy at time zero, then they choose the optimal policy at time one (taking into account the optimal policy at time zero). The equilibrium productions, forward trading and price are represented by (3.1)-(3.3), where $\overline{C}$ stands for the average marginal cost of the plants in the industry. [The derivation details can be found in the appendix].

$$Q_{0,i} = \frac{N}{b}[P_0 - C_i] \tag{3.1}$$

$$F_{1,i} = \frac{(N-1)}{b}[P_0 - C_i] \tag{3.2}$$

$$P_0 = \frac{a + N^2\overline{C}}{N^2 + 1} \tag{3.3}$$

Let us now analyze the basic properties of the *N*-suppliers game represented by the equilibrium equations (3.1)-(3.3). These results, for the duopoly (N = 2) in the symmetric case ($C_i = C$), are the same as in Allaz and Vila (2003).

Let N^+ the maximum (integer) number of firms that can enter the industry, then Proposition 3.1 follows.

Proposition 3.1: *Equations* (3.1)-(3.3) *are valid only when the number of suppliers is such that* $N^+ < \sqrt{\dfrac{a - C_i}{C_i - \overline{C}}}$. [Proof in Appendix.]

Proposition 3.1, and in opposition to the symmetric case, states that this model is applicable only when the maximum number of firms N^+ who can operate in the industry is respected. Moreover, the higher the asymmetry (measured by the difference between the maximum and average marginal costs) the lower the number of firms allowed in the industry. This result is important as, in the absence of capacity constraints, and contrary to the symmetric case, we cannot have an infinite number of firms entering the industry and, for this reason, for every firm in the industry, the level of production and forward contracting is always positive.

Proposition 3.2. If *N is at its maximum,* N^+*: a)* $P_0 \geq C_i$*; b)* $Q_{0,i} \geq 0$*; c)* $F_{1,i} \geq 0$. [Proof in Appendix.]

The main argument supporting Proposition 3.2 is that firms cannot produce negative quantities and, for this reason, we cannot use an analysis ad infinitum to study the implications of new entry on individual performance. From Proposition 3.2 we conclude that, contrary to the symmetric case, the production of the individual firms does not converge to zero with the entry of new firms: this production is actually higher for the most efficient firms and the market price tends to converge, from above, on the highest marginal cost, which means that firms in an industry with heterogeneous technologies remain profitable.

We now examine the asymmetric *T*-stage game (played from time zero to time *T*), for the case in which the suppliers are allowed to re-adjust their forward positions. This analysis differs from Allaz and Vila (1993) as they assume that the forward positions are binding pre-commitments within the forwards market. (We assume that positions in forward contracts are revisable as this is the case of the electricity and freight forward agreements). In mathematical terms this means that the $f_{t,i}$ in the Allaz-Vila model are always *positive* (a supplier can only buy in the forward market) whereas we have unconstrained $f_{t,i}$, which means that the suppliers can both buy or sell using forward contracts. Proposition 3.3 shows that, in this case, time, i.e., the number of sequential forward contracts, has no impact on prices, production and forward trading. These results are corroborated by real markets such as electricity markets (e.g., Wolfram, 1999; Bunn and Oliveira, 2008), telecommunications (e.g., Antelo and Bru, 2002), and freight services (e.g., Fox, 1994; Sys, 2009), where the continuous forward trading does not lead to perfect competition.

Proposition 3.3: *When there is no precommitment within forward transactions*, *in the absence of uncertainty, and under perfect foresight, the Nash-equilibrium in the T-period strategic procurement game is the same as in the* 2*-period game.* [Proof in Appendix]

This result complements the analysis by Allaz and Vila (1993), and Erhun et al. (2008) and it is important to explain why, in general, the increase in the number of opportunities of forward trading does not improve consumer welfare, as is the case in most markets in which such contracts are available. Therefore, the enforcement, or not, of binding pre-commitments is essential to determine the impact of forward contracts on the equilibrium solution of the industry. Our proof, available in the appendix, was produced by backward induction, in order to allow a full comparison of both results. Our analysis shows that the number of trading periods in the forward market is not strategically important to increase social welfare in the absence of pre-commitment in these trades. This result has major implications for market regulators as in markets where forward positions can be revised the continuous trading of these contracts does not lead prices to converge to marginal costs.

Given the importance of the asymmetries in determining the possibility of new entry in the industry and its equilibrium prices, we now analyze the issues of capacity constraints and regulation and study how they influence the equilibrium solution. As proved in Proposition 3.3 the solution of the *T*-period and of the 2-period games are the same, as the former reduces to the latter in the absence of pre-commitment, therefore they are equivalent and, for this reason, we use the 2-period model as the base framework in the rest of the article.

4. The Two-stage Procurement Game with Capacity Constraints and Regulation

In this section we analyze how the presence of capacity constraints and regulation influence the suppliers' procurement strategy. The novel contribution, and main advantage of our approach, is to provide (1) a clear description of the main factors influencing the production and contracting by different players, (2) an exact solution, without having to use any numerical approach to solve the system of equations, when there is *a priori* knowledge of which firms are price takers (which may be the case in most practical applications), and (3) an explanation for the interaction between the shadow prices of the different players' capacity constraints and possible price caps.

In order to make the exposition and analysis simpler, we modify the notation by introducing a new variable, $Z_{0,i}$ which represents the total spot trade by supplier *i,* and, therefore, $Q_{0,i} = Z_{0,i} + F_{1,i}$. Each one of the constraints has an associated shadow price, representing the value of a unit of the scarce resources: λ_i^M, i = 1,…, N, for the minimum capacity constraint; λ_i^K, i = 1,…, N, for the maximum capacity constraint; $\lambda^{\underline{P}}$, for the minimum price cap; $\lambda^{\overline{P}}$, for the maximum price cap.

