

Light control by scattering cancellation in ordered and disordered non-Hermitian media, direct and inverse design

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ABSTRACT

Non-Hermitian Physics has emerged as a fertile ground for a smart control of waves. Here, we present direct and inverse-design strategies to achieve ‘on demand’ dynamical manipulation of light by non-Hermitian potentials. The direct approach is based on our recently proposed generalized Hilbert Transform relating the real and imaginary distributions of the complex permittivity to induce spatial symmetry breaking to control scattering, widening the concept Kramers Kronig relations in space. A recipe to design complex potentials to tailor the propagation of light following any vector field, or to generate invisible potentials where light propagates as in free space. The procedure may be applied on any given arbitrary background permittivity distribution being regular or random, extended or localized. Moreover, it is possible to keep the design parameters within realistic limits, even avoiding gain. Beyond this fundamental approach, we also we also present supervised and unsupervised learning techniques for knowledge acquisition in non-Hermitian systems which accelerate the inverse the “on demand” design process. The different proposals may have direct applications to control the wave dynamics in semiconductor lasers or other linear and nonlinear physical systems including cloaking sensors and arbitrary shaped objects.

Keywords: Non-Hermitian Photonics, Metamaterials, PT-symmetry, Hilbert transform

1. INTRODUCTION

The pursuit of artificial materials to control and shape light is possibly one of the most attracting research areas in photonics in the last decades. Non-Hermitian materials offer renewed possibilities for the smart manipulation of waves as compared to close-conservative Hermitian systems that arise from PT-symmetry breaking [1-2].

Time symmetry breaking is at the basis of causality. The temporal response of a linear time-invariant system may be expressed as the convolution: $Response(t) = \int \chi(t-t') \cdot Signal(t-t') \cdot dt' = \chi(t) * Signal(t)$, where the response function, $\chi(t)$, is the kernel in the integral expression of the response of any physical system. Trivially, the system does not anticipate the future when its response is zero before the arrival of the signal, i.e $\chi(t) = 0$ for $t < 0$, see Fig. 1a.

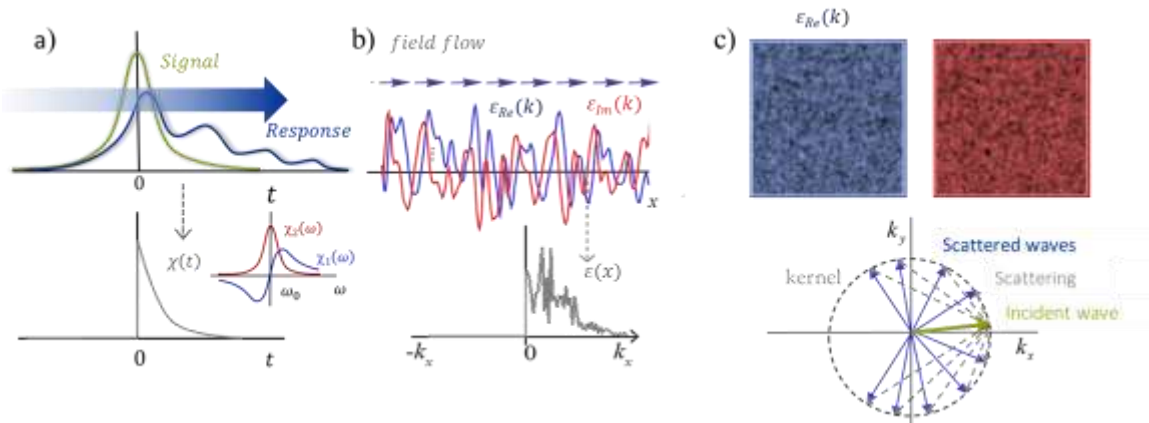


Figure 1. a) Illustration of causality imposing the cancellation of the response before the arrival of the signal; the inset depicts the spectral dependence of the real and imaginary components of the response function as related by a HT. b) The cancellation of the backward scattering in a 1D system is warranted by an asymmetric response function $\epsilon(k_x) = 0$ for $k_x < 0$. This may be achieved by any complex random potential which real and imaginary parts are related by a Hilbert Transform,

ensuring $\varepsilon(k_x)=0$ for $k_x<0$. c) Generalization to a random potential in two spatial dimensions, where the kernel of the HT uncouples incident radiation from particular backward scattering directions, $\varepsilon(k_x,k_y)=0$, full backscattering cancellation would correspond to complete cancellation in the wavevector domain, $\varepsilon(k_x<0,k_y)=0$.

