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Determination of uncertainty of coordinate measurements on the basis of the formula for EL,MPE

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ABSTRACT

A technique of estimating the uncertainty of coordinate measurements, called the sensitivity analysis method according to ISO/TS 15530-1, developed at University of Bielsko-Biała, is presented. Measurement uncertainty is estimated on the basis of information contained in the formula for the maximum permissible error ($\boldsymbol{E}_{\mathrm{L,MPE}}$) of the applied coordinate measuring system (CMS) and on the basis of its acceptance or reverification test results. Measurement models are of the nature of close mathematical dependencies expressing the measured characteristic in the form of a distance which is a function of coordinates differences of a low number of essential points, properly selected on the workpiece. Measurement models for dimensions and various geometrical deviations were developed. Thanks to the applied vector notation the models are in the form of cross and dot products and they are easily programmable in software such as Matlab, Maple or Python. Detailed examples of the uncertainty analysis for two characteristics (position deviations of the axes of the holes in relation to the datum system) of a car steering knuckle are provided.

1. Introduction

The coordinate measuring technique has for many years been the basic technique applied in product quality control in the broader machinery industry, and above all in the automotive and aviation industry. The task of coordinate measurements uncertainty estimation has a long history. Already in 2001 in [1], by using the term"task-specific uncertainty", attention was drawn to the specificity of coordinate measurements consisting in the fact that particular characteristics (dimensions, geometrical deviations) are measured with various uncertainties [2,3]. Attention was also drawn to a large number of factors influencing the accuracy of the coordinate measurement. Attempts at classification of the uncertainty estimation techniques were immediately made. In [1] methods such as"sensitivity analysis","expert judgements","experimental method using calibrated objects","statistical estimations from measurement" or"computer simulations", and"virtual CMM" as well as"simulation by constraints" in particular, are mentioned. The bibliography of [1] contains 124 items. In this as well as in other publications and documents (among others also in Guide to the expression of uncertainty in measurement (GUM) [4] and EA-4/02M [5]) terms such as"type A" and"type B evaluation" (distinguishing that information about uncertainty components are obtained experimentally or in another way),"a priori" and"a posteriori" methods (uncertainty estimated already before or after performance of measurements), and finally"analytical method","GUM uncertainty framework" and"Monte Carlo method (MCM)" appear as methods of uncertainty components propagation.

As a result of standardisation works conducted by ISO TC 213, the standard ISO 15530-3:2011 [6] and two technical specifications ISO/TS 15530–1:2013 [7] and ISO/TS 15530-4:2008 [8] were established. The specification [7] lists three techniques of coordinate measurements uncertainty determination:"use of calibrated workpieces or standards","simulation" and"sensitivity analysis". The ISO/TS 15530-3:2004 reported also the term"use of multiple measurement strategies". In [9] authors reports also the terms" multiple orientation technique/measurement". In the VDI/VDE 2617, Part 11 [10] one of the uncertainty

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estimation techniques is called"using uncertainty budget". At the same time in some institutes of metrology the software called"Virtual CMM (VCMM)" is used, whose provider is PTB [11,12]. Numerous scientific and research centres indicate that they apply proprietary uncertainty analysis techniques for measuring tasks performed on CMS [13–16] (only some of many of them were mentioned).

The technique described in the ISO 15530-3 [6] can be treated as a reference method, that is, a method which does not require validation. Other uncertainty estimation techniques should be validated, mainly because they use different simplifying assumptions. ISO/TS 15530-4 [8] points out the need of validation, in particular with regard to simulation software, but also to any"uncertainty evaluation software (UES)".

In recent years at University of Bielsko-Biała research has been conducted to develop a simple, easy-to-use and, above all, easy to comprehend for an average user technique of coordinate measurements uncertainty estimation. First results were presented in the paper [17]. The developed technique refers to classic methods of measurements uncertainty estimation in the manufacturing metrology, and according to the current terminology contained in the GUM it is *GUM uncertainty framework* (GUF). The attention was drawn there to the fact that it is enough to model a coordinate measurement as an indirect measurement with the use of a minimum number of properly selected points of the measured workpiece. The necessity to express the measured characteristic (dimension or particular geometrical deviation) as a function of coordinates differences of the mentioned points was identified as the other important fact. The CMM kinematic model was used to determine the measurement uncertainty of particular coordinates differences (like at VCMM). To break free from a simulation, which requires the application of the CMM software, the B-type method was applied, determining the greatest possible differences of particular geometrical errors. Appropriate software allowing to estimate the measurement uncertainty offline was developed. Coordinates of essential points of the workpiece and CMM dimensions and geometrical errors constitute the input information [18]. There remains the unsolved problem of how to easily acquire information about these errors (the PTB concept of applying the ball plate standard and KALKOM software was used).

This problem was solved in the paper [19] by noting that the measurement uncertainty of the coordinates differences can be easily estimated based on the knowledge of the basic information about CMS accuracy contained in the $\boldsymbol{E}_{\mathrm{L,MPE}}$ parameter defined in ISO 10360-2 [20].

Detailed information concerning this approach was published in, e.g. [21,22]. On the basis of the classification of coordinate measurements uncertainty estimation methods contained in ISO/TS 15530–1 the developed technique was called the sensitivity analysis method. This technique was validated in the years 2019–2022 within the framework of the EUCoM project [23].

2. Models of the coordinate measurement

The model of measurement in the developed technique is a formula expressing the measured deviation as a function of differences of coordinates of a minimum number of points allowing its definition. The simplest models of measurement are:

- distance of two points,
- distance of a point from a straight line defined by 2 points,
- distance of a point from a plane defined by 3 points,
- distance of two straight lines (each defined by 2 points).

The formulae provided below have been adopted from ISO 17450-1 [24].

In case of the point-point distance the relevant formula has the following form

$$l(x_{AB}, y_{AB}, z_{AB}) = |\boldsymbol{AB}| \tag{1}$$

where:

l – distance,

$\boldsymbol{AB} = [x_{AB}, y_{AB}, z_{AB}]$ – a vector defined by essential (characteristic) points A and B.

In case of the point-straight line distance the relevant formula has the following form (Fig. 1)

$$l(x_{AS}, y_{AS}, z_{AS}, x_{AB}, y_{AB}, z_{AB}) = \left|\boldsymbol{AS} \times \frac{\boldsymbol{AB}}{|\boldsymbol{AB}|}\right| \tag{2}$$

where:

l – distance of the point S from the straight line AB,

$\boldsymbol{AS} = [x_{AS}, y_{AS}, z_{AS}]$ and $\boldsymbol{AB} = [x_{AB}, y_{AB}, z_{AB}]$ – vectors defined by essential points A, B and S.

