FACTA UNIVERSITATIS
Series: **Architecture and Civil Engineering** ONLINE FIRST
https://doi.org/10.2298/FUACE230630029M

Original Scientific Paper

LARGE DEFORMATION THEORY IN GEOMECHANICS - INFLUENCE OF KINEMATIC NONLINEARITY ON THE RESULTS OF SOME CHARACTERISTIC GEOTECHNICAL CALCULATIONS

UDC 624.042.7/.8:621.17

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Abstract. *The geotechnical engineering calculations are usually carried out according to the small deformation and displacement theory (infinitesimal strain theory) i.e. first-order theory. A linear relationship between componental displacements and deformations is adopted. The well-known conditions for equilibrium are defined for an undeformed system i.e. undeformed structure. Therefore, the geometric and static linearity assumptions are usually valid in geotechnical engineering calculations. These linearities are collectively referred to as kinematic linearity. In other words, engineers believe that results of quite satisfactory accuracy are obtained if only material nonlinearity is taken into account in the engineering calculations, regardless of the type of geotechnical problem being analysed. Therefore, it is not necessary to apply the large (finite) deformation theory with the assumption of material nonlinearity. The main aim of this paper is to verify the previous statement in the case of some characteristic problems of Geotechnics. In the first part of this paper, the large deformation theory, which is mostly unknown to the wider professional public, is briefly presented. After that, simple numerical analyses of some characteristic problems of Geotechnics were carried out in the well-known software FLAC 2D software with the aim of comparing the results obtained for the cases of kinematic linearity and kinematic nonlinearity. The obtained results point to the fact that kinematic nonlinearity should not always be ignored in the usual geotechnical engineering calculations. Therefore, engineers are urged to be careful.*

Key words: *geotechnics, large deformation theory, kinematic nonlinearity.*

Received June 30, 2023 / Revised August 1, 2023 / Accepted August 15, 2023
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*Selected paper presented at the International Conference Sinarg 2023 held in Niš, Serbia on 14-15 September 2023.

1. INTRODUCTION

In engineering practice geotechnical calculations are most often carried out according to the small (infinitesimal) deformation (strain) theory (hereinafter SD theory) with the assumption of material nonlinearity of the soil. Therefore, material nonlinearity and kinematic linearity assumptions are adopted (’’material nonlinearity only’’). The kinematic linearity consists of geometric and static linearity. The geometric linearity implies the adoption of the small deformation assumption. It means that the squares and higher degrees of these deformations, as well as the squares and higher degrees of their derivatives, can be completely ignored in the geotechnical engineering calculations. Adopting this assumption, a linear relationship between displacements and deformations is actually provided. The static linearity implies the adoption of the small displacement assumption. It means that the squares and higher degrees of these displacements, as well as the squares and higher degrees of their derivatives, can be completely ignored in the geotechnical engineering calculations. Adopting this assumption, the conditions for equilibrium can be defined for an initial or undeformed configuration i.e. undeformed structure.

Civil engineers consider it unnecessary to carry out usual engineering calculations according to the very complicated large (finite) deformation theory (hereinafter LD theory), because the deviations of the obtained results would be negligible in relation to the results of the calculations carried out according to the SD theory. This statement can be considered quite acceptable in the case of steel or concrete structures. However, its acceptability is more or less debatable in Geotechnics. In this paper, with the application of adequate software, an attempt will be made to verify this statement for some characteristic problems of Geotechnics. The well-known software Flac 2D (Fast Lagrangian Analysis of Continuum) was used. This software makes it possible to carry out stress-strain analysis according to the LD or SD theory for many geotechnical problems.

Generally, in continuum mechanics (mechanics of continuous media), when analyzing the motion and/or deformation of solids, or flow of fluids, two different methods (approaches, formulations, descriptions) can be used. The first formulation is so-called Eulerian formulation. The second formulation is so-called Langrangian formulation, which is directly implemented in the software Flac 2D. Therefore, only the Langrangian formulation will be described in more detail in this paper.

2. LAGRANGIAN FORMULATION IN CONTINUUM MECHANICS

The starting point for Langrangian formulation is well-known principle of virtual displacements i.e. principle of virtual work. This principle will be briefly presented in the following part of this paper. Due to the adopted material nonlinearity assumption, first the principle of virtual displacements and then Langrangian formulation will be briefly presented in the incremental form in the way it was done in [1-5].

2.1 Principle of virtual displacements

Fig. 1 shows a continuum body (hereinafter body) in the space x_i (i=1,2,3) before applying the first load increment (configuration ${}^0\Omega$) and the same body after applying the nth load increment (configuration ${}^n\Omega$). The displacement vectors for surface point E and body (internal) point F are defined for the given body after applying the nth load increment. These are the vectors ${}^n\mathbf{u}_i^E$ i ${}^n\mathbf{u}_i^F$. According to the LD theory, the displacement of any point of the given body can be separated into two independent components. The first component represents the displacement of the point due to translation and/or rotation of the body after applying the nth load increment without changing its shape and dimensions (rigid-body displacement). The second component represents the displacement of the point due to distortion and/or volumetric deformations of the body after applying the nth load increment.