As a consequence, the real and imaginary components of such response function in frequency domain, $\chi_{\text{Re}}(\omega)$ and $\chi_{\text{Im}}(\omega)$, are related particular integral relations if the form of a Hilbert Transform (HT): $\chi_{\text{Re}}(\omega)=1/\pi P[\int \chi_{\text{Im}}(\omega')/(\omega-\omega') d\omega']$; $\chi_{\text{Im}}(\omega)=-1/\pi P[\int \chi_{\text{Re}}(\omega')/(\omega-\omega') d\omega']$; referred as Kramers-Kronig dispersion relations as derived from the Drude model. Analogously, a 1D system with a particular permittivity distribution in space, such that the real/imaginary components are related by a Hilbert Transform (HT), completely cancels backscattering by uncoupling the incident radiation and backward scattered waves in the reciprocal wavevector domain (since $\varepsilon(k_x)=0$ for $k_x<0$), see Fig.1b. Indeed, this was nicely proposed and experimentally demonstrated in 1D [3,4,5].

2. LIGHT CONTROL BY SCATTERING CANCELLATION

2.1 Direct design of scattering cancellation

In the same way that the HT in frequency domain ensures invisibility of the future, spatial symmetry breaking and the HT in wavevector domain are at the basis of spatial invisibility (scattering cancellation). Going one step further, we recently proposed a generalized HT, in higher spatial dimensions, to engineer complex media to induce a non-isotropic response for different purposes.

In the first Born approximation, if permittivity $\varepsilon(\mathbf{k})=0$ for all $\mathbf{k}=(k_x,k_y,k_z)$ on the entire left half space $k_x<0$ in the wave-vector domain, all plane waves propagating from left to right $k_x>0$, will not be. Yet, this condition can be very demanding in terms the scattering potential and we proposed a generalized HT with a scattering cancellation kernel to achieve invisibility on demand under realistic perspective. A procedure to modify the scattering from any potential (being periodic, random or a localized object with any arbitrary shape and size), so that the kernel of determines invisibility for given directions and a range of frequencies since the cancellation of $\varepsilon(\mathbf{k})$ uncouples incident radiation from given scattered waves, as illustrated in Fig.1c [6]. Moreover, choosing symmetric kernels it is possible to achieve such invisibility by scattering cancellation with purely dielectric materials [7].

In turn, since the suppression of backscattering favors light to propagate in a particular direction, we uncovered a procedure to design a complex permittivity spectrum to generate sinks using symmetric complex potentials in 1D [8]. Generalizing this former proposal to 2D, we showed it is possible to design non-Hermitian systems that favor the flow of fields following arbitrary vector fields [9]. This is achieved by a HT which kernel is defined differently in the neighborhood of every point, determining the field flow. In turn, it is possible to restrict the generalized HT to the use of all-dielectric materials or within a given range by iterative methods [10]. The proposal has applications has the potential to contribute to the development of new technologies in optics, improving the performance of VCSELs [11], broad area semiconductor lasers [12] and semiconductor laser arrays [13].

2.2 Inverse design of non-Hermitian systems

As the degrees of freedom increase or if we intend to achieve a selectively multifrequency light transportation, such methods may become computationally expensive. Therefore, beyond the above fundamental approach inverse-design strategies may also achieve 'on demand' manipulation of light by non-Hermitian potentials. As a first step, proposed a general inverse-design strategy based on genetic algorithm optimization to achieve 'on demand' manipulation of light in 1D and 2D non-Hermitian systems. The optimization process faithfully creates non-Hermitian potentials from any given arbitrary real (or imaginary) permittivity distribution for the desired frequency selective and broadband asymmetric response in 1D multilayer structures. As a demonstration in 2D, we designed periodic and aperiodic complex permittivity spatial distributions to create "sink-type" concentrators of light around a desired area. Moreover, inspecting the resulting permittivity distributions uncovers underlying fundamental insights about the designed structures, and the connection to the KK relation in 1D or the HT in higher dimensions [14].

Just recently, supervised and unsupervised learning techniques for knowledge acquisition in non-Hermitian systems have shown to accelerate the inverse design process. A deep learning model has been proposed relating the transmission and asymmetric reflection in non-conservative settings and propose sub-manifold learning to recognize non-Hermitian features from transmission spectra. The developed deep learning framework determines the feasibility of a desired

spectral response for a given structure and uncovers the role of effective gain-loss parameters to tailor the spectral response [15].

3. CONCLUSIONS

To conclude, we reviewed the different phenomena arising from the recently proposed generalized HT connecting the real and imaginary parts of the permittivity of a system to induce spatial symmetry breaking on demand. We demonstrate that such unique feature of non-Hermitian systems allows tailoring the field flows in arbitrary dimensions or designing invisible potentials where the propagation of light is indistinguishable from the one in free space. The approach allows restricting the dimensionality of the complex susceptibility within practical limits for a feasible realization or even avoiding the use of gain. Moreover, the reported findings trigger a route for intelligent inverse design and contribute to the understanding of physical mechanisms in general non-Hermitian systems. The different proposals may have direct applications to control the wave dynamics in semiconductor lasers or other linear and nonlinear physical systems, including cloaking sensors and arbitrary shaped objects. Both the direct and inverse design approaches are not restricted to optical systems, and it can be applied directly to find accurate solutions to other kinds of waves or including acoustics, elasticity, or plasmonics.

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