



# Transmission line parameters estimation in the presence of realistic PMU measurement error models

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## ABSTRACT

Proposals have been presented in literature to estimate line parameters and monitor their changes. Synchrophasor measurements from phasor measurement units (PMUs) have appeared as a possible breakthrough for accurate estimation. However, few methods consider a realistic measurement chain including PMUs and instrument transformers and their systematic and random error contributions. This paper proposes an improved method to simultaneously estimate line parameters and systematic measurement errors on multiple network lines. The algorithm is designed to deal with realistic PMU measurement errors and, in particular, with phase-angle errors caused by common time-base errors on multiple PMU channels. The impact of PMU measurement errors is investigated to achieve a comprehensive view of the performance under realistic conditions. The results obtained on an IEEE test network prove the advantages of the proposed method with respect to other recent methods and its robustness in the presence of mismatches in the error model.

## 1. Introduction

When dealing with power system management, several applications are involved. Among others, it is possible to mention state estimation (e.g., [1,2]) and fault location methods (see for instance [3,4]). In these applications, network models play a fundamental role and line parameters are the basis to build such models and thus to perform any further processing or evaluation. Nevertheless, actual parameter values can significantly differ from data available to Transmission System Operator (TSO) [5] because of manufacturing tolerance, environmental conditions or aging (see, for example, [6]). This can introduce significant mismatches that can result in Energy Management System issues.

Phasor Measurement Units (PMUs) appear a promising tool to overcome some of the difficulties associated with conventional monitoring systems, thanks to their ability to provide accurate measurements referred to an absolute time reference (the coordinated universal time, UTC) [7], and, in particular, to measure phase angles in a synchronized way. Specifically, PMUs compute with a high reporting rate the so-called synchrophasor measurements and are able to monitor voltage and currents simultaneously. Then, PMU measurements from different locations can be aligned and coordinated based on the same timescale. In the last decades, TSOs worldwide have been installing PMUs to build the so-called Wide Area Monitoring Systems (WAMSs), i.e., the new generation of distributed monitoring infrastructures for power systems.

WAMSs are the appropriate context to collect and use synchrophasor measurements for every monitoring and management task. PMUs, thanks to the aforementioned peculiarities, can contribute significantly in the critical and challenging task of evaluating accurately the values of line parameters. In this perspective, a realistic PMU measurement error model is essential to properly take account of the inaccuracy of each component of the measurement chain, which also includes Instrument Transformers (ITs). In the literature, different approaches to PMU-based parameters estimation have been introduced recently.

In [8], only PMU error is considered for a positive sequence line parameters estimation, as well as in [9], where a line parameters estimation in the presence of series and shunt compensators is performed. Also in [10], the identification of susceptances and reactances is addressed assuming only the presence of PMU errors in the measurement chain. In [11], the identification of line and power transformer sequence parameters is carried out. The measurement data are obtained corrupting the phasors with PMU errors considering different values of uncertainty. In [12], line parameters estimation in an augmented state estimation framework is proposed, assuming the presence of random errors due to PMUs.

In other researches, both PMU and IT errors are considered. In [13,14], multiple time instants are considered to perform the estimation, but IT errors are modeled as zero-mean random noise, thus

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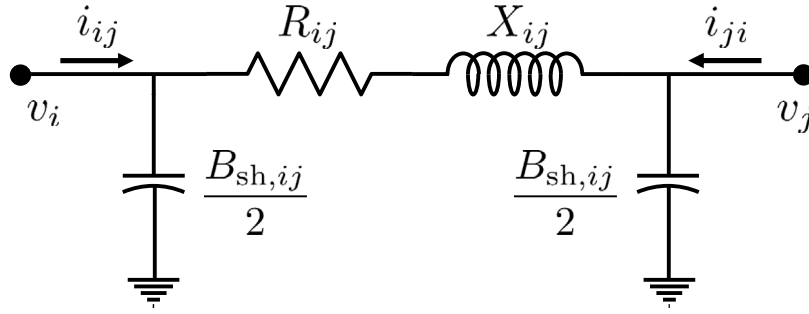


Fig. 1.  $\pi$ -model of a transmission line.

neglecting systematic errors. Also [15] presents a line parameters estimation method considering zero-mean Gaussian noise in the magnitude and phase angle for both ITs and PMUs. In [16], a linear least square method for single line parameters estimation based on real measurements is presented; also in this case the presence of systematic measurement errors in the estimation model is not considered.

In literature, few papers focus on the problem of systematic errors introduced by ITs. Among these, in [17] the estimation performances of three different least square methods have been compared using field measurements and highlighting the problems due to systematic IT errors. In [18], a robust line parameters estimation method is presented using on-site measurements; also in this paper it is emphasized that, in real grid scenarios, ITs accuracy must be considered. In [19], calibrated transducers are used to propagate the accuracy while estimating line parameters in a network. In [20], systematic errors of current amplitudes and voltage phase angles are assumed negligible to simultaneously estimate line parameters and current phase-angle and voltage-amplitude systematic errors. In [21], presenting the detection of uncalibrated ITs in a preliminary way, a simultaneous estimation of single line parameters and IT errors is addressed, assuming calibrated ITs at the sending node. When IT systematic errors are not fully compensated, which is a typical situation, line parameter estimation is strongly affected. In [22], a direct computation method based on the application of PMU current and voltage measurements is used to calculate the uncertainty bounds of transmission line parameters. A direct method, applied to real PMU measurements, is also used in [23] to evaluate the variability of line parameters (without a previous calibration of ITs and without compensating the systematic errors). Such evaluation shows a discrepancy with respect to the theoretical bounds found under the hypothesis of uncorrelated measurements. The presence of correlation in real measurements is highlighted via statistical analysis.

To reduce the impact of the systematic errors, in [24], an algorithm to estimate simultaneously line parameters and systematic errors introduced by ITs is presented. An enhanced version of this algorithm is proposed in [25], where the method is designed to properly deal with multiple lines at the same time and with different operating conditions of the network. In particular, the method relies on the definition of a measurement model that considers both systematic and random measurement errors and on prior knowledge on line parameters, IT and PMU uncertainty. The method, as commonly assumed in the literature, considers that the errors from different PMUs and PMU channels are independent random variables. In [26], systematic errors in PMU measurements have been investigated to assess their impact on the estimation performance.

In this paper, based on the research in [26], the analysis considers other potential issues arising in presence of a mismatch between the assumed PMU error model and actual instrument behavior. In particular, the role of possible common errors (typically due to time-base errors of the instrument) in the phase-angle measurements of different

channels of the same PMU is deeply investigated. This paper proposes an improved algorithm that is designed to evaluate and include in real-time correlation information about phase-angle errors within the estimation framework, thus helping the estimation of both line parameters and measurement chain systematic errors on multiple branches of the network. The new algorithm is validated through simulation on the IEEE 14 bus test system. Obtained results prove its enhanced performance with respect to other methods and its robustness against model mismatches that can occur in real applications on the field.

The paper is organized as follows: Section 2 presents the estimation framework and the proposed improvement is introduced in Section 3; Section 4 explains the test cases and illustrates the analysis of the estimation performance; Section 5 reports the final remarks and concludes the paper.

## 2. Estimation framework

In this section, the algorithm proposed in [25] to address the simultaneous estimation of line parameters and systematic errors in the synchrophasor measurement chain in presence of multiple network branches and several load scenarios is briefly illustrated with a clear focus on its assumptions and on the measurement model.

### 2.1. Measurement problem

First, an equivalent single-phase model of each transmission line is assumed (see [21]), represented by the  $\pi$ -model in Fig. 1. The measurement configuration, corresponding to the widespread installation of WAMS, is also shown in Fig. 1. Four synchrophasor measurements are considered for the generic branch  $(i, j)$ :  $v_i$ ,  $v_j$ ,  $i_{ij}$  and  $i_{ji}$  are the synchrophasors of the start-node voltage of the line (node  $i$ ), of the end-node voltage (node  $j$ ), of the branch current measured from node  $i$  and of that from node  $j$ , respectively. All measurements are assumed to be provided by two PMUs installed at both ends of the line (one for each node) and measuring node voltage and all branch currents of the adjacent edges. Thanks to PMU timestamps, all measurements corresponding to the same time instant  $t = nT_{RR}$  ( $T_{RR}$  is the PMU reporting interval) are considered all together as a time-tagged measurement set.

Fig. 1 shows that the line parameters to estimate are the line resistance  $R_{ij}$ , the line reactance  $X_{ij}$  and the shunt susceptance  $B_{sh,ij}$  (which is equally split into the two sides of the model).

