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Sensitivity of Blood Flow Dynamics Simulations in the Pulmonary Arteries to Inflow Boundary Conditions

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This project is submitted for the
Elective Erasmus Exchange Project at the
Department of Fluid Mechanics, Universitat Politécnica de Catalunya

Spring 2023
EEBE, UPC

Abstract

The Pulmonary Artery (PA) is an important part of the cardiovascular system. The PA drives the oxygenation of blood by transporting it from the Right Ventricle (RV) to the lungs. Blood flow first go through the Main Pulmonary Artery (MPA) and then splits into the Right Pulmonary Artery (RPA) and the Left Pulmonary Artery (LPA), which divides into smaller vessels inside the human lungs.

The blood flow in the PA have biological and mechanical characteristics that are sensitive to physiological and anatomical changes. The objective of this study is to understand the sensitivity of the flow patterns to the inflow boundary condition using a combination of in-vivo data and Computational Fluid Dynamics (CFD).

A population of N = 35 subjects (reduced to 28 after analysis) composed of healthy adults is considered for this study. Geometries are extracted from magnetic resonance imaging (MRI), and velocity fields from corresponding 4D flow data. Numerical simulations are conducted on an cohort-averaged shape using the software SimVascular. Inlet conditions consist of an average velocity profile (obtained from the mean of the patient-specific in-vivo velocity data); sensitivity is studied superimposing a fraction of the standard deviation.

Results were post-processed in the open-source software ParaView. The inlet profile showed significant variations across the cohort. Accordingly, flow patterns demonstrated a non-negligible sensitivity to inflow conditions in terms of flow topology and overall characteristics. The results emphasize the importance of using patient-specific boundary conditions for reliable simulations of blood flow in the pulmonary arteries.

Contents

1 Context

1.1 Computational Fluid Dynamics

Computational fluid dynamics (CFD) is a simulation tool used in the fluid mechanics branch by employing numerical analysis to solve fluid dynamic flow problems. Recently, CFD has beginning to be applied in the biomedical field to study blood flow dynamics in the human body. The simulations contributes to understand disease processes, the exchange of hemodynamic forces with vascular tissue, plan surgical procedures, and in the end improve diagnosis and optimize treatment by medicine for the cardiovascular system.

In this study CFD is adapted with the interactive software application SimVascular. The application is used to create patient-specific geometric models of human vasculature from medical imaging data to perform blood flow simulations using various numerical methods.

1.2 Objectives and process description

1.2.1 Objectives

To reach the aim of this study are the following objectives stated:

- Create a simulation set-up for an average shaped model of the pulmonary artery to perform numerical simulations.
- Sensitivity analysis of numerical predictions to inflow boundary conditions by modifying simulation files.
- Preform blood flow simulations and obtain results to be further discussed and analysed.

1.2.2 Process description

The path from a modelled shape of the pulmonary arteries, to simulations results, is listed below and can be seen as a description of the computational fluid dynamics process of the study:

1. Importing pulmonary arteries model. Filling holes for each cap and create a face for them by using the "Fill holes w/o ID" option. The new faces represent the inlet and outlets.

2. Local linear subdivision on each face to generate points needed for the remesh. The number of divisions is set to 2. Select all new faces and remesh.

3. Creating a Volume Mesh and Surface Mesh of the model.

4. Building a average inlet condition based on the velocity contribution for every patient data. Five different boundary conditions (BC) using the standard deviation, scripted in Matlab. Calculating the flow rates to evaluate the patient specific data and interpolate the velocity points to fit the model in SimVascular.

5. Set up of parameters affecting the simulation based on papers and the SimVascular Simulation Guide. Create Data files for simulation, mainly bct.dat file.

7. Data file bct.dat is regenerated and modified to represent the new conditions of flow rate, inlet velocities, time steps, etc, (boundary conditions). Scripted in Matlab.

8. Simulating with the modified bct.dat data file for each BC case. Collecting results to be post-processed in ParaView by converting the simulations results.

9. Analysing the results and comparing the velocity profiles and flow distributions for each simulation BC case.

2 Introduction

In this section of the report is the background of pulmonary arteries described, an literary study presented, and the aim of the project introduced to give a greater understanding of this study.

2.1 Background

2.1.1 Cardiovascular system

The Pulmonary artery (PA) is part of the Cardiovascular system which provide blood circulation in the human body [5]. The cardiovascular system's main components are the heart, that works as a muscular pumping device, and a closed system of vessels that include arteries, veins and capillaries. The main role of the cardiovascular system is to maintain homeostasis by controlling the blood flow and keep it continuous throughout the hole body. The blood flow transport various products through the body and have the composition of white blood cells, red blood cells, plasma, and platelets.