It should be noted here that there is another possibility (another version of the formula) to calculate the distance l, different from the first one in that there is $\boldsymbol{BS}$ instead of $\boldsymbol{AS}$ in the formula. Of course, the distances calculated from these two formulae are the same, but uncertainties are usually different and the lower one is selected as the result [21].

In case of the point-plane distance the relevant formula has the following form (Fig. 2)

$$l(x_{AS}, y_{AS}, z_{AS}, x_{AB}, y_{AB}, z_{AB}, x_{AC}, y_{AC}, z_{AC}) = \left|\boldsymbol{AS} \bullet \frac{\boldsymbol{AB} \times \boldsymbol{AC}}{|\boldsymbol{AB} \times \boldsymbol{AC}|}\right| \tag{3}$$

where:

l – distance of the point S from the plane ABC,

$\boldsymbol{AS} = [x_{AS}, y_{AS}, z_{AS}]$ $\boldsymbol{AB} = [x_{AB}, y_{AB}, z_{AB}]$ and $\boldsymbol{AC} = [x_{AC}, y_{AC}, z_{AC}]$ – vectors defined by essential points A, B, C and S.

It should be noted here that there are eight other possibilities (9 versions of the formula in total) to calculate the distance l, different from the first one in that there is $\boldsymbol{BS}$ or $\boldsymbol{CS}$ instead of $\boldsymbol{AS}$ in the formula. What is more, instead of $\boldsymbol{AB} \times \boldsymbol{AC}$ there can be $\boldsymbol{BA} \times \boldsymbol{BC}$ or $\boldsymbol{CA} \times \boldsymbol{CB}$. Of course, the distances calculated from these 9 formulas are the same, but uncertainties are usually different and the lowest one is selected as the result [22].

In case of the straight line-straight line distance the relevant formula has the following form (Fig. 3)

$$l(x_{AB}, y_{AB}, z_{AB}, x_{CD}, y_{CD}, z_{CD}, x_{AC}, y_{AC}, z_{AC}) = \left|\boldsymbol{AC} \bullet \frac{\boldsymbol{AB} \times \boldsymbol{CD}}{|\boldsymbol{AB} \times \boldsymbol{CD}|}\right| \tag{4}$$

where:

l – distance of the straight line AB from the straight line CD,

$\boldsymbol{AC} = [x_{AC}, y_{AC}, z_{AC}]$ $\boldsymbol{AB} = [x_{AB}, y_{AB}, z_{AB}]$ and $\boldsymbol{CD} = [x_{CD}, y_{CD}, z_{CD}]$ – vectors defined by essential points A, B, C and D.

It should be noted here that there are three other possibilities (4 versions of the formula in total) to calculate the distance l, different from the first one in that there is $\boldsymbol{BC}$, $\boldsymbol{AD}$ or $\boldsymbol{BD}$ instead of $\boldsymbol{AC}$ in the formula. Of course, the distances calculated from these 4 formulas are the same, but uncertainties are usually different and the lowest one is selected as the result.

It is also worth pointing out that the same model can be used to evaluate the measurement uncertainty for many characteristics [2,21,22].

The described method of determining the uncertainty of coordinate measurements assumes that the applied measurement strategy (including the sampling strategy) is consistent with good measurement

Fig. 1. Model of measurement"distance of the point S from the straight line AB".

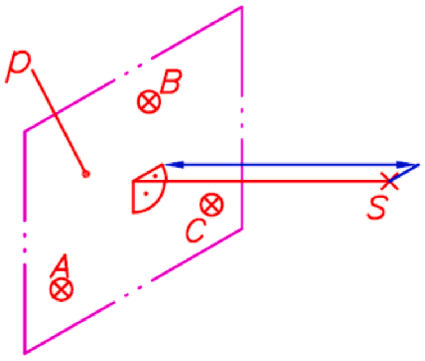

Fig. 2. Model of measurement"distance of the point S from the plane ABC".

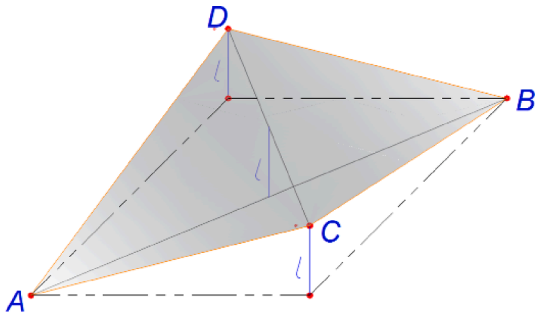

Fig. 3. Model of measurement"distance of the straight line AB from the straight line CD".

practice. General rules are described in guides, e.g. [26], CMM manufacturer manuals, car manufacturer standards (e.g. [27], manuals (e.g. [28]) but also in scientific publications (e.g. [29]).

It should be noted that essential points are sometimes points of integral elements (surfaces) and sometimes of derived elements (e.g. axes). In the first case these points could be sampled directly or they require some reduction. In the example in Fig. 4 the plane is in fact sampled in 15 points, but the three which are correctly arranged (on the edges of the integral element), read from the drawing or e.g. selected from sampling points, are used for the model of measurement.

In the case of a cylinder axis, at least two situations are possible. In the first, 2 circles are actually measured near the ends of the workpiece, and then their centres can be used in the measurement model. In second (the example presented in Fig. 5) in fact, a cylinder is measured but –as essential points could be used 2 extreme points of the axis read from the drawing or calculated, e.g. as points of intersection of the axis of the measured cylinder with additionally measured planes limiting its length.

In view of the fact that differences of coordinates of pairs of points have been assumed as input values, measurement uncertainties of these differences are necessary to establish the uncertainty budget. In accordance with ISO 10360 for every CMS there is known formula for the MPE of length measurement, and thus for every length the $E_{L,MPE}$ value is known. In accordance with the GUM, the length measurement uncertainty can be estimated with the type B method as the product of the greatest possible measurement error a of this length and the coefficient b dependent on the assigned error probability distribution. Assuming for a the $E_{L,MPE}$ value and the uniform distribution ($b = 0.577$) the following is obtained

$$u = ab = E_{L,MPE} \bullet 0.577 \tag{5}$$

Results of acceptance or reverification test can be used to evaluate the CMS error probability distribution, and then the value of the b coefficient will be different [24], and especially for new CMMs used in good environmental conditions can be lower.

Based on the developed models the measurement uncertainty of a given characteristic (that is, the so-called combined measurement uncertainty) u_c is calculated according to the formula (assuming that there is no correlation) [4]:

$$u_c = \sqrt{\sum_{i=1}^{k} \left(\frac{\partial l}{\partial x_i} u_{xi} \right)^2} \tag{6}$$

in which:

l – function expressing the measured characteristic,

x_i – input values of the model (differences of the pair of points coordinates),

u_{xi} – measurement uncertainties of input values calculated according to the formula (5).