We derive the forward-spot equilibrium conditions for the two-stage N-supplier asymmetric game with capacity constraints and price caps represented by (4.1)-(4.4). In this section we explicitly consider the price and capacity constraints in the model by using the Lagrangian multipliers. For this reason, the problem regarding the limit on the number of firms in the industry, addressed in section 3, does not arise. Therefore, in this section, by explicitly incorporating the capacity constraints, any player with a cost structure such that it optimally produces in a corner solution, will either produce at capacity or have no production. These results arise directly from (4.1) and (4.4) which are derived from the KKT conditions, as explained in the appendix [See Appendix for the derivation of (4.1)-(4.4)].

$$Z_{0,i} = \frac{a-\left(N^2+1\right)C_i + N\sum_{j=1}^{N} C_j - \left(N^2+1\right)\left(\lambda_i^K - \lambda_i^M\right) + (N+1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right) - (N-1)b\left(\lambda^{\bar{P}} - \lambda^{\underline{P}}\right)}{\left(N^2+1\right)b} \tag{4.1}$$

$$F_{1,i} = \frac{(N-1)a-\left(N^2+1\right)(N-1)C_i + N(N-1)\sum_{j=1}^{N} C_j - \left(N^2+1\right)N\left(\lambda_i^K - \lambda_i^M\right) + (N+1)(N-1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right)}{\left(N^2+1\right)b} + \\ + \frac{2Nb\left(\lambda^{\bar{P}} - \lambda^{\underline{P}}\right)}{\left(N^2+1\right)b} \tag{4.2}$$

$$Q_{0,i} = \frac{Na - N\left(N^2+1\right)C_i + N^2\sum_{j=1}^{N} C_j - \left(N^2+1\right)(N+1)\left(\lambda_i^K - \lambda_i^M\right) + (N+1)N\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right)}{\left(N^2+1\right)b} + \\ + \frac{(N+1)b\left(\lambda^{\bar{P}} - \lambda^{\underline{P}}\right)}{\left(N^2+1\right)b} \tag{4.3}$$

$$P_0 = \frac{a + N\sum_{j=1}^{N} C_j + (N+1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right) - (N+1)Nb\left(\lambda^{\bar{P}} - \lambda^{\underline{P}}\right)}{N^2+1}. \tag{4.4}$$

These equilibrium conditions provide a (partial) closed form-solution for the problem (a full solution would still require the resolution of the system of non-linear equations in which the complementarity constraints, available in the appendix, are incorporated to compute the shadow prices for the price and capacities). Nonetheless, equations (4.1)-(4.4) provide a complete description of how production, forward trading, and prices, depend not only on each other, on the level of demand, and on the number of firms in the industry, but also on the suppliers' constraints (and associated shadow prices).

Corollary 4.1: *From (4.1)-(4.3) we conclude that a binding price cap (a) decreases spot trading, (b) increases forward trading and (c) increases production.*

Moreover, it follows, taking into account the interactions between (4.4) and the other equations, that a supplier's production, spot and forward trading increase when a competitor produces at full capacity and decreases when a competitor is not producing. Another major result relates to the conditions under which a supplier buys in the forward market, as proved in Proposition 4.1.

Proposition 4.1: *When the price cap is not binding, a supplier producing at maximum capacity buys forward to sell spot if and only if* $K_i < \frac{P_0 - C_i}{Nb}$. [Proof in the appendix].

This result complements Allaz and Vila (1993) as it shows that in the presence of capacity constraints the suppliers can buy in the forward market, increasing the spot price, as defended by Greenstone (1981), Antelo and Bru (2002), Mahenc and Salanie (2004). Moreover, as shown in Proposition 4.2, inefficient players (who have a marginal cost higher than market price) still participate in the industry, selling forward and buying spot.

Proposition 4.2. *When the price cap is not binding every supplier such that* $P_0 < C_i$ *sells forward and buys spot*. [Proof in the appendix.]

Furthermore, our model is able to capture the impact of regulatory actions on the production of a specific supplier, when the regulator controls the minimum quantities produced, for example to ensure that the supplier is not abusing market power by under producing. We can analyze which factors, and how, influence the minimum production shadow price (and, therefore, profit), based on (4.3) and Proposition 4.3. (An increase in the shadow price, due to an imposition of a constraint, corresponds to a reduction in profit.)

Proposition 4.3. *When the price cap is not binding the shadow price associated with the minimum production constraint equals* $\lambda_i^M = \dfrac{b\underline{Q}_{0,i} - N(P_0 - C_i)}{N+1}$. [Proof in the appendix].

From Proposition 4.3 we conclude that the impact on the profits of a regulatory intervention, by imposing a minimum on the production of a supplier *i* ($\underline{Q}_{0,i}$), increases with the minimum production and with the slope of the inverse demand function, and decreases with the number of firms in the industry. It is further evident from Proposition 4.3 that if the number of firms is very large then the threat of regulatory intervention is negligible as the shadow price associated to the minimum production converges on zero, which means that a regulator cannot influence prices, production and profits strongly enough to justify an intervention.

Finally, we analyze how the imposition of an upper and lower bounds on price influences the respective shadow prices, Proposition 4.4.

Proposition 4.4. *When there is a binding upper bound on price, the respective shadow price equals* $\lambda^{\overline{P}} = \dfrac{a - \overline{P_0}(N^2+1) + N\sum_{j=1}^{N} C_j + (N+1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right)}{(N+1)Nb}$, *and when there is a binding lower bound on price the respective shadow price equals*

$$\lambda^{\underline{P}} = \frac{-a + \underline{P}_0(N^2+1) - N\sum_{j=1}^{N} C_j - (N+1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right)}{(N+1)Nb}.$$ [Proof in the appendix].

