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Light robust co-optimization of energy and reserves in the day-ahead electricity market[☆]

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ABSTRACT

To accommodate the stochasticity of variable renewable energy sources (VRES) while efficiently dispatching generation resources and procuring adequate reserves, previous research proposed co-optimizing energy and reserves in the day-ahead (DA) using various uncertainty-based mechanisms. However, the co-optimized markets based on these mechanisms exhibit implementation limitations related to their high computational burden, complex customized solution algorithms, and over-conservative solutions. To address these shortcomings, this paper proposes a practical light robust optimization (LR) approach for the DA co-optimization of energy and reserves. The method results in a linear market clearing mechanism that easily enables the control of the robustness level of the solution through a tunable conservativeness parameter. In addition, the paper explores three different formulations for specifying the system reserve requirements considering, namely, fixed reserve requirements (LRF1), variable reserve requirements based on system uncertainty (LRF2), and a combined approach (LRF3). The formulations integrate the uncertainty from VRES in the market setting using a new bid format called uncertainty bid. The three formulations are then compared using a case study. The numerical results show the effects of the variation of the conservativeness parameter and the reserve requirements on the total socio-economic welfare (SEW), dispatched energy quantities, anticipated activation costs, and procured reserves. Moreover, the analyses showcase that sizing reserves based on system uncertainty (in LRF2) results in a 27%–61% decrease in reserve procurement costs when compared with LRF1, while the combined approach (in LRF3) results in a better performance than LRF2 in terms of reserve activation costs, with costs 61%–263% lower than in LRF2.

1. Introduction

The electricity sector is expected to provide an important contribution to the achievement of climate goals [1]. For this to be realized, a large transformation of the sector has been taking place in recent years, including the high penetration of variable renewable energy sources (VRES). Given the uncertain and variable nature of VRES, additional efforts are needed to safely maintain the balance between generation and demand in a market originally designed for dispatchable, fully-controllable power plants. In this regard, having generation or demand willing to alter their production or consumption as a balancing resource has proven to be a fundamental tool to handle uncertainty. To ensure the availability of adequate capacity to perform these balancing functions, reserve procurement processes have been implemented by transmission system operators (TSO) [2,3].

In its most traditional form, and as applied in most European markets[1], reserves and energy are traded independently in two sequential markets. In the markets where this setup is used, the interdependence of energy and reserve procurement, in terms of the total resulting cost to the system, as well as in terms of the constraints of the players participating in both markets, are not considered when clearing the markets [4]. As a consequence, sub-optimal energy dispatches and

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[1] Note that this work primarily focuses on European electricity markets and reserve procurement settings. However, the proposed framework extends beyond the European scope to any future implementation of co-optimization of energy and reserves.

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Nomenclature

Indexes

d	Demand agent.
g	Conventional producer.
s	Stochastic producer.

Sets

$\mathcal{D}$	Demand agents.
$\mathcal{S}$	Stochastic producers.
$\mathcal{G}$	Conventional producers.

Parameters

m_s^S, m_g^G, m_d^D	Bid: offered energy quantity by agent $s/g/d$.
$\hat{m}_s^n, \hat{m}_s^p$	Bid: maximum anticipated negative/positive deviation by agent s.
p_s^S, p_g^G, p_d^D	Bid: offered energy price by agent $s/g/d$.
$R_g^{uG,max}, R_g^{dG,max}$	Bid: maximum upward/downward reserve quantities in bid by agent g.
$R_s^{uS,max}, R_s^{dS,max}$	Bid: maximum upward/downward reserve quantities by agent s.
p_g^{uRG}, p_g^{dRG}	Bid: upward/downward reserve capacity prices by agent g.
p_s^{uRS}, p_s^{dRS}	Bid: upward/downward reserve capacity prices by agent s.
p_g^{urG}, p_g^{drG}	Bid: anticipated upward/ downward activation prices by agent g.
p_s^{urS}, p_s^{drS}	Bid: anticipated upward/ downward activation prices by agent s.
z^*	Optimal solutions to the nominal problem.
ρ	Conservativeness parameter.
R_{fixed}^u, R_{fixed}^d	Predefined fixed upward/downward reserve requirements.
R_{req}^u, R_{req}^d	Upward/downward reserve requirements.