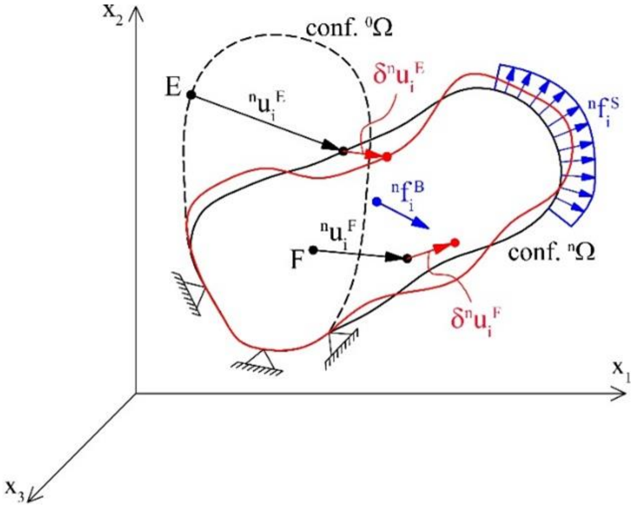

Fig. 1 Configuration of body before and after applying nth load increment in space x_i (i=1,2,3). Virtual displacement field for body after applying the nth load increment.

The configuration of the given body after applying the nth load increment ${}^n\Omega$ (hereinafter body configuration ${}^n\Omega$) is assigned a completely arbitrary virtual displacement field (red line in the image above) which satisfies displacement boundary conditions. This is just one of many possible ways of virtual displacement of the given body. The position of points E and F is additionally defined by vectors marked with $\delta^n\mathbf{u}_i^E$ and $\delta^n\mathbf{u}_i^F$. The basic equation of the principle of virtual displacements can be written in the following incremental form:

$$ {}^n\Re_{int} = {}^n\Re_{ext} \tag{1}$$

$$ \int_{{}^nV} {}^n\boldsymbol{\sigma}_{ij} \cdot \delta_n \mathbf{e}_{ij} \cdot d^nV = \int_{{}^nV} {}^n\mathbf{f}_i^B \cdot \delta^n\mathbf{u}_i^B \cdot d^nV + \int_{{}^nS} {}^n\mathbf{f}_i^S \cdot \delta^n\mathbf{u}_i^S \cdot d^nS \tag{2}$$

where:

${}^n\Re_{int}$ - is virtual work done by all internal forces for the given virtual displacement field of the body configuration ${}^n\Omega$,

${}^{n}\Re_{ext}$ - is virtual work done by all external forces for the given virtual displacement field of the body configuration ${}^{n}\Omega$,

$\delta^{n}\mathbf{u}_{i}^{B}$ - is given virtual displacement vector for all internal points for the body configuration ${}^{n}\Omega$,

$\delta^{n}\mathbf{u}_{i}^{S}$ - is given virtual displacement vector for all surface points for the body configuration ${}^{n}\Omega$,

$\delta_{n}\mathbf{e}_{ij}$ - is matrix (second-order tensor) of virtual deformations corresponding to the given virtual displacement field of the body configuration ${}^{n}\Omega$,

${}^{n}\boldsymbol{\sigma}_{ij}$ - is Cauchy stress tensor,

${}^{n}\mathbf{f}_{i}^{B}$ - is vector of body forces defined per unit volume of body configuration ${}^{n}\Omega$,

${}^{n}\mathbf{f}_{i}^{S}$ - is vector of surface forces defined per unit surface area of the body configuration ${}^{n}\Omega$,

${}^{n}V$ - is volume of the given body after applying nth load increment,

${}^{n}S$ - is surface area of the given body after applying nth load increment,

i, j - are Cartesian axes for which it is valid to i=1,2,3 and j=1,2,3.

It is important to note two things. Firstly, Cartesian axes x_1, x_2 and x_3 remain stationary whereas loaded body moves through those axes. Secondly, at the end of each load increment the three fundamental conditions of Continuum mechanics must be satisfied. These are: conditions for equilibrium, compatibility conditions and stress-strain laws. All three mentioned conditions are implemented in Eq. 2.

Since virtual displacements are actually infinitesimal displacements, the well-known SD theory laws can be used for the relation between virtual displacements and their corresponding virtual deformations. In this particular case, the following equation is used to define each member of the matrix of virtual deformations of the body configuration ${}^{n}\Omega$:

$$\delta_{n}e_{ij} = \frac{1}{2}\cdot\left(\frac{\partial\delta^{n}u_{i}}{\partial^{n}x_{j}} + \frac{\partial\delta^{n}u_{j}}{\partial^{n}x_{i}}\right) \tag{3}$$

Eq. 2 must be rewritten because the volume ${}^{n}V$, surface area ${}^{n}S$ and Cauchy stress tensor of the given body after applying n^{th} load increment are unknown.