The line parameter estimation algorithm is based on a set of equations involving the unknown line parameters and the measurement errors. The equations can be written for each timestamp  $t$  since they give constraints on the unknowns and on the measurement snapshot at  $t$ . For each timestamp, four synchrophasor measurements are thus available for each line and it is possible to impose Kirchhoff's Laws constraints. In particular, this leads to writing two complex-valued equations: the first one expresses the voltage drop across the line (a network branch) while the second corresponds to the current balance at

the branch ends. In detail, the following two equations are considered for the generic branch  $(i, j)^2$ :

$$v_i^R - v_j^R = (R_{ij} + jX_{ij}) \left( i_{ij}^R - j \frac{B_{sh,ij}}{2} v_i^R \right) \quad (1)$$

$$i_{ij}^R + i_{ji}^R = j \frac{B_{sh,ij}}{2} (v_i^R + v_j^R) \quad (2)$$

where superscript  $R$  indicates the reference value of the corresponding measured quantity. Eqs. (1) and (2) define thus constraints for the line parameters based on actual values of voltage and current phasors at a given time.

Since voltages and currents are monitored through a measurement process, the available data correspond to measurements and thus reference values defining (1) and (2) can be rewritten as functions of the measured synchrophasors, including systematic and random errors in the measurement chain, as follows:

$$v_h^R = \frac{V_h}{(1 + \xi_h^{sys} + \xi_h^{rnd})} e^{j(-\alpha_h^{sys} - \alpha_h^{rnd})} \approx V_h e^{j\varphi_h} (1 - \xi_h^{sys} - \xi_h^{rnd} - j\alpha_h^{sys} - j\alpha_h^{rnd}) \quad (3)$$

$$i_{ij}^R = \frac{I_{ij}}{(1 + \eta_{ij}^{sys} + \eta_{ij}^{rnd})} e^{j(-\psi_{ij}^{sys} - \psi_{ij}^{rnd})} \approx I_{ij} e^{j\theta_{ij}} (1 - \eta_{ij}^{sys} - \eta_{ij}^{rnd} - j\psi_{ij}^{sys} - j\psi_{ij}^{rnd}) \quad (4)$$

where  $h \in \{i, j\}$ ,  $V_h$  and  $I_{ij}$  are the measured voltage and current magnitudes, respectively, while  $\xi_h$  and  $\eta_{ij}$  indicate the corresponding relative measurement errors.  $\varphi_h$  and  $\theta_{ij}$  are the voltage and current absolute phase angles, respectively, which are measured by the PMUs, and  $\alpha_h$  and  $\psi_{ij}$  are the corresponding errors. All errors are split into a systematic contribution, indicated by superscript  $^{sys}$ , and a random one, labeled by superscript  $^{rnd}$ . The measurement errors in the chain are small (their absolute value is always  $\ll 1$ ) and thus the approximated expressions in (3) and (4) are obtained considering a first order approximation, thus neglecting terms, even multivariate, with a degree  $> 1$ . An equation analogous to (4) can be written also for the current in the opposite direction from node  $j$ .

Replacing (3) and (4) into (1) and (2), two complex equations can be written. In addition to the line parameters, the equations also involve all the measurement errors. Indeed, considering multiple pairs of equations corresponding to different timestamps (e.g.,  $t_1 \dots, t_{N_t}$ ) and possibly to different operating conditions of the network, a set of equations can be defined. In all equations line parameters and systematic errors can be assumed as the unknowns to be found whereas the random errors can be considered as disturbances in the constraint definitions. Following this approach, it is possible to define the estimation problem, which aims at finding the state composed of all the unknowns starting from the measured values.

## 2.2. Background on the estimation framework

In [25], systematic errors are attributed to ITs, i.e., to voltage transformers (VTs) and current transformers (CTs), whereas PMUs are considered as affected mainly by random errors. To simplify the estimation task, line parameters in (1) and (2) are represented through the following equations:

$$\begin{aligned} R_{ij} &= R_{ij}^0 (1 + \gamma_{ij}) \\ X_{ij} &= X_{ij}^0 (1 + \beta_{ij}) \\ B_{sh,ij} &= B_{sh,ij}^0 (1 + \delta_{ij}) \end{aligned} \quad (5)$$

where superscript  $0$  indicates the known values that are already available to the TSO, and  $\gamma_{ij}$ ,  $\beta_{ij}$  and  $\delta_{ij}$  are the unknown relative deviations

<sup>2</sup> From hereon the timestamp will be reported in the equations only when needed for clarity.

from them, which represent the lack of knowledge. By estimating  $\gamma_{ij}$ ,  $\beta_{ij}$  and  $\delta_{ij}$ , it is possible to estimate the line parameters too.

Replacing then (3) and (4) into (1) and (2), translating the complex equations into their real and imaginary parts, and considering first order approximation ( $|\gamma_{ij}|$ ,  $|\beta_{ij}|$  and  $|\delta_{ij}|$  are also  $\ll 1$ ), a linear system of equations for the generic branch  $(i, j)$  can be written as

$$\mathbf{b}_{ij} = \mathbf{H}_{ij} \begin{bmatrix} \xi_i^{sys} \\ \alpha_i^{sys} \\ \xi_j^{sys} \\ \alpha_j^{sys} \\ \eta_{ij}^{sys} \\ \eta_{ji}^{sys} \\ \psi_{ij}^{sys} \\ \psi_{ji}^{sys} \\ \gamma_{ij} \\ \beta_{ij} \\ \delta_{ij} \end{bmatrix} + \mathbf{E}_{ij} \begin{bmatrix} \xi_i^{rnd} \\ \alpha_i^{rnd} \\ \xi_j^{rnd} \\ \alpha_j^{rnd} \\ \eta_{ij}^{rnd} \\ \eta_{ji}^{rnd} \\ \psi_{ij}^{rnd} \\ \psi_{ji}^{rnd} \end{bmatrix} = \mathbf{H}_{ij} \mathbf{x}_{ij} + \mathbf{E}_{ij} \mathbf{e}_{ij} = \mathbf{H}_{ij} \mathbf{x}_{ij} + \mathbf{e}_{ij} \quad (6)$$

where  $\mathbf{b}_{ij}$  is the real-valued vector of constant terms that can be considered as the equivalent measurements of the formulation of the estimation problem. As mentioned above,  $\mathbf{b}_{ij}$  includes multiple sets of equivalent measurements corresponding to (1) and (2) for each considered timestamp. Vector  $\mathbf{x}_{ij}$  is the vector of unknowns that includes all the deviations in line parameters and the systematic errors to estimate.  $\mathbf{H}_{ij}$  is the measurement matrix defining the equivalent measurements as a function of  $\mathbf{x}_{ij}$ . Vector  $\mathbf{e}_{ij}$  represents the contribution of random errors affecting the equivalent measurements in  $\mathbf{b}_{ij}$ .  $\mathbf{e}_{ij}$  is the result of a transformation through matrix  $\mathbf{E}_{ij}$  of all the random errors in synchrophasor measurement chains, which are given by random vector  $\mathbf{e}_{ij}$ .

Assuming  $N_t$  timestamps available and four real-valued equations for each timestamp, there are  $4N_t$  equations in (6). This corresponds to the definition of the problem when a single branch is considered. However, multiple branches (e.g.,  $N_{br}$  branches) can be treated together in the same estimation process. In this case, for a given timestamp, the voltage and current measurements of all the considered branches corresponding to the same instant are used, thus extending (6) into an augmented problem with  $4N_t N_{br}$  equations, described as:

$$\mathbf{b} = \begin{bmatrix} \mathbf{b}_{i_1 j_1} \\ \vdots \\ \mathbf{b}_{i_{N_{br}} j_{N_{br}}} \end{bmatrix} = \mathbf{H} \mathbf{x} + \mathbf{E} \mathbf{e} = \mathbf{H} \mathbf{x} + \mathbf{e} \quad (7)$$

where  $\mathbf{b}_{i_k j_k}$  includes the equivalent measurements of the  $k$ th considered branch  $(i_k, j_k)$  with  $k = 1, \dots, N_{br}$ .  $\mathbf{H}$  and  $\mathbf{E}$  are the measurement and transformation matrices obtained considering all the branches and the corresponding equations like those in (6). Vector  $\mathbf{e}$  is composed of the random errors for all the measured synchrophasors. In this case, the new vector of unknowns  $\mathbf{x}$  ( $N$ -size vector) includes all the parameter deviations of all the involved lines ( $3N_{br}$  unknowns if all the branches have the same model as in Fig. 1) and all the systematic errors of the measured voltage and current synchrophasors. Since joint branches share the same node voltage measurements, the number of systematic errors in  $\mathbf{x}$  is typically  $< 8N_{br}$ , thus improving the equations to unknowns ratio.