2.1.2 Pulmonary artery

As part of the cardiovascular system, the pulmonary artery make up vital part of the heart and its anatomy [7]. The pulmonary artery transport blood with low oxygen content from the right side of the heart, called right ventricle, to the lungs for oxygenation. It consists of three parts, the Main Pulmonary artery (MPA), the Right Pulmonary Artery (RPA), and the Left Pulmonary Artery (LPA), see figure 1. The MPA is connected to the heart, after the pulmonary valve, and is the inlet of the PA. The MPA has a bifurcation, divided into two parts, which are the RPA and the LPA that supply blood to the right and the left lung respectively as outlets. The PA vessel is short, about 5 cm, and has a diameter of 2-3 cm.
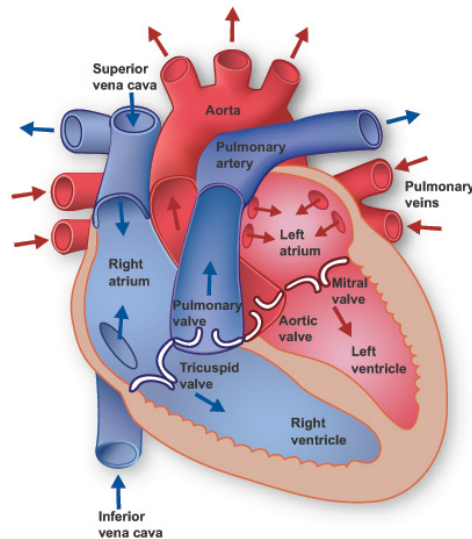

Figure 1: Anatomy of the heart. Pulmonary artery illustrated in the middle with a similar shape as the capital letter T [7].

2.1.3 Magnetic resonance imaging

To provide the in-vivo data to enable CFD simulations is a magnetic resonance imaging (MRI) technique used[1]. The MRI uses magnetic fields to generate detailed imagines of blood vessels. The images can be post-processed in CFD programs, through techniques such as segmentation, to obtain data for creating accurate structural geometries. In this study is the the model already produced as a average shaped pulmonary artery. The study therefore begins by generating a mesh for the model.

2.2 Literary studies

The following sections provide information from articles about similar projects on blood flow simulations, in-vivo simulations, in-plane velocity measurements, and sensitivity analysis. The information lay ground for the approach of the method of this study.

2.2.1 Normal Three-Dimensional Pulmonary Artery Flow using MRI

In a study made by Morgan *et al.* was a characterization of to measure the blood flow in the main, left and right pulmonary arteries in three-dimensions using phase contrast magnetic resonance imaging (MRI) [8]. The study was based on seven healthy patients. The measured parameters were the flow, mean velocity, kinetic velocity, and cross section area in different phases of the cardiac cycle. The study also investigated the axial, circumferential, and radial shear stresses near the wall of the vessel using in-plane velocities. The three-dimensional characterization of the blood flow in the vessel lay ground for an evaluation of flow patterns comparing healthy and unhealthy patients to better understand how vascular diseases can affect the pulmonary arteries. The main findings were the lower velocity profiles along the inferior wall of the main, right and left pulmonary arteries, thus allover resulting in a relatively constant shear stresses along the vessel wall and the cardiac cycle.

A similar study using a similar methodology as presented in the paper above was made a few years earlier by Sloth *et al*, also studying the blood flow velocity profile in the human pulmonary artery (PA) [1]. Data was obtained for 12 volunteers to investigate the axial velocities in the pulmonary vessel and how the velocity profile was distributed. The method used here MR measurements to quantifying the shape of the velocity profiles by using a spatial distribution index for the peak systolic velocity, this was calculated by taking the spatial maximum velocity divided by the spatial mean velocity. The results presented was similar to the findings made by Morgan *et al*. The results showed that axial velocities was lower in the right inferior region of the MPA during systole.

2.2.2 Blood Flow Dynamics at the Pulmonary Artery Bifurcation

In a study made by Capuano *et al.* the purpose of was to characterize flow patterns in the pulmonary artery bifurcation of healthy pediatric subjects using direct numeric simulations [2]. The population contained five healthy patients. The data form the patients generated a reference average 3D surface representation geometry of the pulmonary artery bifurcation using statistical shape modeling. Boundary conditions are set to mimic physiological conditions at rest. This meant a peak Reynolds number equal

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[1]A. Guala and L. Dux-Santoy from the Vall D'Hebron Research Institute (Barcelona, Spain) are gratefully acknowledged for providing the data.

to 3400 and a Womersley number equal to 15. In the inlet plane was a realistic flow rate curve obtained by averaging the in-vivo measured flow rates and the volumetric data was a truncated sum of 10 Fourier harmonics to ensure periodicity. This was mapped onto the inlet face as a plug velocity profile. Results showed a apperance of a transient and localized vortical structure forming on the inner wall of the MPA, resulting in a elongation especially in the RPA.