Variables

q_d^D, q_s^S, q_g^G	Energy dispatch quantities of agent $d/s/g$.
R_s^{uS}, R_s^{dS}	Procured upward/downward reserve capacity from agent s.
R_g^{dG}, R_g^{uG}	Procured upward/downward reserve capacity from agent g.
r_g^{uG}, r_g^{dG}	Anticipated activation of upward/downward reserves from agent g.
r_s^{dS}, r_s^{uS}	Anticipated activation of upward/downward reserves from agent s.
γ_s^n, γ_s^p	Negative/positive slack variable for agent s.
λ	Energy prices.
λ^u	Upward reserve prices.
λ^d	Downward reserve prices.

reserve scheduling are obtained, resulting in inefficient use of generation and demand resources [5]. To improve this efficiency, a shift towards a simultaneous, joint clearing of energy and reserves, hereafter referred to as co-optimization of energy and reserves, is proposed in the literature [6]. Real-life applications of co-optimization of energy and reserves (in, for example, some markets in the U.S., Australia, New Zealand, and Singapore) have shown a reduction in the overall costs of reserves procurement when compared to a previously used sequential independent market clearing [5,7–9]. These results are supported by a number of analyses that indicate that the co-optimization of energy and reserves is more efficient than the sequential mechanism [4,10].

Even though such co-optimization approaches enable a more efficient energy and reserves procurement, they do not account for the uncertainty present in the system. The presence of high amounts of uncertainty – driven, for example, by the large-scale VRES integration – increases the likelihood of last-minute expensive response actions to maintain the system's safe operation. Therefore, to ensure security of supply and reduce operation costs, it is necessary to design and implement mechanisms that allow to better account for the underlying uncertainty when co-optimizing energy and reserves.

1.1. Literature review

Different methods using uncertainty-based approaches have been proposed in the literature [9,11], which go beyond deterministic approaches that procure fixed reserves to meet, for example, pre-set reliability criteria. Among the uncertainty-based approaches, stochastic programming (SP) and adaptive robust optimization (ARO) have been widely proposed.

In SP, the uncertainty is typically represented by a large set of scenarios and their corresponding probability of occurrence [12,13]. In its particular application for the co-optimization of energy and reserves in electricity markets, two-stage SP is commonly used. Under this approach, the first stage represents the co-optimized energy and reserves dispatch (i.e., what is referred to as the "here-and-now" decisions), while the second stage corresponds to the real-time operation (i.e., the "wait-and-see" decisions) [14]. In this regard, a linear two-stage SP market-clearing formulation for the co-optimization of energy and reserves is proposed in [14], while the works in [15,16] propose and analyse a number of pricing mechanisms for electricity markets cleared using two-stage SP. A comparison between electricity markets using the two-stage SP formulation and the deterministic sequential clearing is presented in [12]. Similarly, the work in [13] compares the aforementioned formulations while identifying the possible incentives for strategic behaviour.

Other electricity market applications of two-stage SP focus on solving the unit commitment (UC) problem, for example, the work in [17] formulates a two-stage SP security-constrained UC problem that optimizes the pre-contingency social welfare and the expected costs of reserves deployment. Moreover, the work in [18] proposes a nonlinear programming model using SP that considers the network constraints via an AC power flow formulation.