In [25], it is proposed to integrate the problem in (7) with prior knowledge on the unknowns, thus defining an overall model as follows:

$$\mathbf{b}_{tot} = \begin{bmatrix} \mathbf{b} \\ \mathbf{0}_{N \times 1} \end{bmatrix} = \begin{bmatrix} \mathbf{H} \\ \mathbf{I}_N \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{e} \\ \mathbf{e}_{prior} \end{bmatrix} = \mathbf{H}_{tot} \mathbf{x} + \mathbf{e}_{tot} \quad (8)$$

where  $\mathbf{0}_{N \times 1}$  is the  $N$ -size zero vector and  $\mathbf{I}_N$  is the  $N$ -size identity matrix, meaning that prior assumption for every unknown in  $\mathbf{x}$  is given by a zero value (best assumption on deviations and systematic errors

without further information). Vector  $\mathbf{e}_{\text{prior}}$  includes the corresponding prior errors. Prior errors represent lack of knowledge and can thus be treated as random variables as discussed in Section 3.

Starting from the model defined by (8), it is possible to estimate  $\mathbf{x}$ , i.e., to achieve an estimation of line parameters and systematic errors simultaneously for all the lines and measurement channels without requiring a preliminary calibration. A weighted least squares (WLS) solution of (8) is used, solving the following linear system:

$$(\mathbf{H}_{\text{tot}}^T \mathbf{W}_{\text{tot}} \mathbf{H}_{\text{tot}}) \hat{\mathbf{x}} = \mathbf{H}_{\text{tot}}^T \mathbf{W}_{\text{tot}} \mathbf{b}_{\text{tot}} \quad (9)$$

where  $\hat{\cdot}$  indicates the estimate,  $T$  is the transpose operator and  $\mathbf{W}_{\text{tot}}$  is the weighing matrix.

### 3. Measurement errors and proposed approach

In the solution of (9),  $\mathbf{W}_{\text{tot}}$  is chosen as the inverse of the covariance matrix  $\Sigma_{\epsilon_{\text{tot}}}$  of random vector  $\epsilon_{\text{tot}}$ .

Assuming prior information on the unknowns and random errors of PMU measurements as uncorrelated, it is possible to write:

$$\Sigma_{\epsilon_{\text{tot}}} = \begin{bmatrix} \Sigma_{\epsilon} & \mathbf{0} \\ \mathbf{0} & \Sigma_{\epsilon_{\text{prior}}} \end{bmatrix} \quad (10)$$

where symbol  $\Sigma_{\epsilon}$  and  $\Sigma_{\epsilon_{\text{prior}}}$  are the covariance matrix of  $\epsilon$  and  $\mathbf{e}_{\text{prior}}$ , respectively, while  $\mathbf{0}$  stands for a generic zero matrix of suitable size.

To define  $\Sigma_{\epsilon_{\text{prior}}}$ , two different considerations can be made:

- Prior variances  $\sigma_{r_{ik,jk}}^2$ ,  $\sigma_{\beta_{ik,jk}}^2$  and  $\sigma_{\delta_{ik,jk}}^2$ , for all the considered branches  $(i_k, j_k)$ , can be chosen based on general considerations on the uncertainty of line parameters (e.g., relying on the TSO experience). Line parameter deviations are assumed uncorrelated (if further information is available, it can be integrated seamlessly). A mismatch between actual uncertainty and assumed values can occur and in [27] such issue was thus investigated.
- Systematic errors in the measurement chain are considered uncorrelated. Also in this case, if any prior knowledge is available it can be integrated. As mentioned above, in [25] systematic errors were attributed to ITs and thus the variance of each error was derived from the IT class specification.

From the law of propagation of uncertainty it follows

$$\Sigma_{\epsilon} = \mathbf{E} \Sigma_{\epsilon} \mathbf{E}^T \quad (11)$$

and  $\Sigma_{\epsilon}$  (the covariance matrix of random measurement errors) needs to be defined considering all the random errors  $\xi_{ik}^{\text{rnd}}$ ,  $\alpha_{ik}^{\text{rnd}}$ ,  $\xi_{jk}^{\text{rnd}}$ ,  $\alpha_{jk}^{\text{rnd}}$ ,  $\eta_{ik,jk}^{\text{rnd}}$ ,  $\psi_{ik,jk}^{\text{rnd}}$ ,  $\eta_{jk,i_k}^{\text{rnd}}$  and  $\psi_{jk,i_k}^{\text{rnd}}$ .<sup>3</sup> In [25], these errors were assumed uncorrelated and associated with PMU uncertainty. Thus  $\Sigma_{\epsilon}$  was assumed diagonal and included all the square standard uncertainties derived from PMU specifications. The standard uncertainties were computed assuming uniform distributions and choosing maximum magnitude and phase-angle errors from instrument datasheets.

The presented assumptions allow computing  $\Sigma_{\epsilon}$  and  $\Sigma_{\epsilon_{\text{prior}}}$  and thus solving (9), but they might lead to possible issues in the algorithm configuration. Indeed, the measurement error of PMUs in realistic conditions can be actually composed of both systematic and random errors and this would result in a transfer of uncertainty from  $\Sigma_{\epsilon}$  representing random error only to  $\Sigma_{\epsilon_{\text{prior}}}$ . The amount of each error contribution is difficult to predict. In [26], the realistic presence of residual systematic errors in PMUs was investigated and the impact of a possible mismatch on the corresponding prior quantities was assessed. Robustness of method against such mismatch was found considering different PMU uncertainty scenarios. In this paper, a more complete error model for PMU errors is considered. Indeed, although the considerations in [26]

<sup>3</sup> The symbols here can be interpreted analogously to those in (3) (4), and (6).

are reasonably valid for magnitude measurements, phase-angle errors should be treated more carefully.

Consider the generic phase-angle error  $\lambda^{\text{PMU}}$  of a PMU installed at node  $i$ . It can represent indifferently a voltage phase-angle error or the phase-angle error of one of the current synchrophasors measured by the same PMU. The following expression can be used to illustrate its components:

$$\lambda^{\text{PMU}} = \lambda_s + \lambda_c + \lambda_r \quad (12)$$

where  $\lambda_s$  is the systematic error contribution brought by the PMU and represents the average of the phase-angle error,  $\lambda_c$  is the random phase error that is common to all the channels of the same PMU and  $\lambda_r$  is the portion of the random error contribution that is independent from other errors (specific of the considered PMU channel). This error split represents a realistic behavior of PMUs. In fact, phase-angle error, particularly for high-quality instruments, is strongly related to the time-base error (see also the discussion in [28]), which is in turn often reported in PMU datasheet. Such error is common to all the channels in a PMU and can thus result in a significant component of  $\lambda^{\text{PMU}}$  that cannot be neglected. In Section 4, the impact of such contribution on the estimation process will be deeply investigated.

It is thus interesting to analyze theoretically the effect of the decomposition in (12) on the definition of  $\Sigma_{\epsilon}$ , which is the first step in  $\mathbf{W}_{\text{tot}}$  computation. The variance of the error  $\lambda^{\text{PMU}}$  (i.e.,  $\sigma_{\lambda^{\text{PMU}}}^2$ ) is given by the sum of the variances of the two random contributions since  $\lambda_s$  is constant while  $\lambda_c$  and  $\lambda_r$  can be assumed uncorrelated. Considering two phase-angle measurements,  $\lambda_1^{\text{PMU}}$  and  $\lambda_2^{\text{PMU}}$ , of two generic channels in the same PMU, the following covariance needs to be considered ( $\mathbf{E}[\cdot]$  indicates the expectation operator):

$$\begin{aligned} & \mathbf{E} \left[ \left( \lambda_1^{\text{PMU}} - \mathbf{E} \left[ \lambda_1^{\text{PMU}} \right] \right) \left( \lambda_2^{\text{PMU}} - \mathbf{E} \left[ \lambda_2^{\text{PMU}} \right] \right) \right] \\ &= \mathbf{E} \left[ (\lambda_c + \lambda_{r1}) (\lambda_c + \lambda_{r2}) \right] = \mathbf{E} \left[ \lambda_c^2 \right] = \sigma_c^2 \end{aligned} \quad (13)$$

where  $\lambda_{r1}$  and  $\lambda_{r2}$  indicate the independent random contribution of the first and second considered channels, respectively. The last but one equality is obtained based on the null correlation between  $\lambda_c$  and the independent errors and between  $\lambda_{r1}$  and  $\lambda_{r2}$ . From (13)  $\sigma_c^2$  is thus defined as the variance of  $\lambda_c$ . Eq. (13) shows how, differently from the assumption in [25], correlation arises in the errors of the channels when a realistic common contribution is present.