Proposition 4.4 shows that the shadow price associated with the upper (lower) bound increases (decreases) with the level of the inverse demand (*a*) and with the average marginal cost, $\overline{C}$. This means that the higher the level of demand and the higher the average marginal cost the higher (lower) the loss associated with an upper (lower) bound on price. And it decreases (increases) with a higher price cap, meaning the lower an upper cap, and the higher a lower cap, the higher the respective associated shadow price and, therefore, the stronger the negative impact of the price control on the suppliers' profits.

Most interesting is the relationship between the shadow prices for the price caps and the shadow prices for the capacity constraints: these values tend to be positively and linearly dependent, which means that a lack of capacity leads to an increase in the shadow price of the upper bound and a decrease in the shadow price of the lower bound, and vice-versa. Therefore, regulatory intervention by imposition of an upper bound on price, when capacity is constrained for some suppliers, has a higher impact on the firms' profits (as they are not able to benefit from the lack of capacity). Moreover, the shadow price associated with a lower bound on price is *higher* in industries where some of the competitors are producing at the minimum level, and consequently, the unconstrained supplier is unable to profit from increasing production and lowering prices even further.

5. Computational Experiments

In this section we illustrate, with a computational example, some of the results we proved on the impact of capacity constraints and regulation on strategic procurement. The different parameters used are summarized in Table 5.1. In all these simulations we use an inverse linear demand function, $P_0 = a - bQ_0$.

Table 5.1: **Suppliers' Cost Structure**

Suppliers	One	Two	Three	Four	Five
Marginal Cost	5	10	15	20	50
Capacity	1000	500	250	150	100

In Table 5.2 we use a value of parameter "*a*" equal to 3000, 2000, 1000, 500, and no price cap. These results show that, when there is excess capacity, the proportion of production traded ahead is about 80%. When all capacity is traded (in the scenario *a* equal to 3000), some of the suppliers (Supplier-4 and Supplier-5) *buy* forward and *sell* in the spot market, i.e., they sell in the spot market more than their installed capacity (having balanced their accounts by buying forward), as shown in Proposition 4.1. This illustrates how our analysis complements Allaz and Vila's (1993) when we have capacity constraints and suppliers are asymmetric. Moreover, Supplier-5, in the experiment with *a* equal to 500, produces nothing, behaving as an arbitrageur (selling forward and buying spot), as proved in Proposition 4.2. One should note that all the results in this section were computed using equations (4.1)-(4.4), i.e., with capacity constraints and price caps, when relevant.

The equilibrium price is the same in the forward contracts and spot market, due to the no-arbitrage constraint. The price increases monotonically with the level of demand and the highest price occurs when all the industry's capacity is used. The production levels decrease with cost and the same is true for the volume traded forward (when some of the expensive suppliers are capacity constrained.). Finally, the presence of binding capacity constraints (at the suppliers' level) decreases the proportion of forward trading.

In Figure 5.1 we test a parameter "*a*" equal to 2000 and price caps 100, 50, and 30, illustrating how these caps lead to an increase in the proportion traded forward. The minimum price used was 0. It illustrates, as proved in Corollary 4.1, that price caps lead to an increase of the proportion of production traded forward.

Table 5.2: Computational Results for the Heterogeneous Suppliers Example

Level of Demand and Equilibrium Prices				
Inverse Demand Parameter: *a*	3000	2000	1000	500
Equilibrium Price	1000	170.8	59.5	35.7

Volume of Production per Supplier				
Supplier-1	1000.0	829.2	272.6	153.6
Supplier-2	500.0	500.0	247.6	128.6
Supplier-3	250.0	250.0	222.6	103.6
Supplier-4	150.0	150.0	150.0	78.6
Supplier-5	100.0	100.0	47.6	0.0
Industry	2000.0	1829.2	940.5	464.3

Volume of Forward Contracts Sold per Supplier				
Supplier-1	667.5	663.3	218.1	122.9
Supplier-2	251.7	389.9	198.1	102.9
Supplier-3	44.2	182.4	178.1	82.9
Supplier-4	-38.3	99.9	118.4	62.9
Supplier-5	-75.0	63.2	38.1	2.4
Industry	850.0	1398.6	750.8	373.8

Percentage of Production Traded Forward per Supplier				
Supplier-1	66.8	80.0	80.0	80.0
Supplier-2	50.3	78.0	80.0	80.0
Supplier-3	17.7	72.9	80.0	80.0
Supplier-4	-25.6	66.6	78.9	80.0
Supplier-5	-75.0	63.2	80.0	
Industry	42.5	76.5	79.8	80.5

