An Economic Analysis of Subscription Sharing of Digital Services

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Abstract

Subscription sharing, where one shares her premium digital services subscription with other users, has become common due to subscription-sharing platforms like Togetherprice, Gowd, and Sharesub. This raises a question: Does it still make economic sense to offer a menu of subscription plans (e.g., an individual plan as well as a discounted family plan)? In this study, we look at a monopolist service provider that offers both plans but faces the potential threat of subscription sharing. We analyze the optimal prices and the impact of sharing on profit, customer surplus, and overall society benefits. Our results indicate that even with subscription sharing, offering both plans is at least as profitable as only offering individual plans. Under certain conditions, subscription sharing can even boost profits. Furthermore, our numerical analysis suggests that subscription sharing can benefit society. These findings suggest that subscription sharing is not necessarily as troublesome as one would have expected.

Keywords: subscription sharing, family plan, streaming service

Introduction

Subscription-based business models, where customers pay a recurring fee (typically on a monthly or annual basis) in exchange for access to digital products and/or services, are ubiquitous nowadays. This business model is commonly used in the streaming service industry. Some of the most popular streaming services include Netflix, Amazon Prime Video, Hulu, Disney+, and Apple TV. While individual subscription plans are the most commonly offered options for a consumer to choose from, some of the service providers also roll out family plans in addition to the individual plans. By introducing family plans, service providers intend to allow multiple family members to stream content simultaneously. For example, Netflix offers a basic plan, a standard plan, and a premium plan. The basic plan allows for streaming on one screen at a time, while the standard plan and premium plan allow for streaming content up to two and four screens respectively. In this study, we categorize those offerings that permit several concurrent devices as family plans, and those only allow one device as individual plans.

Sharing a subscription (with non-family members) was exacerbated in recent years by the rise of online communities, social media platforms, and sharing platforms. In particular, there are a growing number of subscription-sharing platforms (e.g., Togetherprice, Gowd, Sharesub, and ShareIt) emerging in the last few years with an aim of saving subscription costs for consumers. The sharing platforms are a marketplace for individuals to team up together and jointly share a family plan even they are not a family. These subscription-sharing websites work as follows: subscription owners share their subscriptions on the platform and participants join the groups that still have free slots. For instance, the owner of a Netflix premium plan can share three free slots (excluding the one slot being used by the owner), and each joiner will pay an evenly-split subscription fee plus a small service fee to the platform (see Figure 1). With the service fee, the subscription-sharing websites will guarantee the success of operations and transactions. Therefore, these platforms considerably reduce consumer search costs and the implicit transaction costs (e.g., the security of transactions, privacy concerns), widen the sharing reach of subscriptions, and make sharing among non-family members more common.

While sharing a subscription might be appealing to consumers and subscription-sharing platforms, some firms worry that it might be a potential threat for the service providers as it could drastically affect profitability. Though service providers can leverage concurrent device limit tools to restrict the number of concurrent streaming screens to no more than the prespecified device limit, it does not guarantee that the consumer will not share the subscription with people living outside their households. This issue seems so widespread that some of the firms (e.g., Netflix) want to put a stop to sharing. Starting in 2023, Netflix curbs sharing by charging an extra fee to those who share outside their household in some countries. Despite Netflix's implementation of a crackdown on subscription sharing, most of the streaming service providers still tacitly allow subscription sharing. Hence, it remains an open question whether subscription sharing will indeed erode profits and benefit consumers. In this study, we aim to answer the following research questions: *1. Is adding family plans more profitable? 2. With the presence of subscription sharing, how does the family-plan strategy affect profit, consumer surplus, and social welfare?*

To answer these questions, we develop an analytical model, in which a monopolist service provider considers implementing an individual-plan strategy (only offers a basic plan) and a family-plan strategy (offers both a basic plan and a family plan). We assume there are two types of consumers: individual consumers (i.e., who are unable to partner with others to jointly share a family plan) and family-type consumers. We first consider a baseline case where there is no subscription sharing and compare the performance of the individual-plan strategy and the family-plan strategy. We then extend the baseline case to allow subscription sharing and derive the optimal prices for the family-plan strategy. In the case of subscription sharing, we note two migrating behaviors: (1) individual plan subscribers team up and switch to share a family plan, which is the cannibalization effect, and (2) potential consumers who otherwise do not subscribe to the service now can join a family plan, which is the market expansion effect. We analyze how switching behaviors affect profits, consumer surplus, and social welfare.

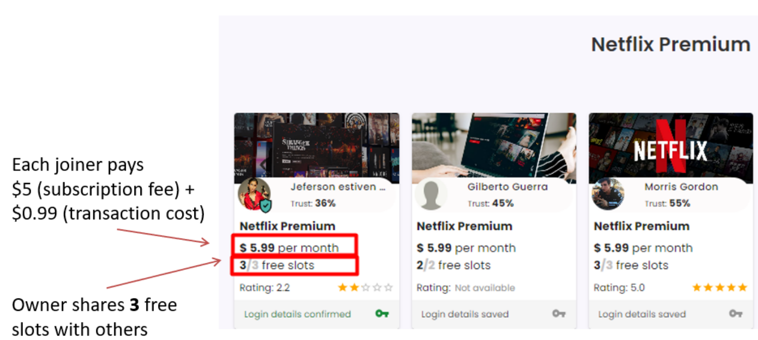

Figure 1. Netflix Premium Shared on Togetherprice.com

We highlight some preliminary findings. First, we find that the family-plan strategy is more profitable compared with the individual-plan strategy. Second, even in the presence of subscription sharing, the profit of the family-plan strategy is always no worse than that of the individual-plan strategy. We also derive conditions based on the tradeoff between the countervailing forces of cannibalization effect and market expansion effect, under which the family-plan strategy is more desirable for the firm. Lastly, subscription sharing results in higher social welfare, but surprisingly, we show that consumer surplus can be lower when the market expansion effect dominates.

Literature Review

Our paper pertains to the extant literature on the sharing of information goods, consumer bundling, and the gray market as a result of price discrimination. To demonstrate how this paper differs from previous literature, we first summarize (in Table 1) the distinct features of subscription sharing captured by our model.

Much of the research about the sharing of information goods has focused on digital piracy (Besen & Kirby, 1989; Jain, 2008; Peitz & Waelbroeck, 2006; Wu & Chen, 2008), and the role of network effects of sharing, demonstrating that software piracy may boost profits when markets exhibit network externalities (e.g., Conner and Rumelt, 1991; Shy and Thisse,1999; Givon, et al. 1995). There are two studies (Bakos et al., 1999; Galbreth et al., 2012) that investigate similar contexts and are more related to this research. Bakos et al.(1999) examine the small-scale sharing of information goods (such as CDs, and journal articles) taking place among friends and families. They identify two competing factors that affect profitability: the aggregation effect of individual valuations that boosts profit and the diversity effect of group sizes that diminishes profits. More recently, Galbreth et al. (2012) adopt a graph-theoretic approach and analyze the implications of the sharing of digitalized information goods (e.g., newspaper subscriptions) for profits under different network structures. They find that whether sharing is favorable depends on the distribution of group sizes and the group decision mechanism. Extending this line of research, this paper also looks at the sharing of information goods. However, subscription sharing in this research is an outcome of the content providers' price discrimination strategy. It happens when the firm rolls out a family plan, thus rendering the firm's price discrimination practice less effective. In addition, the emergence of intermediary sharing platforms has exacerbated the sharing issue, resulting in a wider sharing scale. We capture these new characteristics (e.g., price discrimination and wider scale reach) of sharing in our modeling framework.