To address this interesting condition, in this paper, it is proposed to introduce, for each PMU  $i$  (installed at node  $i$ ), a Pearson correlation coefficient between the channels by including it in the definition of  $\Sigma_{\epsilon}$ . The correlation coefficient  $\rho_{12}$  of the two aforementioned channels 1 and 2 depends on the ratio of  $\sigma_c^2$  to  $\sigma_{\lambda_1^{\text{PMU}}} \sigma_{\lambda_2^{\text{PMU}}}$  (where  $\sigma_{\lambda_1^{\text{PMU}}}$  and  $\sigma_{\lambda_2^{\text{PMU}}}$  are the standard deviations of the errors in the two channels). However, such index is not known in advance and needs to be estimated. When considering the  $N_i$  timestamps, and the associated measurement sets, it is important to highlight that they can correspond to repeated measurements of the same load condition of the network (the high RR of a PMU can guarantee to have multiple snapshots) but also to different cases, i.e., to different configurations of loads and generators. Thus,  $N_i = MC$ , where  $M$  is the number of repeated measurements for each case and  $C$  is the number of monitored load conditions. Based on the different nature of these measurements, a specific approach is proposed to estimate the correlation coefficient  $\rho_{12}$ . The generic timestamp  $t_n$  can be indicated also as  $t_{\chi,m} = [(\chi - 1)M + m - 1]T_{\text{RR}}$  with  $m = 1, \dots, M$  and  $\chi = 1, \dots, C$ . The estimation is performed as explained in what follows.<sup>4</sup>

The reference phase angle is different for each case  $\chi$  and each channel. In order to compute correlation between the errors of different channels, a pre-processing of phase-angle measurements is needed.

<sup>4</sup> The described procedure applies when  $M > 1$ .

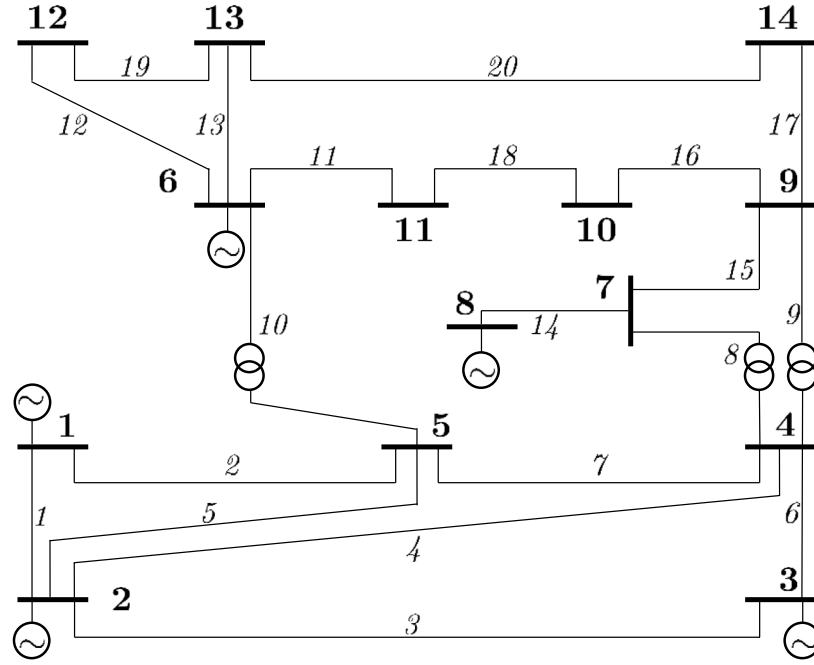


Fig. 2. IEEE 14 bus system with node and branch indices.

Considering channel 1, all the measured phase angles of case  $\chi$ , indicated with  $v_{1,t_{\chi,m}}$ , are made unbiased by subtracting their average value

$$v_{1,t_{\chi,*}} \triangleq \frac{\sum_{m=1}^M v_{1,t_{\chi,m}}}{M} \quad (14)$$

This corresponds to applying the following formula:

$$\tilde{\lambda}_{c+r1,t_{\chi,m}} = v_{1,t_{\chi,m}} - v_{1,t_{\chi,*}} \quad (15)$$

where  $\tilde{\lambda}_{c+r1,t_{\chi,m}}$  is an estimate of  $\lambda_{c,t_{\chi,m}} + \lambda_{r1,t_{\chi,m}}$ , i.e., of the overall random contribution at timestamp  $t_{\chi,m}$  for channel 1. Similar definitions and computations are used for channel 2. Finally, it is possible to estimate  $\rho_{12}$  as:

$$\hat{\rho}_{12} = \frac{\sum_{\chi=1}^C \sum_{m=1}^M \tilde{\lambda}_{\chi+r1,t_{\chi,m}} \cdot \tilde{\lambda}_{\chi+r2,t_{\chi,m}}}{\sqrt{\sum_{\chi=1}^C \sum_{m=1}^M \tilde{\lambda}_{\chi+r1,t_{\chi,m}}^2 \cdot \sum_{\chi=1}^C \sum_{m=1}^M \tilde{\lambda}_{\chi+r2,t_{\chi,m}}^2}} \quad (16)$$

When the accuracy of phase-angle measurements is the same for all the channels in a PMU, it is possible to consider a unique correlation coefficient  $\hat{\rho}^{\text{PMU}_i}$  for the PMU  $i$  computed as:

$$\hat{\rho}^{\text{PMU}_i} = 2 \frac{\sum_{ch_1=1}^{N_{ch,i}} \sum_{ch_2=ch_1+1}^{N_{ch,i}} \hat{\rho}_{ch_1 ch_2}}{N_{ch,i}(N_{ch,i} - 1)} \quad (17)$$

where  $N_{ch,i}$  is the number of channels of PMU  $i$  and  $\hat{\rho}_{ch_1 ch_2}$  is the estimated correlation coefficient between its generic channels  $ch_1$  and  $ch_2$ . Once all the correlation coefficients have been estimated,  $\Sigma_e$  can be built accordingly and then the estimation procedure follows the same steps as in Section 2.2.

It is important to underline that the main source of correlation in the above discussion can be considered the synchronization process inside the PMU. Nonetheless, the proposed method can easily be extended to consider also phase-angle errors shared between PMUs (this might happen, e.g., in case of PMUs of the same type fed by the same time source).

## 4. Tests and results

### 4.1. Test assumptions

The proposed method assessment has been performed through simulations in MATLAB environment considering the IEEE 14 bus system (Fig. 2, [29]). Two networks have been used: the network limited to the first 6 branches (involving the first 5 buses) and the whole network, then considering 20 branches (involving 14 buses). The algorithm is configured to work on all the branches simultaneously, considering two scenarios with  $N_t \in \{100, 1000\}$  measurement timestamps for each estimation. In particular,  $M = 10$  repeated measurements of the same load condition are used for each test. This corresponds to defining two measurement scenarios with a different number of cases  $C$ , i.e.,  $C \in \{10, 100\}$ .

To assess the performance in each scenario,  $N_{\text{MC}} = 5000$  Monte Carlo (MC) trials are used. In each trial, starting from a reference load condition, a powerflow is computed considering the actual line parameters to obtain the reference value of each measured quantity.

For each MC trial, the following conditions are considered:

1. The line parameters  $R_{ij}$ ,  $X_{ij}$  and  $B_{sh,ij}$  are extracted from a uniform distribution with a maximum deviation of  $\pm 15\%$  from  $R_{ij}^0$ ,  $X_{ij}^0$  and  $B_{sh,ij}^0$ , respectively (i.e., from the nominal values of the network).
2. All ITs are of Class 0.5 and thus maximum voltage and current magnitude errors are 0.5%; maximum phase-angle displacements are 0.6 crad for VTs and 0.9 crad for CTs, respectively. Actual IT errors in each trial are extracted from uniform distributions.
3. The PMUs are compliant with the synchrophasor standard IEC/IEEE 60255-118-1:2018 [30]. Maximum errors for magnitudes ( $\Delta_{\text{mag}}$ ) and phase angles ( $\Delta_{\text{ang}}$ ) for both voltages and currents are assumed to vary from  $\Delta_{\text{mag}} = 0.1\%$  and  $\Delta_{\text{ang}} = 0.1$  crad (indicated as PMU accuracy A, in the following) to  $\Delta_{\text{mag}} = 0.707\%$  and  $\Delta_{\text{ang}} = 0.707$  crad (PMU accuracy B),



depending on the test.<sup>5</sup> For each test,  $\Delta_{\text{mag}} = \Delta_{\text{ang}}$  and thus, for the sake of brevity, their value is referred to as ‘PMU accuracy’ in the presentation below.

