QUANTUM TELEPORTATION OF ENTANGLED STATES VIA GENERALIZED PHOTON-ADDED PAIR COHERENT STATE

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Abstract

In this paper, we study the quantum teleportation of an unknown atomic state based on the two-photon Jaynes-Cummings model, consisting of an effective two-level atom with a two-mode field in the generalized photon-added pair coherent state (GPAPCS). By applying the detecting method, we use a scheme that includes two two-level atoms and a cavity field to teleport the unknown atomic state from a sender to a receiver. The results show that the number of photons added to the field and the intensity of the initial field influence the average fidelity and success probability of the teleportation process. The time-evolution dependence of the average fidelity is also considered and compared for the field in the pair coherent state and in the GPAPCS.

Keywords: Entanglement; Jaynes-Cummings model; Photon-added pair coherent state; Quantum channel; Quantum teleportation.

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1. INTRODUCTION

The concepts of quantum information and the quantum computer were developed by scientists in the mid-twentieth century and have led to many desirable applications recently. Quantum teleportation is one of the important applications that contributed to a turning point in transmitting and receiving information and to changing our communication methods (Truong et al., 2020). Quantum teleportation is a method to transmit an unknown state from a sender (Alice) to a receiver (Bob) over any distance in space. To do this, Alice and Bob need to share a quantum channel that is *n* entangled state and a classical information channel. The information Bob receives from Alice is completely secret and cannot be copied, which is important and necessary in quantum communication (Bennett et al., 1993).

Bennet and his partners were the first to offer a theoretical scheme to perform quantum teleportation (Bennett et al., 1993). The scheme includes three particles, where Alice keeps particles 1 and 2, and Bob keeps particle 3. Particle 1 maintains a state $|\varphi\rangle$ needed for teleportation, and particles 2 and 3 form a quantum channel in which their state can be described via four Bell states. Alice performs a joint measurement between particle 1 and particle 2 in the Bell basis system and sends the measurements via the classical channel to Bob. Based on these measurements, Bob performs a measurement on particle 3 to recreate particle 1 so that the received state is the state $|\varphi\rangle$. This process is called the Bell state measurement (BSM) method (Bennet et al., 1993). Bouwmeester and his partners proved the correctness of Bennet's scheme with an experiment using a photon pair as a quantum-entangled resource to teleport the polarized state of an arbitrary photon (Bouwmeester et al., 1997). Later, many schemes were offered to teleport the states of atoms, fields, or ions in which the quantum channel is a pair of entangled atoms, an atom entangled with a field, or a phonon entangled with an ion (Cirac & Parkins, 1994; Cardoso et al., 2005; dSouza et al. 2009). Therefore, the generation of entanglement resources for the quantum channel plays a primary role in the teleportation process (Tran et al., 2018).

Various studies have suggested entanglement resources as quantum channels in teleportation processes (Nguyen & Truong, 2016; Truong et al., 2020, 2021; Tran & Truong, 2022; Ho & Truong, 2022). In these studies, the class of photon-added and (or) -subtracted fields are used as entanglement resources to teleport the field states, which significantly improved the average fidelity and success probability of the teleportation. However, these studies only refer to teleporting the field states via the nonclassical fields with the BSM method. This method is very complex for experimental realization, and thus it has been an obstacle to teleportation (dSouza et al., 2011; Pakniat et al., 2017).

However, another system that can be used for the quantum channel is cavity quantum electrodynamics (QED). The Jaynes-Cummings model (JCM) is a simple model that describes the interaction between atoms and radiation fields, which is also a useful choice as the quantum channel to teleport entangled states, such as unknown atomic states or field states (Zheng, 1999; Xue et al., 2006; Pakniat et al., 2017). In addition, these schemes avoid the difficulty of the BSM method by proposing a detecting method based on cavity QED and approximate conditions in the teleportation processes (Metwally et al., 2004, 2005; Zheng, 2004; Liu & Weng, 2006). The authors showed that the schemes using the JCM are more successful than previous ones and have higher fidelity (Zheng, 2004; Xue et al., 2006). In addition, our studies showed that by using the field in the generalized photon-added pair coherent state (GPAPCS) in the two-photon JCM, the atom-field entanglement is significantly improved (Le & Truong, 2022). Therefore, we expect better results in the teleportation process using JCMs with the field in the GPAPCS.