This research is also related to the stream of second-degree price discrimination, specifically, consumer bundling and quantity discount. Consumer bundling refers to the setting where a group of consumers jointly purchase a product, and groups usually qualify for a discount (Desai et al., 2018; Chen & Zhang, 2015). Group buying (Anand & Aron, 2003; Chen & Zhang, 2015; Jing & Xie, 2011) and family plan (Desai et al., 2018) are common examples of consumer bundling. Anand & Aron (2003)'s analytical investigation of the group buying phenomenon is the first of its kind. Their results reveal that the group buying mechanism outperforms the conventional pricing mechanism under demand uncertainty, production postponement, and scale economies. Jing & Xie (2011) examine the phenomenon from another perspective: the information-sharing role of group buying whereby informed consumers are motivated to disseminate product information to uninformed consumers. They conclude that group-buying is superior when the information sharing is efficient and the valuation of the less-informed segment is high. Building on the previous two studies, Chen & Zhang (2015) approach the group buying problem using a general bundling framework, notably group-discount schemes with minimum or maximum group sizes. Family plan, on the other hand, is a small-scale of consumer bundling and usually is restricted to family customers. Desai et al., (2018) find that firms are better off by adding family plans to a menu of individual plans as family plans enable the firm to price discriminate more effectively by charging a higher price to individual high-valuation consumers who are unable to be part of a family. Following this stream, we investigate how does the introduction of a family plan, in the presence of subscription sharing services, affect the firm's effectiveness of price discrimination strategy based on group size. Although the sharing of goods shares some similarities with group buying and family plan, subscription sharing of a family plan is beyond the firms' control, while group buying and family plan are intended for "sharing."

	Context	Limited sharing[1]	Scale reach[2]	Price discrimination[3]	A menu of offerings	Resale[4]	Digital goods
Bakos et al., (1999)	Pirating computer program, copying CDs, duplicating journals	√	×	×	×	×	×
Galbreth et al., (2012)	Sharing of digitalized information goods (e.g., newspaper subscription)	×	×	×	×	×	√
Chen & Zhang, (2015)	Group buying (Groupon)	√	√	√	√	×	×
Desai et al., (2018)	Mobile data family plan	√	×	√	√	×	√
Jiang & Tian, (2018)	Group buying (RelayRides, NeighborGoods)	√	√	×	×	×	×
This research	Sharing of streaming content subscription	√	√	√	√	√	√

Table 1. Literature Review and Comparison

Lastly, the study is related to the gray market literature of physical goods. Price discrimination will be ineffective in the face of arbitrage, where consumers offered lower prices can buy and resell these goods to consumers facing higher prices (Zhang & Feng, 2017). One stream of research looks at the gray market as a result of price discrimination across geographic regions by a single firm (Zhang & Feng, 2017; Szymanski & Valletti, 2005; Ahmadi & Yang, 2000; Raff & Schmitt, 2007). Ahmadi and Yang (2000) develop a sequential game-theoretical model where parallel importers can enter the higher priced market. Their results demonstrate that under some circumstances parallel imports boosts the manufacturer's profits. In another study, Raff and Schmitt (2007) also show that there are circumstances where allowing retailers' parallel trade will increase a manufacturer's profits. Zhang & Feng (2017) show that the price gap between the two separate authorized markets affects gray market sales and the firm profit; the authors also find empirical evidence using sales data of physical goods.

The Basic Model

In this section, we develop a basic model and compare the profitability of the individual-plan strategy and the family-plan strategy (offering both an individual plan and family plans). We consider the cases where the firm offers one family plan as well as multiple-tier family plans if it chooses the family-plan strategy.

One Family Plan

Suppose that a monopolistic service provider offers both a family plan and an individual plan, and they are the same in terms of service quality, including video/music quality, resolution, whether offline streaming is permitted, streaming device type, the size of the streaming library, etc. The only difference is the number of concurrent streaming devices allowed. We refer to the family plan as the option that permits more than one concurrent device.

[1] The goods can't be used unlimitedly either because it is a physical goods or because there are some tools to control unlimited sharing.
[2] The scope of sharing is widened with a sharing platform, enabling consumers to share with strangers.
[3] Streaming service providers practice discrimination by segmenting the consumers and offering different plans targeted at different consumer segments.
[4] Subscription sharing can be viewed as family plan owners reselling free slots to individual consumers.

Two types of consumers constitute the market: α family consumers (F) with family size of N, and $(1-\alpha N)$ individual consumers (I) with family size of 1, where $N \geq 2 (N \in \mathbb{N})$ and $0 < \alpha < \frac{1}{N}$. The total market size is, therefore, 1. The maximum utility level gained from streaming by an individual consumer is V_I, which is her reservation price for a basic plan or a family plan. For family consumers, the total utility attained by the family increases linearly with the prespecified device limits L, and reach its maximum utility level V_F when $L = N$. Thus, the average utility of each family member is $\frac{V_F}{N}$, which is also the utility gained if family consumers purchase one basic plan.

Provided that there are two customer segments in the market, the firm considers offering a family plan and a basic plan to attract more customers. The family plan, designed specifically for families, comes at price P_F and allows L concurrent devices at most. The basic plan, targeted at individual customers, has price P_B and allow one concurrent device. The family plan and basic plan are written as (P_F, L) and $(P_B, 1)$, respectively. The firm determines the prices P_B, P_F and device limit L. In this paper, we mainly focus on price per *user* $\frac{P_F}{min\{L,N\}}$ (i.e., $\frac{P_F}{L}$ when $L < N$ and $\frac{P_F}{N}$ when $L \geq N$)[5], as it is comparable with P_B, the price for one slot. The notation is presented in Table 2 and Table 3. Note that it is unnecessary to offer a family plan if $L^* = 1$ or $P_F = min\{L, N\}P_B$.

Customer Type	Market proportion	Reservation Price	Family Size
Family	α	V_F	N
Individual	$1-\alpha N$	V_I	1

Table 2. Consumer Segment

	Basic plan	Family plan
Price	P_B	P_F
Device limit	1	L

Table 3. Pricing Scheme

When two plans(P_F, L) and $(P_B, 1)$ are offered, individual consumers will subscribe to the basic plan $(P_B, 1)$ only if the net surplus yielded from doing so is greater or equal than that from not purchasing any plans (i.e., personal rationality constraint) and that from purchasing the family plan (i.e., incentive compatibility constraint). That is, for the individual consumers to choose $(P_B, 1)$, the following constraints must hold

(1) $V_I - P_B \geq 0 \Rightarrow P_B \leq V_I$ — I prefers $(P_B, 1)$ to not buying

(2) $V_I - P_B \geq V_I - P_F \Rightarrow P_B \leq P_F$ — I prefers $(P_B, 1)$ to (P_F, L)

Similarly, to ensure that family consumers subscribe to family plans, the pricing strategy is such that the net surplus realized from purchasing a family plan is greater than that from not buying and that from purchasing at most N units of basic plans:

(3) $V_F min\left\{\frac{L}{N},\ 1\right\} - P_F \geq 0 \Rightarrow P_F \leq V_F min\left\{\frac{L}{N}, 1\right\}$ — F prefers (P_F, L) to not buying

(4)$V_F min\left\{\frac{L}{N},\ 1\right\} - P_F \geq \frac{V_F}{N} min\{L,\ N\} - P_B min\{L,\ N\} \Rightarrow P_F \leq P_B min\{N, L\}$ — F prefers (P_F, L) to $(P_B, 1)$

Constraints (3) and (4) can be decomposed to: when $L < N$, $P_F \leq V_F \frac{L}{N}$ and $P_F \leq LP_B$; when $L \geq N$, $P_F \leq V_F$ and $P_F \leq NP_B$.The streaming service provider then decides P_F, L and P_B to maximize its profit subject to the above constraints (1) to (4), $P_B, P_F \geq 0$, and $L \in \mathbb{N}$. We derive the optimal prices for two cases: (a) $\frac{V_F}{N} \leq V_I < V_F$ and (b) $V_I \leq \frac{V_F}{N}$, and compare the results with those in the individual-plan strategy.