2.2 Total Lagrangian formulation

The problem of unknown geometry (volume and surface area) of the given body after applying nth load increment can be overcome if, instead of Cauchy stress tensor, some other type of stress tensor and an energetically compatible strain tensor is applied. In the case of Total Langrangian formulation (hareinafter TL formulation), instead of Cauchy stress tensor for unknown body configuration ${}^{n}\Omega$, a second-order Piola-Kirchhoff stress tensor ${}_{0}^{n}\mathbf{S}$ which is referred (defined) to the initial body configuration ${}^{0}\Omega$ (before first load increment) is used. This stress tensor is energetically compatible with the Green-

Lagrange strain tensor ${}_0^n\boldsymbol{\varepsilon}$ which is also referred to the initial body configuration ${}^0\Omega$. By introducing new stress and strain tensors into the calculation with some other minor corrections, the following equation of the principle of virtual displacements according to the TL formulation was obtained for body configuration ${}^n\Omega$ shown in Fig. 1:

$$\int_{{}^0V} {}_0^n\mathbf{S}_{ij} \cdot \delta_0^n\boldsymbol{\varepsilon}_{ij} \cdot d^0V = \underbrace{\int_{{}^0V} {}_0^n\mathbf{f}_i^B \cdot \delta^n\mathbf{u}_i^B \cdot d^0V + \int_{{}^0S} {}_0^n\mathbf{f}_i^S \cdot \delta^n\mathbf{u}_i^S \cdot d^0S}_{{}^n\Re_{ext}\text{ - known}} \tag{4}$$

where:

$\delta_0^n\boldsymbol{\varepsilon}_{ij}$ - is variation of the Green-Lagrange strain tensor.

The 2nd Piolla-Kirchhoff stress tensor ${}_0^n\mathbf{S}$ is the symmetrical tensor that has no real (physical) meaning, so it is not suitable for practical application. It is important to note that the values of the members of this stress tensor change only in the case of deformation of the loaded body. Therefore, the members of this stress tensor do not change in the case of rigid-body motion. The Green-Lagrange strain tensor ${}_0^n\boldsymbol{\varepsilon}$ is also a symmetrical tensor. The members of this tensor are partial derivatives of displacements after applying n^{th} load increment with respect to the original coordinates i.e. coordinates of initial body configuration ${}^0\Omega$.

Assuming that all static and kinematic quantities are known for body configurations ${}^0\Omega$, ${}^1\Omega$, ${}^2\Omega$,..., ${}^{n-1}\Omega$, in order to define the Piolla-Kirchhoff stress tensor and the variation of the Green-Lagrange strain tensor, the principle of incremental decomposition is applied to the following way:

$$ {}^n\mathbf{u}_i = \underbrace{{}^{n-1}\mathbf{u}_i}_{\text{known}} + \underbrace{\Delta\mathbf{u}_i}_{\text{unknown}} \tag{5}$$

$$ {}_0^n\mathbf{S}_{ij} = \underbrace{{}_0^{n-1}\mathbf{S}_{ij}}_{\text{known}} + \underbrace{{}_0\mathbf{S}_{ij}}_{\text{unknown}} \tag{6}$$

$$ {}_0^n\boldsymbol{\varepsilon}_{ij} = \underbrace{{}_0^{n-1}\boldsymbol{\varepsilon}_{ij}}_{\text{known}} + \underbrace{{}_0\boldsymbol{\varepsilon}_{ij}}_{\text{unknown}} \tag{7}$$

where:

${}^n\mathbf{u}_i$ - is unknown displacement vector of given body after applying nth load increment,

${}^{n-1}\mathbf{u}_i$ - is known displacement vector of given body after applying $(n-1)^{th}$ load increment,

${}_0\mathbf{S}_{ij}$ - is unknown increment of the 2nd Piolla-Kirchhoff stress tensor for body configuration change from ${}^{n-1}\Omega$ to ${}^n\Omega$, which is also referred to the initial body configuration ${}^0\Omega$,

${}_0\boldsymbol{\varepsilon}_{ij}$ - is unknown increment of the Green-Lagrange strain tensor for body configuration change from ${}^{n-1}\Omega$ to ${}^n\Omega$, which is also referred to the initial body configuration ${}^0\Omega$.

Using the principle of incremental decomposition, the problem is reduced to determining the unknown increments of the corresponding stress and strain tensor. According to Eq. 3 and Eq. 5, the increment of the Green-Lagrange strain tensor is:

$$ {}_{0}\boldsymbol{\varepsilon}_{ij} = {}^{n}_{0}\boldsymbol{\varepsilon}_{ij} - {}^{n-1}_{0}\boldsymbol{\varepsilon}_{ij} \tag{8} $$

$$ {}_{0}\boldsymbol{\varepsilon}_{ij} = \underbrace{\frac{1}{2}\cdot\left(\overbrace{{}_{0}\mathbf{U}_{i,j}}^{\text{unknown}} + \overbrace{{}_{0}\mathbf{U}_{j,i}}^{\text{unknown}} + \overbrace{{}^{n}_{0}\mathbf{U}_{k,i}}^{\text{known}} \cdot \overbrace{{}_{0}\mathbf{U}_{k,j}}^{\text{unknown}} + \overbrace{{}_{0}\mathbf{U}_{k,i}}^{\text{unknown}} \cdot \overbrace{{}^{n}_{0}\mathbf{U}_{k,j}}^{\text{known}} \right)}_{\text{linear part by increment } \Delta\mathbf{u}_{i}} + \underbrace{\frac{1}{2}\cdot\left(\overbrace{{}_{0}\mathbf{U}_{k,i}}^{\text{unknown}} \cdot \overbrace{{}_{0}\mathbf{U}_{k,j}}^{\text{unknown}} \right)}_{\text{nonlinear part by increment } \Delta\mathbf{u}_{i}} = {}_{0}\mathbf{e}_{ij} + {}_{0}\boldsymbol{\eta}_{ij} \tag{9} $$

