Efficient models for low thrust collision avoidance in space

Juan Luis Gonzalo[1,a *], Camilla Colombo[1], Pierluigi Di Lizia[1], Andrea De Vittori[1], Michele Maestrini[1], Pau Gago Padreny[2], Marc Torras Ribell[2], Diego Escobar Antón[2]

[1]Politecnico di Milano, Department of Aerospace Science and Technology, Via la Masa 34, 20158 Milan (Italy)

[2]GMV, Calle de Isaac Newton 11, 28760 Tres Cantos, Madrid (Spain)

[a]juanluis.gonzalo@polimi.it

Keywords: Space Collision Avoidance, Low Thrust Propulsion, Spaceflight Mechanics, Analytical Models, Space Traffic Management

Abstract. A family of analytical and semi-analytical models for the characterization and design of low thrust collision avoidance manoeuvres (CAMs) in space is presented. The orbit modification due to the CAM is quantified through the change in Keplerian elements, and their evolution in time is described by analytical expressions separating secular and oscillatory components. Furthermore, quasi-optimal, piecewise-constant control profiles are derived from impulsive CAM models. The development of these models is part of an ESA-funded project to advance existing tools for collision avoidance activities.

Introduction

The number of objects in orbit around the Earth is growing at an accelerating pace, due to both the increasing number of public entities and private companies leveraging space assets for diverse applications and the accumulation of space debris. Regarding active satellites, their numbers are soaring owing to the deployment of large constellations like Starlink, and the democratization of access to space enabled by lower launch costs and smaller, cost-effective platforms like CubeSats. The increasing congestion in space presents multiple challenges for Space Traffic Management and Space Situational Awareness, and demands the implementation of space debris mitigation actions like end-of-life deorbiting and collision avoidance (COLA) to curb the increase of space debris. However, an increasing portion of satellites are equipping low thrust propulsion systems, which, although more efficient, present a smaller control authority and complicate the design of disposal and collision avoidance manoeuvres (CAMs), particularly for last-time scenarios. As a result, new models and tools are needed to support low thrust COLA activities in congested orbital regions like low Earth orbit (LEO) and geostationary orbit (GEO).

In this context, GMV, UC3M and Politecnico di Milano are developing the ELECTROCAM project, funded by the European Space Agency to advance their models and tools for the analysis of low thrust COLA activities. The project covers several aspects of COLA activities, including the assessment of current capabilities of low thrust satellites [1], propagation of uncertainties [2,3], efficient analytical and semi-analytical models for CAMs [4], and update of ESA's operational software tool ARES.

This work deals with some of the analytical and semi-analytical CAM models developed within the ELECTROCAM project. The models are focused on computational efficiency, to serve as initial guesses for more accurate, higher-cost numerical algorithms, and to perform sensitivity analyses over large sets of data. To this end, a single-averaging of the thrust-perturbed equations of motion is performed to express the evolution of the Keplerian elements as the combination of linear and oscillatory contributions, both expressed through exact or approximated analytical expressions. A simplified control law is also proposed, leveraging analogous models for the impulse propulsion case. The performance of the models has been assessed through sensitivity analyses in relevant COLA scenarios, and some selected results are presented here.

Single-averaged low-thrust CAM model

Let us consider a predicted close approach (CA) at a given time of closest approach (TCA) between a low-thrust-capable spacecraft and a debris (in general, a non-collaborative object). The CAM is modelled following the scheme proposed in [5], composed of three steps. First, the orbit modification due to the CAM is quantified through the change $\delta\boldsymbol{\alpha}$ of its vector of Keplerian elements $\boldsymbol{\alpha}=[a,e,i,\Omega,\omega,M]$, whose components are, respectively, semimajor axis, eccentricity, inclination, right ascension of the ascending node, argument of pericentre, and mean anomaly. Second, the change in orbital elements is mapped to changes in position and velocity at the TCA using a linearized relative motion model with the nominal trajectory as reference orbit. Because the displacements associated to CAMs in practical scenarios are typically small, the loss of accuracy due to the use of a linearized model is limited. Finally, the outcome from the CAM is projected and characterized in the nominal encounter plane at TCA, and the change in collision probability quantified.