2.2.3 Effects of the Distribution in Space of the Velocity-Inlet Condition in Hemodynamic Simulations of the Thoracic Aorta

In a paper made by Antonuccio *et al.* is a similar analysis to what is done in this project presented, but for the aorta [4]. The technique of quantifying the sensitivity of the inlet conditions, called Generalized polynomial chaos, is preformed. Generalized polynomial chaos (gPC) is a strategy to obtain a continuous response surface in the parameter space. This starts from a few deterministic numerical simulations. The approach is based of the projection of a given stochastic response in terms of an orthogonal polynomial basis. The selected parameters of the gPC affect the velocity distribution in the model. The strategy is investigating the sensitivity of the hemodynamic outlet quantities, such as velocity, pressure, wall-shear stress, etc., to the velocity distribution in the inlet section. A function is determined for the inlet velocity distribution and resembles as a truncated cone shape. The cone is a result of a function of the ratio between the upper and lower base. The ratio can in numerical terms be described as $r = A_{top}/A_{base}$ and is defined as a uncertain parameter in the range of r = [0.2 - 1], were $A_{top}/A_{base} = 1$ is the plug flow condition.

2.3 Aim

The aim of this study is to investigate the sensitivity of numerical predictions to inflow boundary conditions for the pulmonary arteries. This will be done through numerical simulation of blood flow in the pulmonary arteries based on a realistic inflow condition and associated variations, using various software packages: SimVascular, ParaView, and MATLAB.

3 Methodology

Presented in this section, the methodology lays the groundwork for how the study is realized. The steps here, from model to simulation, will carry to the following sections.

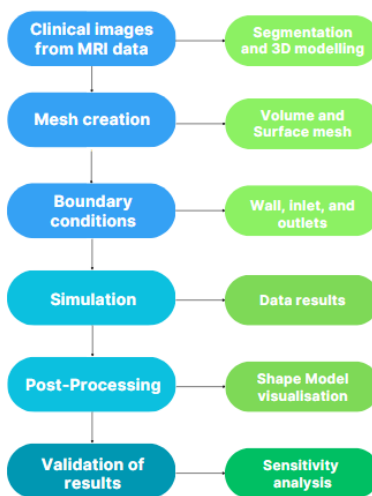The stages of a medical CFD simulation i illustrated to get an overview of the study, see figure 2.

Figure 2: Overview of the CFD methodology.

3.1 Simulation tool

The mainly used simulation tool in this study is the medical adaption software called SimVascular that simulate blood flow with computational fluid dynamics methods. The simulations are made on a average shaped model of the pulmonary artery. The model is conducted by using the shape of several patients pulmonary arteries to generate one average sized model. To evaluate the simulations results is the post-processing application called ParaView used. ParaView visualize the data results from SimVascular and create imagines showing the blood flow distribution in the model. Managing the patient-specific data and creating desired boundary conditions files are possible by using scripts programmed in the software Matlab.

3.2 Model of the Pulmonary artery

The model used in SimVascualar is an average 3D sized model created by taking every subjects pulmonary artery into reference and generating a average shaped model for surface representation, inspired by the method used by Capuano *et al.* described in section 2.2.2. The model could therefore represent each subject without the need of preforming a simulation for every subjects artery alone. The model is illustrated in figure 3.
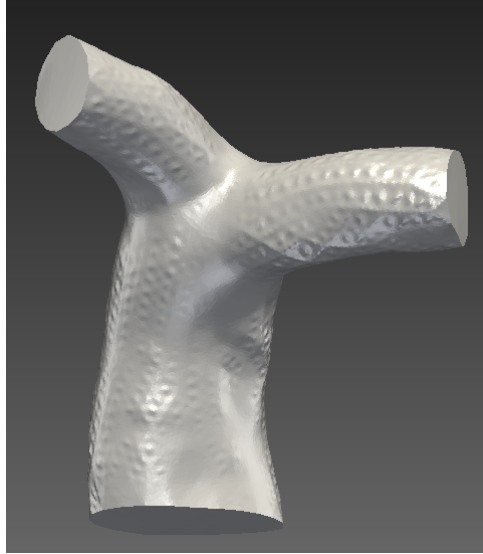
Figure 3: Average shape of pulmonary artery model used in SimVascular simulations. With the right and left pulmonary arteries as outlets and the main pulmonary artery as inlet.

3.3 Mesh

Based on the model presented in section 3.2 could a mesh be generated in SimVascular, see figure 4. A volume- and surface meshing are used with a linear subdivison on each face. Global Max Edge Size is set to 1.5. The number of divisions is set to 2 for a more precise mesh result at the capillaries outlets.
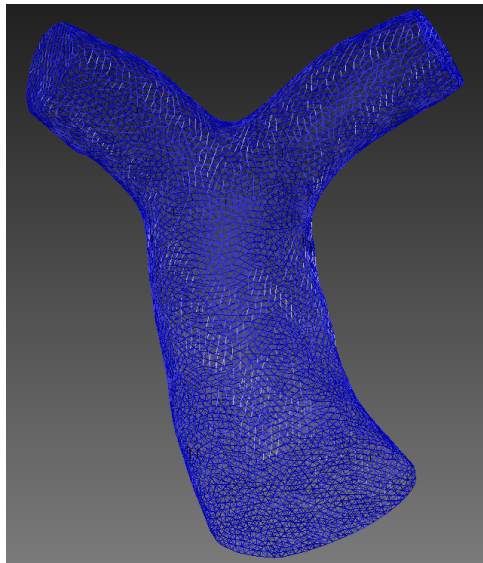

Figure 4: Mesh of the pulmonary artery used to preform the siumulations.