where ${}_{0}\mathbf{e}_{ij}$ and ${}_{0}\boldsymbol{\eta}_{ij}$ are linear and nonlinear part of the strain tensor increment ${}_{0}\boldsymbol{\varepsilon}_{ij}$ respectively. The nonlinear part ${}_{0}\boldsymbol{\eta}_{ij}$ is usually neglected in SD theory in contrast to LD theory. By taking into account the nonlinear part of the strain tensor increment ${}_{0}\boldsymbol{\varepsilon}_{ij}$ in engineering calculations, the geometric nonlinearity assumption is actually adopted. In Eq. 9, there are unknown tensors of the displacement increments. The members of these tensors are actually partial derivatives of displacement increments with respect to the original coordinates. According to the previously mentioned equations, the variation of the Green-Lagrange strain tensor for the body configuration ${}^{n}\Omega$, which is referred to the initial body configuration ${}^{0}\Omega$, is:

$$ \delta^{n}_{0}\boldsymbol{\varepsilon}_{ij} = \delta\left({}^{n-1}_{0}\boldsymbol{\varepsilon}_{ij} + {}_{0}\boldsymbol{\varepsilon}_{ij}\right) = \delta_{0}\boldsymbol{\varepsilon}_{ij} = \delta_{0}\mathbf{e}_{ij} + \delta_{0}\boldsymbol{\eta}_{ij} \tag{10} $$

Now Eq. 4 can be written in the following form:

$$ \int_{{}^{0}V} {}^{n-1}_{0}\mathbf{S}_{ij}\cdot\delta_{0}\boldsymbol{\varepsilon}_{ij}\cdot d^{0}V + \int_{{}^{0}V} {}_{0}\mathbf{S}_{ij}\cdot\delta_{0}\boldsymbol{\varepsilon}_{ij}\cdot d^{0}V = {}^{n}\Re_{ext} \tag{11} $$

$$ \int_{{}^{0}V} {}^{n-1}_{0}\mathbf{S}_{ij}\cdot\delta_{0}\mathbf{e}_{ij}\cdot d^{0}V + \int_{{}^{0}V} {}^{n-1}_{0}\mathbf{S}_{ij}\cdot\delta_{0}\boldsymbol{\eta}_{ij}\cdot d^{0}V + \int_{{}^{0}V} {}_{0}\mathbf{S}_{ij}\cdot\delta_{0}\boldsymbol{\varepsilon}_{ij}\cdot d^{0}V = {}^{n}\Re_{ext} \tag{12} $$

In order to define the unknown increment of the 2nd Piolla-Kirchhoff stress tensor ${}_{0}\mathbf{S}_{ij}$, the following relation of stress and strain tensor increment is adopted:

$$ {}_{0}\mathbf{S}_{ij} = {}_{0}\mathbf{D}_{ijrs}\cdot{}_{0}\boldsymbol{\varepsilon}_{rs} \approx {}_{0}\mathbf{D}_{ijrs}\cdot{}_{0}\mathbf{e}_{rs} \tag{13} $$

where ${}_{0}\mathbf{D}_{ijrs}$ is tangential elastic tensor which is referred to the initial body configuration ${}^{0}\Omega$ and r,s are Cartesian axes for which it is valid to r=1, 2, 3 and s=1, 2, 3. Eq. 13 is obtained by writing the increment ${}_{0}\mathbf{S}_{ij}$ as a Taylor series in ${}_{0}\boldsymbol{\varepsilon}_{rs}$ neglecting higher-order members and nonlinear part of increment of the Green-Lagrange strain tensor for body configuration change from ${}^{n-1}\Omega$ to ${}^{n}\Omega$ which is referred to the initial body configuration ${}^{0}\Omega$. Now, the product of the increment ${}_{0}\mathbf{S}_{ij}$ and a variation of the strain tensor increment $\delta_0 \boldsymbol{\varepsilon}_{ij}$ is:

$$ {}_{0}\mathbf{S}_{ij} \cdot \delta_0 \boldsymbol{\varepsilon}_{ij} = {}_{0}\mathbf{D}_{ijrs} \cdot {}_{0}\mathbf{e}_{rs} \cdot \left(\delta_0 \mathbf{e}_{ij} + \delta_0 \boldsymbol{\eta}_{ij}\right) = \overbrace{{}_{0}\mathbf{D}_{ijrs} \cdot {}_{0}\mathbf{e}_{rs} \cdot \delta_0 \mathbf{e}_{ij}}^{\text{linear part by } \Delta\mathbf{u}_i} + \\ + \underbrace{\overbrace{{}_{0}\mathbf{D}_{ijrs} \cdot {}_{0}\mathbf{e}_{rs} \cdot \delta_0 \boldsymbol{\eta}_{ij}}^{\text{nonlinear part by } \Delta\mathbf{u}_i}}_{\text{neglected}} \approx {}_{0}\mathbf{D}_{ijrs} \cdot {}_{0}\mathbf{e}_{rs} \cdot \delta_0 \mathbf{e}_{ij} \tag{14} $$