More recently and taking a different perspective, SP has been extensively used in the literature to optimize the participants' decision-making process. The work in [19] proposes a two-stage SP model to optimize the participation of wind producers in the day-ahead (DA), intraday, balancing and reserve markets. The work presented in [20] proposes a data-driven method to derive meaningful scenarios and uses a two-stage SP model to optimize the participation of wind and storage technologies in the wholesale energy markets. Optimal intraday bids of VRES considering generation and price uncertainty are derived using a risk-aware two-stage SP model in [21]. A multi-stage SP model to optimize the scheduling of conventional and virtual power plants participating in the Italian DA and ancillary services market is proposed in [22], while the intraday market is also considered in the work in [23]. The optimal bidding strategies and operation of energy hubs and wind-solar-hydro complementary systems are determined using two-stage SP in [24,25], respectively. Moreover, two-stage SP has also been used to optimize the operation of microgrids. For example, the work in [26] proposes a two-stage SP method to minimize the operation costs of a microgrid in overloading conditions, while a two-stage SP with piecewise affine correction rules is proposed in [27] to solve the UC problem in a microgrid with large amounts of uncertainty. Moreover, the work in [28] uses a two-stage SP approach to determine the optimal bidding strategy of a microgrid dealing with price and generation uncertainty.

Though SP (and two-stage SP for co-optimization applications) has proven to provide economic efficiency in expectation, it has concerning implementation limitations related to, for example, the high computational burden associated with the large number of required scenarios [29,30]. Traditional decomposition techniques e.g., Benders decomposition are commonly used to reduce the large computational time, as in [31]. The work in [22] proposes a clustering-based two-stage decomposition method for multi-stage SP problems. However, optimality and feasibility of the solution cannot be always guaranteed, and the proposed methods can lead to mechanisms that are complex to understand and implement, which can obstruct their practical implementation potential in actual markets.

In ARO, the uncertainty is represented by an uncertainty set instead of scenarios. Under this approach, the optimal solution is feasible for any realization within the uncertainty set and optimal for the worst-case realization of the uncertain parameters in the real-time stage [32]. In this respect, the works in [14,32] account for the uncertainty in the co-optimization process by proposing market clearing formulations based on ARO.

ARO has also been used to solve the UC problem in electricity markets. In this regard, an ARO formulation is proposed in [33] to solve the UC problem in a market with thermal and wind generation. The work in [29] proposes an ARO formulation to solve the security-constrained UC problem under the presence of demand and wind generation uncertainty. A comparison of a number of robust security-constrained UC models is presented in [34] to assess the impact of different worst-case definitions. Moreover, the work in [35] proposes an ARO UC model that incorporates data-driven disjunctive uncertainty sets in a multi-level optimization structure for energy systems under uncertainty. Recently, ARO has additionally been used for multiple applications, including, for example, multi-energy systems, microgrids operations, and generation expansion planning. The work in [36] proposes a so called affinely ARO model to solve the UC problem for a microgrid with multi-energy systems under uncertainty. A similar application is considered in [37] where an ARO formulation is proposed to co-optimize the electricity and heat systems for energy and reserves under wind uncertainty. An ARO formulation is proposed in [38] to optimize the operation of an integrated energy service provider which clears a multi-energy market. Furthermore, the work in [39] proposes a multistage ARO generation expansion planning model that takes into account the uncertainty of the demand and VRES generation through bounded intervals.

The applications of ARO have shown that it provides a feasible solution for any realization within the uncertainty set but, since ARO focuses on the worst-case realization, its results are considered, in general, highly conservative. Indeed, as investigated in [14], when used for the co-optimization of energy and reserves problems, ARO was shown to yield conservative scheduling of reserves, which translates into high operational costs. A number of articles in the literature have aimed to address the over-conservativeness aspect of the ARO formulation. For example, the authors in [32,34] propose to use a so-called uncertainty budget, which restricts the worst-case realization within a defined uncertainty set. Even though these proposals hold key scientific contributions, ARO results in a tri-level formulation (min–max–min) that requires complex specific solution algorithms [32], which hinder its practical implementation. In addition, a consistent mechanism to derive market prices when using ARO methods remains a key challenge.

1.2. Contributions

To address the above shortcomings, this work introduces a novel light robust (LR) uncertainty-based market formulation for the co-optimization of energy and reserves in the DA market. Unlike the traditional ARO, the proposed LR co-optimization of energy and reserves results in a linear market-clearing mechanism that can be efficiently solved, not requiring complex solution algorithms. Different from SP, the non-complex nature of this approach rapidly yields traceable solutions that can be clearly understood by market participants, hence improving transparency, encouraging market participation, and enabling the formulation's practical implementation. Moreover, this formulation accepts uncertainty bids [40], which enable stochastic (renewable) producers to reflect their uncertainty space in their submitted bids for energy and reserves. More specifically, this work makes the following key contributions.