The most complex step is the derivation of (semi-)analytical expressions for $\delta\boldsymbol{\alpha}$ under a continuous thrust acceleration $\mathbf{a}_t$. The models considered here are based on the single-averaging of Gauss's planetary equations over one revolution, under the assumption of small thrust acceleration. This assumption allows to linearize the equations of motion in thrust acceleration and study the tangential and normal components separately (denoted by superscripts t and n):

$$\dot{\boldsymbol{\alpha}} = \mathbf{G}(\boldsymbol{\alpha}, t; \boldsymbol{a}_t) \approx \mathbf{G}^t(\boldsymbol{\alpha}_0 + \delta\boldsymbol{\alpha}^t, t; a_t^t) + \mathbf{G}^n(\boldsymbol{\alpha}_0 + \delta\boldsymbol{\alpha}^n, t; a_t^n) + \dot{\boldsymbol{\alpha}}_0 = \delta\dot{\boldsymbol{\alpha}}^t + \delta\dot{\boldsymbol{\alpha}}^n + \dot{\boldsymbol{\alpha}}_0. \quad (1)$$

Where $\mathbf{G}$ represents Gauss's planetary equations, and $\dot{\boldsymbol{\alpha}}_0$ is the evolution of mean anomaly for the unperturbed orbit. The solutions for $\delta\boldsymbol{\alpha}^t$ and $\delta\boldsymbol{\alpha}^n$ from Eq. 1 fall into three categories. First, some Keplerian elements are unaffected by the corresponding thrust component (Ω and i for tangential, a, i and Ω for normal). Then, some elements have only oscillatory behaviours, and their expressions can be integrated directly (ω for tangential, e for normal). Finally, the rest of elements combine secular behaviours with time scale proportional to the thrust magnitude, and oscillatory components with period linked to the orbital one. This is the case for a and e under tangential thrust, and ω under normal thrust. The secular expressions are obtained changing the independent variable in the equations of motion from time to eccentric anomaly E, and averaging the ODEs over 1 revolution. The resulting secular terms are linear, with slopes function of complete elliptic integrals of the first and second kind of the reference eccentricity e_{ref}. The oscillatory terms are obtained as a series expansion in e_{ref} and involve harmonics of the orbital period in E. The detailed derivation of the single-averaged analytical models and resulting expressions can be found in [5,6,7,4].

The proposed expressions for $\delta\boldsymbol{\alpha}$ have E as independent variable. A time law for $E(t)$ can be obtained from the differential equation for E, introducing the analytical approximation obtained for $\boldsymbol{\alpha}(E)= \boldsymbol{\alpha}_0+ \delta\boldsymbol{\alpha}(E)$. The resulting time law is explicit to compute time as function of anomaly, $t(E)$, but implicit to compute $E(t)$. This situation is analogous to that of the Kepler's equation, to which it reduces for $\mathbf{a}_t$=0, and prevents the solution from being fully analytical

Quasi-optimal control law

The previous models allow to evaluate the outcome of a CAM without a numerical integration, but to optimize the CAM they should be used within an iterative numerical optimizer. A more computationally efficient approach is proposed based on analogous models developed for impulsive CAMs [8]. The optimization problem for impulsive CAMs can be reduced to an eigenproblem, where the eigenvector associated to the largest eigenvalue determines the optimal direction of thrust. Then, a piecewise-constant low-thrust CAM can be defined by dividing the thrust arc in segments and assigning to each of them the orientation of the impulsive CAM at its middle point. Furthermore, given that optimal fuel manoeuvres follow bang-bang structures, the magnitude of thrust at each segment will be either maximum or 0. To define in which segments is more convenient to thrust, it can be proven that the eigenvalue of the impulsive CAM at each segment serves as proxy for the local efficiency of the CAM compared to the other segments.