- The aforementioned maximum errors are considered as limits of confidence intervals with a probability  $>0.97$ . In fact, differently from [26], here PMU phase-angle errors are described as in (12) whereas magnitude errors are split only into systematic and random contributions. In both cases, however, instead of splitting the maximum values, for a fairer comparison of the different error model combinations, an approach based on variance splitting is used here.

Focusing on phase-angle errors, in the different tests, the total error variance (considering also the MC trials and thus the variability of systematic error), derived from the datasheets as described in Section 3, is divided as:

$$\sigma_{\lambda^{\text{PMU}}}^2 = \frac{\sigma_s^2 + \sigma_c^2 + \sigma_r^2}{100} = \frac{p_s \sigma_{\lambda^{\text{PMU}}}^2 + p_c \sigma_{\lambda^{\text{PMU}}}^2 + p_r \sigma_{\lambda^{\text{PMU}}}^2}{100} \quad (18)$$

where  $\sigma_s^2$  is the variance of the systematic error contribution across different trials and  $\sigma_r^2$  is the variance of  $\lambda_r$ .  $p_s$ ,  $p_c$ , and  $p_r$  are the percentages of variance split. The three percentages can range from 0% to 100% depending on the tests. Regardless the different nature of the errors, they are all extracted from uniform distributions in the tests. This means that  $\lambda^{\text{PMU}}$  has a different distribution depending on the configured error mix, but, as mentioned above, a coverage factor is used to guarantee the given confidence level to the error. In the tests, magnitude errors are treated following a similar approach but without common errors.

- The active and reactive power of loads and generators vary within  $\pm 10\%$  (uniform distribution) of nominal value for all  $C$  cases in a trial.

In each MC trial, the systematic errors of the measurements are then the sum of two contributions, from IT and PMU. The tests have been conducted to assess the performance of the method in [25], reported as ‘Method A’, and of the proposed method, which can be generally applied but is well-suited for more realistic PMU behaviors (labeled with ‘Proposed’ in the following figures and tables). Furthermore, for the sake of comparison, the method presented in [21, Sec. IV] is also applied, since it considers IT systematic errors at least on one of the branch nodes and benefits from considering measurements related to more operational cases. Such method is referred to as ‘Method B’ in the following test descriptions. For completeness, the estimation is compared also with a simplified version of the proposed method that considers, for all the channels of the same PMU, and for all PMUs, an a priori fixed Pearson correlation coefficient of  $\rho^{\text{PMU}} = 0.75$  (referred to as ‘Method C’ in the following tests). In order to assess the effect of the presence of systematic errors in PMU measurements (quantified through  $p_s$ ) and of a phase-angle random error common to every channel of the same PMU (quantified through  $p_c$ ), the tests have been mainly conducted isolating each type of contribution. Finally, an example of the estimation performance obtained when both contributions are considered is also provided.

#### 4.2. Tests in the presence of systematic PMU errors

The first series of tests has been carried out considering the presence of systematic errors in PMU measurements, using different values of  $p_s$  (with  $p_c = 0\%$ ), and performing the estimation on the first six branches of the grid. Fig. 3 reports the average percent root mean square errors (RMSEs) of voltage magnitude systematic error estimation, i.e., the

<sup>5</sup> PMU accuracy B corresponds to about 1% maximum total vector error (TVE) for synchrophasor measurement, i.e., the standard limit for steady-state conditions.

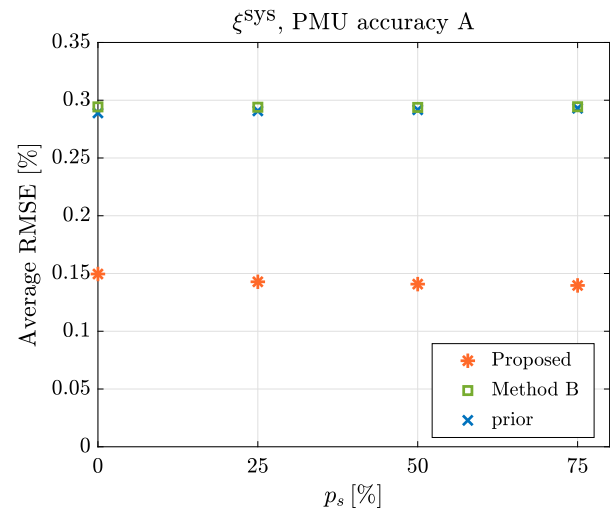


Fig. 3. Average RMSE of voltage synchrophasor magnitude estimation as a function of PMU systematic error percentage. PMU accuracy A,  $C = 100$ .

average RMSE of  $\xi_h^{\text{sys}}$  ( $h = 1, \dots, 5$ ) estimates across the considered nodes. This quantity represents also the root mean square residual compensation error averaged on the nodes and thus gives an idea of the capability to estimate and compensate the systematic component of the measurement chain error. The tests have been carried out using Method A, the proposed method and Method B and Fig. 3 shows the comparison of the estimation results obtained with PMU accuracy A and using  $C = 100$  operative cases. The proposed method and Method B are indicated with orange asterisks and green squares, respectively. The proposed method, thanks to the flexibility brought by the introduction of the estimated correlation factor  $\hat{\rho}_{ch_1 ch_2}$ , achieves the same results of Method A even though  $p_c = 0\%$ . This highlights its capability to generalize and, for this reason, Method A results are not reported in the figures.

In Fig. 3, the RMSE values are also compared with prior standard deviations, i.e., with the total standard uncertainty of  $\xi^{\text{sys}}$  (the subscripts indicating the nodes are often dropped from hereon when referring to a generic systematic error), which is computed across all MC trials based on the extracted systematic errors and then averaged for the nodes. First of all, it has to be observed that prior values only slightly increase with  $p_s$  because, with  $\Delta_{\text{mag}} = 0.1\%$ , the additional contribution brought by the PMU to the systematic error is much lower than IT contribution  $\sigma_{s_{\text{sys,VT}}} = 0.5/\sqrt{3}\%$ . For the proposed method, the average RMSE slightly decreases because PMU random errors decrease with higher  $p_s$ , thus confirming that, notwithstanding the mismatch in prior definition, the proposed method (and Method A) is still able to estimate the overall systematic error (which is reduced with respect to prior of about 48% in absence of systematic PMU error and of about 52% when  $p_s = 75\%$ ). Similar considerations hold true also for voltage phase angles, and for current magnitudes and phase angles. Method B results, instead, even if they improve for higher  $p_s$ , are always slightly above prior values.

Fig. 4 shows the same type of results of Fig. 3 but obtained with PMU accuracy B. The estimation results of Method B are not reported because they are far beyond the prior values. In this case, the contribution of PMU systematic errors is much larger, as proven by the increasing prior values. Nevertheless, the algorithm is still able to reduce significantly the overall systematic error and the RMSE reduction with respect to prior is even larger with higher values of  $p_s$ , thanks to the reduced random contribution. In particular, the RMSE reduction is of about 40% for  $p_s = 0\%$ , then it increases with  $p_s$  reaching the maximum improvement of about 45% for  $p_s = 75\%$ .

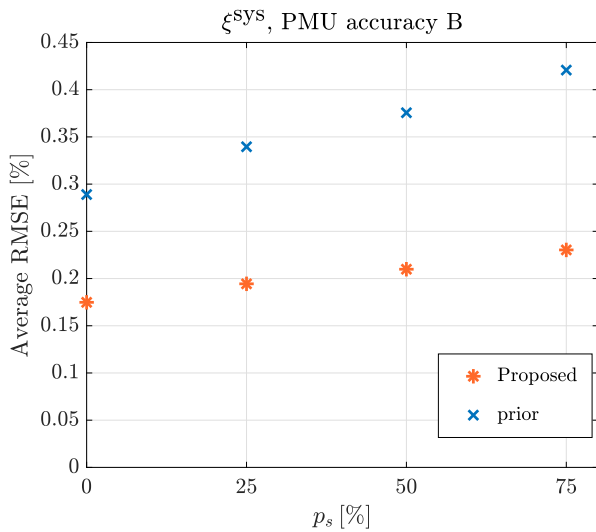


Fig. 4. Average RMSE of voltage synchrophasor magnitude estimation as a function of PMU systematic error percentage. PMU accuracy B,  $C = 100$ .