3.4 Boundary conditions and analyse approach

To evaluate the uncertainty quantification of the inflow for the pulmonary arteries and preform a sensitivity analysis for different numerical boundary conditions is an average inlet condition generated. The average normal inlet velocity, the in-plane velocity and variance from all the patient specific data is included. By doing this could several velocity points for each specific patient be taken into reference. This is scripted in Matlab.

The average template and inlet condition can then be modelled in SimVascular for CFD simulations. The method adapted here is mainly inspired by the Three-dimensional characterization of blood flow studies in section 2.2.1 and the Generalized polynomial chaos (gPC) strategy described in section 2.2.3.

The boundary conditions are assumed to analyse different inlet velocity profiles and in-plane velocities. To investigate how the distribution of the flow changes with different conditions is the following variations selected and scripted:

- An average template for inlet velocity vectors for every subject corresponding to inlet points and time points. This is made using the mean value of the collected data for all the subjects.

- Four simulations where the standard deviation is added. The standard deviation is added times 1 and 2, and subtracted times 0.2 and 04 (to prevent opposed flow direction), ending up with 4 additional cases. The standard deviation is a measure of how dispersed the input is in relation to the mean.

3.5 Simulation

The following set-up and parameters are used to create the simulations in SimVascular. First, the patient-specific data was managed and evaluated by using a data-sheet containing the spatial dimensions, inlet points, time points and subjects respectively. This became a matrix, (3, 137, 40, 35). The matrix indicated 35 number of patients with individual data. The velocity template frame for each subject is also taken into reference. The data are tested by preforming calculations of the flow rate and plot it for all subjects. This visualised which patient-specific data that couldn't be used for the simulations due to incomplete features. Following subjects could be excluded: 5, 8, 10, 17, 24, 30, and 32. Leaving 28 number of subject data to be simulated.

To preform the simulations, data files are created in SimVascular. The unit used is [mm] and the simulation set-up had the listed parameters below, see table 1. Presented parameters was obtained from the SimVascular Simulation Guide [6].

Table 1: Simulation parameters

Characteristic	Value
Global Max Edge Size	1.5
Fluid density	0.00106
Dynamic viscosity	0.004
BC	Assigned: In-vivo driven
LPA	Resistance: 0.0155
RPA	Resistance: 0.01
Wall (BC)	Rigid
Number of Timesteps	1000
Time Step Size	0.001
Number of Timesteps between restarts	10
Step Construction	5

The Simvascular simulation data files contains a bct.dat file that illustrate the boundary inlet conditions of the flow. To perform the sensitivity analysis and simulate the five different velocity templates, as stated above, the files are modified in an Matlab script. The script interpolates the programmed points representing the patient-specific data with the SimVascular model points to integrate the boundary conditions into the simulation. After this´step could the simulations be ran. The results are then converted to be available for post-processing in the visualization tool ParaView.

3.5.1 Converting results

The hearts anatomy include the so called cardiac cycle, also known as a single heartbeat and its phases [3]. To understand how the blood flow simulation results should be converted is the understanding of the cardiac cycle important. The cycle can be divided into two phases: diastole and systole. Diastole is when the heart expand and fills with blood, and systole is when the heart pumps the blood out to the body. The systole phase contain three specific moments: acceleration phase, peak systole and deceleration. The systole phase is illustrated in figure 5 as a inlet flow rate profile.

During the acceleration phase of the systole is the heart starting to contract. This raises the pressure in the Right Ventricle (RV), see figure 1 for the hearts anatomy. This cause a lower pressure in the PA that allows the pulmonary valve to open. The blood now flows into the PA with increasing velocity.

The peak systole, or Systolic peak, is reached after the acceleration phase. Here, the maximum contraction and blood ejection from the ventricles is reached. Also described as the point were the RV has pumped the greatest possible amount of blood during the contraction and the velocity of the blood in the PA starts to decrease.

Deceleration phase starts after the two above stated phases, when the heart starts to relax. Pressure in RV decrease which also decrease the flow rate in the PA. When the pressure in the RV is lower than

the pressure in the PA, the pulmonary valve closes to prevent blood from flowing back into the RV. The systole phase is finished and the diastole phase starts again.
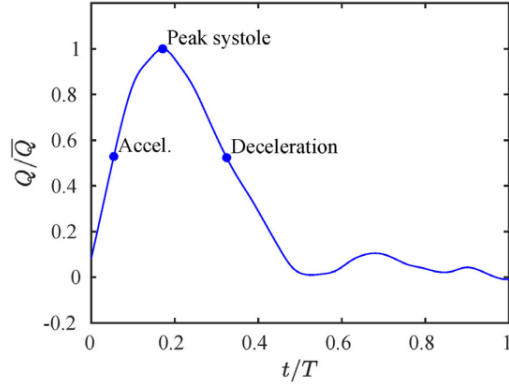

Figure 5: Inlet flow rate profile for the systole phase. Showing the flow rate in cardiac duration time.