It is worth presenting the calculations in the form of a classic uncertainty budget.

3. An example of a new measurement model definition

Fig. 6 presents a design drawing of a steering knuckle with marked axis position tolerances of:

- hole Ø12,
- 3 holes Ø12.5.

Fig. 7 shows the steering knuckle during the measurement on a CMM.

To calculate the measurement uncertainty a measurement model needs to be constructed and coordinates of the necessary essential points need to be determined.

The datum system is common for all positions and it is constituted by 3 mutually perpendicular planes. The primary datum X is constituted by a plane that can be defined by datum targets X1, X2 and X3. In this case they are3 planes PLA1, PLA2 and PLA3 (datum target areas). In measurement model the datum X can be formally represented by the point A, B or C and a normal vector u of the ABC plane defined as a cross product of the vectors ***AB*** and ***AC***, ***BA*** and ***BC*** or ***CA*** and ***CB***. A relevant formula for the first case is:

$$u = \boldsymbol{AB} \times \boldsymbol{AC} \tag{7}$$

The secondary datum is constituted by a plane perpendicular to the primary datum, containing the point D of the large hole axis (CYL0) and the point E of the plane of symmetry of planes PLA11 and PLA12. This plane is formally defined by the point D or the point E and a normal vector v that can be defined as a cross product of the vector ***DE*** and the

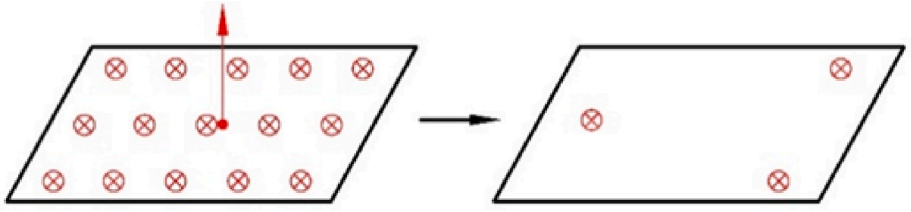

Fig. 4. An example of the origins of 3 - essential points belonging to the integral element (the plane).

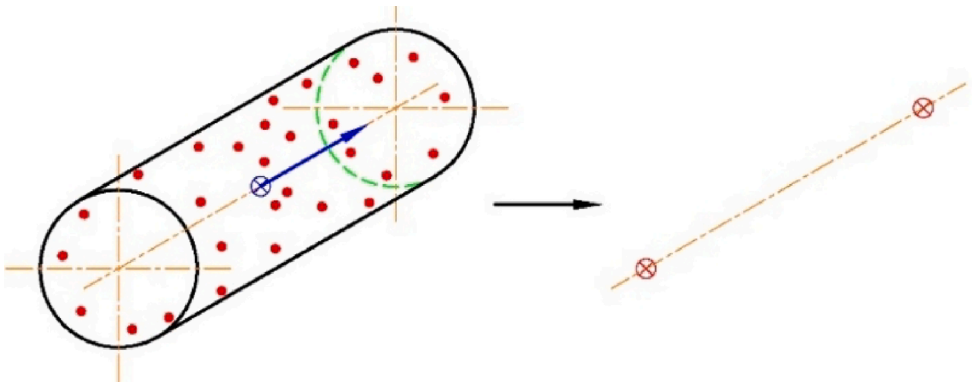

Fig. 5. An example of the origins of 2 - essential points belonging to the derived element (axis).

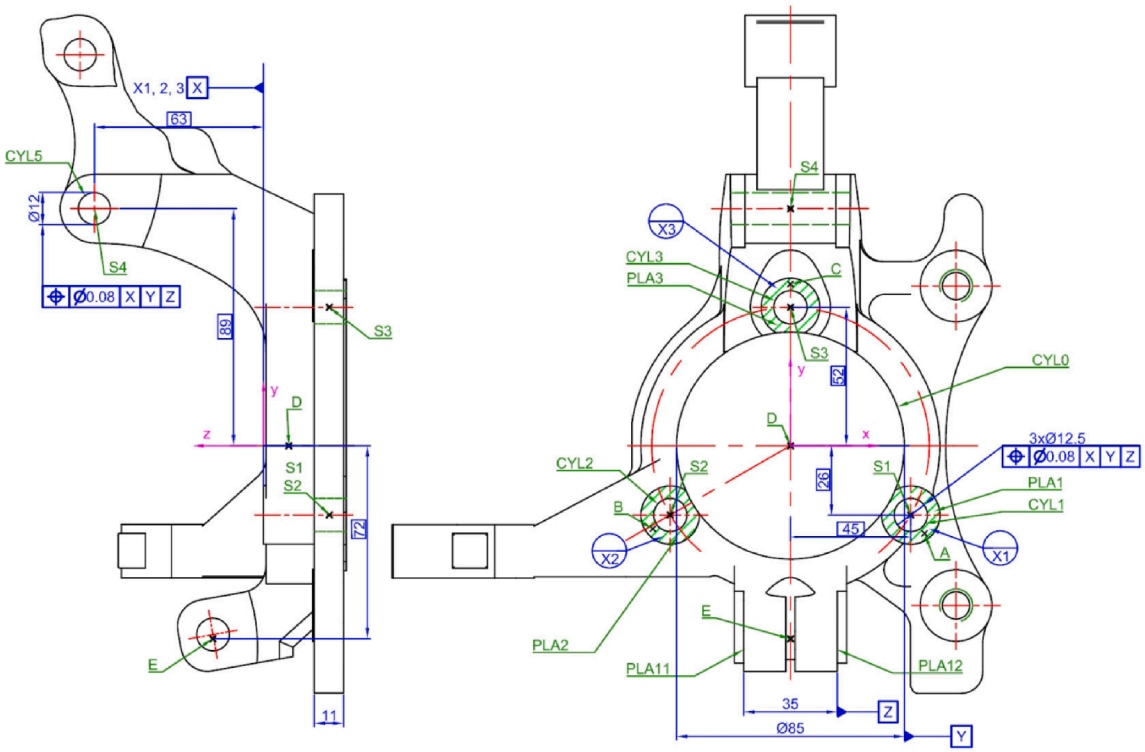

Fig. 6. A steering knuckle; an example of defining models for position deviations of 4 holes in relation to the datum system.

previously defined vector u:

$$v = \boldsymbol{DE} \times u = \boldsymbol{DE} \times (\boldsymbol{AB} \times \boldsymbol{AC}) \tag{8}$$

The tertiary datum is constituted by a plane perpendicular to the two previous ones. This plane is formally defined by the point D and a normal vector w that can be defined as a cross product of the vectors u and v:

$$w = u \times v = (\boldsymbol{AB} \times \boldsymbol{AC}) \times (\boldsymbol{DE} \times (\boldsymbol{AB} \times \boldsymbol{AC})) \tag{9}$$

In the presented example, in both cases we are dealing with cylindrical tolerance zone. That's why it was used two-stage measurement model [30]. In the first stage, the uncertainty of measuring the distance of toleranced elements (axis) from two datums (planes) will be determined, and in the second - the uncertainty of measuring position deviations.

