

## Assessment and optimization of dynamic stall semi-empirical model for pitching aerofoils

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**Keywords:** Dynamic Stall, Helicopter Rotors, Optimization

**Abstract.** Dynamic stall is a phenomenon affecting aerofoils in unsteady flows which is particularly relevant in the helicopter field. Semi-empirical models are reliable tools to simulate this phenomenon, especially during preliminary design phases and for aeroelastic assessments. However, they need a large number of tuning parameters to provide reliable estimations of unsteady airloads. To face this problem, a parameter identification procedure based on sequential resolutions of optimization problems by means of a Genetic Algorithm is developed and it is applied to the state-space formulation of a modified version of the so-called "Second Generation" Leishman-Beddoes model. The effects of the optimal parameters on the model prediction capabilities are discussed and the variability of the parameters with reduced frequency is studied. The estimations of the unsteady airloads obtained by applying the optimization of parameters show a great improvement in the correlation of the experimental data if compared to the predictions obtained by using the parameters provided in literature, especially for pitching moments where the negative peaks are very well described. These improvements justify the need for optimization to set the parameters.

### Introduction

Dynamic stall is a very impacting phenomenon in the helicopter field because it is the main cause of the occurrence of large blade torsional airloads and rotor in-plane vibrations which are usually limiting factors both for design purposes and for performance evaluation. So, a reliable prediction of unsteady airloads is paramount.

Mathematical tools, aiming at modelling the effects of dynamic stall, range from semi-empirical models to sophisticated CFD methods. Despite the continuously increasing applications of the CFD methods for research purposes and their capability of providing physically realistic simulation of the rotor flow field, their daily use during the design process is still impractical nowadays due to the huge computational costs, the large memory requirements and the numerical issues.

It is here that semi-empirical models, developed since the eighties and the nineties of the last century, return as competitive and trustworthy prediction tools. Indeed, although they are based on simplified nonlinear equations, they can provide very quickly reliable predictions of unsteady airloads if properly tuned. Therefore, their application could be fundamental, especially in those cases where many predictions of unsteady airloads are needed in a short time, as is the case of aeroelastic assessments.



Among all the semi-empirical models available in the literature, the Leishman-Beddoes model is chosen for the present investigation because it is one of the most used by the industries and it is one of those which has known a great number of modifications and improvements over the years. Moreover, differently to the models developed starting from the ONERA model [1] or from the Peters model [2], the Leishman-Beddoes is more physically based since it is easier giving a physical interpretation to the involved parameters. The state-space formulation of the modified version of the "Second Generation" of the LB model which is considered in the present investigation is extensively explained in [3].

One of the main drawbacks of semi-empirical models is the necessity to identify many parameters that have to be extracted from experimental measurements or very reliable numerical simulations. Unfortunately, there are a couple of issues about the available parameters:

- the parameters can be applied with confidence only for the combinations of aerofoil shapes, Mach and Reynolds numbers used for their identification, limiting the extrapolation capability of semi-empirical models.
- the ranges of the parameters reported in the literature have a very limited extension in terms of aerofoil shapes and aerodynamic conditions. So, the number of experimental test cases where they can be used decreases.

To solve this issue, it has been decided to focus on the development of a parameter identification procedure based on optimization problems that aims at identifying the proper tuning parameters of the considered version of the Leishman-Beddoes model. The goal of this investigation is to provide a general methodology which could be applied routinely to aerodynamic conditions and aerofoil shapes different from those already assessed in the past, allowing to enlarge the range of effective applicability of the Leishman-Beddoes model.

### **Optimization of tuning parameters for the Leishman-Beddoes model**

The considered version of the model needs sixteen parameters depending on aerofoil shapes, Mach and Reynolds numbers which can be extracted from the static curves of normal force and pitching moment coefficients and from the unsteady airloads of a few selected pitching motions performed at different combinations of mean angle, oscillation amplitude and reduced frequency. The developed parameter identification procedure has some characteristics:

- it is a sequential procedure because the parameters are not identified simultaneously by means of a unique optimization, instead, they are divided into smaller groups and then they are identified performing successive optimizations. Indeed:
  - the slope of the static normal force coefficient ( $C_{N/\alpha}$ ), the maximum of the normal force coefficient ( $C_{N_{max}}$ ), the angle of attack at zero lift ( $\alpha_{zL}$ ) and the pitching moment coefficient at null incidence ( $C_{M_0}$ ) are easily extracted from the static curves of the coefficients and therefore they are not identified by means of any optimization,
  - the first optimization allows to identify parameters  $\alpha_1$ ,  $S_1$ ,  $S_2$  which are used for the approximation of the curve of the static trailing edge separation point,
  - then, the second optimization allows to identify parameters  $m$ ,  $K_0$ ,  $K_1$ ,  $K_2$  which approximates the static curve of the pitching moment coefficient,
  - finally, the last optimizations are performed to extract parameters  $T_p$ ,  $T_f$ ,  $T_v$ ,  $T_{v1}$ ,  $\delta_{\alpha_1}$  which account for all the delays occurring during dynamic stall due to: evolution of the unsteady pressure distribution ( $T_p$ ), evolution of the unsteady boundary layer ( $T_f$ ), vortex formation ( $T_v$ ), vortex travel over aerofoil upper surface ( $T_{v1}$ ) and flow reattachment ( $\delta_{\alpha_1}$ ).