In summary, the blood velocity is highest during the systole phase and specifically during the systole peak. At this point the blood is flowing through the pulmonary valve with the highest velocity, filling the PA, and can be seen as a good measurement point for the inflow blood conditions. The simulation results are therefore converted at this point. To extract these results are the flow rate of the cardiac cycle plotted based on the patient specific data. This resulted in a systole peak after approximately 7 time points, out of a period of 40 time points. As the simulation has 1000 number of time steps and a time step size of 10, the systole peak can be found at the 170th time step. Files, increasing with 10 steps each time, for the time steps are created during simulation. The 170th file are converted for post-processing.

4 Results

In this section are the results of the SimVascular simulation presented. Five variations of velocity boundary conditions are analysed, as described in section 3.4, and are listed as cases in table 2. The purpose of the sensitivity analysis was to investigate if different boundary conditions for the blood inflow affects the velocity distribution in the pulmonary artery. The simulation results were post-processed in ParaView by three-dimensional imaging creating Shaped Models (SM).

The selected analyse approach is to create inlet velocity profiles for each simulation by using the Slice and Warp By Vector modes in ParaView. This resulted in an inflow SM visualizing the inlet velocity distribution.

Another approach to understand the allocation of the blood flow in the model is the Stream Tracer mode. The velocity of the blood flow is presented as streamlines creating a flow pattern running through the SM.

Table 2: The five different boundary conditions, (velocity profiles).

| Velocity profile	Boundary condition
1	average mean velocity
2	average mean velocity + standard deviation
3	average mean velocity + standard deviation*2
4	average mean velocity - standard deviation*0.2
5	average mean velocity - standard deviation*0.4

4.1 Inlet velocity profiles

The inlet velocity profiles for each BC is presented below, see figure 6. The profiles are positioned close to the inlet of the PA.

The velocity profile shape of case 1 shows higher velocity in the middle, with lower velocities on te left and right part of the MPA. Velocity are lower in the left part. Velocity range reach to around 770 mm/s.

The velocity distribution for case 2 shows higher velocity in the front and middle part of the MPA, compared to case 1. The velocity seems to be lower in the left and right part here also. The velocity is high in the peaks (tops), which more create a pointy mountain shape than a plug shape. Noted is the data range in the color legend, peak range of approx. 1200 mm/s.

For case 3 are the velocity magnitude range higher, peak range 1900 mm/s, indicating that the velocities are higher when the standard deviation is included in the boundary conditions. This as case 1 have the lowest velocity range. The velocity distribution for case 3 are similar to case 2, with higher velocity in the middle and front of the MPA and much on the left, right, and back part.