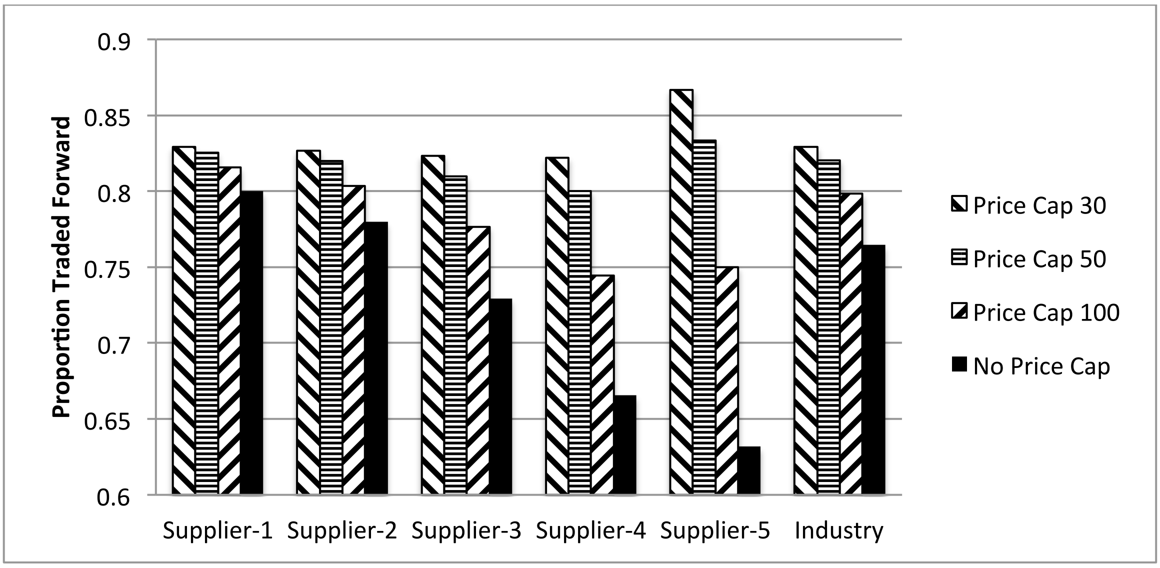

Figure 5.1: Proportion of forward contracts as a function of the price cap, *a* = 2000

Similarly, in Figure 5.2 we illustrate the effect of a minimum price constraint on the the proportion traded forward. We use a parameter "*a*" equal to 500 (to consider a low demand scenario) and minimum prices of 0, 40, 50, and 60. We did not define a binding price cap. As expected, an increase in the minimum price leads to a decrease in the the proportion of production traded forward. In this scenario, Supplier-5 does not sell spot and only trades in the futures as a arbitrageur (buying when the minimum price is less than 50 and selling when it is 60.)

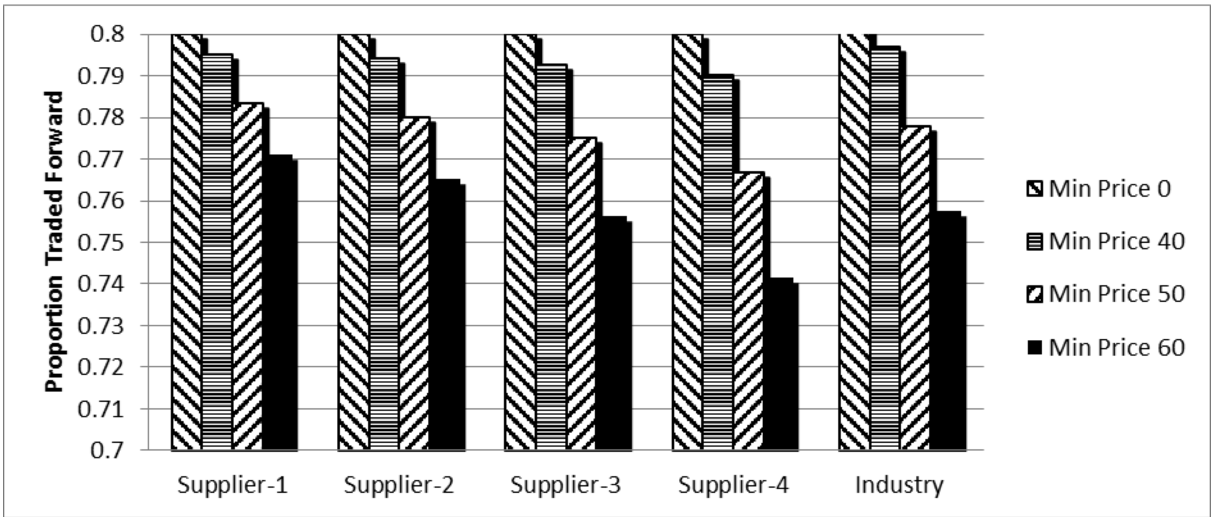

Figure 5.2: Proportion traded forward as a function of the minimum price, *a* = 500.

Finally, to understand the importance of capturing the heterogeneity of the production function, we analyze the same market structure but assuming instead that all five suppliers have a marginal production cost of 20, and an installed capacity of 400, each (the means of the production costs and capacities in Table 5.1). The results are summarized in Table 5.3.

Table 5.3: Computational Results for the Homogeneous Suppliers Example

Inverse Demand Parameter: *a*	3000	2000	1000	500
Equilibrium Price	1000.0	96.15	57.69	38.46
Production per Supplier	400.0	380.8	188.5	92.3
Forward Contracts per Supplier	170.0	304.6	150.8	73.8
Prop. of Production Traded Forward	42.5	80.0	80.0	80.0

For a value of parameter a equal to 3000 we need to use the system of equations (4.1)-(4.4) to solve the problem as the suppliers are producing at full capacity, and the results are equivalent to the ones reported in Table 5.2. In the solutions for a value of parameter a between 2000 and 500 both equations (3.1)-(3.3) and (4.1)-(4.4) can be used to solve the problem as these are interior point solutions. The proportion traded forward is always approximately equal to 80% when there is no constraint binding the solution, and significantly lower when the capacity constraints are binding, as proved in section 4.