In case when the toleranced element is the axis of the hole Ø12 represented by the point S_4, then to define the measurement model of the axis position CYL4 (for the case of a cylindrical tolerance zone) the distance $l_{S4,1}$ of the point S_4 from the primary datum and the distance $l_{S4,2}$ from the tertiary datum need to be determined. The distance $l_{S4,1}$ of the point S_4 from the primary datum (from the plane defined by the point A and a unit normal vector) is calculated according to the following formula (in the following formulae, the first index of the symbol l is the name of the point defining the toleranced feature, in the following examples S_1, S_2, S_3 or S_4; the second index, 1 or 2, denotes one of the two calculated distances of the toleranced element from the datum):

$$l_{S4,1} = \left| \boldsymbol{AS}_4 \bullet \frac{\boldsymbol{AB} \times \boldsymbol{AC}}{|\boldsymbol{AB} \times \boldsymbol{AC}|} \right| \tag{10}$$

Variants in which cross products $\boldsymbol{BA} \times \boldsymbol{BC}$ and $\boldsymbol{CA} \times \boldsymbol{CB}$ were used to define the normal vector are also to be considered:

$$l_{S4,1} = \left| \boldsymbol{AS}_4 \bullet \frac{\boldsymbol{BA} \times \boldsymbol{BC}}{|\boldsymbol{BA} \times \boldsymbol{BC}|} \right| \tag{11}$$

$$l_{S4,1} = \left| \boldsymbol{AS}_4 \bullet \frac{\boldsymbol{CA} \times \boldsymbol{CB}}{|\boldsymbol{CA} \times \boldsymbol{CB}|} \right| \tag{12}$$

What is more, variants in which the point B or C was used to define

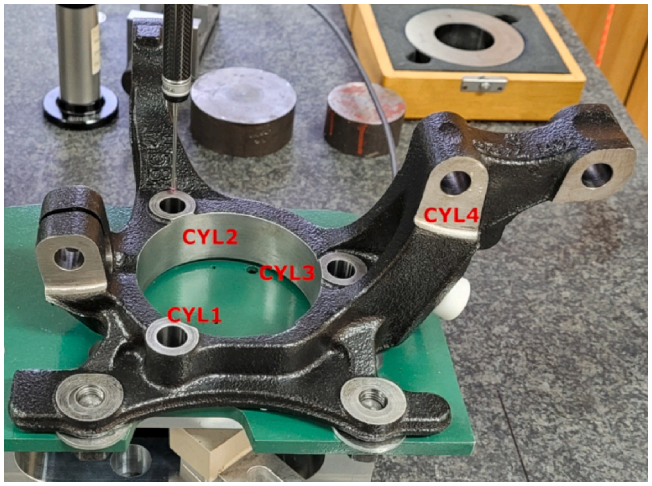

Fig. 7. A steering knuckle in time of measurement.

the plane instead of the point A (9 variants in total), are to be considered:

$$l_{S4,1} = \left| \boldsymbol{BS}_4 \bullet \frac{\boldsymbol{AB} \times \boldsymbol{AC}}{|\boldsymbol{AB} \times \boldsymbol{AC}|} \right| \tag{13}$$

$$l_{S4,1} = \left| \boldsymbol{BS}_4 \bullet \frac{\boldsymbol{BA} \times \boldsymbol{BC}}{|\boldsymbol{BA} \times \boldsymbol{BC}|} \right| \tag{14}$$

$$l_{S4,1} = \left| \boldsymbol{BS}_4 \bullet \frac{\boldsymbol{CA} \times \boldsymbol{CB}}{|\boldsymbol{CA} \times \boldsymbol{CB}|} \right| \tag{15}$$

$$l_{S4,1} = \left| \boldsymbol{CS}_4 \bullet \frac{\boldsymbol{AB} \times \boldsymbol{AC}}{|\boldsymbol{BA} \times \boldsymbol{AC}|} \right| \tag{16}$$

$$l_{S4,1} = \left| \boldsymbol{CS}_4 \bullet \frac{\boldsymbol{BA} \times \boldsymbol{BC}}{|\boldsymbol{BA} \times \boldsymbol{BC}|} \right| \tag{17}$$

$$l_{S4,1} = \left| \boldsymbol{CS}_4 \bullet \frac{\boldsymbol{CA} \times \boldsymbol{CB}}{|\boldsymbol{CA} \times \boldsymbol{CB}|} \right| \tag{18}$$

Distance $l_{S4,2}$ of the point S_4 from the tertiary datum (from the plane defined by the point D and a unit normal vector) is calculated according to the formula:

$$l_{S4,2} = \left| \boldsymbol{DS}_4 \bullet \frac{(\boldsymbol{AB} \times \boldsymbol{AC}) \times (\boldsymbol{DE} \times (\boldsymbol{AB} \times \boldsymbol{AC}))}{|(\boldsymbol{AB} \times \boldsymbol{AC}) \times (\boldsymbol{DE} \times (\boldsymbol{AB} \times \boldsymbol{AC}))|} \right| \tag{19}$$

Variants in which instead of the cross product $\boldsymbol{AB} \times \boldsymbol{AC}$ there is $\boldsymbol{BA} \times \boldsymbol{BC}$ or $\boldsymbol{CA} \times \boldsymbol{CB}$ (3 variants in total) are also to be considered:

$$l_{S4,2} = \left| \boldsymbol{DS}_4 \bullet \frac{(\boldsymbol{BA} \times \boldsymbol{BC}) \times (\boldsymbol{DE} \times (\boldsymbol{BA} \times \boldsymbol{BC}))}{|(\boldsymbol{BA} \times \boldsymbol{BC}) \times (\boldsymbol{DE} \times (\boldsymbol{BA} \times \boldsymbol{BC}))|} \right| \tag{20}$$

$$l_{S4,2} = \left| \boldsymbol{DS}_4 \bullet \frac{(\boldsymbol{CA} \times \boldsymbol{CB}) \times (\boldsymbol{DE} \times (\boldsymbol{CA} \times \boldsymbol{CB}))}{|(\boldsymbol{CA} \times \boldsymbol{CB}) \times (\boldsymbol{DE} \times (\boldsymbol{CA} \times \boldsymbol{CB}))|} \right| \tag{21}$$