5 Users are those who can have simultaneous access to the service. For families, when $L < N$, only L members are users, and when $N \geq L$, N members are users.

Case (a): $\frac{V_F}{N} \le V_I < V_F$

If $\frac{V_F}{N} \le V_I < V_F$, an individual has a higher average valuation than those belong to a family, but has a smaller willingness-to-pay than a whole family. Individual consumers can be considered as high-valuation types at the individual level. To ensure the individual consumers choose the basic plan, constraints (1) and (2) should be binding; to ensure the family consumers choose the family plan, constraints (3) and (4) should be binding. When $\frac{V_F}{N} \le V_I < V_F$, by proof of contradiction, only constraints (1) and (3) are binding. It is in the firm's interest to raise the price as much as possible; therefore, prices are given by:

$$P_B = V_I$$

$$P_F = V_F min\left\{\frac{L}{N}, 1\right\}$$

The purpose of adding a family plan to the menu is to cover more market segments. Therefore, we lay emphasis on analyzing the scenario where both families and individuals will buy the product targeted at them (i.e., the market size is 1). When all above constraints are met, individuals will subscribe to the basic plan while families will choose the family plan; the firm's profit function is thus given by:

$$\pi = \alpha P_F + (1 - \alpha N)P_B = \alpha V_F min\left\{\frac{L}{N}, 1\right\} + (1 - \alpha N)V_I$$

Then the company chooses L to maximize its profit. When $L \le N$, $\frac{\partial \pi_F}{\partial L} = \alpha \frac{V_F}{N} > 0$, $L^* = N$; when $L > N$, L can be any natural number greater than N. Thus, the optimal decisions are $L^* \ge N$ ($L^* \in \mathbb{N}$), $P_F^* = V_F$ and $P_B^* = V_I$. It is worth noting that the service provider can charge the reservation prices (i.e., V_F and V_I) for both consumer groups under the assumption of $\frac{V_F}{N} \le V_I < V_F$. By substituting the optimal prices into the expressions for profit (π), consumer surplus (CS), and the social welfare (SW), $\pi^* = \alpha V_F + (1 - \alpha N)V_I$, consumer surplus $\text{CS}^* = 0$. Social welfare (SW) simply equals the provider surplus: $\text{SW}^* = \pi^* = \alpha V_F + (1 - \alpha N)V_I$.

To illuminate why the introduction of a family plan is the best course of action, we provide the optimal pricing (P_B) for offering only the basic plan (i.e., the individual-plan strategy). If the firm only offer a basic plan, catering to the whole market by setting a lower basic plan price is profitable when proportion of family consumers (low-valuation consumer) is sufficiently large; otherwise, it should only accommodate individual consumers and set a high price. The optimal solutions are thus: i) if α is large (specifically, $\alpha_0 < \alpha \le \frac{1}{N}$, where $\alpha_0 = \frac{1}{N}\left(1 - \frac{V_F/N}{V_I}\right)$), $P_B^* = \frac{V_F}{N}$, $\pi^* = \frac{V_F}{N}$, $\text{CS}^* = (1 - \alpha N)\left(V_I - \frac{V_F}{N}\right)$, and $\text{SW}^* = \frac{V_F}{N} + (1 - \alpha N)\left(V_I - \frac{V_F}{N}\right)$; ii) if α is small ($0 < \alpha \le \alpha_0$), $P_B^* = V_I$, $\pi^* = (1 - \alpha N)V_I$, $\text{CS}^* = 0$, and $\text{SW}^* = (1 - \alpha N)V_I$.

Strategy		L^*	P_B^*	P_F^*	π^*	CS^*	SW^*	**Optimal Strategy**
FP		$L^* \ge N$ ($L \in \mathbb{N}$)	V_I	V_F	$\alpha V_F + (1 - \alpha N)V_I$	0	$\alpha V_F + (1 - \alpha N)V_I$	FP
IP	i)	/	$\frac{V_F}{N}$	/	$\frac{V_F}{N}$	$(1 - \alpha N)\left(V_I - \frac{V_F}{N}\right)$	$\alpha V_F + (1 - \alpha N)V_I$	
	ii)	/	V_I	/	$(1 - \alpha N)V_I$	0	$(1 - \alpha N)V_I$	

Note: FP stands for the family-plan strategy and IP stands for the individual-plan strategy. i) $\boldsymbol{\alpha_o < \alpha \le \frac{1}{N}}$, ii) $\boldsymbol{0 < \alpha \le \alpha_o}$, where $\boldsymbol{\alpha_o = \frac{1}{N}\left(1 - \frac{V_F/N}{V_I}\right)}$

Table 4. Optimal Solutions for the Family-Plan Strategy and Individual-Plan Strategy if $\frac{V_F}{N} \le V_I < V_F$

The comparison of the family-plan strategy and the individual-plan strategy is presented in Table 4. In the family-plan strategy, the profit is $\alpha V_F + (1 - \alpha N)V_I$, which is greater than the profit in the individual-plan strategy $max\left\{(1 - \alpha N)V_I, \frac{V_F}{N}\right\}$ regardless of α. Therefore, the family-plan strategy is optimal if $\frac{V_F}{N} \le V_I < V_F$. This is because, as opposed to the individual-plan strategy, adding a family plan allows the service provider to price discriminate and extract all consumer surplus by charging each consumer type their reservation prices. The result is consistent with that of Desai et al. (2018). In the family-plan strategy, consumer surplus could be smaller but social welfare is no worse than that in the individual-plan.

Case (b): $\frac{V_F}{N} \geq V_I$

If $\frac{V_F}{N} \geq V_I$, the constraints (1) and (4) are binding through proof by contradiction:

$$P_B = V_I$$

$$P_F = P_B min\{N, L\}$$

The firm's profit function is thus:

$$\pi = \alpha P_F + (1 - \alpha N)P_B = \alpha V_I min\{N, L\} + (1 - \alpha N)V_I$$

Then the company chooses L to maximize its profit. When $L \leq N$, As $\frac{\partial \pi}{\partial L} = \alpha V_I > 0, L^* = N$; when $L > N$, L can be any natural number greater than N. Thus, the optimal decisions are $L^* \geq N$ $(L \in \mathbb{N})$, $P_B^* = V_I$, $P_F^* = NV_I$. By substituting the optimal prices into the expressions for profit (π), consumer surplus (CS), and the social welfare (SW), we get $\pi^* = V_I$, $\text{CS}^* = \alpha(V_F - NV_I)$, $\text{SW}^* = \text{CS}^* + \pi^* = \alpha(V_F - NV_I) + V_I$.