Finally, according to the TL formulation, the following basic equation of the principle of virtual displacements for body configuration ${}^{n}\Omega$ is obtained:

$$ \int_{{}^{0}V} {}^{n-1}_{0}\mathbf{S}_{ij} \cdot \delta_0 \mathbf{e}_{ij} \cdot d^0V + \int_{{}^{0}V} {}^{n-1}_{0}\mathbf{S}_{ij} \cdot \delta_0 \boldsymbol{\eta}_{ij} \cdot d^0V + \int_{{}^{0}V} {}_{0}\mathbf{D}_{ijrs} \cdot {}_{0}\mathbf{e}_{ijrs} \cdot \delta_0 \mathbf{e}_{ij} \cdot d^0V = {}^{n}\Re_{ext} \tag{15} $$

By solving Eq. 15, the values of all members of the displacement increment vector Δui for body configuration change from ${}^{n-1}\Omega$ to ${}^{n}\Omega$ are obtained. After that, the values of all members of the total displacement vector for a body after applying n[th] load increment ${}^{n}\mathbf{u}_i$ are simply calculated. However, it is very important to note that Eq. 15 is actually a linearized equation since the tangential elastic tensor defined for the initial body configuration ${}_{0}\mathbf{D}_{ijrs}$ is used. This means that the correct or approximately correct solution of Eq. 15 comes iteratively. At the end of each iteration, the level of error i.e. the level of ''out-of-balance'' virtual work must be checked as follows:

$$ \text{Error=balance virtual work} = {}^{n}\Re_{ext} - \int_{{}^{0}V} {}^{n}_{0}\mathbf{S}_{ij} \cdot \delta {}^{n}_{0}\boldsymbol{\varepsilon}_{ij} \cdot d^0V \tag{16} $$

2.3 Updated Lagrangian formulation

In the case of the TL formulation, all static and kinematic variables are determined based on the initial body configuration ${}^{0}\Omega$. However, in the case of the Updated Langrangian formulation (hareinafter UL formulation), all static and kinematic variables for the current body configuration ${}^{n}\Omega$ are determined based on the previously defined body configuration ${}^{n-1}\Omega$. For the body configuration ${}^{n}\Omega$ shown in Fig. 1, according to the UL formulation, the following basic equation of the principle of virtual displacements for body configuration is obtained:

$$ \underbrace{\int_{{}^{n-1}V} {}_{n-1}\boldsymbol{\sigma}_{ij} \cdot \delta_n \boldsymbol{\eta}_{ij} \cdot d^{n-1}V + \int_{{}^{n-1}V} {}_{n-1}\mathbf{D}_{ijrs} \cdot {}_{n-1}\mathbf{e}_{rs} \cdot \delta_n \mathbf{e}_{ij} \cdot d^{n-1}V + \int_{{}^{n-1}V} {}_{n-1}\boldsymbol{\sigma}_{ij} \cdot \delta_{n-1} \mathbf{e}_{ij} \cdot d^{n-1}V}_{{}^{n}\Re_{int}} = \\ = \underbrace{\int_{{}^{n-1}V} {}^{n}_{n-1}\mathbf{f}^{B}_{i} \cdot \delta^{n} \mathbf{u}^{B}_{i} \cdot d^{n-1}V + \int_{{}^{n-1}S} {}^{n}_{n-1}\mathbf{f}^{S}_{i} \cdot \delta^{n} \mathbf{u}^{S}_{i} \cdot d^{n-1}S}_{{}^{n}\Re_{ext}\text{ - known}} \tag{17} $$

where:

$$ {}_{n-1}^{n}\mathbf{S}_{ij} = {}_{n-1}^{n-1}\mathbf{S}_{ij} + {}_{n}\mathbf{S}_{ij} = \underbrace{{}_{n-1}\boldsymbol{\sigma}_{ij}}_{\text{known}} + \underbrace{{}_{n}\mathbf{S}_{ij}}_{\text{unknown}} \quad (18) $$

$$ {}_{n-1}^{n}\boldsymbol{\varepsilon}_{ij} = \underbrace{{}_{n-1}^{n-1}\boldsymbol{\varepsilon}_{ij}}_{=0} + {}_{n}\boldsymbol{\varepsilon}_{ij} = \underbrace{\frac{1}{2}\cdot\left(\overbrace{{}_{n-1}\mathbf{U}_{i,j}}^{\text{known}} + \overbrace{{}_{n-1}\mathbf{U}_{j,i}}^{\text{unknown}}\right)}_{{}_{n}\mathbf{e}_{ij}\text{ - linear part by increment }\Delta\mathbf{u}_{i}} + \underbrace{\frac{1}{2}\cdot\left(\overbrace{{}_{n-1}\mathbf{U}_{k,i}}^{\text{known}} \cdot \overbrace{{}_{n-1}\mathbf{U}_{k,j}}^{\text{unknown}}\right)}_{{}_{n}\boldsymbol{\eta}_{ij}\text{ - nonlinear part by increment }\Delta\mathbf{u}_{i}} \quad (19) $$

It is important to note that, in the case of the UL formulation, functions are integrated over a known volume ${}^{n-1}V$ and known surface area ${}^{n-1}S$ of the deformed body (body configuration ${}^{n-1}\Omega$). This actually means that, the static nonlinearity assumption of the problem is adopted in the calculation. This assumption together with the geometric nonlinearity assumption, which is implemented in Eq. 19, provides kinematic nonlinearity of the problem. Eq. 17 is also a linearized equation and it solved iteratively. At the end of each iteration, the level of error i.e. the level of ''out-of-balance'' virtual work must be checked.