Position deviation for the cylindrical tolerance zone is calculated as twice the value of the geometrical sum of the distance from the theoretically exact location (marked generally as l_1 and l_2):

$$POS = 2\sqrt{(l_1 - l_{1TED})^2 + (l_2 - l_{2TED})^2} \tag{22}$$

This means that in accordance with the GUF the measurement uncertainty of position deviation u_{POS} for the cylindrical tolerance zone is equal to:

$$u_{POS} = \sqrt{\left(\frac{\partial POS}{\partial l_1} u_{l1}\right)^2 + \left(\frac{\partial POS}{\partial l_2} u_{l2}\right)^2} \tag{23}$$

Relevant partial derivatives are calculated according to the formula:

$$\frac{\partial POS}{\partial l_i} = \frac{2(l_i - l_{1TED})}{\sqrt{(l_1 - l_{1TED})^2 + (l_2 - l_{2TED})^2}}, i = 1, 2 \tag{24}$$

and as a result we obtain:

$$u_{POS} = \frac{2\sqrt{(l_1 - l_{1TED})^2 u_{l1}^2 + (l_2 - l_{2TED})^2 u_{l2}^2}}{\sqrt{(l_1 - l_{1TED})^2 + (l_2 - l_{2TED})^2}} \tag{25}$$

The above formula is cumbersome in use, since the values l_1 and l_2 are not known prior to the measurement. All we know is that the values $(l_1 - l_{1TED})$ and $(l_2 - l_{2TED})$ are low. However, since $u_{l1} \approx u_{l2}$ this formula can be converted to the form:

$$u_{POS} \approx \frac{2u_l\sqrt{(l_1 - l_{1TED})^2 + (l_2 - l_{2TED})^2}}{\sqrt{(l_1 - l_{1TED})^2 + (l_2 - l_{2TED})^2}} \approx 2u_l \tag{26}$$

where $u_l = max(u_{l1}, u_{l2})$ or.$u_l = (u_{l1} + u_{l2})/2$

In case when the toleranced element is the axis of the hole ⌀12.5 represented by the point S_1 (analogically for S_2 and S_3), then to define the measurement model of the axis position deviation CYL1 (for the case of the cylindrical tolerance zone) the distance $l_{S1,1}$ of the point S_1 from the secondary datum and the distance $l_{S1,2}$ from the tertiary datum need to be determined. The distance $l_{S1,1}$ from the secondary datum is calculated according to the formula:

$$l_{S1,1} = \left| \boldsymbol{DS}_1 \bullet \frac{\boldsymbol{DE} \times (\boldsymbol{AB} \times \boldsymbol{AC})}{|\boldsymbol{DE} \times (\boldsymbol{AB} \times \boldsymbol{AC})|} \right| \tag{27}$$

Variants related to cross products (instead of $\boldsymbol{AB} \times \boldsymbol{AC}$ there can be $\boldsymbol{BA} \times \boldsymbol{BC}$ or $\boldsymbol{CA} \times \boldsymbol{CB}$) and definitional point of the secondary datum (instead of D there can be E) are also to be considered (6 variants in total):

$$l_{S1,1} = \left| \boldsymbol{DS}_1 \bullet \frac{\boldsymbol{DE} \times (\boldsymbol{BA} \times \boldsymbol{BC})}{|\boldsymbol{DE} \times (\boldsymbol{BA} \times \boldsymbol{BC})|} \right| \tag{28}$$

$$l_{S1,1} = \left| \boldsymbol{DS}_1 \bullet \frac{\boldsymbol{DE} \times (\boldsymbol{CA} \times \boldsymbol{CB})}{|\boldsymbol{DE} \times (\boldsymbol{CA} \times \boldsymbol{CB})|} \right| \tag{29}$$

$$l_{S1,1} = \left| \boldsymbol{ES}_1 \bullet \frac{\boldsymbol{DE} \times (\boldsymbol{AB} \times \boldsymbol{AC})}{|\boldsymbol{DE} \times (\boldsymbol{AB} \times \boldsymbol{AC})|} \right| \tag{30}$$

$$l_{S1,1} = \left| \boldsymbol{ES}_1 \bullet \frac{\boldsymbol{DE} \times (\boldsymbol{BA} \times \boldsymbol{BC})}{|\boldsymbol{DE} \times (\boldsymbol{BA} \times \boldsymbol{BC})|} \right| \tag{31}$$

$$l_{S1,1} = \left| \boldsymbol{ES}_1 \bullet \frac{\boldsymbol{DE} \times (\boldsymbol{CA} \times \boldsymbol{CB})}{|\boldsymbol{DE} \times (\boldsymbol{CA} \times \boldsymbol{CB})|} \right| \tag{32}$$

The distance $l_{S1,2}$ of the point S_1 from the tertiary datum is calculated according to the formula:

$$l_{S1,2} = \left| \boldsymbol{DS}_1 \bullet \frac{(\boldsymbol{AB} \times \boldsymbol{AC}) \times (\boldsymbol{DE} \times (\boldsymbol{AB} \times \boldsymbol{AC}))}{|(\boldsymbol{AB} \times \boldsymbol{AC}) \times (\boldsymbol{DE} \times (\boldsymbol{AB} \times \boldsymbol{AC}))|} \right| \tag{33}$$

Variants related to cross products used to determine a normal vector of the plane (3 variants in total) are also to be considered:

$$l_{S1,2} = \left| \boldsymbol{DS}_1 \bullet \frac{(\boldsymbol{BA} \times \boldsymbol{BC}) \times (\boldsymbol{DE} \times (\boldsymbol{BA} \times \boldsymbol{BC}))}{|(\boldsymbol{BA} \times \boldsymbol{BC}) \times (\boldsymbol{DE} \times (\boldsymbol{BA} \times \boldsymbol{BC}))|} \right| \tag{34}$$

$$l_{S1,2} = \left| \boldsymbol{DS}_1 \bullet \frac{(\boldsymbol{CA} \times \boldsymbol{CB}) \times (\boldsymbol{DE} \times (\boldsymbol{CA} \times \boldsymbol{CB}))}{|(\boldsymbol{CA} \times \boldsymbol{CB}) \times (\boldsymbol{DE} \times (\boldsymbol{CA} \times \boldsymbol{CB}))|} \right| \tag{35}$$

The position deviation and measurement uncertainty of this deviation for the cylindrical tolerance zone are calculated in the way described earlier.

4. Examples of the measurement uncertainty budgets

The following data follows from Fig. 6: A = [50,–32,0], B = [–50, –32, 0], C = [0, 61, 0], D = [0, 0, –15], E = [0, –72, 19], S_1 = [45, –26, –15], S_2 = [–45, –26, –15], S_3 = [0, 52, –15], S_4 = [0, 89, 63].