- A novel linear LR co-optimization of energy and reserves market clearing formulation under uncertainty is introduced, in which the robustness level of the co-optimized solution can be dynamically specified through the control of a conservativeness parameter. The latter allows the balance of the robustness and economic efficiency of the co-optimized solution.
- To account for the stochasticity of VRES, this work introduces three different formulations for the specification and sizing of the reserve requirements. The proposed formulations consider either a fixed reserve volume or a variable requirement based on the anticipated reserve needs arising from the uncertainty of VRES and the anticipated associated activation costs — the latter reserve requirement is a novel formulation resulting from the developed light-robust optimization framework. A hybrid approach is also proposed considering the two (fixed and variable) requirements.
- A novel pricing mechanism is proposed, from which the market clearing price (MCP) for energy and reserves for the proposed market formulations can be readily derived. This is a feature that is currently missing in robustness-based market clearing formulations based on ARO.

A previous work [40] was the first to propose an LR formulation for the market clearing of the DA electricity market. However, it focused solely on the energy dispatch problem and did not consider the reserve procurement dimension. This work expands on [40] not only through the integration of the co-optimized reserve procurement stage in the DA market, but also by proposing three different formulations for the specification of the reserve requirements. Moreover, in the present work, the maximum anticipated *positive* and *negative* deviations of stochastic producers from their most probable production levels are considered, whereas in [40] only *negative* deviations were included. Considering both sides of the uncertainty range allows the market to optimize the procurement of upward and downward reserves.

This paper is structured as follows. We first introduce the deterministic nominal problem, which co-optimizes, in the DA, the energy and reserve procurement (Section 2). Then, the three alternative LR formulations co-optimizing energy and reserves under uncertainty are presented (Section 3). Next, a pricing scheme for the proposed formulations is introduced, from which dual prices can be readily derived (Section 4). Finally, a case study based on an updated version of the IEEE 24-bus system is presented (Section 5). The case study shows and compares the effects of the proposed LR formulations and the conservativeness parameter on the total socio-economic welfare (SEW), the anticipated activation costs in real-time (RT), and the dispatched quantities and procured reserves. These results show that significant economic benefits are obtained when the uncertainty of the stochastic producers (captured via the uncertainty bids) is used endogenously in the sizing of the reserve requirements of the system. Lastly, the conclusions and future work are presented in Section 6.

2. Co-optimization of energy and reserves: the nominal problem

In most European markets, the short-term electricity markets are composed of a DA energy market, an intraday market, and a balancing market (each cleared independently). One day before delivery, producers and consumers submit their price-quantity bids to the market operator (MO). The MO clears the DA energy market resulting in generation and demand hourly schedules for the next operating day along with the hourly MCP. In the intraday market, market participants are allowed to modify their DA positions by trading (bilaterally) in discrete auctions or continuous trading schemes. The balancing market is run by the TSO to maintain the balance between production and demand. The balancing market enables the system balance by accounting for near RT variations in generation and demand, whose occurrence is expected to increase as more VRES production is deployed [14]. To ensure enough resources are available in RT, the TSO procures reserve capacity well in advance (from one day to months ahead). This work focuses on the co-optimization of energy trading and reserve procurement in the DA, due to the efficiency gains it introduces as compared to the sequential market clearing [11] while anticipating reserve activation through the balancing market.