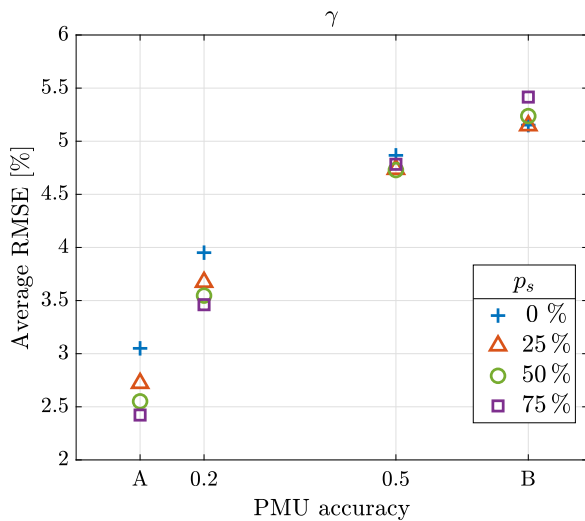


Fig. 5. Proposed method. Average RMSE of line resistance deviation as a function of PMU systematic error percentage and of PMU accuracy,  $C = 100$ .

Previous results have shown that systematic errors in the measurement chain can be reduced significantly regardless of their origin and of a possible lack of prior knowledge. However, it is important to understand the effect of the occurring mismatch between prior information on systematic errors and actual measurement chain behavior also on the main target of the estimation, i.e., line parameters.

Figs. 5 and 6 show an example of the results on line parameters estimation obtained with the proposed method (as mentioned before almost the same results have been found for Method A, thus not reported in the figures) and Method B, respectively. Average percent RMSE values are reported for the estimates of  $\gamma_{i_k-j_k}$  ( $k = 1, \dots, 6$ ) when PMU accuracy and  $p_s$  vary. As a term of comparison, prior values are always the same and equal to  $15/\sqrt{3} = 8.66\%$  (with negligible variations due to random extractions during MC trials). As expected, all the methods have better estimates in the presence of better PMU accuracy; nevertheless, it is important to highlight the differences in the response of the two methods to the variations in PMU accuracy and  $p_s$ .

Fig. 5 shows the proposed method results: even if estimation errors become larger with a decreasing PMU accuracy, they are well below

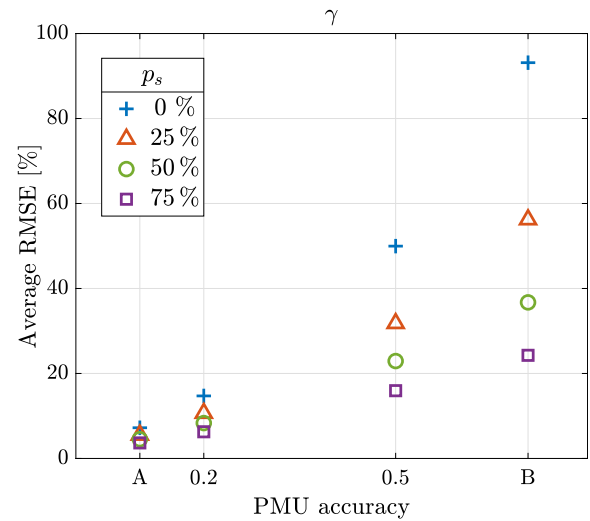


Fig. 6. Method B. Average RMSE of line resistance deviation as a function of PMU systematic error percentage and of PMU accuracy,  $C = 100$ .

the prior in each considered measurement scenario. It is also possible to notice that, given the best PMU accuracy, increasing systematic errors correspond to better estimations of  $\gamma$ . In agreement with the consideration drawn from Fig. 3, the proposed method is able to compensate the slight increase in systematic errors due to PMU and takes advantage of the reduction of random error. When the PMU accuracy B is used, instead, the contribution of PMU systematic error is more significant and better estimates are obtained for lower  $p_s$ . When PMU accuracy degrades, the contribution of the systematic error introduced by the instrument becomes more relevant and comparable with IT contribution. For this reason, for example, resistance estimation starts to degrade with higher  $p_s$ , even if the random contribution is reduced (see PMU accuracy B, Fig. 5). However, the maximum RMSE increase is less than 5% with  $p_s = 75\%$  and worst PMU accuracy, thus confirming the robustness of the method also with respect to measurement model tuning degradation (higher mismatch between prior assumptions on systematic errors and actual errors).

The behavior of Method B, reported in Fig. 6, is substantially different and determined by its great sensitivity to random errors. The estimates get considerably worse quickly and even with a PMU accuracy of 0.2 they become basically unreliable. The estimation errors increase as the PMU accuracy degrades and the random contribution in PMU error increases.

#### 4.3. Tests in the presence of phase-angle common error in PMU channels

Other tests have been carried out to investigate the impact of a phase-angle random error common to all the channels of the same PMU. The investigation has been conducted on the whole network in Fig. 2, considering PMU accuracy A and B and having different percentages  $p_c$  of common phase-angle error. In the tests,  $p_c$  goes from 0% up to 95% and  $p_s$  is kept equal to 0% to highlight the impact of the common phase-angle error. In this context, Figs. 7 and 8 show the line resistance RMSE results, averaged considering the whole network, for PMU accuracy A and B, respectively. Figs. 7 and 8 compare the estimation results of three methods with different assumptions on the correlation among the phase-angle errors of the PMU channels: the proposed method (orange asterisks) implements the methodology presented in Section 3, Method A (green diamonds) assumes uncorrelated phase-angle errors (as in [25]), whereas Method C (blue squares) uses a fixed correlation coefficient of 0.75.

The two figures display the same trends, showing that the impact of  $p_c$  is similar when PMUs with different accuracy are considered.

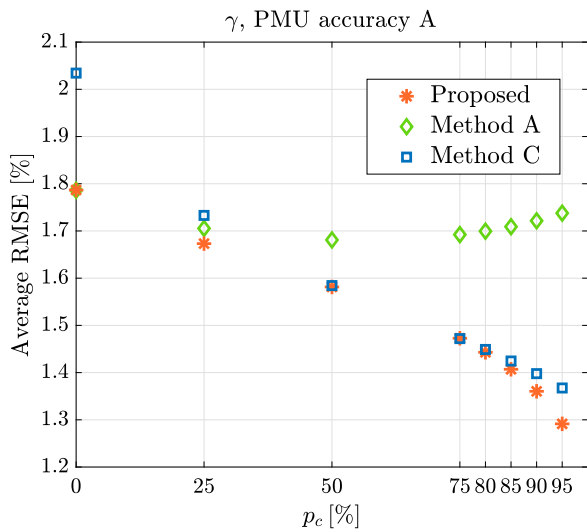


Fig. 7. Average RMSE of line resistance deviation as a function of the PMU common phase-angle error percentage. Entire network, PMU accuracy A,  $C = 10$ .

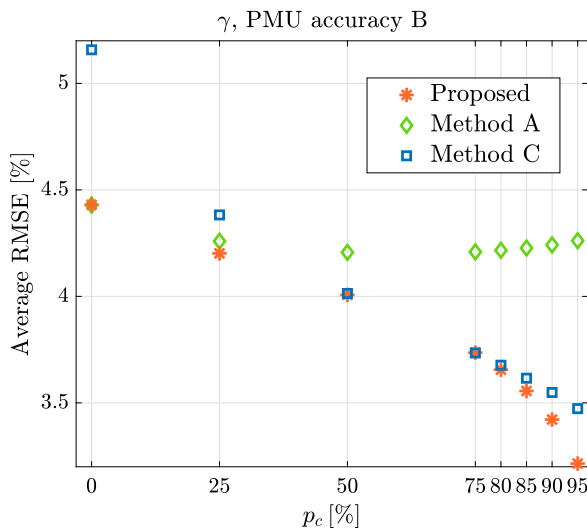


Fig. 8. Average RMSE of line resistance deviation as a function of the PMU common phase-angle error percentage. Entire network, PMU accuracy B,  $C = 10$ .

In particular, Method A RMSEs decrease in the presence of a phase-angle common error, reaching their minimum value approximately for  $p_c = 50\%$ , to get then slightly worse with higher  $p_c$  values, but still below the results obtained with  $p_c = 0\%$ . At first glance this fact appears counter-intuitive and deserves to be explained in detail.