For the points S_1 and S_2 l_{1TED} = 45 mm and l_{2TED} = 26 mm, for the point S_3 l_{1TED} = 0 mm and l_{2TED} = 52 mm, for the point S_4 l_{1TED} = 63 mm and l_{2TED} = 89 mm.

Results of the analyses for CMM of the accuracy described with the formula $E_{L,MPE} = 2 + L/250$ and for $b = 1/3$ are discussed below.

For the measurement uncertainty of the distance $l_{S4,1}$ of the point S_4 from the primary datum the variant in which the cross product $\boldsymbol{CA} \times \boldsymbol{CB}$ was used to define the normal vector of the plane, and the point C was assumed as the definitional point of the plane turned out to be suitable (formula 18). The relevant uncertainty budget is presented in Table 1.

The value of the standard uncertainty of the distance $l_{S4,1}$ is 1.91 μm. To arrange the points as in Fig. 6 this value is combined of 3 elements, whereby the key one is the element related to the component z of the distance of the point S_4 from the datum amounting to 63 mm, since it is included in the budget with the weight 1. The other two non-zero elements are related to the components z of the distance of the points A and C and B and C, but they are included in the budget with weights 0.15 and do not have a significant impact on the value of the combined uncertainty.

For the measurement uncertainty of the distance $l_{S4,2}$ of the point S_4 from the secondary datum the variant with the cross product $\boldsymbol{CA} \times \boldsymbol{CB}$ also turned out to be suitable (formula 21). The relevant uncertainty budget is presented in Table 2.

The value of the standard uncertainty of the distance $l_{S4,2}$ is 2.19 μm. This value is composed of 3 elements, whereby the key one is the element related to the component y of the distance of the point S_4 from the datum equal to 89 mm, since it is included in the budget with the weight 1. The other two non-zero elements are related to zero components z of the distance of the points C and A and C and B and they are included in the budget with the weights 0.42.

In accordance with the formula (26) the standard measurement uncertainty of the position deviation of the axis of the S_4 hole (the cylindrical tolerance zone) amounts to u_{POS} = 5.82 μm, so the expanded measurement uncertainty is U_{POS} = 11.5 μm.

For the measurement uncertainty of the distance $l_{S1,1}$ of the point S_1 from the secondary datum (from the plane containing the points D and E and perpendicular to the ABC plane) the variant in which to define a normal vector of the plane the cross product $\boldsymbol{AB} \times \boldsymbol{AC}$ was used, and the point D (formula 27) was assumed as the definitional point of the secondary datum, turned out to be suitable. The relevant uncertainty budget is presented in Table 3.

The value of the standard uncertainty of the distance $l_{S1,1}$ is 1.95 μm. Only 3 out of 12 elements are non-zero. The first one, with the weight 1, is related to the component x of the distance of the point S_1 from the secondary datum amounting to 45 mm. The second most important one with the weight 0.361 is related to the zero component x of the distance of the points D and E, and the last one, with the weight 0.123, is related to the zero component z of the distance of the points A and B.

For the measurement uncertainty of the distance $l_{S1,2}$ of the point S_1 from the tertiary datum (from the plane containing the point D and perpendicular to the planes constituting the primary and the secondary datums) the variant in which the cross product $\boldsymbol{AB} \times \boldsymbol{AC}$ was used to define a normal vector of the plane turned out to be suitable (33). The relevant uncertainty budget is presented in Table 4.

The value of the standard uncertainty of the distance $l_{S1,2}$ is 2.12 μm. Once again, only 3 out of 12 elements are non-zero. The first one, with the weight 1, is related to the component y of the distance of the point S_1 from the tertiary datum amounting to 26 mm. The second most important one, with the weight 0.625, is related to the zero component x of the distance of the points D and E and the last one, with the weight 0.212, is related to the zero component z of the distance of the points A and B.

In accordance with the formula (26) the standard measurement

Table 1
The uncertainty budget for the distance $l_{S4,1}$ of the point S_4 from the primary datum.

	xi, mm	*∂l/∂xi*	*ui*, μm	*∂l/∂xi·ui*, μm
x_{CS4}	0.01	0	1.73	0
y_{CS4}	28	0	1.80	0
z_{CS4}	63	1	1.88	1.88
x_{CA}	50	0	1.85	0
y_{CA}	−93	0	1.95	0
z_{CA}	0	0.150	1.73	0.260
x_{CB}	−50	0	1.85	0
y_{CB}	−93	0	1.95	0
$_B$	0	0.151	1.73	0.261
			uc, μm	1.91

Table 2
The uncertainty budget for the distance $l_{S4,2}$ of the point S_4 from the secondary datum.

	xi, mm	*∂l/∂xi*	*ui*, μm	*∂l/∂xi·ui*, μm
x_{DS4}	0.01	0	1.73	0
y_{DS4}	89	1	1.94	1.94
z_{DS4}	78	0	1.91	0
x_{CA}	50	0	1.85	0
y_{CA}	−93	0	1.95	0
z_{CA}	0	−0.419	1.73	−0.726
x_{CB}	−50	0	1.85	0
y_{CB}	−93	0	1.95	0
z_{CB}	0	−0.419	1.73	−0.726
x_{DE}	0	0	1.73	0
y_{DE}	−72	0	1.90	0
z_{DE}	34	0	1.81	0
			uc, μm	2.19

Table 3
The uncertainty budget for the distance $l_{S1,1,}$ of the point S_1 from the secondary datum.

	xi, mm	*∂l/∂xi*	*ui*, μm	*∂l/∂xi·ui*, μm
x_{DS1}	45	1	1.83	1.83
y_{DS1}	−26	0	1.79	0
z_{DS1}	0	0	1.73	0
x_{AB}	−100	0	1.96	0
y_{AB}	0	0	1.73	0
z_{AB}	0	0.123	1.73	0.213
x_{AC}	−50	0	1.85	0
y_{AC}	93	0	1.95	0
z_{AC}	0	0	1.73	0
x_{DE}	0	−0.361	1.73	−0.625
y_{DE}	−72	0	1.90	0
z_{DE}	34	0	1.81	0
			uc, μm	1.95

Table 4
The budget for the distance $l_{S1,2,}$ of the point S_1 from the tertiary datum.

	xi, mm	*∂l/∂xi*	*ui*, μm	*∂l/∂xi·ui*, μm
x_{DS1}	45	0	1.83	0
y_{DS1}	−26	−1	1.79	−1.79
z_{DS1}	0	0	1.73	0
x_{AB}	−100	0	1.96	0
y_{AB}	0	0	0.6671.73	0
z_{AB}	0	−0.212	1.73	−0.368
x_{AC}	−50	0	1.85	0
y_{AC}	93	0	1.95	0
z_{AC}	0	0	1.73	0
x_{DE}	0	0.625	1.73	1.08
y_{DE}	−72	0	1.90	0
z_{DE}	34	0	0.711.812	0
			uc, μm	2.12

uncertainty of the position deviation of the axis of the S_1 hole (the cylindrical tolerance zone) amounts to $u_{POS} = 5.77$ µm.