Strategy		L^*	P_B^*	P_F^*	π^*	CS*	SW*	Optimal Strategy
FP		$L^* \geq N$ $(L \in \mathbb{N})$	V_I	NV_I	V_I	$\alpha(V_F - NV_I)$	$\alpha(V_F - NV_I) + V_I$	IP
IP	i)	/	V_I	/	V_I	$\alpha(V_F - NV_I)$	$\alpha(V_F - NV_I) + V_I$	
	ii)	/	$\frac{V_F}{N}$	/	αV_F	0	αV_F	

Note: i) $0 < \alpha \leq \frac{V_I}{V_F}$, ii) $\frac{V_I}{V_F} < \alpha \leq \frac{1}{N}$.

Table 5. Optimal Solutions for the Family-Plan Strategy and Individual-Plan Strategy if $V_I < \frac{V_F}{N}$

If the firm only offers a basic plan, it will set a low price if the proportion of individual consumers is large (i.e., α is small). The optimal pricing scheme and outcomes are: i) if α is small ($0 < \alpha \leq \frac{V_I}{V_F}$), $P_B^* = V_I$, $\pi^* = V_I$, $\text{CS}^* = \alpha(V_F - NV_I)$, and $\text{SW}^* = \text{CS}^* + \pi^* = \alpha(V_F - NV_I) + V_I$; ii) if α is large($\frac{V_I}{V_F} < \alpha \leq \frac{1}{N}$), $P_B^* = \frac{V_F}{N}$, $\pi^* = \alpha V_F$, $\text{CS}^* = 0$, $\text{SW}^* = \alpha V_F$.

In the family-plan strategy, to cater to both family and individual consumers, the firm could only set a low price for the basic plan while the price of family plan is simply N times the price of basic plan. This result is equivalent to implementing individual-plan strategy when α is small. However, if α is large, the firm is better off leaving the individual consumers out of the market. Therefore, if $V_I < \frac{V_F}{N}$, the individual-plan strategy is as good as or superior to the family-plan strategy.

To conclude, price discrimination makes economic sense only when families contain low-valuation members, which pull down the average utility. In streaming services, family consumers usually include children and elderly, whose valuations are low. Thus, case (a) $\frac{V_F}{N} \leq V_I < V_F$ is closer to reality, and adding family plan for streaming service provider is optimal.

Proposition 1. *If* $\frac{V_F}{N} \leq V_I < V_F$, *adding a family plan to the menu is the optimal strategy for the streaming service provider; the pricing scheme is* $(P_F^*, L^*) = (V_F, L^* \geq N)$, *where* $L \in \mathbb{N}$, *and* $(P_B^*, 1) = (V_I, 1)$. *If* $V_I < \frac{V_F}{N}$, *individual-plan strategy is optimal, and the pricing scheme is* $P_B^* = \frac{V_F}{N}$ *or* $P_B^* = V_I$.

Two Family Plans

In this section, we analyze the case where the firm offers two family plans and one basic plan. We further segment the family consumers (F) into two types: α_1 family 1 (F1) with family size N_1 and α_2 family 2 (F2) with family size N_2, where $1 < N_1 < N_2 (N_1, N_2 \in \mathbb{N})$, $0 < \alpha_1, \alpha_2 < \frac{1}{N}$ and $1 - \alpha_1 N_1 - \alpha_2 N_2 <$

1. The service provider releases two family plans and one individual plan: (P_1, L_1), (P_2, L_2) and $(P_B, 1)$, and these plans only differ in the number of concurrent device limits. Family plan 1 (P_1, L_1), Family plan 2 (P_2, L_2), and Basic plan$(P_B, 1)$, are for F1, F2, and I, respectively. The market segments and price schemes are given in Table 6 and Table 7.

	Basic plan	Family plan 1	Family plan 2
Price	P_B	P_1	P_2
Device limit	1	L_1	L_2

Table 6. The Pricing Scheme for the Two-Tier Family-Plan Strategy

Customer Type	Market proportion	Maximum Valuation	Family Size
Individual (I)	$1 - \alpha_1 N_1 - \alpha_2 N_2$	V_I	1
Family 1 (F1)	α_1	V_{F1}	N_1
Family 2 (F2)	α_2	V_{F2}	N_2

Table 7. Market Segments for the Two-Tier Family-Plan Strategy

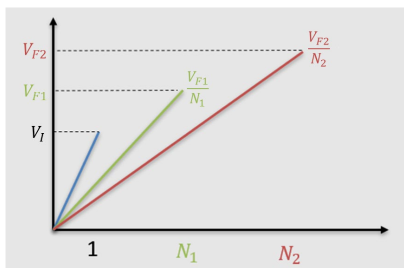

Figure 3. Consumer Valuation Under the Assumption $\frac{V_{F2}}{N_2} < \frac{V_{F1}}{N_1} < V_I$

We focus on the case where $\frac{V_{F2}}{N_2} < \frac{V_{F1}}{N_1} < V_I < V_{F1} < V_{F2}$ (A large family has a greater total valuation, but a smaller average valuation per member, compared to a small family), as it is more in line with reality. For example, F2 are families including parents and two children, while F1 are couples. The larger the family size, the more low-valuation users there are, and thus the lower *average* valuation. Additionally, a family's marginal utility is likely to be diminishing with the number of devices allowed, which will reduce the average utility. We thus assume that family with a larger family size has a lower average willingness-to-pay.

The profit maximization problem for the service provider is given by:

$$\max_{L_1, L_2, P_B, P_1, P_2} \pi = (1 - \alpha_1 N_1 - \alpha_2 N_2) P_B + \alpha_1 P_1 + \alpha_2 P_2$$

s.t.

(1) $V_I - P_B \geq 0 \Rightarrow P_B \leq V_I$ — I prefers $(P_B, 1)$ to not buying

(2) $V_I - P_B \geq V_I - P_1 \Rightarrow P_B \leq P_1$ — I prefers $(P_B, 1)$ to (P_1, L_1)

(3) $V_I - P_B \geq V_I - P_2 \Rightarrow P_B \leq P_2$ — I prefers $(P_B, 1)$ to (P_2, L_2)

(4) $\frac{V_{F1}}{N_1} - \frac{P_1}{min\{N_1, L_1\}} \geq 0 \Rightarrow P_1 \leq \frac{V_{F1}}{N_1} min\{L_1, N_1\}$

F1 prefers (P_1, L_1) to not buying

(5) $\frac{V_{F1}}{N_1} - \frac{P_1}{min\{L_1, N_1\}} \geq \frac{V_{F1}}{N_1} - P_B \Rightarrow P_1 \leq P_B min\{L_1, N_1\}$

F1 prefers (P_1, L_1) to $(P_B, 1)$

(6) $\frac{V_{F1}}{N_1} - \frac{P_1}{min\{L_1,N_1\}} \geq \frac{V_{F1}}{N_1} - \frac{P_2}{min\{L_2,N_1\}} \Rightarrow P_1 \leq \frac{P_2}{min\{L_2,N_1\}} min\{L_1, N_1\}$

F1 prefers (P_1, L_1) to (P_2, L_2)

(7) $\frac{V_{F2}}{N_2} - \frac{P_2}{min\{N_2,L_2\}} \geq 0 \Rightarrow P_2 \leq \frac{V_{F2}}{N_2} min\{N_2, L_2\}$

F2 prefers (P_2, L_2) to not buying

(8) $\frac{V_{F2}}{N_2} - \frac{P_2}{min\{N_2,L_2\}} \geq \frac{V_{F2}}{N_2} - P_B \Rightarrow P_2 \leq P_B min\{N_2, L_2\}$