In this paper, we study the teleportation of an unknown atomic state by using the detecting method (dSouza et al., 2009) with the scheme considered by Liu and Weng (2006). The paper is organized as follows. In Section 2, we introduce the quantum channel created by the two-photon Jaynes-Cummings model, including an effective two-level atom and the field in the GPAPCS. Section 3 describes the teleported atomic state found using the detecting method. Section 4 gives the results and a discussion. Finally, conclusions are presented in Section 5.

2. ATOM-FIELD INTERACTION: A QUANTUM CHANNEL

As mentioned in Section 1, to teleport an atomic state from Alice to Bob, we first need to create a quantum channel. In this paper, the quantum channel we consider is the atom-field interaction in the JCM. This model includes an effective two-level atom interacting with a field in the generalized photon-added pair coherent state (Le & Truong, 2022). The effective Hamiltonian of the total system in the rotating-wave approximation without Stark shift has the following form (Le & Truong, 2022),

$$\hat{H}=\omega\hat{S}_z+\omega_1\hat{a}^+\hat{a}+\omega_2\hat{b}^+\hat{b}+\lambda\left(\hat{a}^+\hat{b}^+|g\rangle\langle e|+\hat{a}\hat{b}|e\rangle\langle g|\right), \tag{1}$$

where $\omega=\omega_e-\omega_g$ is the atomic frequency, $\omega_i\ \{i=1,\ 2\}$ is the corresponding frequency of the *a*(*b*) mode of the field, λ is the effective coupling constant, $\hat{a},\ \hat{b}\left(\hat{a}^+,\ \hat{b}^+\right)$ are photon annihilation (creation) operators of the *a*, *b* modes, and $|e\rangle\ \left(|g\rangle\right)$ is the excited (ground) state of the atom.

A pair coherent state (PCS) is the state of a two-mode boson field. This is an eigenstate of both the pair annihilation operator $\hat{a}\hat{b}$ and the particle difference operator $\hat{N}_b-\hat{N}_a$ with eigenvalues ξ and *q*, respectively. In the representation of particle number states, the PCS has the form (Le & Truong, 2022),

$$|\xi,\ q\rangle=C_q\sum_{n=0}^{\infty}\frac{\xi^n}{\sqrt{n!(n+q)!}}|n+q,\ n\rangle, \tag{2}$$

where $|n+q,\ n\rangle=|n+q\rangle_a|n\rangle_b$ is a two-mode field, *n* is a real positive number that gives the number of photons in the field, q is the eigenvalue of the particle difference operator, and the variance of the photons between modes *a*, *b* and $\xi=|\xi|\,e^{i\varphi}$ is a complex number, amplitude $|\xi|$ ranges from 0 to ∞, φ lies in the range from 0 to 2π (rad), and the coefficient C_q is determined as

$$C_q=\left[\sum_{n=0}^{\infty}\frac{|\xi|^{2n}}{n!\,(n+q)!}\right]^{-1/2}. \tag{3}$$

By repeatedly applying the photon creation operators to two modes *a*, *b* of the PCS, the generalized photon-added pair coherent state is produced with *m* photons added to mode *a* and *k* photons added to mode *b* of the field, as follows

$$|\xi,\, q;\, m,\, k\rangle=C_{q;\,m,\,k}\sum_{n=0}^{\infty}R_n|n+q+m,\, n+k\rangle, \tag{4}$$

in which *m*(*k*) is the number of photons added to mode *a*(*b*) of the field, *q* is the difference of photons between the two modes, and the coefficients are given by

$$C_{q;\,m,\,k}=\left(\sum_{n=0}^{\infty}\frac{|\xi|^{2n}(n+q+m)!(n+k)!}{[n!(n+q)!]^2}\right)^{-1/2},\ R_n=\frac{\xi^n}{n!(n+q)!}\sqrt{(n+q+m)!(n+k)!}. \tag{5}$$

From Equation (4), if we choose *m=k=0,* the field $|\xi,\, q;\, m,\, k\rangle$ becomes the pair coherent state, which means Equation (4) reduces to Equation (2). This matter will be taken care of when we select conditions of the parameters *m* and *k* in the figures below. Thus, we will indicate the effect of adding photons to two modes of the field in the course of creating entanglement and teleporting atomic states when comparing the field in both the PCS and GPAPCS cases.