- The optimization problems are bounded problems, and the design spaces of the variables are defined by exploiting the physical interpretation of the parameters. The parameters reported in [4] are used as guess values around which the design spaces are set.
- The objective functions of the optimization problems are error functions computed between the static curves of the aerodynamic coefficients or the unsteady airloads from pitching motions and their numerical approximations.
- The bounded optimization problems are solved by means of the Genetic Algorithm Toolbox provided by Matlab [5]. A genetic algorithm is chosen to solve the problems due to the high non-linearity and the complexity of the objective functions.

**Approximation static trailing edge separation point curve.** The LB model uses the Kirchhoff-Helmholtz theory to account for the nonlinear airloads due to flow separation at the trailing edge. In particular, the static normal force coefficient is related to angle of attack  $\alpha$  and to the effective trailing edge separation point,  $f$ :

$$C_N = C_{N/\alpha}(\alpha - \alpha_{zL}) \left( \frac{1+\sqrt{f}}{2} \right)^2 \tag{1}$$

By measuring the experimental static behavior of the normal force coefficient and by manipulating Eq.(1), it is possible to obtain an experimentally derived distribution for the static trailing edge separation point. Then, this distribution can be fitted by the following exponential approximation:

$$f(\alpha) = \begin{cases} 1 - 0.3e^{-\frac{\alpha - \alpha_1}{S_1}}, & \alpha \leq \alpha_1 \\ 0.04 + 0.66e^{-\frac{\alpha_1 - \alpha}{S_2}}, & \alpha > \alpha_1 \end{cases} \tag{2}$$

To extract  $\alpha_1$ ,  $S_1$  and  $S_2$ , the fitting problem of Eq.(2) is considered as an optimization problem where the optimal set of parameters is the one which minimizes the fitting error against the distribution of the separation point  $f$  previously obtained by inverting Eq.(1).

**Approximation pitching moment static curve.** The LB model suggests the following expression for the approximation of the static curve of the pitching moment where the non-linearity of the airloads is accounted by means of the effective separation point,  $f$ :

$$\frac{C_M - C_{M_0}}{C_N} = K_0 + K_1(1 - f) + K_2 \sin(\pi f^m) \tag{3}$$

Since the parameters for the exponential approximation of Eq. (2) and the static curves of the aerodynamic coefficients are available from experimental measurements, then the parameters for the pitching moment approximation can be extracted by considering the problem of Eq. (3) as an optimization problem where the optimal set of parameters is the best one over many optimization problems. The total error to be minimized is the sum of a fitting error between static pitching moment curve and its numerical approximation, the error between the maxima and the minima of the experimental curve and its numerical approximation.

**Time constants computation.** To identify the non-dimensional time constants, the bounded optimization problems are performed by considering the time history of the unsteady airloads

coming from pitching oscillations. The objective function to be minimized is complex and it is the sum of a total error on the lift coefficient and a total error on the pitching moment coefficient. These total errors are themselves the sum of a global error between the time history of the coefficient and its numerical approximation and an error between the maximum and an error between the minimum over the pitching cycle. To limit the interdependent effects between the variables, the identification is further split in two successive steps:

- At first, a unique optimization is performed by considering the unsteady airloads of a pitching motion in a condition of moderate stall with mean angle lower than the static stall one and a large variation of incidence both under and above static stall angle ( $\pm 10^\circ$ ) at high reduced frequency ( $k = 0.1$ ). This specific condition is assumed to be appropriate because all the stages of dynamic stall can have enough time to develop, all the involved delays can occur and the vortex shedding effects are not yet too much strong. This first optimization is performed by using all the five variables but only  $T_p$ ,  $T_f$  and  $\delta_{\alpha_1}$  are correctly computed.
- Then, to identify parameters  $T_v$  and  $T_{v1}$ , which describe vortex-induced effects, many optimizations are performed by considering the unsteady airloads from pitching motions at different conditions of deep stall characterized by a mean angle greater than the static stall one, a large variation of incidence both under and above static stall angle ( $\pm 10^\circ$ ) and different values of reduced frequency smaller than the previous one. The condition of deep stall is considered suitable to compute vortex-related parameters because in this case the vortex is very strong. These computations are performed by keeping fixed the values of  $T_p$ ,  $T_f$  and  $\delta_{\alpha_1}$  obtained by previous resolution and allowing only parameters  $T_v$  and  $T_{v1}$  to vary. It is interesting to note that performing the optimizations considering the unsteady airloads at different reduced frequencies allows to study the variability of  $T_v$  and  $T_{v1}$  with this parameter which is defined as  $k = \frac{\omega c}{2V}$  (where  $\omega$  is the frequency of the pitching oscillation of the aerofoil,  $c$  is the chord length and  $V$  is the constant free stream velocity). So, by modifying  $k$ , it is possible to explore the widest possible range of conditions that a blade section could encounter on the rotor disk during operative conditions.