In engineering practice, Eq. 15 and Eq. 17 are usually solved numerically. In that case, the members of the vector $\Delta\mathbf{u}_i$ represent the displacement increments of nodes of the adopted numerical grid. In the case of the UL formulation, at the beginning of each load increment, the numerical grid is regenerated (updated). This means that, at the beginning of each load increment, the coordinates of all nodes of the adopted numerical grid are corrected based on the values of the displacement increments which are calculated for the previous load increment

3. CHARACTERISTIC GEOTECHNICAL CALCULATIONS

3.1 Example 1 – embankment settlement

In the first example, the settlements of the embankment (Fig. 2), which are calculated according to SD theory (''material nonlinearity only'') and LD theory (material+kinematic nonlinearity) using the previously described UL formulation, will be compared between each other.

3.1.1 Input data

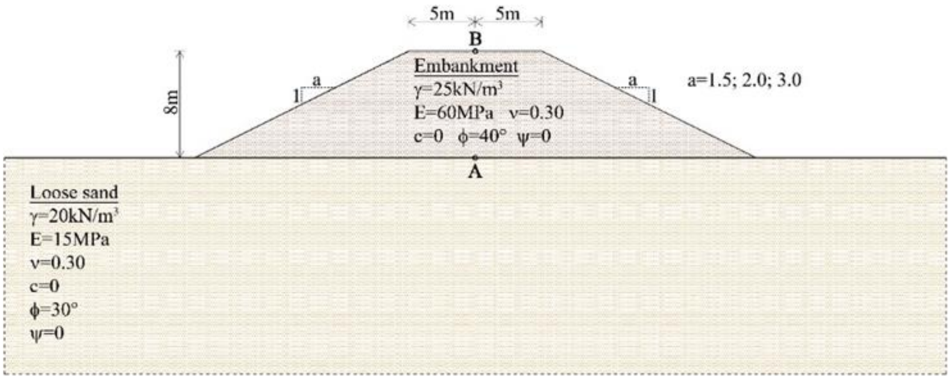

Fig. 2 Geometrical and material characteristics of the analysed embankment

3.1.2 Numerical model and calculation procedure

The formation of an appropriate numerical model in the software Flac 2D, in order to calculate the embankment settlements according to SD and LD theory, is divided into two phases. In the first phase, a numerical model is formed in order to perform a linear-elastic analysis of the foundation soil (loose sand) before the construction of the embankment. This analysis is performed to generate the initial stress state in the foundation soil. The second phase implies ’’adding’’ an embankment to the numerical model from the first phase, in order to perform the desired elasto-plastic analysis. MC material model with appropriate characteristics is used for the foundation soil and embankment in the second phase. All nodal displacements from the first phase must be canceled at the beginning of the second phase. For this cancellation, there is a corresponding option in the software Flac 2D.

3.1.3 Results and discussion

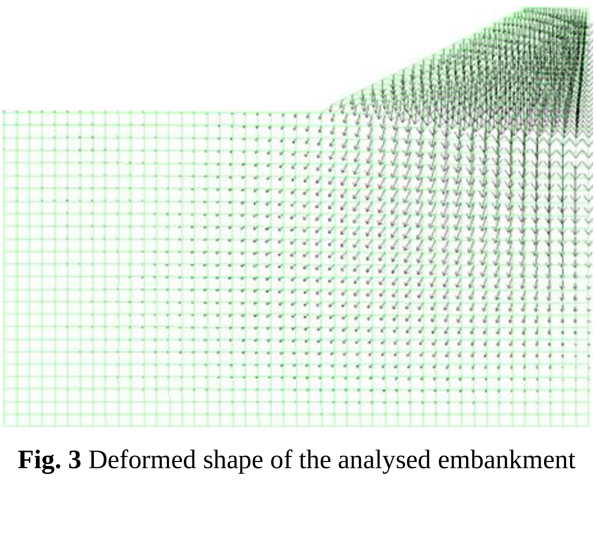

Fig. 3 Deformed shape of the analysed embankment

Table 1 Settlements of selected points of the embankment expressed in millimeters

Point	a=1.5			a=2.0			a=3.0		
	SD	LD	Δ (%)	SD	LD	Δ (%)	SD	LD	Δ (%)
A	222.8	225.6	1.25	229.1	231.4	1.00	233.4	236.6	1.37
B	176.3	170.3	3.52	154.1	148.3	3.91	120.8	117.6	2.72

According to the presented results, it can be concluded that there are negligible differences between the settlement values of the selected embankment points, which were calculated according to the SD and LD theory in the software Flac 2D.

3.2 Example 2 – deformation of the tunnel opening without lining

In the second numerical example, the displacements of the points on the contour of a circular tunnel opening without lining (Fig. 4), which are calculated according to SD theory (''material nonlinearity only'') and LD theory (material+kinematic nonlinearity) using the previously described UL formulation, will be compared between each other. In the calculation, the values of the Young's elastic modulus of rock mass are varied in the interval from 15 MPa to 500 MPa.