In the two-photon JCM under consideration, the time evolution of the unitary operator has the form

$$\hat{U}(n,\, t)=\begin{pmatrix} U_{ee}(n,\, t) & U_{eg}(n,\, t)\\ U_{ge}(n,\, t) & U_{gg}(n,\, t)\end{pmatrix}, \tag{6}$$

where the matrix elements are

$$U_{\mathrm{ee}}(n,\, t)=U_{\mathrm{gg}}(n,\, t)=\frac{1}{2}\left(e^{-i\Gamma_n^{+}}+e^{-i\Gamma_n^{-}}\right),\ U_{\mathrm{eg}}(n,\, t)=U_{\mathrm{ge}}(n,\, t)=\frac{1}{2}\left(e^{-i\Gamma_n^{+}}-e^{-i\Gamma_n^{-}}\right), \tag{7}$$

with $\Gamma_n^{\pm}$ being the eigenvalues of the Hamiltonian $\hat{H}$ (1), which has the form

$$\Gamma_n^{\pm} = \omega_1 (n+q+m) + \omega_2 (n+k) + \frac{1}{2}\omega \pm \lambda\beta_n, \; \beta_n = \sqrt{(n+q+m)(n+k)}. \tag{8}$$

We also calculated in detail and gave conclusions about the dynamical and entanglement properties between the atom and the field in the GPAPCS. We showed that the entanglement between the atom and the field as well as one mode of the field and the subsystem including the atom and the remaining mode has periodicity over time (Le & Truong, 2022). Therefore, the model of the atom-field interaction proposed in this paper is an effective quantum channel for teleporting the unknown atomic state.

3. ATOMIC STATE TELEPORTATION

Zheng (2004) offered a simple method to teleport an arbitrary atomic state. In this scheme, Alice keeps a qubit of a quantum field and a qubit of atom 1 that needs to be teleported, and Bob keeps a qubit of atom 2. The qubit of atom 2 and the quantum field are an entangled quantum channel created by the interacting atom-field in the Jaynes-Cummings model. Instead of performing the BSM method, we use a detector to detect when atom 1 returns from the input state to the excited state. At that time, the 3-qubit superposition system will immediately collapse to a new state, whereby a qubit of atom 2 has the same state as the original qubit of atom 1. This is called the detecting method (Zheng, 2004). Later, many schemes were proposed to improve the experimental models (Liu & Weng, 2006; dSouza et al., 2009; Pakniat et al., 2017). In this section, we use the detecting method to teleport an unknown state of atom 1 that has the form

$$|\varphi\rangle_{in} = |\varphi_A\rangle_1 = \mu|e\rangle_1 + \upsilon|g\rangle_1, \tag{9}$$

where the coefficients μ, υ satisfy the condition $|\mu|^2 + |\upsilon|^2 = 1$. Atom 2 is initially prepared in the excited state $|\varphi_A\rangle_2 = |e\rangle_2$. This atom interacts with the field in the two-photon JCM, as mentioned in Section 2. At the initial time, the state of the system including atom 2 and the field has the form

$$|\psi_{A_2F}(0)\rangle = |A_2\rangle \otimes |F\rangle = C_{q;m,k} \sum_{n=0}^{\infty} R_n |e, n+q+m, n+k\rangle_2, \tag{10}$$

with $|F\rangle$ given in Equation (4). The state of the system after interaction time t_1 has the form

$$\begin{aligned} |\psi_{A_2F}(t)\rangle &= \hat{U}(n,t)|\psi_{A_2F}(0)\rangle = e^{-i\hat{H}t}|\psi_{A_2F}(0)\rangle \\ &= C_{q;m,k} \sum_{n=0}^{\infty} R_n \Big[\mathrm{Cos}(\lambda\beta_n t_1)|e, n+q+m, n+k\rangle_2 \\ &\quad - i\mathrm{Sin}(\lambda\beta_n t_1)|g, n+q+m+1, n+k+1\rangle_2 \Big], \end{aligned} \tag{11}$$

where the time evolution operator $\hat{U}(n,t)$ and β_n are given in Equations (6) and (8).