6. Conclusions

In this article we derive the equilibrium solution for the strategic procurement N-player game (e.g., Boer et al., 2002; Chen and Liu, 2007) in non-storable commodity markets (such as electricity and freight services), taking into account the interaction between forward contracts and spot markets (e.g., Allaz and Vila, 1993), and including capacity constraints and regulation. We study the procurement problem with asymmetric suppliers, determining under which conditions the equilibrium holds, and analyzing how market structure influences the suppliers' technology mix. Moreover, we show that the multi-period strategic procurement problem is equivalent to the two-period problem when the positions in forward contracts are not binding, as it is the case in electricity and freight services markets.

From a managerial perspective, our results contribute to clarify the conditions under which suppliers engage in strategic procurement in the spot and forward markets. The availability of both markets tends to decrease prices and profits (when compared with a situation in which only one market is available) and this phenomenon is more important in industries with high competitive intensity. Additionally, the proved equivalence between the 2-period and the T-period solution, in the absence of binding pre-commitment in the forward markets, as important implications for suppliers as they can engage in non-binding forward contracting without fearing prices to be driven to marginal cost.

Moreover, our analysis shows that, in the uncapacitated case, when there are asymmetries in the production technology, there is a limit on the number of firms that can enter the market (this number decreases with the heterogeneous degree of the technology), explaining how technological diversity influences the long-term dynamics of the industry. Moreover, in this case, and contrary to expectations, the cheaper technology does not expel the most expensive one and, in fact, the clearing price converges to the marginal cost of the most expensive technology. In this situation, the suppliers can continue producing an important share of the market (directly proportional to difference between the maximum marginal cost and the actual marginal cost of the firm) with a profit.

Furthermore, we have shown that suppliers' procurement in spot and forward markets is conditioned by binding capacity constraints: these change the proportion produced by each firm, increasing the proportion procured with forward contracts and, under certain conditions, leading some suppliers to *buy* forward and to sell in spot (when they are producing at full capacity) and driving the inefficient suppliers to act as arbitrageurs, selling forward and buying spot.

Finally, we have addressed the impact of regulatory action, via price and production controls, on strategic procurement. We have shown the price caps have a negative impact on suppliers' profits: the lower (higher) the upper (lower) price cap the stronger the impact on profits, and the higher (lower) the suppliers' marginal costs the stronger (weaker) the impact of the upper (lower) cap on price. We have also shown that the regulation of suppliers' minimum production influences their profit: the higher the minimum generation the higher the suppliers' loss, and the higher the number of firms in the industry the less important production controls are. We have also proved that the capacity shadow price depends on the shadow price associated with price caps: which means that the upper (lower) cap is more effective in states in which some suppliers produce at full (minimum) capacity.

Appendix

Derivation of Equations (3.1)-(3.3) - From the optimality conditions, at time zero, $\frac{du_{0,i}}{dQ_{0,i}} = 0$, and we get the reaction functions $Q_{0,i} = \frac{a + bF_{1,i} - C_i - b\sum_{\forall j \neq i} Q_{0,j}}{2b}$. Then by solving the system of equations, at time 0, with a different number of players, it can be obtained the analytical solution of the Cournot-Nash equilibrium, conditioned on the number of players used. Then, using a proof by induction, and by comparing the results obtained with the different number of players, the general Cournot-Nash equilibrium productions is generalized for all N, deriving $Q_{0,i} = \frac{a - NC_i + NbF_{1,i} + \sum_{\forall j \neq i} C_j - b\sum_{\forall j \neq i} F_{1,j}}{(N+1)b}$. The price was derived using a similar method and it is represented by $P_0 = \frac{a + \sum_{i=1}^{N} C_i - b\sum_{i=1}^{N} F_{1,i}}{N+1}$. From the profit function $\Pi_i = (P_1 - P_0)F_{1,i} + (P_0 - C_i)Q_{0,i}$ and as in equilibrium there are no arbitrage opportunities, it follows that $P_1 = P_0$ and $\Pi_i = (P_0 - C_i)Q_{0,i}$. Replacing $Q_{0,i}$ and P_0, in the profit function and calculating the optimal forward trading, i.e., imposing that $\frac{d\Pi i}{dF_{1,i}} = 0$, and using the proof by induction, we derive the reaction functions for the forward contracts in $F_{1,i} = \frac{(N-1)a - N(N-1)C_i + (N-1)\sum_{\forall j \neq i} C_j - (N-1)b\sum_{\forall j \neq i} F_{1,j}}{2Nb}$. It follows that the equilibrium solution for production, forward contracting and prices is represented by equations:

$$Q_{0,i}=\frac{Na-N\left(N^2+1\right)C_i+N^2\sum_{j=1}^{N}C_j}{\left(N^2+1\right)b},F_{1,i}=\frac{(N-1)a-\left(N^2+1\right)(N-1)C_i+N(N-1)\sum_{j=1}^{N}C_j}{\left(N^2+1\right)b},$$ and

$P_0=\frac{a+N^2\overline{C}}{N^2+1}$. After some simple algebraic manipulations we derive equations (3.1)-(3.2):

$$Q_{0,i}=\frac{N}{b}[P_0-C_i],\ F_{1,i}=\frac{(N-1)}{b}[P_0-C_i].$$ ■

Proposition 3.1 - Proof. Since for every firm in the industry production is always positive, from (3.1) we get $\frac{N}{b}[P_0-C_i]>0$ and from (3.3 it follows that $\frac{a+N^2\overline{C}}{N^2+1}>C_i$ and equivalently $N^+<\sqrt{\frac{a-C_i}{C_i-\overline{C}}}$ for all i in the industry. ■