When the standard deviation is subtracted, as for case 4 and 5, is also a reduction in velocity range obtained. This is logical, as the inlet velocity data becomes lower. These boundary conditions are tricky. It's important not to generate data that shows negative velocity. These can be seen in case 5. The profile illustrate backward streams. It seems that the majority of the velocity data have become negative due to the subtraction of the standard deviation.
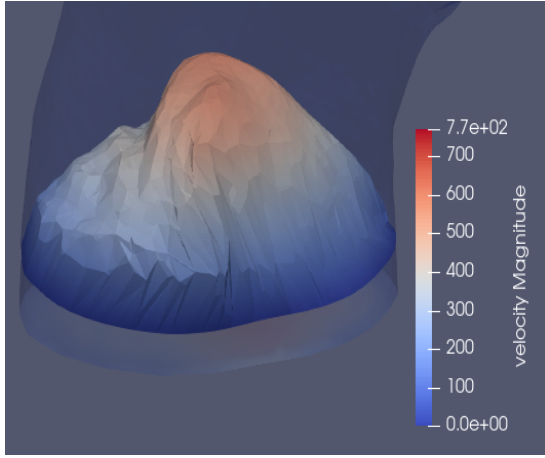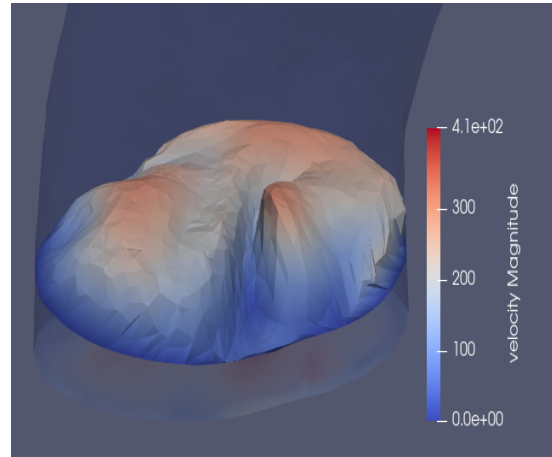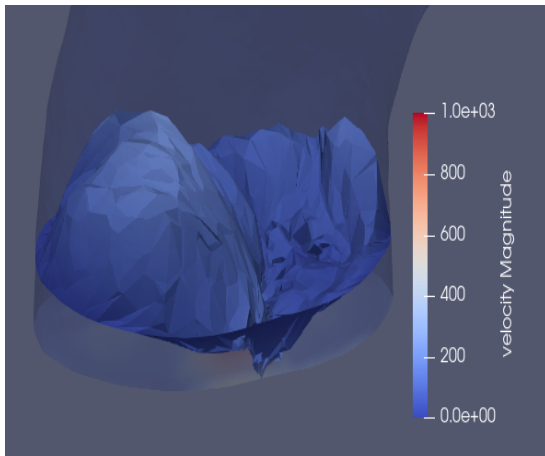
(a) Case 1
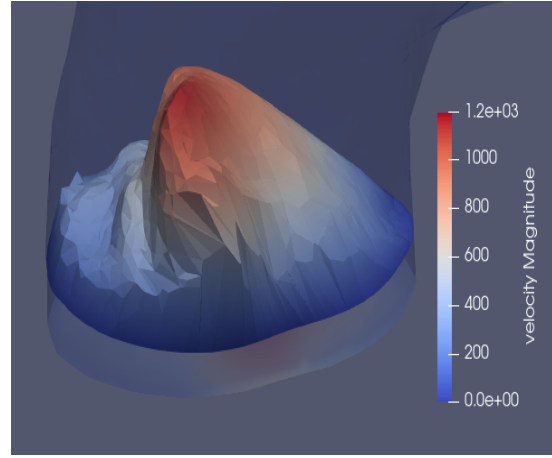
(b) Case 2
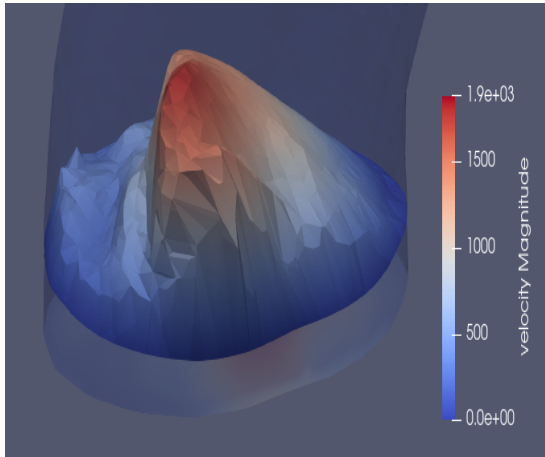
(c) Case 3

(d) Case 4

(e) Case 5

Figure 6: Inlet velocity profiles for BC-case number 1-5.

## 4.2 Blood flow velocity distribution in model

Blood flow patterns and distributions is illustrated by visualizing streamlines, with coloring representing the velocity. The five cases of BC is presented in figure 7.

Case 1 has a stream with higher velocity in the middle part of the MPA, near the inflow. The velocity becomes lower at the bifurcation. Flow velocity then becomes higher in the RPA compared to the LPA. Some streams move almost horizontally along the walls of the MPA, otherwise a laminar flow pattern.

For case 2 are similar flow characteristics shown, except more horizontal flow lines. Streamlines from the back right part of the vessel flow along the right wall to the front of the MPA and continues into the RPA. This becomes even more common is case 3. Case 3 also have higher velocity distribution towards the LPA.