In its most standard form, the deterministic co-optimization of energy and reserves (hereafter referred to as "the nominal problem") is formulated as Problem (1). In addition to achieving an optimal DA energy market dispatch, Problem (1) aims at concurrently procuring *predefined* volumes of upward and downward reserves, as described in e.g.,[14].

$$\max_{\boldsymbol{\Psi}} \sum_{d\in D}(q_d^D p_d^D) - \sum_{s\in S}(q_s^S p_s^S + R_s^{dS} p_s^{dRS} + R_s^{uS} p_s^{uRS}) - \sum_{g\in\mathcal{G}}(q_g^G p_g^G + R_g^{dG} p_g^{dRG} + R_g^{uG} p_g^{uRG}), \tag{1a}$$

subject to:

$$\sum_{d\in D} q_d^D - \sum_{s\in S} q_s^S - \sum_{g\in G} q_g^G = 0, \tag{1b}$$

$$\sum_{s\in S} R_s^{dS} + \sum_{g\in G} R_g^{dG} = R_{req}^d, \tag{1c}$$

$$\sum_{s\in S} R_s^{uS} + \sum_{g\in G} R_g^{uG} = R_{req}^u, \tag{1d}$$

$$q_d^D \le m_d^D \ \forall d \in \mathcal{D}, \tag{1e}$$

$$q_s^S + R_s^{uS} \le m_s^S \ \forall s \in \mathcal{S}, \tag{1f}$$

$$q_g^G + R_g^{uG} \le m_g^G \ \forall g \in \mathcal{G}, \tag{1g}$$

$$q_s^S - R_s^{dS} \ge 0 \ \forall s \in \mathcal{S}, \tag{1h}$$

$$q_g^G - R_g^{dG} \ge 0 \ \forall g \in \mathcal{G}, \tag{1i}$$

$$R_s^{uS} \le R_s^{uS,max} \ \forall s \in \mathcal{S}, \tag{1j}$$

$$R_s^{dS} \le R_s^{dS,max} \ \forall s \in \mathcal{S}, \tag{1k}$$

$$R_g^{uG} \le R_g^{uG,max} \ \forall g \in \mathcal{G}, \tag{1l}$$

$$R_g^{dG} \le R_g^{dG,max} \ \forall g \in \mathcal{G}, \tag{1m}$$

$$q_d^D, q_s^S, q_g^G, R_s^{uS}, R_g^{uG}, R_s^{dS}, R_g^{dG} \ge 0 \ \forall d \in \mathcal{D}, \forall s \in \mathcal{S}, \forall g \in \mathcal{G}, \tag{1n}$$

where the demand (d), and the stochastic (s) and conventional (g) producers participating in the market lie in the sets $\mathcal{D}$, $\mathcal{S}$, and $\mathcal{G}$, respectively. Moreover, $\boldsymbol{\Psi}$ is the set of decision variables $q_d^D, q_s^S, q_g^G, R_s^{dS}, R_s^{uS}, R_g^{dG}, R_g^{uG}$ corresponding to the accepted demand (q_d^D), the quantities of the dispatched generation from stochastic (q_s^S) and conventional (q_g^G) producers, and the procured upward and downward reserves from stochastic (R_s^{uS}, R_s^{dS}) and conventional (R_g^{uG}, R_g^{dG}) producers. To participate in the market, participants submit in their bids energy quantities (m_s^S, m_g^G, m_d^D), energy prices (p_s^S, p_g^G, p_d^D), maximum reserve quantities ($R_s^{uS,max}, R_s^{dS,max}, R_g^{uG,max}, R_g^{dG,max}$), and reserve capacity prices (p_s^{dRS}, $p_s^{uRS}, p_g^{dRG}, p_g^{uRG}$). The reserve capacity prices are parameters that can be adjusted or excluded to suit different market settings (e.g., in the case where a certain type of reserve is not explicitly procured).

As expressed in (1a), the objective function aims at maximizing the total SEW from the trading of energy and reserves in the DA market. Constraint (1b) corresponds to the energy balance constraint, while constraints (1c) and (1d) guarantee that enough upward and downward reserves are procured to meet the reserve requirements R_{req}^u and R_{req}^d [2]. The maximum bid boundary constraints for the demand and generation (including both energy and upward reserves) are represented in (1e)–(1g), while constraints (1h) and (1i) ensure that enough dispatched generation is available to provide the procured downward reserves. Additionally, the maximum bid boundary constraints for the procurement of reserves are represented in (1j)–(1m). Finally, in (1n), the non-negativity constraints for all the decision variables are included. This general formulation reflects an emerging setup, expected to be predominant in a future in which stochastic producers can provide different types of ancillary services [41–45]. The formulation can be easily adjusted to limit the provision of reserves per technology if needed, either through the formulation itself (i.e., dropping respective decision variables) or through the set of bids accepted (reflecting product requirements set by the TSO).