Proposition 3.2 - Proof: (*a*) From (3.3), $P_0=\frac{a+N^2\overline{C}}{N^2+1}$ and, therefore, from Proposition 3.1, if N equals N^+, which is less or equal to $\sqrt{\frac{a-C_i}{C_i-\overline{C}}}$, it follows that for all i $P_0\geq\frac{a+\left(\frac{a-C_i}{C_i-\overline{C}}\right)\overline{C}}{\left(\frac{a-C_i}{C_i-\overline{C}}\right)+1}=C_i$. (b) From (3.1) and Proposition 3.2.a) follows that $Q_{0,i}\geq 0$. (*c*) From (3.2) we have $F_{1,i}=\frac{(N-1)}{b}[P_0-C_i]$, then from Proposition 3.1, and proposition 3.2.a), we get $F_{1,i}\geq 0$.■

Proposition 3.3 - Proof: From equation (2.1) we know that *i*'s profit function is equal to $\Pi_i=P_0\left(Q_{0,i}-F_{1,i}\right)-C_iQ_{0,i}+\sum_{t=1}^{T}P_tf_{t,i}$. Following a backward induction process, we start by computing the equilibrium at stage zero (spot market), then at stage 1, stage 2,…, stage K, …, stage T. The profit functions for each one of these stages can be derived from (2.1). Let $\Pi_{t,i}$ represent the profit function to be maximized by supplier i at time t, conditioned on its previous decisions and on the previous and future decisions of the other suppliers: $\Pi_{0,i} = P_0(Q_{0,i} - F_{1,i}) - C_i Q_{0,i}$, $\Pi_{1,i} = P_0(Q_{0,i} - F_{1,i}) - C_i Q_{0,i} + P_1 f_{1,i}$, …, $\Pi_{K,i} = P_0(Q_{0,i} - F_{1,i}) - C_i Q_{0,i} + \sum_{t=1}^{K} P_t f_{t,i}$ and $\Pi_{T,i} = P_0(Q_{0,i} - F_{1,i}) - C_i Q_{0,i} + \sum_{t=1}^{T} P_t f_{t,i}$.

The solution procedure has T steps. (1) $\underset{Q_{0,i}}{Max}\, \Pi_{0,\mathrm{i}}$, (2) $\underset{f_{1,i}}{Max}\, \Pi_{1,\mathrm{i}}$ s.t. $\frac{\mathrm{d}\Pi_{0,\mathrm{i}}}{dQ_{0,\mathrm{i}}} = 0$, … (K) $\underset{f_{K,i}}{Max}\, \Pi_{\mathrm{K,i}}$ s.t. $\frac{\mathrm{d}\Pi_{0,\mathrm{i}}}{dQ_{0,\mathrm{i}}} = 0,. \frac{\mathrm{d}\Pi_{1,\mathrm{i}}}{df_{1,\mathrm{i}}} = 0,..., \frac{\mathrm{d}\Pi_{\mathrm{K\text{-}1,i}}}{df_{\mathrm{K\text{-}1,i}}} = 0, \ldots,$

(T) $\underset{f_{T,i}}{Max}\, \Pi_{\mathrm{K,i}}$ s.t. $\frac{\mathrm{d}\Pi_{0,\mathrm{i}}}{dQ_{0,\mathrm{i}}} = 0,. \frac{\mathrm{d}\Pi_{1,\mathrm{i}}}{df_{1,\mathrm{i}}} = 0,..., \frac{\mathrm{d}\Pi_{\mathrm{T\text{-}1,i}}}{df_{\mathrm{T\text{-}1,i}}} = 0.$ Let us now derive the necessary optimality conditions for each step.

(1) $\underset{Q_{0,i}}{Max}\, \Pi_{0,\mathrm{i}}$. From $\frac{\mathrm{d}\Pi_{0,\mathrm{i}}}{dQ_{0,\mathrm{i}}} = 0$ we have $\frac{dP_0}{dQ_{0,i}}\left(Q_{0,i} - \sum_{t=1}^{T} f_{t,i}\right) + (P_0 - C_i) = 0$, and as $P_0 = a - b\sum_{j=1}^{N} Q_{0,j}$ we get $Q_{0,i} = \frac{a - NC_i + NbF_{1,i} + \sum_{\forall j \neq i} C_j - b\sum_{\forall j \neq i} F_{1,j}}{(N+1)b}$.

(2) $\underset{f_{1,i}}{Max}\, \Pi_{1,\mathrm{i}}$ s.t. $\frac{\mathrm{d}\Pi_{0,\mathrm{i}}}{dQ_{0,\mathrm{i}}} = 0.$ From $\underset{f_{1,i}}{Max}\, \Pi_{1,\mathrm{i}}$, and as $P_0 = P_1$, we get $\frac{dP_0}{df_{1,i}}\left(Q_{0,i} - \sum_{t=2}^{T} f_{t,i}\right) + (P_0 - C_i)\frac{dQ_{0,i}}{df_{1,i}} = 0$. As $P_0 = a - b\sum_{j=1}^{N} Q_{0,j}$ we have $P_0 = \frac{a + \sum_{i=1}^{N} C_i - b\sum_{i=1}^{N} F_{1,i}}{N+1}$. Therefore, from (1) and (2) we get $\frac{dP_0}{df_{1,i}} = -\frac{b}{(N+1)}$, and

$\frac{dQ_{0,i}}{df_{1,i}} = \frac{N}{(N+1)}$ which can be plugged into the optimality condition and we solve it in order

to get $f_{1,i}$, obtaining: $f_{1,i} = \frac{(N-1)a - N(N-1)C_i + (N-1)\sum_{j \neq i} C_j}{2Nb} - \frac{(N-1)F_{2,i} + (N-1)\sum_{j \neq i} F_{1,j}}{2N}$

.