After atom 2 and the field become entangled together, the atomic-field system is considered a quantum channel in which Alice keeps the qubit of the field and Bob keeps the qubit of atom 2. Now, the total system includes atom 1, needed to teleport the state, and the subsystem involving atom 2 and the field, which is described by a state vector

$$\begin{aligned}|\psi\rangle &= |\varphi_{A_1}\rangle \otimes |\psi_{A_2F}(t)\rangle \\ &= \left(\mu|e\rangle_1 + \upsilon|g\rangle_1\right) C_{q;\,m,\,k} \sum_{n=0}^{\infty} R_n \Big[\mathrm{Cos}\left(\lambda\beta_n t_1\right)|e,\, n+q+m,\, n+k\rangle_2 \\ &\quad - i\mathrm{Sin}\left(\lambda\beta_n t_1\right)|g,\, n+q+m+1,\, n+k+1\rangle_2\Big].\end{aligned} \tag{12}$$

After time t_2, atom 1 interacts with the field and the total state of the system is defined as follows

$$\begin{aligned}|\psi'\rangle &= C_{q;\,m,\,k} \sum_{n=0}^{\infty} R_n \Big\{\mu\mathrm{Cos}\left(\lambda\beta_n t_1\right)\Big[\mathrm{Cos}\left(\lambda\beta_n t_2\right)|e,\, e,\, n+q+m,\, n+k\rangle_{12} \\ &\quad - i\mathrm{Sin}\left(\lambda\beta_n t_2\right)|g,\, e,\, n+q+m+1,\, n+k+1\rangle_{12}\Big] \\ &\quad - i\mu\mathrm{Sin}\left(\lambda\beta_n t_1\right)\Big[\mathrm{Cos}\left(\lambda\beta_{n+1} t_2\right)|e,\, g,\, n+q+m+1,\, n+k+1\rangle_{12} \\ &\quad - i\mathrm{Sin}\left(\lambda\beta_{n+1} t_2\right)|g,\, g,\, n+q+m+2,\, n+k+2\rangle_{12}\Big] \\ &\quad + \upsilon\mathrm{Cos}\left(\lambda\beta_n t_1\right)|g,\, e,\, n+q+m,\, n+k\rangle_{12} \\ &\quad - i\upsilon\mathrm{Sin}\left(\lambda\beta_n t_1\right)\Big[\mathrm{Cos}\left(\lambda\beta_n t_2\right)|g,\, g,\, n+q+m+1,\, n+k+1\rangle_{12} \\ &\quad - i\mathrm{Sin}\left(\lambda\beta_n t_2\right)|e,\, g,\, n+q+m,\, n+k\rangle_{12}\Big]\Big\}.\end{aligned} \tag{13}$$

Now, if atom 1 is detected in the excited state $|e\rangle_1$, the subsystem including atom 2 and the field collapses to the state

$$\begin{aligned}|\Phi\rangle &= NC_{q;\,m,\,k} \sum_{n=0}^{\infty} R_n \Big\{\mu\mathrm{Cos}\left(\lambda\beta_n t_1\right)\mathrm{Cos}\left(\lambda\beta_n t_2\right)|e,\, n+q+m,\, n+k\rangle_2 \\ &\quad - i\mu\mathrm{Sin}\left(\lambda\beta_n t_1\right)\mathrm{Cos}\left(\lambda\beta_{n+1} t_2\right)|g,\, n+q+m+1,\, n+k+1\rangle_2 \\ &\quad - \upsilon\mathrm{Sin}\left(\lambda\beta_n t_1\right)\mathrm{Sin}\left(\lambda\beta_n t_2\right)|g,\, n+q+m,\, n+k\rangle_2\Big\} \\ &= NC_{q;\,m,\,k} \sum_{n=0}^{\infty} R_n \Big[\gamma|e,\, n+q+m,\, n+k\rangle_2 \\ &\quad + i\chi|g,\, n+q+m+1,\, n+k+1\rangle_2 + \eta|g,\, n+q+m,\, n+k\rangle_2\Big],\end{aligned} \tag{14}$$

in which

$$\begin{aligned}\gamma &= \mu\mathrm{Cos}\left(\lambda\beta_n t_1\right)\mathrm{Cos}\left(\lambda\beta_n t_2\right), \\ \chi &= -\mu\mathrm{Sin}\left(\lambda\beta_n t_1\right)\mathrm{Cos}\left(\lambda\beta_{n+1} t_2\right), \\ \eta &= -\upsilon\mathrm{Sin}\left(\lambda\beta_n t_1\right)\mathrm{Sin}\left(\lambda\beta_n t_2\right),\end{aligned} \tag{15}$$

and normalized constant N has the form

$$N = \left[\left| C_{q;\,m,\,k} \right|^2 \sum_{n=0}^{\infty} \left| R_n \right|^2 \left(\gamma^2 + \chi^2 + \eta^2 \right) \right]^{-1/2}. \tag{16}$$