The presence of phase-angle error correlation among the PMU channels is indeed in contrast with Method A assumption; nevertheless, its performance improves because correlation somehow limits the error variability and thus makes the Kirchhoff's constraints that are at the basis of the estimation more effective. Method C manifests the limits of a fixed assumption on error correlation particularly when  $p_c = 0\%$ : RMSEs are about 12% worse for PMU accuracy A and about 14% worse for PMU accuracy B with respect to Method A and the proposed method. Then, it is worth highlighting that the error results show a decreasing trend as  $p_c$  increases also beyond  $p_c = 75\%$  which corresponds to the assumption of Method C (thanks to the reduction of the error variability given by the correlation). The proposed method, leveraging the dynamic measurement error correlation model described in Section 3, achieves always the best estimation performance. In particular, it has the same performance as Method A when  $p_c = 0\%$ ,

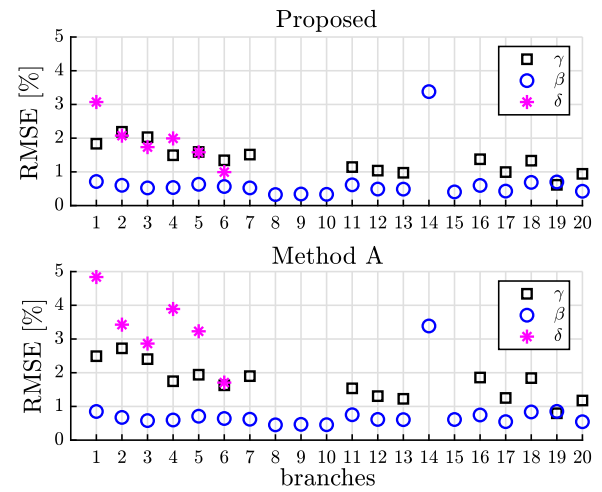


Fig. 9. RMSE of line parameters deviation considering 90% of PMU common phase-angle error for each channel of the same PMU. Entire network, PMU accuracy A,  $C = 10$ .

the same as Method C when  $p_c = 75\%$ , and overcomes both in the other conditions. For example, the proposed method performs better than Method A of about 25% when PMU accuracy B and  $p_c = 95\%$  are considered. The estimation results of reactance and transversal susceptance present similar trends and thus lead to similar conclusions.

To investigate the impact of common phase-angle error in PMU channels also on the line parameter estimation, more results are presented in what follows. Fig. 9 shows  $\gamma$ ,  $\beta$  and  $\delta$  estimation results (when available depending on the branch model) obtained by proposed method (at the top) and by Method A (at the bottom) considering PMU accuracy A and  $p_c = 90\%$ .

It can be observed that the proposed method has lower RMSEs than Method A for all the considered parameters, confirming the behavior found for systematic error estimation in previous results. In particular, the RMSEs of the proposed method have an average improvement with respect to Method A of about 21%, 18% and 43% for  $\gamma$ ,  $\beta$  and  $\delta$ , respectively. Similar results can be observed considering the 99th percentile of the errors (in absolute value): the average improvement of the proposed method with respect to Method A is of about 23% for  $\gamma$ , 19% for  $\beta$  and 41% for  $\delta$ .<sup>6</sup> Another interesting outcome is that the proposed method and Method A have basically the same estimation performance for  $\beta_{14}$ . This behavior can be explained observing the topology of the IEEE 14 bus test system shown in Fig. 2. Indeed, branch 14, i.e., the branch between nodes 7 and 8, is a terminal branch. Such branch is monitored by a PMU at node 8 that is isolated and therefore monitors only one branch.

In order to have a comprehensive view of the estimation performance, other tests have been carried out considering two different values of PMU accuracy (A and B), two different numbers of operating conditions ( $C = 10$  and  $C = 100$ ), and three different  $p_c$  values (0, 50, and 90%). Table 1 shows the most significant estimation results for the series parameters of branches 4 and 5, the systematic phase-angle errors of their end-node voltages (nodes 4 and 5) and the systematic phase-angle errors of their reverse currents (relating to the measurements of  $I_{42}$  and  $I_{52}$ ). Results obtained by the proposed method, Method A, and Method B are compared. It is possible to observe that (coherently with Figs. 7 and 8), when  $p_c = 0\%$ , for each PMU accuracy and for every considered value of  $C$ , the proposed method, updating the  $\rho^{\text{PMU}_i}$  estimation as described in Section 3, achieves the same results as Method A, which assumes no correlation among PMU measurements.

<sup>6</sup> In the same test, the corresponding improvements computed based on maximum errors are 22%, 15%, and 38%.



**Table 1**  
Estimation results in the presence of different accuracies, errors, and cases.

Method	PMU accuracy	C	$p_c$ [%]	RMSE							
				$\gamma_4$ [%]	$\beta_4$ [%]	$\gamma_5$ [%]	$\beta_5$ [%]	$\alpha_4$ [crad]	$\psi_{42}$ [crad]	$\alpha_5$ [crad]	$\psi_{52}$ [crad]
Proposed	A	10	0	1.81	0.61	2.09	0.73	0.09	0.38	0.09	0.38
			50	1.68	0.57	1.87	0.68	0.09	0.37	0.09	0.37
			90	1.49	0.54	1.59	0.63	0.09	0.36	0.09	0.36
		100	0	1.34	0.37	1.41	0.40	0.09	0.37	0.09	0.36
			50	1.29	0.35	1.32	0.37	0.09	0.36	0.09	0.35
			90	1.21	0.34	1.20	0.37	0.08	0.36	0.08	0.35
	B	100	0	3.13	2.00	3.74	2.47	0.12	0.41	0.13	0.42
			50	2.88	1.75	3.41	2.17	0.11	0.40	0.12	0.41
			90	2.51	1.47	2.81	1.82	0.10	0.39	0.11	0.38

**Table 2**  
Estimation results considering  $p_s = 25\%$  and  $p_c = 70\%$ , in the presence of different accuracies and cases.

Method	PMU accuracy	C	RMSE							
			$\gamma_4$ [%]	$\beta_4$ [%]	$\gamma_5$ [%]	$\beta_5$ [%]	$\alpha_4$ [crad]	$\psi_{42}$ [crad]	$\alpha_5$ [crad]	$\psi_{52}$ [crad]
Proposed	A	10	1.35	0.44	1.41	0.51	0.09	0.36	0.09	0.36
		100	1.17	0.31	1.17	0.34	0.09	0.36	0.09	0.35
	B	100	2.42	1.21	2.69	1.49	0.11	0.41	0.11	0.40
		100	3.06	1.52	3.77	1.88	0.13	0.43	0.13	0.43
Method A	A	10	1.61	0.49	1.74	0.58	0.09	0.37	0.09	0.37
		100	1.29	0.32	1.34	0.33	0.09	0.37	0.09	0.36
	B	100	3.06	1.52	3.77	1.88	0.13	0.43	0.13	0.43
		100	3.06	1.52	3.77	1.88	0.13	0.43	0.13	0.43
Method B	A	10	5.76	1.87	8.53	2.89	0.41	0.56	0.43	0.56
		100	2.56	0.80	3.21	1.30	0.35	0.52	0.36	0.52

Focusing on the impact of different values of  $p_c$ , the increase of  $p_c$  brings, in general, better results for all the considered methods, but, despite the general trends are the same, the behavior is different among the methods. Whereas Method A and Method B improvements are limited, reaching a minimum for  $p_c = 50\%$ , the proposed method improvement increases significantly with  $p_c$ . As an example, focusing on  $\gamma_5$  estimation with PMU accuracy A,  $C = 10$  and  $p_c = 90\%$ , the estimation improvements with respect to  $p_c = 0\%$  for the proposed method, Method A, and Method B are about 24%, 7% and 2%, respectively. It has to be noted that exploiting a large number of operative cases ( $C = 100$ ) improves estimation performance while reducing the impact of larger  $p_c$ . As for the systematic errors, the RMSEs obtained by the proposed method and Method A are significantly lower than prior uncertainty, e.g., for  $\alpha_4$  and  $\alpha_5$  the reduction is up to 72%, whereas the RMSEs of Method B are close or beyond the prior values. Finally, focusing on the impact of PMU accuracy on the estimation performance, it is worth noticing that Table 1 does not report the results for Method B when PMU accuracy B is considered since its estimation errors grow far beyond the prior when PMU accuracy degrades. Results in Table 1 show that the estimation performances of the proposed method and of Method A worsen with a decreasing of PMU accuracy

but are always well below prior values. In particular, it can be noted that the proposed method, under each condition, has results better than or equal to Method A.

To complete the analysis, further tests have been performed considering the simultaneous presence in PMU measurements of both systematic and common errors. Table 2 shows some results of the tests conducted considering  $p_s = 25\%$  and  $p_c = 70\%$ , PMU accuracies A and B, and both  $C = 10$  and  $C = 100$  operative cases.

It is possible to mention that the performance of Method B in line parameters estimation improves significantly when more cases are used, but its RMSE values are always the worst ones while the RMSEs of systematic errors are beyond prior uncertainties. As for the proposed method, the results confirm all the considerations previously drawn: thanks to a better model of measurement error, it achieves the best performance in all the conditions. As an example, focusing on branch 5 estimation results, i.e., longitudinal line parameters  $\gamma_5$  and  $\beta_5$  and systematic voltage and current phase-angle errors  $\alpha_5$  and  $\psi_{52}$ , with  $C = 100$  and PMU accuracy B, the improvements of the proposed method with respect to Method A are of about 28% and 21% for line parameters and of about 15% and 8% for  $\alpha_5$  and  $\psi_{52}$ , respectively.