(3) $\underset{f_{2,i}}{Max}\,\Pi_{2,\mathrm{i}}$ s.t. $\frac{\mathrm{d}\Pi_{0,\mathrm{i}}}{dQ_{0,\mathrm{i}}} = 0, \frac{\mathrm{d}\Pi_{1,\mathrm{i}}}{df_{1,\mathrm{i}}} = 0$. From $\underset{f_{2,i}}{Max}\,\Pi_{2,\mathrm{i}}$, and as $P_0 = P_1 = P_2$, we get $\frac{dP_0}{df_{2,i}}\left(Q_{0,i} - \sum_{t=3}^{T} f_{t,i}\right) + (P_0 - C_i)\frac{dQ_{0,i}}{df_{2,i}} = 0$. Moreover, as there is no pre-commitment between the different periods of the forward contracts, these positions can be revised every period, therefore, the optimal positions at stage 1, given the decisions in the spot market, are the optimal level of forward contracting for the entire planning horizon. Therefore, from $\frac{\mathrm{d}\Pi_{0,\mathrm{i}}}{dQ_{0,\mathrm{i}}} = 0, \frac{\mathrm{d}\Pi_{1,\mathrm{i}}}{df_{1,\mathrm{i}}} = 0$, we get $Q_{0,i} = \frac{a - NC_i + Nbf_{1,i} + \sum_{\forall j \neq i} C_j - b\sum_{\forall j \neq i} f_{1,j}}{(N+1)b}$ and $P_0 = \frac{a + \sum_{i=1}^{N} C_i - b\sum_{i=1}^{N} f_{1,i}}{N+1}$. Consequently, $\frac{dP_0}{df_{2,i}} = 0$, and $\frac{dQ_{0,i}}{df_{2,i}} = 0$. Hence, by plugging in $Q_{0,i}$, $\frac{dP_0}{df_{2,i}}$ and $\frac{dQ_{0,i}}{df_{2,i}}$ into $\frac{dP_0}{df_{2,i}}\left(Q_{0,i} - \sum_{t=3}^{T} f_{t,i}\right) + (P_0 - C_i)\frac{dQ_{0,i}}{df_{2,i}} = 0$ we get 0=0, and the level of contracting in each period t>1 is undetermined. When there is no pre-commitment the $f_{t,i}$ can be negative and the current forward position can always be corrected in latter stages. For this reason, the production depends only on the accumulated position at time 1, which can be optimized, and not on the specific positions at times 2,…, *T*. In states 4, …, T, the same reasoning and equations as in state (3) applies and the level of forward contracting in these

stages is undetermined. Finally, as $F_{1,i}=f_{1,i}$ it follows that the production levels and prices are the same for the model with T and 2 periods. ■

Derivation of Equations (4.1)-(4.4) – We start by defining $S^{\underline{P}}$, i = 1, … , N, for the minimum price constraint; $S^{\overline{P}}$, i = 1,…, N, for the maximum price constraint. Equations $K_i - Z_{0,i} - F_{1,i} \geq 0$, $Z_{0,i} + F_{1,i} - M_i \geq 0$ are the capacity constraints for a firm i. Moreover, $S^{\underline{P}} = a - b\sum_{i=1}^{N}\left(Z_{0,i} + F_{1,i}\right) - \underline{P}_0$ and $S^{\overline{P}} = \overline{P}_0 - a + b\sum_{i=1}^{N}\left(Z_{0,i} + F_{1,i}\right)$ define the price caps using the respective slack variables. The Lagrangian is represented by:

$$L_{0,i} = P_0 Z_{0,i} - C_i\left(Z_{0,i} + F_{1,i}\right) + \lambda_i^K\left(K_i - Z_{0,i} - F_{1,i}\right) + \lambda_i^M\left(Z_{0,i} + F_{1,i} - M_i\right) +$$
$$+\lambda^{\overline{P}}\left(\overline{P_0} - a + b\sum_{j=1}^{N}\left(Z_{0,j} + F_{1,j}\right)\right) + \lambda^{\underline{P}}\left(a - b\sum_{j=1}^{N}\left(Z_{0,j} + F_{1,j}\right) - \underline{P_0}\right)$$

By computing the derivative of the Lagrangian with respect to the quantities, $\frac{dL_{0,i}}{dZ_{0,i}} = 0$, and by replacing $P_0 = a - b\sum_{j=1}^{N} Q_{0,j}$ with $P_0 = a - b\sum_{j=1}^{N}\left(Z_{0,i} + F_{1,i}\right)$ then we obtain $P_0 + \frac{\partial P_0}{\partial Z_{0,i}} Z_{0,i} - C_i - \lambda_i^K + \lambda_i^M + b\lambda^{\overline{P}} - b\lambda^{\underline{P}} = 0$ from which it follows that $a - C_i - 2bZ_{0,i} - b\sum_{\substack{j=1 \\ j\neq i}}^{N} Z_{0,j} - b\sum_{j=1}^{N} F_{1,j} - \lambda_i^K + \lambda_i^M + b\lambda^{\overline{P}} - b\lambda^{\underline{P}} = 0$. Furthermore, we consider the complementarity conditions defining the relationship between capacity and price constraints and the respective shadow prices. Finally, we add the non-negativity constraints. From the first-order optimality conditions in the spot market we get the quantities

$$Z_{0,i} = \frac{a + \left(N+1\right)\left(\lambda_i^K - \lambda_i^M - C_i\right) + \sum_{j=1}^{N}\left(C_j - bF_{1,j} - \lambda_j^K + \lambda_j^M\right) + b\lambda^{\overline{P}} - b\lambda^{\underline{P}}}{\left(N+1\right)b}$$