3.2.1 Input data

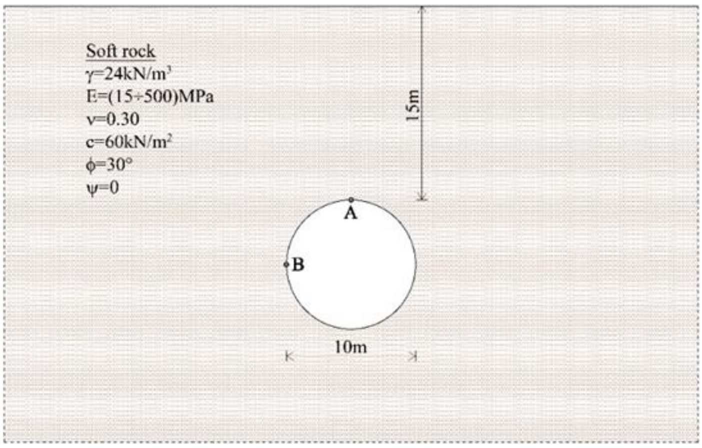

Fig. 4 Geometrical and material characteristics of the analysed problem

3.2.2 Numerical model and calculation procedure

The formation of an appropriate numerical model in the software Flac 2D, in order to calculate the displacements of selected points on the contour of a circular tunnel opening without lining according to SD and LD theory, is divided into two phases. In the first phase, a numerical model is formed in order to perform a linear-elastic analysis of the rock mass as a homogeneous, infinite half-space (continuum) before tunnel excavation. This analysis is performed to generate the initial stress state in the rock mass. The second phase implies ''adding'' a circular tunnel opening to the numerical model from the first phase, in order to perform the desired elasto-plastic analysis. MC material model with appropriate characteristics is used for the rock masse. All nodal displacements from the first phase must be canceled at the beginning of the second phase.

3.2.3 Results and discussion

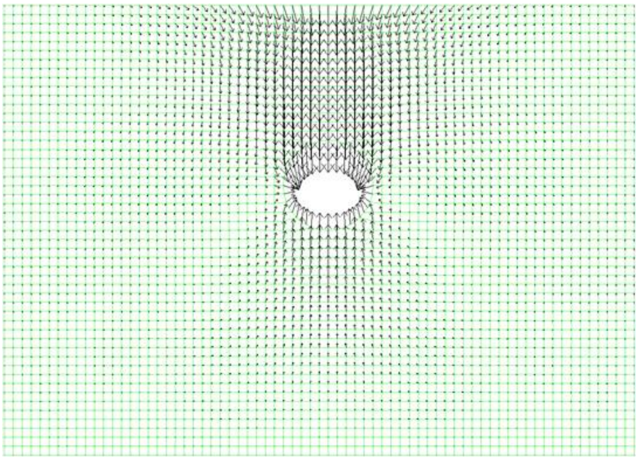

Fig. 5 Displacements of the rock mass around tunnel opening

Table 2 Displacements of selected points on the contour of a circular tunnel opening without lining expressed in millimeters (the plus sign for ↓ and → displacements)

E (MPa)	Point A – vertical displacement			Point B – horizontal displacement		
	SD	LD	Δ (%)	SD	LD	Δ (%)
15	695.1	565.2	22.98	257.8	191.2	34.80
25	416.7	378.3	10.10	154.3	130.7	18.05
50	210.6	201.3	4.61	77.5	71.1	9.06
100	105.6	106.9	1.32	38.7	37.4	3.47
200	51.9	52.1	0.38	19.3	18.9	2.11
500	21.2	21.0	0.95	7.7	7.7	0.00

According to the presented results, it can be concluded that in the case of very soft rocks the difference between displacements of the corresponding points on the contour of the tunnel opening without lining, which were calculated according to SD or LD theory, can be significant (maximum 35%). It is interesting that in these cases the displacements calculated according to the SD theory are significantly larger than the displacements calculated according to the LD theory. Obviously, the previously mentioned regeneration of the numerical grid at the end of each load increment can be quite significant. However, with an increase in the value of the elastic modulus of the rock, the difference between displacements, which were calculated according to the SD and LD theory, become smaller (already for E=100 MPa become negligible).

3.3 Example 3 – soil-slope displacement due to the action of the seismic load

In the third numerical example, the horizontal displacements of the selected points on the contour of a natural and homogeneous soil-slope (Fig. 6) due to the action of the seismic load, which are calculated according to SD theory (’’material nonlinearity only’’) and LD theory (material+kinematic nonlinearity) using the previously described UL formulation, will be compared between each other. In the calculation, the values of the Young’s elastic modulus of soil are varied in the interval from 15 MPa to 10 0MPa. The seismic load is adopted as a pseudo-static horizontal load. Its intensity is defined based on the adopted horizontal acceleration of the ground, which is 0.20·g.