F2 prefers (P_2, L_2) to $(P_B, 1)$

(9) $\frac{V_{F2}}{N_2} - \frac{P_2}{min\{N_2,L_2\}} \geq \frac{V_{F2}}{N_2} - \frac{P_1}{min\{L_1,N_2\}} \Rightarrow P_2 \leq \frac{P_1}{min\{L_1,N_2\}} min\{N_2, L_2\}$

F2 prefers (P_2, L_2) to (P_1, L_1)

The firm decides L_2, L_1, P_B, P_1, P_2 to maximize its profits subject to the above constraints (1) to (9), $P_B, P_1, P_2 \geq 0$, and $L_1, L_2 \in \mathbb{N}$. Since the firm would want price as higher as possible, from constraints (4) and (7), we should raise L_1 and L_2 as high as possible as well, i.e., $L_1 \geq N_1$ and $L_2 \geq N_2$ .Therefore, constraints (4), (6), (7) can be simplified as (4a) $P_1 \leq V_{F1}$,(6a) $P_1 \leq P_2$, and (7a) $P_2 \leq V_{F2}$; constraint (9) can be simplified as (9a) $P_2 \leq \frac{P_1}{L_1} N_2$ if $N_1 \leq L_1 \leq N_2$, and (9b) $P_2 \leq P_1$ if $L_1 > N_2$.

Given $\frac{V_{F2}}{N_2} < \frac{V_{F1}}{N_1} < V_I < V_{F1} < V_{F2}$, (I) when $N_1 \leq L_1 < \frac{V_{F1}}{V_{F2}/N_2}$, constraints (1), (4a), and (7a) are binding. Solving $\max\limits_{L_1,L_2,P_B,P_1,P_2} \pi = (1 - \alpha_1 N_1 - \alpha_2 N_2) P_B + \alpha_1 P_1 + \alpha_2 P_2$ yields the following: $P_B^* = V_I$, $P_1^* = V_{F1}$, $P_2^* = V_{F2}$, $N_1 \leq L_1^* < \frac{V_{F1}}{V_{F2}/N_2}$,$L_2^* \geq N_2$. (II) When $\frac{V_{F1}}{V_{F2}/N_2} \leq L_1 < N_2$, constraints (1), (4a), and (9a) are binding. The optimal solutions are: $P_B^* = V_I$, $P_1^* = V_{F1}$, $P_2^* = V_{F2}$, $L_1^* = \frac{V_{F1}}{V_{F2}/N_2}$, $L_2^* \geq N_2$. (III) When $L_1 \geq N_2$, constraints (1), (4a), and (9b) are binding, $P_B^* = V_I$, $P_1^* = P_2^* = V_{F1}$. However, this result is not optimal as $P_2^* = V_{F1} < V_{F2}$.

Therefore, combining (I) and (II) gives the following optimal solutions:

$$P_B^* = V_I,$$

$$P_1^* = V_{F1},$$

$$P_2^* = V_{F2},$$

$$N_1 \leq L_1^* \leq \frac{V_{F1}}{V_{F2}/N_2} (L_1^* \in \mathbb{N}),$$

$$L_2^* \geq N_2 (L_2^* \in \mathbb{N}).$$

The profit is $\pi^* = \alpha_1 V_{F1} + \alpha_2 V_{F2} + (1 - \alpha_1 N_1 - \alpha_2 N_2) V_I$, and consumer surplus is fully exacted $\mathrm{CS}^* = 0$. Social welfare is all distributed to the service provider: $\mathrm{SW}^* = \pi^* = \alpha_1 V_{F1} + \alpha_2 V_{F2} + (1 - \alpha_1 N_1 - \alpha_2 N_2) V_I$.

Analogous to the pricing scheme in the individual-plan strategy in previous subsection, $P_B^* = V_I$ or V_{F1}/N_1 or V_{F2}/N_2 when there are three segments, depending on the relative magnitude of α_1 and α_2. Regardless of the value of P_B^* in the individual-plan strategy, the family-plan strategy is optimal. This is because in the family-plan strategy, the firm fully extracts all consumer surplus by offering multiple plans to cover all market segments, allowing profits to reach their maximum potential. Social welfare is also no worse than that in the individual-plan strategy. The optimal pricing scheme of the two-tier family-plans strategy is given by the following proposition.

Proposition 2. *If $\frac{V_{F2}}{N_2} < \frac{V_{F1}}{N_1} < V_I < V_{F1} < V_{F2}$, offering two family plans to the menu is superior. The pricing scheme is: Family plan 1 $(P_1^*, L_1^*) = \left(V_{F1}, N_1 \leq L_1^* \leq \frac{V_{F1}}{V_{F2}/N_2}\right)$, Family plan 2 $(P_2^*, L_2^*) = (V_{F2}, L_2^* \geq N_2)$ and Individual plan $(P_B^*, 1) = (V_I, 1)$,where $L_1^*, L_2^* \in \mathbb{N}$.*

The finding suggests that, when the largest family group has the lowest average valuation, offering multi-tier family plans is strictly profit-enhancing. This is due to the fact that offering a larger family plan at a cheaper per-slot price (i.e., $\frac{V_{F2}}{L_2^*}$, where $L_2^* \geq N_2$) can attract consumers of lower average valuation. But similar to the notion of the quantity discount, the discount only applies when the group size is large enough, which is N_2. Smaller families or individual consumers, on the other hand, would be charged a higher price. To put it another way, the firm is able to practice price discrimination more effectively and charge a higher price for consumers who are unable to be part of a large family. However, this finding only holds if $\frac{V_{F2}}{N_2} < \frac{V_{F1}}{N_1} < V_I < V_{F1} < V_{F2}$. We also look at other possible scenarios. Table 8 summarizes the optimal strategy for each possible situation.

From Table 8, we can draw some interesting conclusions. First of all, launching multi-tier family plans is not strictly optimal. Providing multiple-tier family plans is strictly better only when the largest family has the lowest average reservation price, the second largest family has the second lowest average reservation price, and so on. This is because family customers are free to subscribe to multiple individual plans, limiting the price of family plan 1 P_1 to be no more than $L_1 P_B$, and the price of family plan 2 P_2 to be no more than $L_2 P_B$. The basic plan's price P_B^* is always V_I. Even if consumers F1 or F2 have higher average reservation prices than V_I, the firm is unable to charge the families their reservation price but rather $P_1 = L_1 P_B$ and $P_2 = L_2 P_B$, which is equivalent to the individual-plan strategy. Thus, for situations (a) and (b), rolling out family plans is unnecessary.[6]

Second, it may be profit-enhancing to roll out one family plan when there exist family consumers with an average reservation price lower than V_I. In the case of (c), introducing family plan 2 and the individual plan will be preferable, which target consumers F2 and I, respectively, whereas F1 are expected to select N_1 units of basic plans. Which family plan should be offered for conditions (d) and (e), however, depends on the relative percentage of consumers F1 and F2.