For the fourth case is the blood flow even more twisted, with flow directions both horizontal and downstream at the inlet. Flow velocity becomes higher in the back part of the MPA vessel. Compared to case 1-3 is there no horizontal streams along the vessel wall further up in the PA.
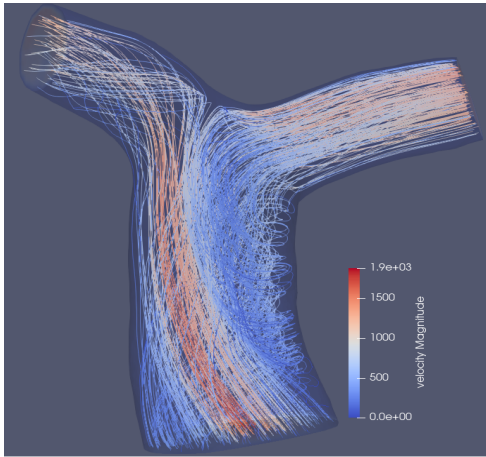
Case 5 are very different from the other cases. The velocity is much lower, and as we could see in the velocity inlet profile in figure 6 (e) is the flow direction negative, backwards, for some streamlines. This probably cause the lower velocity in the MPA.
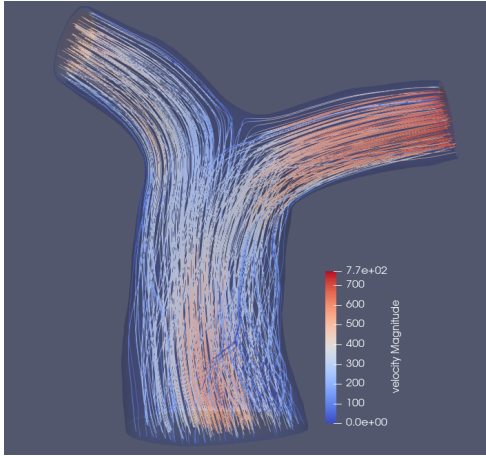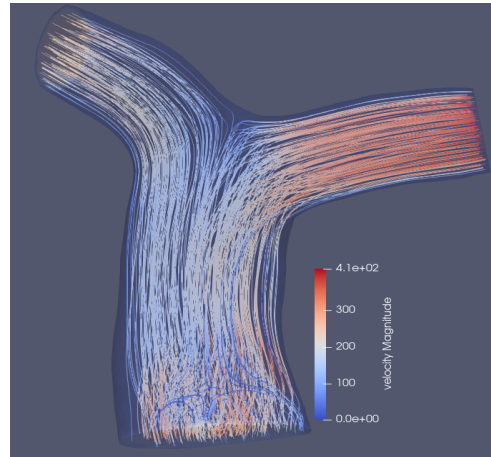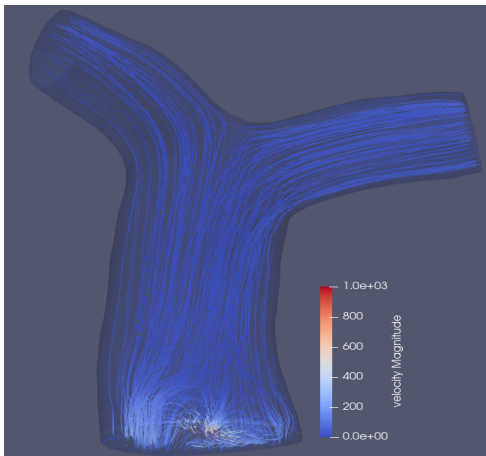
(a) Case 1
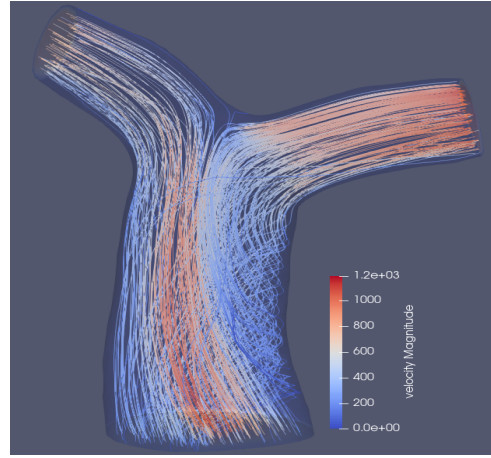
(b) Case 2

(c) Case 3

(d) Case 4

(e) Case 5

Figure 7: Blood flow streams for BC-case number 1-5.

To visualise the horizontal and twisted behavior of the blood stream can the flow be represented by vectors. In figure 8 is case 4 shown, using the Glyph mode in ParaView. Vectors in the inlet part of the MPA seems to be directed parallel or downstream with the other flow patterns. The velocity seems to be high in the back of the vessel, putting pressure on the back wall.
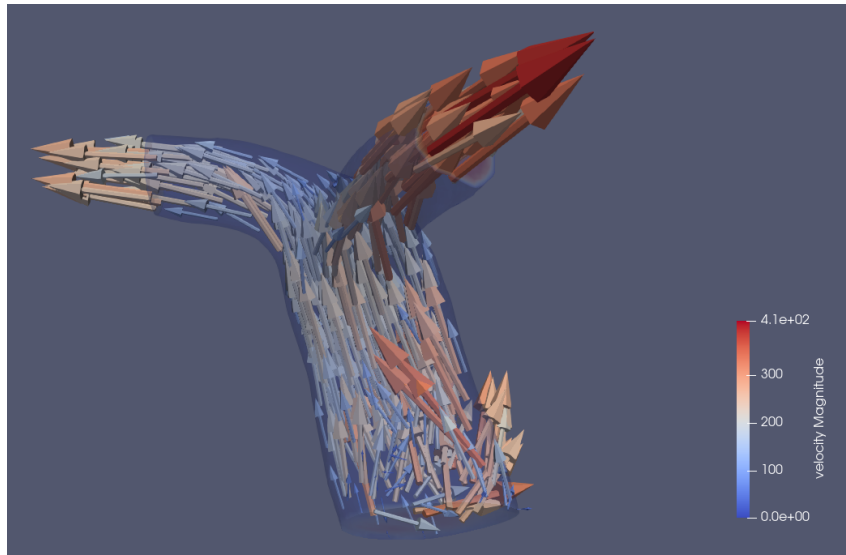

Figure 8: Blood flow stream represented with vectors for case 4.