Test case

Multiple scenarios for low-thrust CAMs have been studied within the ELECTROCAM project. For brevity, a single case is presented here. The Keplerian elements at TCA of the spacecraft are $\boldsymbol{\alpha}_{sc}$ = [7552.1 km, 0.0012, 87.93 deg, 1.95 deg, 127.53 deg, 5.10 deg], while for the debris $\boldsymbol{\alpha}_{deb}$ = [7575.5 km, 0.0100, 89.39 deg, 179.03 deg, 112.64 deg, 295.72 deg]. The elements of the combined covariance matrix in the B-plane (defined as in [5]) are σ_ξ=0.0179 km, σ_ζ=0.0214 km, $\rho_{\xi\zeta}$=-0.0524. Fig. 1 shows the results in terms of probability of collision PoC and displacement inside the B-plane δb, for a single-thrust-arc CAM with fixed thrust magnitude (10^{-8} km/s or 10^{-9} km/s) and different values of the thrust arc duration, Δt_{CAM}, and the coast arc between thrust end and TCA, $\Delta t_{\mathrm{coast}}$. Times are expressed as fractions of the nominal orbital period *T*.

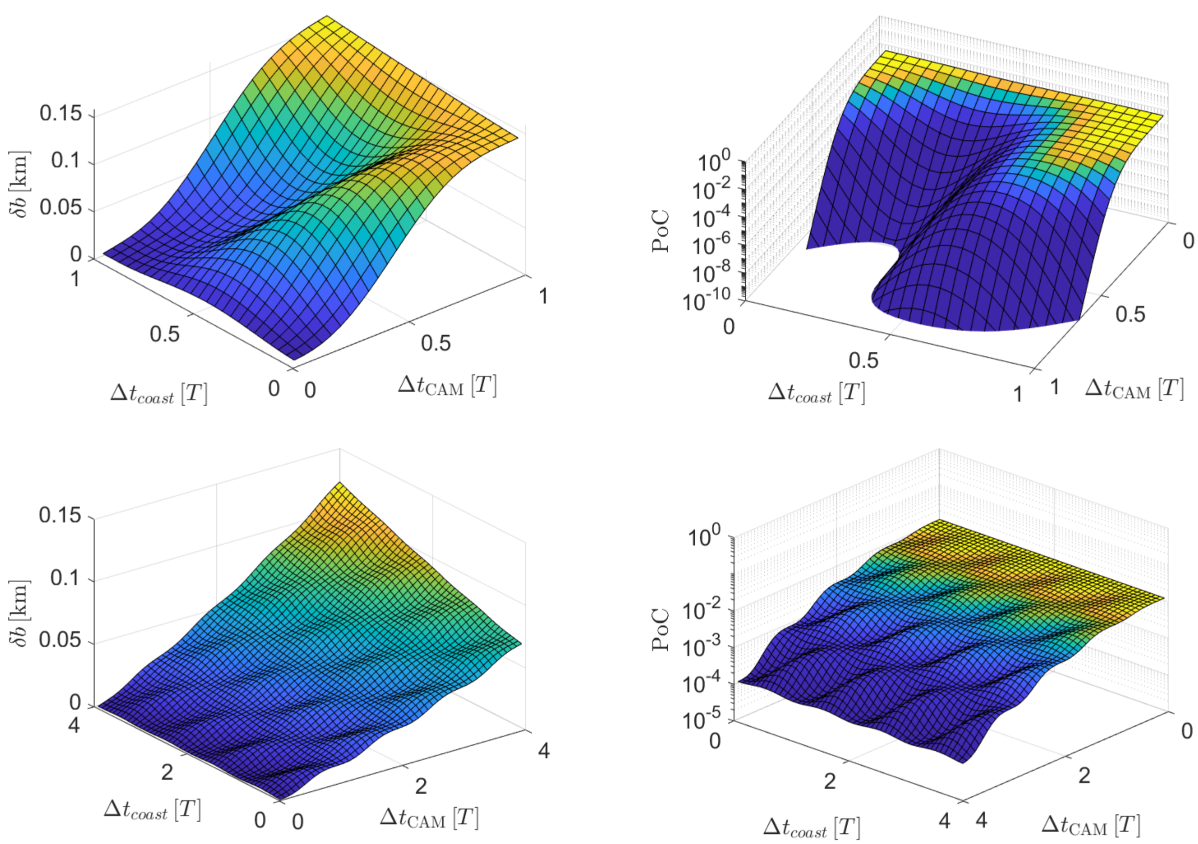

Figure 1. Displacement and PoC for test case in LEO, with a_t=10^{-8} km/s (top) and a_t=10^{-9} km/s (bottom)