Assumptions		**Optimal Strategy**
(a)	$V_I < \frac{V_{F1}}{N_1} < \frac{V_{F2}}{N_2}$	Basic plan
(b)	$V_I < \frac{V_{F2}}{N_2} < \frac{V_{F1}}{N_1}$	Basic plan
(c)	$\frac{V_{F2}}{N_2} < V_I < \frac{V_{F1}}{N_1}$	Basic plan + Family plan 2
(d)	$\frac{V_{F1}}{N_1} < V_I < \frac{V_{F2}}{N_2}$	i) When α_1/α_2 is small, Basic plan ii) When α_1/α_2 is large, Basic plan + Family plan 1
(e)	$\frac{V_{F1}}{N_1} < \frac{V_{F2}}{N_2} < V_I$	i) When α_1/α_2 is small, Basic plan+ Family plan 2 ii) When α_1/α_2 is large, Basic plan+ Family plan 1
(f)	$\frac{V_{F2}}{N_2} < \frac{V_{F1}}{N_1} < V_I$	Basic plan + Family plan 1 + Family plan 2

Table 8. Optimal Strategy Under Different Assumptions

To conclude, which strategy is strictly optimal depends on the assumptions made regarding the average reservation price of the consumer segments. However, in practice, larger families typically include more financially dependent individuals such as children, which may pull down the average valuation. Thus, the firm will be better off by offering multiple-tier family plans to cover more consumer types.

The Impact of Subscription Sharing

Pricing Scheme

In this section, we seek to design pricing schemes for the family-plan strategy with and without subscription sharing. Similar to the simple model outlined in last section, a monopolist offers one family and one basic (or individual) plan, except that consumer heterogeneity within each type is taken into consideration. As before, the market is composed of individual consumers and family consumers. Individual consumers are

[6] We focus on the case where the proportion of individual consumer is large enough to make offering basic plan always profitable. This aligns with the reality that almost all streaming service providers offer the individual plan options.

assumed to follow a uniform distribution U[0,1] and the family consumers with family size N follow U[0, W], where $1 < W < N$. The market size of individual consumers is represented by β and that of the family consumers is $\frac{1-\beta}{N}$, and therefore the total market size sums to 1. The net surplus realized from streaming contents is $u - p$, where u denotes the consumer's valuation or willingness to pay and p represents the subscription fee (p_b for the basic/individual plan and p_f for the family plan). Only consumers whose net surplus is non-negative will make a purchase.

We first analyze a benchmark situation where there is no subscription-sharing opportunity, i.e., individual consumers are unable to team up with other individual consumers to share a family plan. Thus, the demand functions for the individual plan and the family plan are $D_b = \beta(1 - P_b)$ and $D_f = \frac{1-\beta}{N}(W - P_f)$, respectively. The firm has the monopoly power to determine P_b and P_f, and maximizes its profits subject to $p_b \leq 1$, $p_f \leq$ W, and $p_f \leq Np_b$. Assuming zero marginal cost, the profit maximization problem for the monopolist is written as

$$\max_{p_b, p_f} \pi_F = p_b D_b + p_f D_f$$

$s.t.$

$$p_f \leq Np_b$$
$$0 \leq p_b \leq 1$$
$$0 \leq p_f \leq W$$

The optimal prices are thus

$$p_b^* = \frac{1}{2} \text{ and } p_f^* = \frac{W}{2}.$$

The maximum profit at the optimal prices is given by

$$\pi_F^* = \frac{1}{4}\left[\beta + \frac{W^2(1-\beta)}{N}\right]. \tag{1}$$

The price gap is defined as $\Delta p = p_b^* - \frac{p_f^*}{N}$. Note that the subscription fee for the family plan per user is $\frac{p_f^*}{N} = \frac{W}{2N}$, which is less than that of the basic plan $p_b^* = \frac{1}{2}$ as we assume W $< N$. The positive price gap could motivate individual consumers to share a family plan with other individuals to save on subscription expenses.

Now we look at the situation where subscription sharing is feasible. In addition to subscribing to basic plans on authorized websites, individual consumers can also choose to share a family plan with other individuals on subscription-sharing platforms. The presence of subscription-sharing platforms may create two countervailing effects: the canalization effect and the market expansion effect. On one hand, after becoming aware of the subscription-sharing opportunities, individual consumers who are high-valuation and otherwise would subscribe to the basic plan now are incentivized to switch to join a family plan through the platform. The switching behavior (from an individual plan subscriber to a family plan co-subscriber) cannibalizes the profits. On the other, since the family plan per person is cheaper than the basic plan, the subscription-sharing opportunities might draw in those who have a low willingness-to-pay and would otherwise be left out of the market (market expansion effect). Suppose there are γ ($0 \leq \gamma \leq 1$) percentage of potential consumers who previously will not subscribe to the basic plan now become family plan joiners, and δ ($0 \leq \delta \leq 1$) percentage of basic plan subscribers switch to share a family plan.

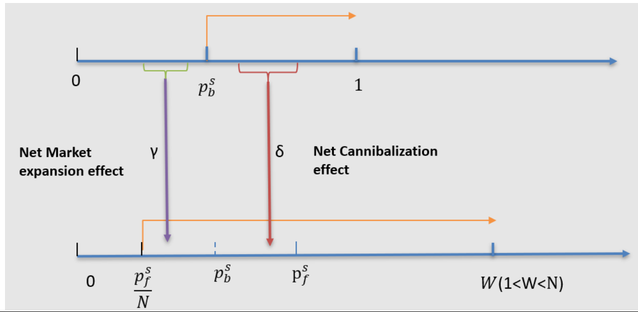

Figure 2. Consumer Valuation and Migrating Behaviors

Taking subscription sharing into consideration, the demand for the basic plan is given by $D_b^s = \beta(1-\delta)(1-p_b^s)$. The demand for the family plan D_f^s contains two parts: the original demand from the authorized channels, captured by $D_f^a = \frac{1-\beta}{N}(W - p_f^s)$, as well as the demand from the subscription sharing websites, represented by $D_f^u = \gamma\beta(p_b^s - \frac{p_f^s}{N})/N + \delta\beta(1\text{-}p_b^s)/N$. The first term of D_f^u comes from new consumers whilst the second term comes from switchers who are previously basic plan buyers. The two parameters γ and δ together depict the severity of subscription sharing. The overall demand for the family plan option is thus given by $D_f^s = \frac{1-\beta}{N}(W - p_f^s) + \gamma\beta(p_b^s - \frac{p_f^s}{N})/N + \delta\beta(1\text{-}p_b^s)/N$. When $\delta = \gamma = 0$, i.e., subscription sharing is absent, the demand for the family plan subsumes to D_f. When $\gamma \neq 0$ and $\delta = 0$, subscription sharing only gives rise to the market expansion effect; When $\delta \neq 0$ and $\gamma = 0$, only the cannibalization effect plays a part, and if $\delta = 1$, no more consumers will subscribe to the basic plan.

We analyze how demands respond to prices where $\delta \neq 0$ or 1 and $\gamma \neq 0$. Note that $\frac{\partial D_b^s}{\partial p_b^s} = -\beta(1-\delta) < 0$ and $\frac{\partial D_b^s}{\partial p_f^s} = 0$, suggesting that the demand for the basic plan decreases with its own price p_b^s but does not vary with p_f^s. $\frac{\partial D_f^s}{\partial p_f^s} = -\frac{1-\beta}{N} - \frac{\beta\gamma}{N^2} < 0$ and $\frac{\partial D_f^s}{\partial p_b^s} = \frac{\beta(\gamma-\delta)}{N}$ suggest D_f^s decreases as p_f^s increases, but the relationship between D_f^s and p_b^s depends on the relative magnitude of γ and δ, leading to the following:

Lemma 1. *Taking subscription sharing into consideration, the demand for the family plan, D_f^s, varies with both p_b^s and p_f^s. D_f^s decreases with its own price p_f^s, but increases (decreases) with p_b^s when $\gamma > \delta$ ($\gamma < \delta$).*

The negative correlation between the demand for the family plan D_f^s and its own price p_f^s is obvious. To understand why the relationship between D_f^s and p_b^s depends on the countervailing force of γ and δ, we look at the extreme case where only the market expansion effect exists. When $\gamma = 1$ and $\delta = 0$, all low-valuation individuals with $V \in \left[\frac{p_f^s}{N}, p_b^s\right]$ will become family plan co-subscribers, and the demand for the family plan is thus: $D_f^s = \frac{1-\beta}{N}(W - p_f^s) + \gamma\beta(p_b^s - \frac{p_f^s}{N})/N$. As p_b^s increases, holding p_f^s constant, the price gap widens, and therefore attracts more new consumers, pulling up the demand for the family plan. In contrast, if the switching behavior is predominant, there are relatively more basic plan subscribers who will switch to share family plans. The more expensive the basic plan, the fewer basic plan buyers, and therefore the fewer switchers, lowering the overall demand for the family plan.