In Equation (14), if we select the condition that $\chi \approx 0$, an output state kept by Bob has the form of atom 1 needing to be teleported first

$$\left| \varphi \right\rangle_{out} = \mu \left| e \right\rangle_2 + \upsilon \left| g \right\rangle_2. \tag{17}$$

The average fidelity F of the teleported state is given by

$$\begin{aligned} F &= \left| \left\langle \Phi | \varphi \right\rangle_{out} \right|^2 \\ &= \left| N \right|^2 \left| C_{q;\,m,\,k} \right|^2 \sum_{n,\,n'=0}^{\infty} R_n^* R_{n'} \Big[\left(\gamma^* \mu + \eta^* \upsilon \right) \left\langle n+q+m,\, n+k \right| - i \chi^* \upsilon \left\langle n+q+m+1,\, n+k+1 \right| \Big] \\ &\times \Big[\left(\gamma \mu^* + \eta \upsilon^* \right) \left| n'+q+m,\, n'+k \right\rangle + i \chi \upsilon^* \left| n'+q+m+1,\, n'+k+1 \right\rangle \Big] \\ &= \frac{\sum_{n=0}^{\infty} \left| R \right|_n^2 \left(\left| \gamma^* \mu + \eta^* \upsilon \right|^2 + \left| \chi^* \upsilon \right|^2 \right)}{\sum_{n=0}^{\infty} \left| R_n \right|^2 \left(\gamma^2 + \chi^2 + \eta^2 \right)}, \end{aligned} \tag{18}$$

where $R_n,\, \gamma,\, \chi,\, \eta$ are given in Equations (5) and (15). The probability P of measuring the atomic state $\left| e \right\rangle_1$ is given by

$$P = \frac{1}{N^2} = \left[\left| C_{q;\,m,\,k} \right|^2 \sum_{n=0}^{\infty} \left| R_n \right|^2 \left(\gamma^2 + \chi^2 + \eta^2 \right) \right]^2. \tag{19}$$

The correlation between input and output states is expressed through the average fidelity function. The value of F is equal to one unit when the output state completely corresponds with the input state. The teleportation process achieves good results when the value of F is greater than 2/3 (Horodecki et al., 1999). Results of the average fidelity and probability of teleportation are discussed in detail in the next section. We will indicate the effect of the parameters, such as the initial field intensity, the amplitude of the teleported state, and the addition of photons to two modes of the field, on teleportation.

4. RESULTS AND DISCUSSION

In Figure 1 the average fidelity F from Equation (18) is plotted as a function of time λt_2 and parameter μ, which is related to the amplitude of the teleported state. The values are selected corresponding to the examined conditions that maximize the entanglement between the atom and the field (Le & Truong, 2022). For the interaction time t_1 to create the initial quantum channel, we select the corresponding value, $\lambda t_1 = \frac{3\pi}{4}$. This value is examined, and it indicates that with circulation over the period, the atom and the field obtain maximum entanglement. Figure 1 is plotted in the case of the field in the PCS (*m=k=0*) and the photon difference between two modes *q=0*.

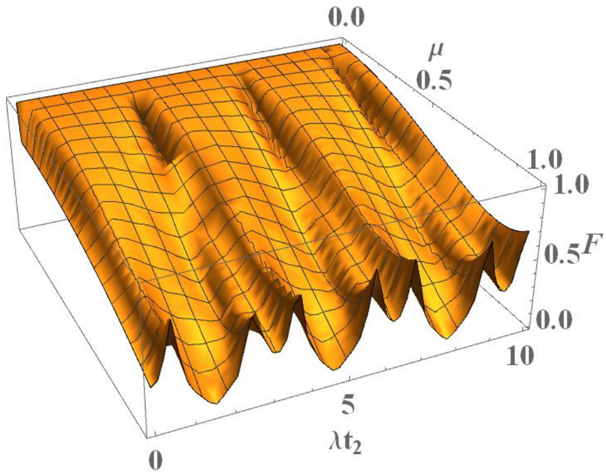


Figure 1. The average fidelity F as a function of λt_2 and μ with $\lambda t_1 = \frac{3\pi}{4}$, $|\xi| = 1$, $q = m = k = 0$

Figure 1 shows that the average fidelity *F* oscillates over time. This corresponds with dynamical properties when the atom interacts over time with the field in the two-photon JCM. Moreover, *F* also depends on parameter μ in the range from 0 to 1. The value of *F* gradually decreases as μ increases. Figure 1 also indicates the values of λt_2 corresponding with the maximum of *F* that will be selected for examination in the figures below.