Conclusions

The latest developments in a novel family of analytical and semi-analytical models for low-thrust CAM computation and design have been presented. These models rely on the single-averaging of the equations of motion in Keplerian elements to derive approximate analytical solutions separating the secular and oscillatory components for the orbit evolution induced by the CAM. For

CAM design, instead, quasi-optimal piecewise-constant thrust profiles are derived from the impulsive counterpart of the models. Part of these developments have been performed within the ESA-funded ELECTROCAM project.

Acknowledgements

This work has received funding from the European Space Agency through the project "ELECTROCAM: Assessment of collision avoidance manoeuvre planning for low-thrust missions" (call AO/1-10666/21/D/SR).

Juan Luis Gonzalo also thanks the funding of his research position by the Italian Ministero dell'Università e della Rierca, Programma Operativo Nazionale (PON) "Ricerca e Innovazione" 2014-2020, contract RTDA – DM 1062 (REACT-EU).

References

[1] C. Colombo, A. De Vittori, M. Omodei, J.L. Gonzalo, M. Maestrini, P. Di Lizia, P. Gago Padreny, M. Torras Ribell, A. Gallego Torrego, D. Escobar Antón, Roberto Armellin, Sensitivity analysis of collision avoidance manoeuvre with low thrust propulsion, Aerospace Europe Conference 2023 (10th EUCASS - 9th CEAS), Lausanne, Switzerland, 9-13 July 2023

[2] M. Maestrini, A. De Vittori, P. Di Lizia, C. Colombo, Dynamics-Based Uncertainty Propagation with Low-Thrust, 2022 AAS/AIAA Astrodynamics Specialist Conference, Charlotte, NC, USA, 7-11 Aug. 2022

[3] M. Maestrini, A. De Vittori, J.L. Gonzalo, C. Colombo, P. Di Lizia, J. Míguez Arenas, M. Sanjurjo Rivo, A. Diez Martín, P. Gago Padreny, D. Escobar Antón, ELECTROCAM: Assessing the Effect of Low-Thrust Uncertainties on Orbit Propagation, 2nd ESA NEO and debris detection conference, Darmstadt, Germany, 24-26 Jan. 2023

[4] J.L. Gonzalo, C. Colombo, P. Di Lizia, Single-averaged models for low-thrust collision avoidance under uncertainties, 73rd IAC, Paris, France, 18-22 Sept. 2022

[5] J.L. Gonzalo, C. Colombo, P. Di Lizia, Introducing MISS, a new tool for collision avoidance analysis and design, Journal of Space Safety Engineering, 7:3 (2020), 282-289. https://doi.org/10.1016/j.jsse.2020.07.010

[6] J.L. Gonzalo, C. Colombo, P. Di Lizia, A semianalytical approach to low-thrust collision avoidance manoeuvre design, 70th IAC, Washington, D.C., USA, 21-25 Oct. 2019. Paper no. IAC-19.A6.2.3

[7] J.L. Gonzalo, C. Colombo, P. Di Lizia, Computationally efficient approaches for low-thrust collision avoidance activities, 72nd IAC, Dubai, UAE, 25-29 Oct. 2021. Paper no. IAC-21-A6.10-B6.5.5

[8] J.L. Gonzalo, C. Colombo, P. Di Lizia, Analytical framework for space debris collision avoidance maneuver design, J. Guid. Control Dyn., 44:3 (2021), 469-487. https://doi.org/10.2514/1.G005398