For the measurement uncertainty of the distance $l_{S2,1}$ of the point S_2 from the secondary datum the variant in which the cross product $\boldsymbol{AB} \times \boldsymbol{AC}$ was used to define a normal vector of the plane turned out to be suitable (formula 27). The relevant uncertainty budget is presented in Table 5.

In relation to the symmetry of the points A and B with respect to the plane constituting the secondary datum both the elements and the combined uncertainty are identical as for the point S_1.

In the Table 6 the uncertainty budget for the distance $l_{S2,2}$ of the point S_2 from the tertiary datum is presented.

Since the point S_2 is located at the same distance from the tertiary datum as the point S_1, both the elements and the combined uncertainty are identical. This also applies to the measurement uncertainty of the position deviation ($u_{POS} = 5.77$ µm).

For the measurement uncertainty of the distance $l_{S3,1}$ of the point S_3 from the secondary datum the variant in which the cross product $\boldsymbol{AB} \times \boldsymbol{AC}$ was used to define a normal vector of the plane, and the point D was assumed as the definitional point of the secondary datum, turned out to be suitable (formula 27). The relevant uncertainty budget is presented in Table 7.

The value of the standard uncertainty of the distance $l_{S3,1}$ is 2.18 µm. Only 3 out of 12 elements are non-zero. The first one, with the weight 1, is related to the zero component x of the distance of the point S_1 from the secondary datum (due to numerical reasons the value 0.01 mm was assumed for calculations). The second most important one, with the weight 0.722, is related to the zero component x of the distance of the points D and E, and the last one, with the weight 0.246, is related to the zero component z of the distance of the points A and B. It is worth noting that both the weight 0.722 and the weight 0.246 are twice as high as the corresponding weights for the points S_1 and S_2, which is related to the twice as great distance of the point S_3 from the tertiary datum in comparison with the points S_1 and S_2,

For the measurement uncertainty of the distance $l_{S3,2}$ of the point S_3 from the tertiary datum the variant in which the cross product $\boldsymbol{AB} \times \boldsymbol{AC}$ was used to define a normal vector of the plane turned out to be suitable (33). The relevant uncertainty budget is presented in Table 8.

The value of the standard uncertainty of the distance $l_{S3,2}$ is 1.85 µm. Here there is only one uncertainty element (with the weight 1) in the budget. It is related to the component y of the distance of the point S_3 from the tertiary datum amounting to 52 mm.

In accordance with the formula (26) the standard measurement uncertainty of the position deviation of the axis of the S_3 hole amounts to $u_{POS} = 2.86$ µm.

5. Discussion

The presented uncertainty budgets contain 9 or 12 input quantities, and when the coordinate system is well defined, most of the uncertainty components take the value equal to zero. With a different coordinate system defined, there will be more non-zero elements, but the end result will not change significantly.

The final results of the conducted analysis are summarised in the Table 9.

The obtained values of measurement uncertainty of deviations of hole axes position differ slightly (from 5.72 µm to 5.82 µm). Some

Table 5
The budget for the distance $l_{S2,1}$ of the point S_2 from the secondary datum.

	xi, mm	$\partial l/\partial xi$	ui, µm	$\partial l/\partial xi \cdot ui$, µm
x_{DS2}	−45	−1	1.83	−1.83
y_{DS2}	−26	0	1.79	0
z_{DS2}	0	0	1.73	0
x_{AB}	−100	0	1.96	0
y_{AB}	0	0	1.73	0
z_{AB}	0	−0.123	1.73	−0.213
x_{AC}	−50	0	1.85	0
y_{AC}	93	0	1.95	0
z_{AC}	0	0	1.73	0
x_{DE}	0	0.361	1.73	0.625
y_{DE}	−72	0	1.90	0
z_{DE}	34	0	1.81	0
			uc, µm	1.95

Table 6
The budget for the distance $l_{S2,2}$ of the point S_2 from the tertiary datum.

	xi, mm	$\partial l/\partial xi$	ui, µm	$\partial l/\partial xi \cdot ui$, µm
x_{DS2}	−45	0	1.83	0
y_{DS2}	−26	−1	1.79	−1.79
z_{DS2}	0	0	1.73	0
x_{AB}	−100	0	1.96	0
y_{AB}	0	0	1.73	0
z_{AB}	0	0.212	1.73	0.368
x_{AC}	−50	0	1.85	0
y_{AC}	93	0	1.95	0
z_{AC}	0	0	1.73	0
x_{DE}	0	−0.625	1.73	−1.08
y_{DE}	−72	0	1.90	0
z_{DE}	34	0	1.81	0
			uc, µm	2.12

Table 7
The uncertainty budget for the distance $l_{S3,1}$ of the point S_3 from the secondary datum.

	xi, mm	$\partial l/\partial xi$	ui, µm	$\partial l/\partial xi \cdot ui$, µm
x_{DS3}	0.01	1	1.73	1.73
y_{DS3}	52	0	1.85	0
z_{DS3}	0	0	1.73	0
x_{AB}	−100	0	1.96	0
y_{AB}	0	0	1.73	0
z_{AB}	0	−0.246	1.73	−0.425
x_{AC}	−50	0	1.85	0
y_{AC}	93	0	1.95	0
z_{AC}	0	0	1.73	0
x_{DE}	0	0.722	1.73	1.25
y_{DE}	−72	0	1.90	0
z_{DE}	34	0	1.81	0
			uc, µm	2.18

Table 8
The budget for the distance $l_{S3,2,}$ of the point S_3 from the tertiary datum.

	xi, mm	$\partial l/\partial xi$	ui, µm	$\partial l/\partial xi \cdot ui$, µm
x_{DS3}	0.01	0	1.73	0
y_{DS3}	52	1	1.85	1.85
z_{DS3}	0	0	1.73	0
x_{AB}	−100	0	1.96	0
y_{AB}	0	0	1.73	0
z_{AB}	0	0	1.73	0
x_{AC}	−50	0	1.85	0
y_{AC}	93	0	1.95	0
z_{AC}	0	0	1.73	0
x_{DE}	0	0	1.73	0
y_{DE}	−72	0	1.90	0
z_{DE}	34	0	1.81	0
			uc, µm	1.85

Table 9
The summary of the measurement uncertainty of the position deviations of the axes of the holes (values in µm).