Even though the results for case 5 can be difficult to evaluate as reasonable is a Glyph generated here also. This mainly illustrate why the velocity range becomes so much lower. In figure 9 can we see vectors with high velocity flowing downstream, inhibits higher flow in the MPA.
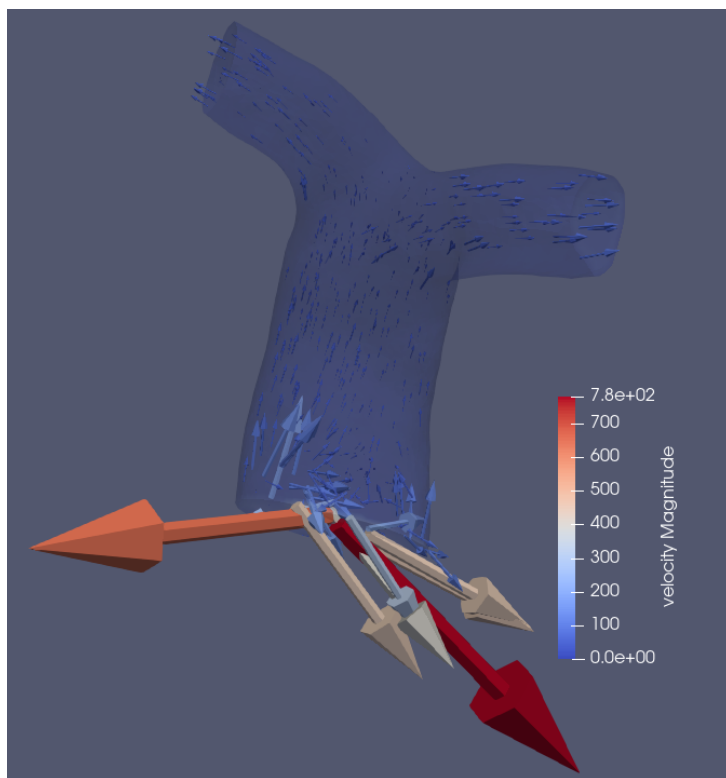
Figure 9: Blood flow stream represented with vectors for case 5.

5 Discussion and conclusion

5.1 Discussion

The velocity profiles for case 1-5 are showing differences, especially between the first three cases and the fourth and fifth. The variation of BC seem to change the inlet velocity profile in several ways. In common for case 1-3 is the peak velocity centralised, with lower velocity on the left and right side of the vessel. Similar finding that was made by Sloth *et al*, were the velocity profile showed a lower velocity area towards the right, but still higher velocity to the left, see figure 10 [1]. In this study can a low velocity be found at both areas.
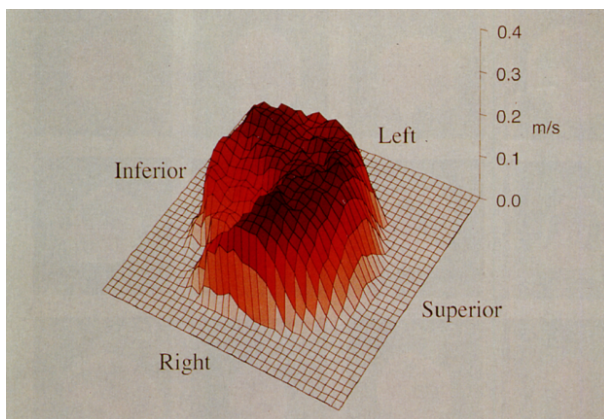

Figure 10: Three-dimensional visualization of mean temporal velocity inlet profile. Showing lower velocity on the right inferior area [1].

The distribution form a plug shape especially for the first case. This changes for case 2 and 3 when the velocities is higher at the peak (middle). Case 4 shows a more even distribution of the velocity. This can also be seen when analysing the streamlines for case 4, were the flow look laminar. Exception for a small twisted behavior at the inflow.

For case 3 are the velocity magnitude range higher, indicating that the velocities are higher when the standard deviation is included in the boundary conditions. This as case 5 have the lowest velocity magnitude maximum when the standard deviation is subtracted.

The formation of parallel or horizontal stream directions and the creation of twisting structures appeared in all cases, especially case 3. This seems to be higher with the variation size of the BC's. In case 5 are the variation largest at the inlet. The streams even go downstream, making this boundary condition simulation less reliable. There was a challenge to structure the BC to create a dependable sensitivity analysis. The conclusion for case 5 is that this BC was to divergent to give reasonable results. The results for this case is included as it could be informative for future work in this area.

The flow velocity is higher in the RPA compared to the LPA for all cases. This is assumed to be because of the higher resistance at the LPA and therefore contributes to a easier path for the blood flow through

the RPA.

5.2 Conclusion

In this study was patient-specific in-vivo data of the pulmonary artery for N = 28 healthy subjects analysed. The study included a complete set-up of a simulation for a average shaped model of the PA based on MRI data. To analyse the how the inflow is affected by various boundary conditions of the blood was inflow properties scripted by using the mean and the standard deviation. This resulted in an sensitivity analysis of the velocity distribution in the PA. Results showed that the blood flow changes with different boundary condition, especially the flow streams. Inlet profiles is assumed to have higher velocities in the middle part of the MPA inlet.

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