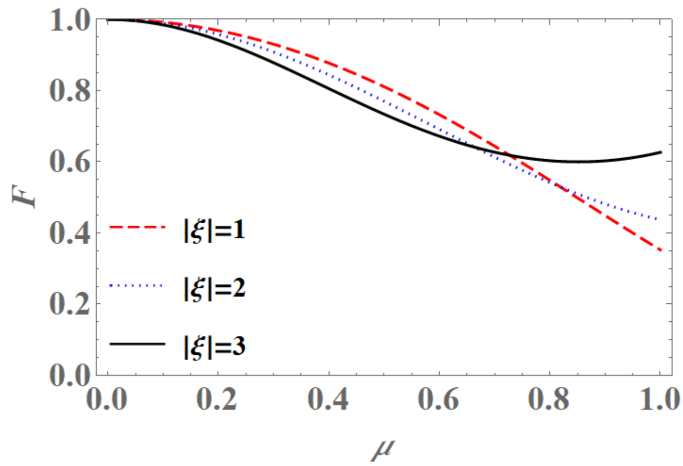


Figure 2. The average fidelity F as a function of μ with $|\xi| = 1$ (red dashed line), $|\xi| = 2$ (blue dotted line), $|\xi| = 3$ (black solid line), and fixed parameters $\lambda t_1 = \frac{3\pi}{4}$, $\lambda t_2 = \frac{5\pi}{4}$, $q = m = k = 0$

Figure 2 expresses the dependence of F on parameter μ for conditions $\lambda t_1 = \frac{3\pi}{4},\ \lambda t_2 = \frac{5\pi}{4},\ q = m = k = 0$, in which $|\xi| = 1$ corresponds with the red dashed line, $|\xi| = 2$ corresponds with the blue dotted line, and $|\xi| = 3$ corresponds with the black solid line. Additionally, Figure 2 is plotted in the case of the field in the PCS and indicates that the average fidelity of the teleportation depends on μ. The value of F decreases linearly when μ increases in the range (0, 0.7). However, in the range $0.7 < \mu < 1$, the value of F increases when the initial field intensity increases $\left(F = \{0.35,\ 0.45,\ 0.65\}\right)$.

Figure 3 describes the dependence of F on parameter μ and (m, k), which is the number of photons added to two modes of the field in the GPAPCS, and with parameters $\lambda t_1 = 3\pi/4,\ \lambda t_2 = 5\pi/4,\ |\xi| = 1,\ q = 0$. Figure 3 also shows that when μ increases in the range (0, 0.7). F gradually decreases. However, in the range $0.7 < \mu < 1$, the maximum value of F is substantially improved (blue dotted line and black solid line) when more photons are added to two modes of the field. The value of F only reaches 0.35 when the field is in the PCS (red dashed line). When simultaneously adding 1 and 2 photons to two modes of the field, the value of F reaches 0.55 and 0.85, respectively. This shows that adding photons to two modes of the field plays an important role in improving the average fidelity of teleportation.

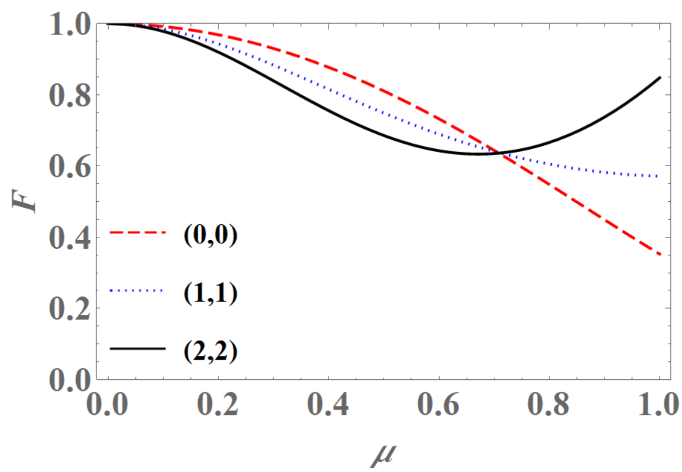
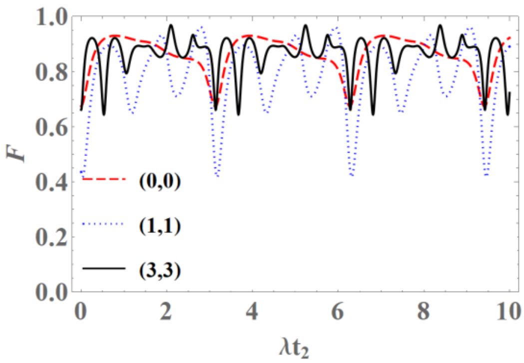