3.3.1 Input data

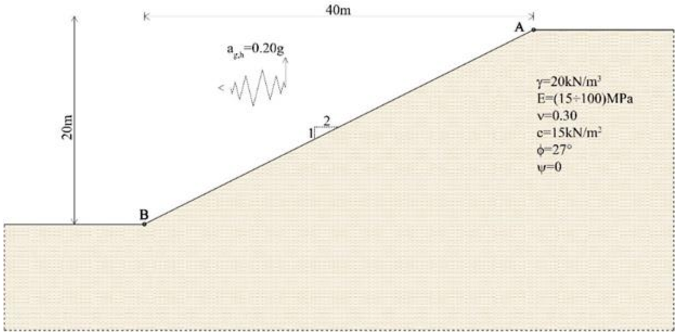

Fig. 6 Geometrical and material characteristics of the analysed soil-slope

3.3.2 Numerical model and calculation procedure

The displacement calculation of selected points on the contour of a natural soil-slope due to the action of the seismic load is divided into two phases. In the first phase, an appropriate numerical model is formed in order to perform a linear-elastic analysis of the natural soil-slope. This analysis is performed to generate the initial stress state in the soil. In the second phase, for the same numerical model, elasto-plastic analysis is performed. In the second phase MC material model with appropriate characteristics is used for the soil. All nodal displacements from the first phase must be canceled at the beginning of the second phase of the calculation. Inertial seismic forces are easy generated in softver Flac 2D by defining a horizontal ground acceleration.

3.3.3 Results and discussion

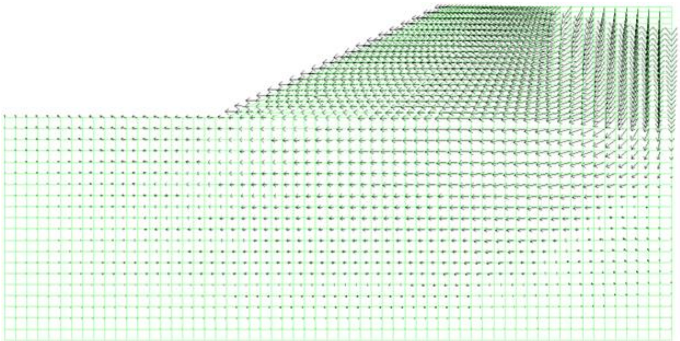

Fig. 7 Deformed shape of the analysed soil-slope due to the action of the seismic load

Table 3 Displacements of selected points on contour of the analysed soil-slope due to the action of the seismic load expressed in millimeters (the plus sign for → displacement)

E (MPa)	Point A			Point B		
	SD	LD	Δ (%)	SD	LD	Δ (%)
15	-678.8	-635.6	6.80	-434.9	-425.9	2.11
25	-403.3	-388.5	3.81	-260.6	-257.5	1.20
50	-201.6	-197.8	1.92	-130.1	-129.8	0.23
100	-100.7	-100.3	0.39	-65.2	-65.1	0.15

According to the presented results, it can be concluded that there are negligible differences between displacements of the corresponding points on the contour of a natural soil-slope due to the action of the seismic load, which were calculated according to the SD and LD theory in the software Flac 2D for different values of the soil elastic modulus.

4. Conclusion

By using LD theory (material+kinematic nonlinearity) in geotechnical engineering calculations, more realistic displacement (deformation) values of geotechnical constructions due to load are obtained. The differences between the amplitudes of these displacements and the corresponding displacements calculated for the same geotechnical problems according to the SD theory (’’material nonlinearity only’’) are mostly negligible. For this reason, SD theory is recommended to engineers for practical application. However, engineers must be careful of some specific situations such as e.g. larger diameter tunneling excavation in the softer materials. The differences between the results obtained in calculations according to SD and LD theory can be significant. In these specific situations, the use of calculations according to LD theory, which simulates the real behavior of the geotechnical structure much more accurately, is recommended.

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TEORIJA VELIKE DEFORMACIJE U GEOMEHANICI - UTICAJ KINEMATIČKE NELINEARNOSTI NA REZULTATE NEKIH KARAKTERISTIČNIH GEOTEHNIČKIH PRORAČUNA

Geotehnički inženjerski proračuni se obično izvode prema teoriji malih deformacija i pomeranja (teorija infinitezimalnih deformacija), odnosno teoriji prvog reda. Usvojen je linearni odnos između pomeranja komponenti i deformacija. Dobro poznati uslovi za ravnotežu definisani su za nedeformisan sistem, odnosno nedeformisanu strukturu. Stoga su pretpostavke geometrijske i statičke linearnosti obično validne u geotehničkim proračunima. Ove linearnosti se zajednički nazivaju kinematička linearnost. Drugim rečima, inženjeri smatraju da se rezultati sasvim zadovoljavajuće tačnosti dobijaju ako se u inženjerskim proračunima uzme u obzir samo nelinearnost materijala, bez obzira na vrstu geotehničkog problema koji se analizira. Stoga nije potrebno primenjivati teoriju velikih (konačnih) deformacija uz pretpostavku nelinearnosti materijala. Osnovni cilj ovog rada je da se potvrdi prethodna tvrdnja u slučaju nekih karakterističnih problema geotehnike. U prvom delu ovog rada ukratko je predstavljena teorija velikih deformacija, koja je uglavnom nepoznata široj stručnoj javnosti. Nakon toga, izvršene su jednostavne numeričke analize nekih karakterističnih problema geotehnike u poznatom softveru FLAC 2D sa ciljem poređenja rezultata dobijenih za slučajeve kinematičke linearnosti i kinematičke nelinearnosti. Dobijeni rezultati ukazuju na činjenicu da u uobičajenim geotehničkim inženjerskim proračunima ne treba uvek zanemariti kinematičku nelinearnost. Zbog toga se inženjeri pozivaju da budu oprezni.

Ključne reči: *geothenika, teorija velikih deformacija, kinematička nelinearnost.*