Regarding network constraints, we note that the formulations proposed in this paper reflect the European DA electricity market setting, which considers a zonal market framework where internal (i.e., intra-zonal) network constraints are not considered when clearing the market. Network topology and operational constraints can be readily included at different levels of detail, using e.g., linearized DC power flow models to preserve the linearity of the formulation.

3. Light robust co-optimization of energy and reserves

Although the nominal problem enables a more efficient energy dispatch and reserves procurement by co-optimizing energy and reserves (compared with the sequential setup), it does not account for the uncertain nature of VRES participating in the market. Therefore, when a mismatch between the actual delivery and the energy dispatch quantities from the DA (i.e., imbalance) occurs, additional more expensive (last-minute) reserves are needed to maintain the system balance, resulting in additional costs for the system. This is the case, especially when the resulting imbalance is greater than initially estimated by the TSO, making the fixed predefined reserve requirements insufficient to maintain the system balance.

To better consider the uncertainty present in the system, we propose a co-optimization of the energy and reserves market using a novel LR approach. LR optimization is a mathematical framework first introduced in [46] to provide a balance between efficiency (i.e., the optimal solution for the nominal, deterministic case) and robustness (i.e., the feasibility of the solution when uncertainty is considered) in linear optimization problems.

Taking into account the principles of LR optimization [46] and the uncertain nature of VRES, we propose three different LR co-optimization formulations that differ in the specification of their reserve requirements. In the first formulation (LRF1), the reserve requirements are fixed and pre-defined by the TSO. In the second formulation (LRF2), the reserve requirements are defined, as a variable by-product, directly from the uncertainty present in the system as given by the uncertainty bids. Lastly, the third formulation (LRF3) combines and incorporates both fixed and variable reserve requirements.

[2] The reserve requirements can be defined in different ways, as proposed in Section 3.

3.1. LRF1-fixed reserve requirements

In most actual systems, the reserve requirements are defined well in advance of the DA energy market clearing. These requirements are usually determined by the TSO, subject to approval by regulating authorities, to ensure the reliable operation of the power system during unforeseen events (e.g., failure of generation units, variable loads, VRES variability) [47]. In alignment with this setup, a LR co-optimization of energy and reserves that considers fixed reserve requirements is presented next in Problem (2). Note that to capture the uncertainty from VRES, stochastic producers participating in LRF1 are allowed to submit uncertainty bids. This bid format considers, in addition to the bid elements required in the nominal problem, the maximum anticipated negative deviation ($\hat{m}_s^n$) from their most probable production m_s^S, as proposed in [40]. Negative deviations are considered due to the high cost associated with the correction of negative imbalances leading to upward regulation needs. In this regard, Fig. 1 illustrates the uncertainty range in the bid format from stochastic producers in LRF1.

$$\min_{\Psi,\gamma_s^n} \sum_{s\in S} \gamma_s^n, \tag{2a}$$

subject to:

$$\begin{aligned}&\sum_{d\in D}(q_d^D p_d^D) - \sum_{s\in S}(q_s^S p_s^S + R_s^{dS} p_s^{dRS} + R_s^{uS} p_s^{uRS})\\ &- \sum_{g\in \mathcal{G}}(q_g^G p_g^G + R_g^{dG} p_g^{dRG} + R_g^{uG} p_g^{uRG}) \geq z^*(1-\rho),\end{aligned} \tag{2b}$$

$$\sum_{d\in D} q_d^D - \sum_{s\in S} q_s^S - \sum_{g\in G} q_g^G = 0, \tag{2c}$$