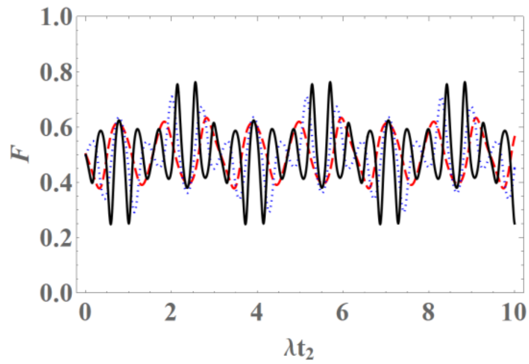
Figure 3. The average fidelity F as a function of μ with $(m, k) = (0, 0)$ (red dashed line), $(m, k) = (1, 1)$ (blue dotted line), $(m, k) = (2, 2)$ (black solid line) and fixed parameters $\lambda t_1 = \frac{3\pi}{4},\ \lambda t_2 = \frac{5\pi}{4},\ |\xi| = 1,\ q = 0$

In Figure 4 the dependence of F on time λt_2 with $\lambda t_1 = \frac{3\pi}{4}$ is plotted in the case of the field in the PCS $(m, k) = (0, 0)$ (red dashed line) and in two cases of the field in the

GPAPCS $(m, k)=(1, 1)$ (blue dotted line) and $(m, k)=(3, 3)$ (black solid line). Figure 4a is plotted in the case of $|\xi|=1, \mu=0.3$ to compare the average fidelity over time with or without adding a photon to two modes in the region $\mu<0.7$. Figure 4b is plotted in the case of $|\xi|=2, \mu=0.7$ to examine the average fidelity over time in the remaining region of μ. In both figures, we also see the periodic oscillation over time of average fidelity in the PCS and the GPAPCS. In the range $\mu<0.7$ (Figure 4a), the maximum value of F substantially increases when photons are added to two modes. F is approximately 0.9 when the field is in the PCS, but it exceeds 0.95 when three photons are simultaneously added to two modes of the field. In the range $0.7<\mu<1$ (Figure 4b), the value of F is smaller than in the range $\mu<0.7$. To improve the average fidelity, we not only increase the initial field intensity ($|\xi|=2$) but also add more photons to two modes of the field. The value of F reaches 0.75 when three photons are added to two modes, but in the case of the PCS, it is approximately 0.6.



(a)



(b)

Figure 4. The average fidelity F as a function of λt_2 with $\lambda t_1=\frac{3\pi}{4}$ and $(m, k)=(0, 0)$ (red dashed line), $(m, k)=(1, 1)$ (blue dotted line), $(m, k)=(3, 3)$ (black solid line) for (a) $|\xi|=1, \mu=0.3$, (b) $|\xi|=2, \mu=0.7$

5. CONCLUSIONS

In conclusion, we have studied the teleportation of an unknown atomic state in the two-photon JCM with the field in the GPAPCS by using the detecting method. The results have shown that the average fidelity of the teleportation oscillates periodically over time. The average fidelity also depends on the number of photons added to the field (m, k), the intensity of the initial field $(|\xi|)$, and the intensity of the teleported state $(|\mu|)$. The value of the average fidelity in the range $0.7<\mu<1$ is smaller than in the remaining range, and it decreases when μ increases. By adding more photons to two modes of the field and by increasing the intensity of the initial field, the average fidelity is significantly improved. Furthermore, the addition of photons has a strong impact on the value of the average fidelity. We also compare the average fidelity between the PCS and the GPAPCS and find that the average fidelity of the GPAPCS is better than that of the PCS. This proves the important role in the teleportation process of adding more photons to the field in the GPAPCS.

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