and price $P_0 = \dfrac{a + \sum_{j=1}^{N} C_j - b\sum_{j=1}^{N} F_{1,j} - \sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right) - Nb\left(\lambda^{\overline{P}} - \lambda^{\underline{P}}\right)}{N+1}$. The derivation of these equations was done by induction, i.e., by computing the solution of the problem for different structures of the industry we induced the solution for an abstract number of N players. After computing the equilibrium solution at time zero for the spot market we compute the solution for the forward contracts by backward induction, continuing to assume perfect foresight and no arbitrage opportunities, i.e., $P_1 = P_0$, by solving $\dfrac{dL_{1,i}}{dF_{1,i}} = 0$ using the Lagrangian in

$$
\begin{aligned}
L_{1,i} &= \left(P_0 - C_i\right)\left(Z_{0,i} + F_{1,i}\right) + \lambda_i^K\left(K_i - Z_{0,i} - F_{1,i}\right) + \lambda_i^M\left(Z_{0,i} + F_{1,i} - M_i\right) + \\
&+ \lambda^{\overline{P}}\left(\overline{P_0} - a + b\sum_{j=1}^{N}\left(Z_{0,i} + F_{1,i}\right)\right) + \lambda^{\underline{P}}\left(a - b\sum_{j=1}^{N}\left(Z_{0,i} + F_{1,i}\right) - \underline{P_0}\right)
\end{aligned},
$$

obtaining equations:

$$
Z_{0,i} = \frac{a - \left(N^2+1\right)C_i + N\sum_{j=1}^{N} C_j - \left(N^2+1\right)\left(\lambda_i^K - \lambda_i^M\right) + (N+1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right) - (N-1)b\left(\lambda^{\overline{P}} - \lambda^{\underline{P}}\right)}{\left(N^2+1\right)b}
$$

$$
\begin{aligned}
F_{1,i} &= \frac{(N-1)a - \left(N^2+1\right)(N-1)C_i + N(N-1)\sum_{j=1}^{N} C_j - \left(N^2+1\right)N\left(\lambda_i^K - \lambda_i^M\right) + (N+1)(N-1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right)}{\left(N^2+1\right)b} + \\
&+ \frac{2Nb\left(\lambda^{\overline{P}} - \lambda^{\underline{P}}\right)}{\left(N^2+1\right)b}
\end{aligned}
$$

$$
\begin{aligned}
Q_{0,i} &= \frac{Na - N\left(N^2+1\right)C_i + N^2\sum_{j=1}^{N} C_j - \left(N^2+1\right)(N+1)\left(\lambda_i^K - \lambda_i^M\right) + (N+1)N\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right)}{\left(N^2+1\right)b} + \\
&+ \frac{(N+1)b\left(\lambda^{\overline{P}} - \lambda^{\underline{P}}\right)}{\left(N^2+1\right)b}
\end{aligned}
$$

$$P_0 = \frac{a + N\sum_{j=1}^{N} C_j + (N+1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right) - (N+1)Nb\left(\lambda^{\bar{P}} - \lambda^{\underline{P}}\right)}{N^2+1}.$$

These equations, after some algebraic manipulation, can be shown equivalent to (4.1)-(4.4).■

Proposition 4.1 - Proof: When the price cap is not binding, from (4.3) we get $K_i = \frac{N(P_0 - C_i) - (N+1)\lambda_i^K}{b}$, which can be re-arranged to obtain $\lambda_i^K = \frac{N}{N+1}(P_0 - C_i) - \frac{bK_i}{N+1}$, which can be plug-in (4.2) to obtain $F_{1,i} = \frac{N}{N+1}K_i - \frac{(P_0 - C_i)}{b(N+1)}$. Therefore, the firm buys using forward contracts, $F_{1,i} > 0$, if and only if $K_i < \frac{P_0 - C_i}{Nb}$. ■

Proposition 4.2 - Proof: When the price cap is not binding, for a firm *i* such that $P_0 < C_i$ from (4.3) we obtain $\lambda_i^M = \frac{N}{N+1}(C_i - P_0)$ which, together with (4.2) leads to $F_{1,i} = \frac{(C_i - P_0)}{b(N+1)}$, a positive term. ■

Proposition 4.3 - Proof: When the price cap is not binding, and there is a minimum production constraint, $\underline{Q_{0,i}}$, from (4.3) we get $\underline{Q_{0,i}} = \frac{N(P_0 - C_i) - (N+1)\left(-\lambda_i^M\right)}{b}$ from which we obtain $\lambda_i^M = \frac{b\underline{Q_{0,i}} - N(P_0 - C_i)}{N+1}$. ■

Proposition 4.4. - Proof: When there is a binding upper bound on price, from (4.4) we get $\overline{P_0} = \frac{a + N\sum_{j=1}^{N} C_j + (N+1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right) - (N+1)Nb\lambda^{\bar{P}}}{N^2+1}$, which after some algebraic manipulation becomes $\lambda^{\overline{P}} = \dfrac{a - \overline{P_0}(N^2+1) + N\sum_{j=1}^{N} C_j + (N+1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right)}{(N+1)Nb}$. When there is a lower bound on price, from (4.4) we get $\underline{P_0} = \dfrac{a + N\sum_{j=1}^{N} C_j + (N+1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right) + (N+1)Nb\lambda^{\underline{P}}}{N^2+1}$, which becomes

$$\lambda^{\underline{P}} = \frac{-a + \underline{P_0}(N^2+1) - N\sum_{j=1}^{N} C_j - (N+1)\sum_{j=1}^{N}\left(\lambda_j^K - \lambda_j^M\right)}{(N+1)Nb} .$$

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