## Results

The previously described identification procedure is applied to NACA 0012 at conditions of  $Ma = 0.3$  and  $Re = 3.8 \cdot 10^6$ . All the static curves of the aerodynamic coefficients and the experimental unsteady airloads come from the database of [6].

The predictions of the unsteady airloads obtained by applying the optimization of parameters are shown in Fig. 1 and Fig. 2. It is possible to see how the predictions obtained by the optimal set of parameters (red curves) allow a great improvement in the correlation of the experimental data (black dashed lines), if compared to the predictions obtained by using the parameters provided in literature (blue curves). The improvement applies especially for pitching moments where the negative peaks are very well described and for small values of the reduced frequency. These improvements justify the need for optimization in the parameters setting.

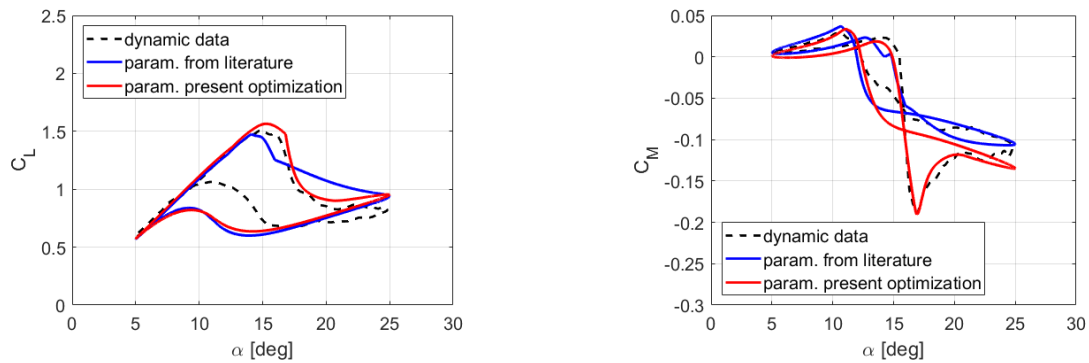


Figure 1: Comparison of the unsteady airloads ( $C_L$  on the left,  $C_M$  on the right) by means of LB model for NACA 0012 oscillating in pitch with  $\alpha = 15^\circ + 10^\circ \sin(\omega t)$  and  $k = 0.025$ .

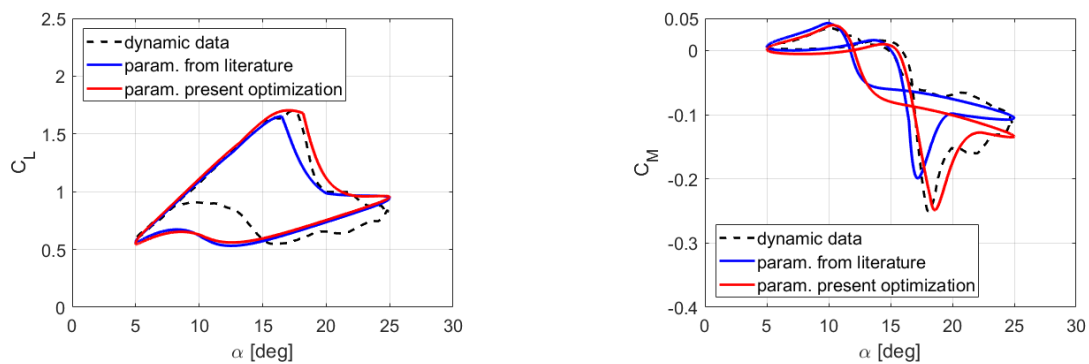


Figure 2: Comparison of the unsteady airloads ( $C_L$  on the left,  $C_M$  on the right) by means of LB model for NACA 0012 oscillating in pitch with  $\alpha = 15^\circ + 10^\circ \sin(\omega t)$  and  $k = 0.05$ .

## References

- [1] C. T. Tran, D. Petot, Semi-empirical model for the dynamic stall of airfoils in view of the application to the calculation of the responses of a helicopter blade in forward flight, *Vertica* 5 (1981) 35-53.
- [2] D. A. Peters, Toward a unified lift model for use in rotor blade stability analysis, *Journal of the American Helicopter Society* 30 (1985) 32-42. <https://doi.org/10.4050/JAHS.30.3.32>
- [3] G. Dimitriadis, Introduction to nonlinear aeroelasticity, first edition, John Wiley & Sons Ltd, Chichester, West Sussex, UK, 2017.
- [4] J. G. Leishman, G. L. Crouse, State-space model for unsteady airfoil behavior and dynamic stall, Technical Report Paper 89-1219, AIAA, 1989. <https://doi.org/10.2514/6.1989-1319>
- [5] Matlab, Genetic Algorithm Toolbox, <https://it.mathworks.com/help/gads/genetic-algorithm.html>.
- [6] W. J. McCroskey, K. W. McAlister, L. W. Carr, S. L. Pucci, An experimental study of dynamic stall on advanced airfoil sections, Volume 1, 2, 3, Technical Memorandum TM-84245, NASA, 1982.