The profit maximization problem with subscription sharing is written as

$$\max_{p_b^s, p_f^s} \pi_F^s = \beta(1-\delta)(1-p_b^s)p_b^s + [\frac{1-\beta}{N}(W - p_f^s) + \gamma\beta(p_b^s - \frac{p_f^s}{N})/N + \delta\beta(1\text{-}p_b^s)/N]p_f^s \qquad (2)$$

s.t.

$$0 \le p_b^s \le 1$$
$$0 \le p_f^s \le \mathrm{W}$$
$$p_f^s \le N p_b^s$$

It should be pointed out that when the constraint $p_f^s \le N p_b^s$ become binding, i.e., the price gap becomes 0, the family-plan strategy reduces to the individual-plan strategy. This is not surprising as the pricing scheme of the family-plan strategy is flexible enough to subsume that of the individual-plan strategy. Notice that the hessian matrix is negative definite (see appendix), solving the problem yields the optimal prices:

when $p_f^s \le N p_b^s$ is not binding,

$$p_b^{s,*} = \frac{W(1-\beta)(\gamma-\delta) + 2N(1-\delta)(1-\beta) + \beta[\gamma(2-\delta) - \delta^2]}{4N(1-\beta)(1-\delta) - \beta[(\gamma+\delta)^2 - 4\gamma]}$$
$$p_f^{s,*} = \frac{N(1-\delta)[2W(1-\beta) + \beta(\gamma+\delta)]}{4N(1-\beta)(1-\delta) - \beta[(\gamma+\delta)^2 - 4\gamma]}$$
$$\Delta p^{s,*} = p_b^{s,*} - \frac{p_f^{s,*}}{N} = \frac{\beta(\gamma-\delta) + 2N(1-\beta)(1-\delta) - W(1-\beta)(2-\gamma-\delta)}{4N(1-\beta)(1-\delta) - \beta[(\gamma+\delta)^2 - 4\gamma]} \quad (3)$$

when $p_f^s = N p_b^s$ is binding,

$$p_b^{s,*} = \frac{W(1-\beta)+\beta}{2[N(1-\beta)+\beta]}$$
$$p_f^{s,*} = \frac{NW(1-\beta)+N\beta}{2[N(1-\beta)+\beta]} \quad (4)$$

We then demonstrate how optimal prices respond to changes in consumer valuation. Proofs are given in the appendix.

Lemma 2. *p_f^s increases with W, the consumer valuation of the family-type consumers. p_b^s increases (decreases) with W when $\gamma > \delta$ ($\gamma < \delta$). Δp^s decreases with W.*

Lemma 2 shows that increases in W, the maximum willingness-to-pay of family consumers, allow the company to charge a higher price for the family plan. This is obvious as the demand for the family plan increases with W, thereby driving up the price p_f^s. Nevertheless, the relationship between p_b^s and W is not straightforward. To see why p_b^s increase with W when $\gamma > \delta$, we look at the price gap Δp^s. When there are relatively more new consumers entering the market ($\gamma > \delta$), a profit-seeking firm would want to maintain a certain level of price gap by raising p_b^s accordingly when W pulls up p_f^s. Despite that, compared with the returns realized from the direct increase in the family plan's price as a result of increased willingness-to-pay, the benefits of having more new customers are negligible. Taken together, the p_f^s increases more than p_b^s, causing the price gap to become smaller as a result. The result implies that when the maximum willingness-to-pay of family-type consumers increases, the firm benefits more from increasing the price p_f^s than maintaining a large price gap to expand the market. On the contrary, if the market has relatively more switchers, higher W will lead the firm to lower the basic plan price.

We now turn our attention to the two key parameters γ and δ and provide the numerical analysis of the optimal prices and the profit as functions of γ and δ. Figure 4 and 5 show how the optimal prices and price gap vary with γ and δ when $W = 1.6$, $N = 2$, and $\beta = 0.7$. As can be seen, both prices are sensitive to the two effects. p_b^s monotonically decreases with δ while p_f^s monotonically increases with δ, which is as expected since a higher switching probability will erode profit margins, and therefore it is in the firm's interest to narrow the price gap. On the contrary, when the market expansion effect γ enlarges, p_b^s increases and p_f^s decreases monotonically, thereby widening the price gap. Notice that when δ is too high while γ is too low, the price functions level off, suggesting that the firm will be better off closing the price gap. In this case, it is equivalent to implementing the individual-plan strategy, and adding a family plan option is no longer strictly superior.

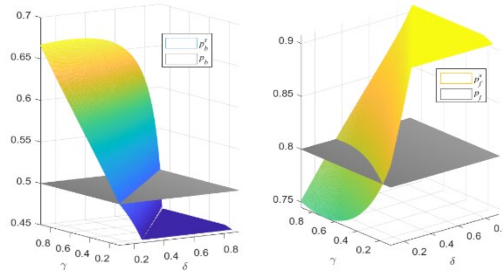

Figure 3. Optimal Prices as a Function of γ and δ ($W = 1.6$, $N = 2$ and $\beta = 0.7$)

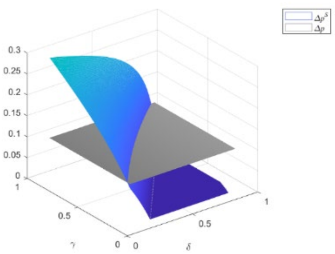

Figure 4. Price Gap as a Function of γ and δ

Profit

In this subsection, we compare the profit of the individual-plan strategy, the family-plan strategy without sharing, and the family-plan strategy with sharing to evaluate the impact of subscription sharing.

When the constraint $p_b^s \geq \frac{p_f^s}{N}$ is not binding, substituting the optimal prices into the profit function, we get

$$\pi_F^{s*} = \frac{(1-\delta)\left[W^2(1-\beta)^2 + \beta\left(\beta\gamma + N(1-\beta)(1-\delta)\right) + W(1-\beta)\beta(\gamma+\delta)\right]}{4N\,(1-\beta)(1-\delta) - \beta[(\gamma+\delta)^2 - 4\gamma]}. \quad (5)$$

When $p_b^s = \frac{p_f^s}{N}$, the optimal profit of the family-plan strategy will become the profit function for the individual-plan strategy, which is given by

$$\pi_S^* = \frac{[W(1-\beta)+\beta]^2}{4N(1-\beta)+4\beta}. \quad (6)$$

As displayed in Figure 6, the profit of the family-plan strategy with the presence of subscription sharing increases with γ and decreases with δ before the constraint becomes binding. Comparing the performance of the family-plan strategy π_F^s with sharing to that without sharing π_F, we notice that the profit in the presence of subscription sharing can be higher when the market expansion effect outweighs the cannibalization effect. In addition, recall that the optimal solution of the profit-maximizing problem for the individual-plan strategy can be derived by simply forcing the constraint $p_b^s \geq \frac{p_f^s}{N}$ to be binding. Therefore, the profit of the family-plan strategy with the presence of sharing π_F^s is invariably no worse than that of the individual-plan strategy π_S.