	S_4	S_1, S_2	S_3
u_1	1.91	1.95	2.18
u_2	2.19	2.12	1.85
u_{POS}	5.82	5.77	5.72

surprises are the differences for holes evenly spaced on a circle with a diameter of 100 mm and centred at the origin (5.72 µm and 5.77 µm). In order to facilitate the analysis of the obtained results, the values of uncertainty and its components are given with an accuracy of 3 decimal places (in practice, 2 significant places are sufficient).

In conclusion, it should be noted that the obtained uncertainty values of the S_1, S_2 and S_3 axes positions are practically the same. This was to expected, since all 3 holes are located within the same distance from the origin of the coordinate system, which corresponds to the datum system. The measurement uncertainty of the position deviation of the axis of the S_4 hole is only slightly greater, which follows from the greater distance from the origin of the coordinate system. Differences are small due to the significant influence of the element A from the $E_{\mathrm{L,MPE}}$ formula (as compared to the other element related to the measured distance).

6. Validation of the developed technique

In ISO/TS 15530-4 a proposal to use calibrated objects to validate the coordinate measurements uncertainty estimation software is presented. The document was created mainly with simulation software in mind, however, a general term of uncertainty evaluation software (UES) is used in it for software for uncertainty estimation.

Advantages of the cylinder square are highlighted there. On this standard (of a very simple form) several various geometrical characteristics can be defined, such as diameter, roundness, perpendicularity of the axis to the plane or the plane to the axis and, what is most interesting, coaxiality for various proportions of the datum length and the distance of the toleranced element from the datum.

Within the framework of the project the cylinder square was used to validate the developed technique on CMM of $E_{\mathrm{L,MPE}} = 4 + 6L/1000$.

The experiment was conducted in accordance with ISO 15530-3. For 17 different circles of the nominal diameter of 80 mm the obtained expanded uncertainty values U_{d} fell within the limits 6.30–8.54 µm, whereas the uncertainty calculated according to the described method amounts to 7.49 µm (Fig. 8). In no case did the applied chi-square test reject the variances equality hypothesis [25].

In case of the coaxiality for 84 different combinations of the datum length and the distance of the toleranced element from the datum the chi-square test rejected the variances equality hypothesis 13 times, whereby in 12 cases the uncertainty evaluation was overestimated, and only in 1 case the uncertainty evaluation was underestimated [25].

In order to validate the described method, an experimental determination of uncertainty was also performed in accordance with ISO 15530-3. The measurements were carried out on the CMM MicroXcel 765 with probing head PH20, for which the MPE formula is: $E_{L,MPE} = (3 + L/250)\mu m(Lwmm)$.

The comparison of the results is presented in Table 10.

The results obtained from the experiment are clearly lower, which may indicate that the assumption of a uniform distribution was too conservative.

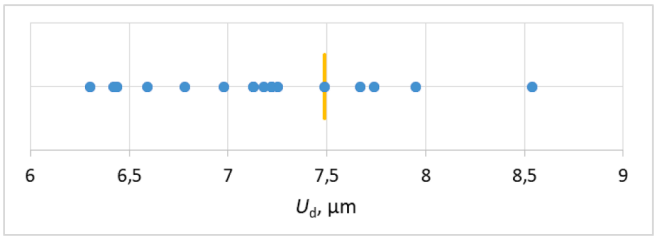

Fig. 8. Validation results for the circle diameter; the points represent the expanded uncertainty of measurement in 17 experiments performed in accordance with ISO 15530-3, line - value obtained according to the described method.

Table 10
The validation results for the position of the axes of the holes (values in µm).

	S_1	S_2	S_3	S_4
sensitivity analysis	5.77	5.77	5.72	5.82
experiment	3.65	2,39	3.49	3.84

7. Conclusions

1. Complete models of measurement of hole axes position deviation (cylindrical tolerance zone) in relation to the datum system are presented (to calculate the uncertainty of measurement, all quoted formulae are needed). The formulae are presented in a matrix notation and the calculations were made in the software developed with Phyton. Scalar notation is possible, but the corresponding formulae are then very complex.
2. Two cases of axis position deviation were considered. In one case, the position is defined by a primary datum and a tertiary datum, in the other by a secondary datum and a tertiary datum. The input quantities are the differences of the essential points' coordinates. Two-stage measuring models were used. At the first stage, the uncertainty of the toleranced element from the datum was determined, at the second - the uncertainty of measuring position of the axis for the case of a cylindrical tolerance zone. Uncertainty budgets contain 9 or 12 input quantities, and when the coordinate system is well defined, most of the uncertainty components take the value equal to zero (with a different coordinate system defined, there will be more non-zero terms, but the end result will remain the same). In order to facilitate the analysis of the obtained results, the values of uncertainty and its components are given with an accuracy of 3 decimal places (in practice, 2 significant places are sufficient).
3. The obtained values of measurement uncertainty of deviations of hole axes position differ slightly (from 1.656 µm to 1.758 µm). Some surprises are the differences for holes evenly spaced on a circle with a diameter of 100 mm and centred at the origin (1.656 µm and 1.677 µm).
4. The technique of estimating the coordinate measurements uncertainty developed at University of Bielsko-Biała is based on close mathematical relations analogous to those applied in the evaluation of the classical measurement uncertainty and applies to any CMS for which the MPE is known and verified. The developed technique belongs to the group of sensitivity analysis techniques according to ISO/TS 15530-1 and it is compatible with the GUM uncertainty framework. All components of uncertainty are determined by the type B method. In this case, validation is not formally required. However, the method can be experimentally validated e.g. using a cylindrical square according to ISO/TS 15530-4 and this is the case here. As the previous experience of the authors shows, the ability to define the concentricity tolerance on a cylindrical square at various combinations of the length of the datum and the distance of the toleranced element from the datum is particularly valuable. Validation using cylindrical square has been performed only on one CMM so far and it has been positive. The validation performed for the example described in this publication indicates that it may be too conservative to assume a uniform distribution for CMM errors.
5. The developed technique is fully universal. A considerable number of models has already been developed and is suitable for direct application. The examples presented in this publication are more complex, in particular, they require the use of a two-stage measurement model. For the remaining complex models it is possible to conduct easily an analysis analogous to the ones described herein.

CRediT authorship contribution statement

Mirosław Wojtyła: Investigation, Resources, Formal analysis, Writing – original draft, Writing – review & editing. **Paweł Rosner:**

Investigation, Resources, Formal analysis, Writing – original draft. **Wojciech Płowucha:** Methodology, Supervision, Formal analysis, Writing – review & editing. **Alessandro Balsamo:** Funding acquisition, Project administration. **Aline Piccato:** Project administration. **Alistair B. Forbes:** Data curation. **Enrico Savio:** Data curation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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