## 5. Conclusions

In this paper, an improved method based on PMU measurements has been proposed for the simultaneous estimation of transmission line parameters and systematic measurement errors. It is based on a flexible estimation framework and the PMU measurement error model is refined to keep realistic effects into account. The PMU errors are now appropriately considered as composed of random and systematic contributions but also phase-angle errors common to all the channels are counted. In particular, the proposed algorithm evaluates and includes at run-time the found correlation information within the estimation framework. All the performed tests have proven that, in the presence of phase-angle measurement error sources as those originated by the device time-base errors, the estimation of both line parameters and measurement chain systematic errors is significantly improved with respect to other methods in the literature.

With the proposed approach the impact of model mismatch proves to be low in the considered realistic conditions thus pointing to the algorithm robustness. Future research studies will tackle the generalization of the current method to other measurement configurations, including, for instance, synchronized smart meters that could help cover a possible lack of PMUs on some nodes.

## Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Paolo Attilio Pegoraro reports financial support was provided by Sardegna Foundation.

## Data availability

Data will be made available on request

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## References

- [1] C. Muscas, P.A. Pegoraro, S. Sulis, M. Pau, F. Ponci, A. Monti, New Kalman filter approach exploiting frequency knowledge for accurate PMU-based power system state estimation, *IEEE Trans. Instrum. Meas.* 69 (9) (2020) 6713–6722.
- [2] D.H. Tungadio, J.A. Jordaan, M.W. Siti, Power system state estimation solution using modified models of PSO algorithm: Comparative study, *Measurement* 92 (2016) 508–523.
- [3] J. Fu, G. Song, B. De Schutter, Influence of measurement uncertainty on parameter estimation and fault location for transmission lines, *IEEE Trans. Autom. Sci. Eng.* 18 (1) (2021) 337–345.
- [4] M.Q. Khan, M.M. Ahmed, A.M.A. Haidar, An accurate algorithm of PMU-based wide area measurements for fault detection using positive-sequence voltage and unwrapped dynamic angles, *Measurement* 192 (2022) 1–19.
- [5] G. Kusic, D. Garrison, Measurement of transmission line parameters from SCADA data, in: *IEEE PES Power Syst. Conf. and Expo.*, 2004, pp. 344–349.
- [6] P. Kumar, A.K. Singh, Optimal mechanical sag estimator for leveled span overhead transmission line conductor, *Measurement* 137 (2019) 691–699.
- [7] A.A. V.V., Phasor Measurement Units and Wide Area Monitoring Systems, first ed., Academic Press, 2016.
- [8] A. Xue, F. Xu, K.E. Martin, H. You, J. Xu, L. Wang, G. Wei, Robust identification method for transmission line parameters that considers PMU phase angle error, *IEEE Access* 8 (2020) 86962–86971.

- [9] J. Lin, J. Song, C. Lu, Synchrophasor data analytics: Transmission line parameters online estimation for energy management, *IEEE Trans. Eng. Manag.* 69 (3) (2022) 671–681.
- [10] H. Haiyan, K. He, G. Lei, M. Jing, X. Feiyang, X. Ancheng, Steady-state PMU data selection for parameter identification of transmission line considering the influence of measurement error, *IET Gener. Transm. Distrib.* 16 (2022) 4549–4562.
- [11] M.R. Rezaei, S.R. Hadian-amrei, M.R. Miveh, Online identification of power transformer and transmission line parameters using synchronized voltage and current phasors, *Electr. Power Syst. Res.* 203 (2022) 107638.
- [12] Y. Wang, M. Xia, Q. Yang, Y. Song, Q. Chen, Y. Chen, Augmented state estimation of line parameters in active power distribution systems with phasor measurement units, *IEEE Trans. Power Del.* 37 (5) (2022) 3835–3845.
- [13] M. Asprou, E. Kyriakides, Identification and estimation of erroneous transmission line parameters using PMU measurements, *IEEE Trans. Power Del.* 32 (6) (2017) 2510–2519.
- [14] A. Wehenkel, A. Mukhopadhyay, J.-Y.L. Boudec, M. Paolone, Parameter estimation of three-phase untransposed short transmission lines from synchrophasor measurements, *IEEE Trans. Instrum. Meas.* 69 (9) (2020) 6143–6154.
- [15] F.P. De Albuquerque, E.C.M. Da Costa, R.F.R. Pereira, L.H.B. Liboni, M.C. De Oliveira, Nonlinear analysis on transmission line parameters estimation from noisy phasorial measurements, *IEEE Access* 10 (2022) 1720–1730.
- [16] E. Satsuk, A. Zhukov, D. Dubinin, I. Ivanov, A. Murzin, Analytical approach to phasor-based line parameter estimation verified through real PMU data, in: *2022 International Conference on Smart Grid Synchronized Measurements and Analytics, SCSMA, 2022*, pp. 1–6.
- [17] D. Franković, S. Vlahinić, M.Ž. Durović, Application of different least square methods for transmission line parameter estimation, in: *2022 IEEE 12th International Workshop on Applied Measurements for Power Systems, AMPSS, 2022*, pp. 1–5.
- [18] V. Milojević, M.V. Aćanski, D. Colangelo, Utilization of PMU Measurements for Three-Phase Line Parameter Estimation in Power Systems, *IEEE Trans. Instrum. Meas.* 67 (10) (2018).
- [19] C. Wang, V.A. Centeno, K.D. Jones, D. Yang, Transmission lines positive sequence parameters estimation and instrument transformers calibration based on PMU measurement error model, *IEEE Access* 7 (2019) 145104–145117.
- [20] Y.G. Kononov, O.S. Rybasova, K.A. Sidirov, Identification of overhead-line parameters from PMU data with compensation of systematic measurement errors, in: *Int. Conf. on Ind. Eng., Appl. and Manuf., ICIEAM, 2018*, pp. 1–5.
- [21] H. Goklani, G. Gajjar, S.A. Soman, Instrument transformer calibration and robust estimation of transmission line parameters using PMU measurements, *IEEE Trans. Power Syst.* 36 (3) (2021) 1761–1770.
- [22] M. Asprou, E. Kyriakides, M.M. Albu, Uncertainty bounds of transmission line parameters estimated from synchronized measurements, *IEEE Trans. Instrum. Meas.* 68 (8) (2019) 2808–2818.
- [23] S. Vlahinić, D. Franković, M.Ž. Durović, N. Stojković, Measurement uncertainty evaluation of transmission line parameters, *IEEE Trans. Instrum. Meas.* 70 (2021) 1–7.
- [24] C. Muscas, P.A. Pegoraro, C. Sitzia, A.V. Solinas, S. Sulis, Compensation of systematic measurement errors in PMU-based monitoring systems for transmission grids, in: *IEEE Int. Instrum. and Meas. Technol. Conf. (I2MTC)*, 2021, pp. 1–6.
- [25] C. Sitzia, C. Muscas, P.A. Pegoraro, A.V. Solinas, S. Sulis, Enhanced PMU-based line parameters estimation and compensation of systematic measurement errors in power grids considering multiple operating conditions, *IEEE Trans. Instrum. Meas.* 71 (2022) 1–12.
- [26] P.A. Pegoraro, C. Sitzia, A.V. Solinas, S. Sulis, Characterization of a method for transmission line parameters estimation with respect to PMU measurement error modeling, in: *25th IMEKO TC4 International Symposium, IEEE, Brescia, Italy, 2022*, pp. 1–6.
- [27] C. Muscas, P.A. Pegoraro, C. Sitzia, A.V. Solinas, S. Sulis, E.M. Carlini, G.M. Giannuzzi, C. Pisani, Characterization of a PMU-based method for transmission line parameters estimation with systematic measurement error modeling, in: *AIEIT Int. Annual Conf.*, 2021, pp. 1–6.
- [28] P. Castello, C. Muscas, P.A. Pegoraro, Statistical behavior of PMU measurement errors: An experimental characterization, *IEEE Open J. Instrum. Meas.* 1 (2022) 1–9.
- [29] R. Abu-Hashim, R. Burch, G. Chang, M. Grady, E. Gunther, M. Halpin, C. Harziadonin, Y. Liu, M. Marz, T. Ortmeyer, V. Rajagopalan, S. Ranade, P. Ribeiro, T. Sim, W. Xu, Test systems for harmonics modeling and simulation, *IEEE Trans. Power Del.* 14 (2) (1999) 579–587.
- [30] IEEE/IEC international standard - measuring relays and protection equipment - part 118-1: Synchrophasor for power systems - measurements, 2018, pp. 1–78, IEC/IEEE 60255-118-1:2018.