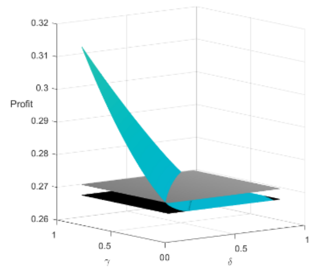

Figure 6. Total Profit as a Function of γ and δ ($W = 1.6$, $N = 2$ and $\beta = 0.7$)

That being said, even if subscription sharing is so severe that the company suffers from massive profit loss due to the cannibalization effect, the family-plan strategy is always a better choice than the individual-plan strategy for the firm. This is because, the family-plan strategy is flexible to mimic the uniform pricing (i.e., individual-plan strategy), and the firm will optimally close the price gap when the cannibalization effect dominates. We believe these results have important implications for streaming service providers who may fear that the unauthorized sales of family plans may be detrimental to profit and hesitate to roll out the family plan.

Lemma 3 *With subscription sharing, the total profit of the family-plan strategy increases with γ and decreases with δ.*

Lemma 4 *The total profit of the family-plan strategy is always no worse than that of the individual-plan strategy even in the presence of subscription sharing.*

Consumer Surplus and Social Welfare

Here, we evaluate the welfare implications of subscription sharing. Absent subscription sharing, by substituting the optimal prices into the expressions for consumer surplus (CS_F) and social welfare (SW_F), we obtain

$$CS_F = \beta \int_{p_b^*}^{1} (v - p_b^*) dv + \frac{1-\beta}{N} \int_{p_f^*}^{W} (v - p_f^*) dv = \frac{W^2 + N\beta - W^2\beta}{8N} \quad (7)$$

$$SW_F = \pi_F^* + CS_F = \frac{3(W^2(1-\beta) + N\beta)}{8N} \quad (8)$$

With the presence of subscription sharing, the expressions for the lower bound of consumer surplus (CS_F^S) and social welfare (SW_F^S) are given by

$$CS_F^S = \beta \int_{p_b^{s,*}}^{1} (v - p_b^{s,*}) dv + \beta\delta(1 - p_b^{s,*})\left(p_b^{s,*} - \frac{p_f^{s,*}}{N}\right) + \beta \int_{\frac{p_f^{s,*}}{N}}^{\frac{p_f^{s,*}}{N} + \left(p_b^{s,*} - \frac{p_f^{s,*}}{N}\right)\gamma} \left(v - \frac{p_f^{s,*}}{N}\right) dv \quad (9)$$
$$+ \frac{1-\beta}{N} \int_{p_f^{s,*}}^{W} (v - p_f^{s,*}) dv$$
$$SW_F^S = \pi_F^{s,*} + CS_F^S$$

The first and the fourth term of equation (9) capture the original consumer surplus (absent the subscription sharing), while the second and the third term represent the surplus gained as a result of switching from individual plan to family plan and drawing in low-valuation consumers, respectively.

The full expressions are attached in the appendix. As CS_F^S and SW_F^S are quartic functions of γ and δ, we lose the analytical tractability. We present the numerical results when $W = 1.6, N = 2, \beta = 0.7$. As can be seen, the figure shows how social welfare and consumer surplus vary with the two key parameters. Interestingly, when the market expansion effect is large and the cannibalization effect is small, consumer surplus with the presence of subscription sharing can be smaller than that without sharing, which seems counterintuitive at the first sight. Note that the firm can adjust the price strategically, and p_b^{s*} is greater than p_b^* when the cannibalization effect outweighs the market expansion effect (see Figure 4). By setting a higher price for the individual plan, the firm can extract more surplus from individual plan subscribers. The consumer surplus lost for individual plan subscribers is sufficiently larger to offset the surplus gain for new individual consumers, leading to smaller total consumer surplus. As for social welfare, social welfare with the presence of subscription sharing is greater than that without sharing.

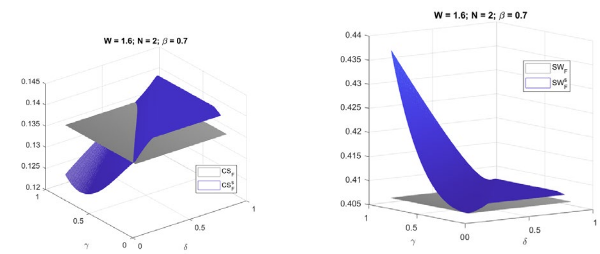

Figure 7. Consumer Surplus and Social Welfare as a Function of γ and δ

Conclusion

Subscription sharing of digital services, fueled by online communities and especially subscription-sharing platforms, has become ubiquitous in recent years. The ability for non-family members to jointly subscribe to a family plan poses potential challenges for streaming service providers. Although streaming service providers are concerned about the potential negative effects of such unauthorized activities on the profitability of family plans, and some even attempted to implement a crackdown on subscription sharing, our results suggest that they can design an optimal pricing scheme to take advantage of subscription sharing.

In this paper, we develop an economic model under which a service provider considers adopting an individual-plan strategy (only offering a basic plan) or a family-plan strategy (offering both a basic plan and a family plan). With the presence of subscription sharing, individual consumers are able to share a family plan via a subscription-sharing channel in the family-plan strategy. We derive the optimal prices and profits for the individual-plan strategy and the family-plan strategy, and examine how subscription sharing influences profit, consumer surplus, and social welfare. Results demonstrate that the profitability of the family-plan strategy is always no worse than that of the individual-plan strategy even with the presence of subscription sharing. Under certain circumstances, i.e., when market expansion effect outweighs the cannibalization effect, allowing subscription sharing can lead to higher profit. For consumers, being able to share a family plan subscription does not necessarily result in higher consumer surplus. Taken together, subscription sharing leads to higher social welfare.

Managerial Implications

The main takeaway of this research is that subscription sharing is not necessarily a threat for the firms. If the subscription-sharing platforms attract more new consumers than originally individual plan subscribers, the firm can leverage subscription sharing and design an optimal pricing scheme in response to subscription sharing.

In fact, the firm can design a proper menu and utilize technical tools to mitigate the negative impact of the cannibalization effect and to enlarge the market expansion effect. In the paper, we assume the homogenous product quality in our model, but in practice, family plan subscribers usually enjoy better services than individual plan subscribers. To discourage switchers, the firm can design subscription plans so that the basic plan is at least as attractive (e.g., offering the same resolution, video/music quality, and allowing video download) as the family plan except for the inability to share with family members. On the other hand, to attract new consumers, the firm can reduce the friction (such as privacy concerns) of subscription sharing of a family plan, for instance, allowing separate accounts for each user and disabling tracking other users' browsing histories.

Limitations

This study has several limitations. First, we model a monopolistic service provider in our setting. It remains to be seen whether our findings will hold in a duopolistic and competitive environment. Second, we can relax some of our modeling assumptions, e.g., the two migrating parameters might be correlated, the maximum willingness-to-pay of family consumers W might be greater than N, the family consumers may purchase more than 1 unit of basic plan in the individual-plan strategy. We intend to examine these extensions in the future.

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