$$\sum_{s\in S} R_s^{dS} + \sum_{g\in G} R_g^{dG} = R^d_{fixed,req}, \tag{2d}$$

$$\sum_{s\in S} R_s^{uS} + \sum_{g\in G} R_g^{uG} = R^u_{fixed,req}, \tag{2e}$$

$$q_d^D \leq m_d^D \;\forall d \in \mathcal{D}, \tag{2f}$$

$$q_s^S + R_s^{uS} \leq m_s^S \;\forall s \in \mathcal{S}, \tag{2g}$$

$$q_s^S - R_s^{dS} \geq 0 \;\forall s \in \mathcal{S}, \tag{2h}$$

$$q_g^G + R_g^{uG} \leq m_g^G \;\forall g \in \mathcal{G}, \tag{2i}$$

$$q_g^G - R_g^{dG} \geq 0 \;\forall g \in \mathcal{G}, \tag{2j}$$

$$R_s^{uS} \leq R_s^{uS,max} \;\forall s \in \mathcal{S}, \tag{2k}$$

$$R_s^{dS} \leq R_s^{dS,max} \;\forall s \in \mathcal{S}, \tag{2l}$$

$$R_g^{uG} \leq R_g^{uG,max} \;\forall g \in \mathcal{G}, \tag{2m}$$

$$R_g^{dG} \leq R_g^{dG,max} \;\forall g \in \mathcal{G}, \tag{2n}$$

$$q_s^S + R_s^{uS} - \gamma_s^n \leq m_s^S - \hat{m}_s^n \;\forall s \in \mathcal{S}, \tag{2o}$$

$$q_d^D, q_s^S, q_g^G, R_s^{uS}, R_g^{uG}, R_s^{dS}, R_g^{dG}, \gamma_s^n \geq 0 \;\forall d \in \mathcal{D},\; \forall s \in \mathcal{S}, \forall g \in \mathcal{G}, \tag{2p}$$

where upward and downward reserves are procured to meet the predefined fixed reserve requirements, as captured in constraints (2d) and (2e). Constraints (2c)–(2n) guarantee feasibility for the nominal case, while (2p) corresponds to the non-negativity constraints for all the decision variables.

Moreover, z^* is the optimal solution to the nominal problem (1), and ρ is the conservativeness parameter defined by the MO (TSO) to balance the robustness levels of the co-optimized solution. This conservativeness parameter is used in (2b) to set the maximum allowed deterioration from the nominal solution. In this regard, $\rho = 0$ corresponds to the optimal solution for the nominal case, while $\rho = \rho_{\max}$ relates to the most robust solution. The most robust solution accounts for the worst possible deviation from the most probable case where, for each stochastic producer, this corresponds to the right-hand side of (2o) (i.e., $m_s^S - \hat{m}_s^n$). The conformity with the robustness goal of each stochastic producer is defined by the dispatched energy quantities (q_s^S), the procured upward reserves (R_s^{uS}) and the slack variable γ_s^n, which is in turn minimized by the objective function (2a). Indeed, the variable γ_s^n in (2o) allows a deviation of the scheduled energy dispatch and reserve levels of stochastic units from the most robust solution. The objective function aims to drive the solution toward meeting the robustness goal, while constraint (2b) maintains the required level of economic efficiency, hence, striking a balance between robustness and efficiency. This balance is controllable by the MO, through ρ, allowing the formulation to span different possible settings (from most robust to least robust) through a change in ρ. The link between the variables and parameters in (2o) is illustrated in Fig. 2.

3.2. LRF2-variable reserve requirements

Fixed reserves requirements, as captured in LRF1, allow meeting preset reliability goals, by ensuring the availability of a fixed amount of reserve capacity. This formulation, however, does not account for the influence of variations in stochastic generation on the optimal procurement of reserves (reserve volumes and allocation among reserve providers). Differently from LRF1, the next formulation introduces a co-optimized market clearing problem that aims to link the reserve requirements to the reserve needs from the uncertainty of the dispatched stochastic producers (captured by the uncertainty bids that the stochastic producers submit).

Due to the uncertainty ranges revealed by the stochastic units, any dispatch of a stochastic producer implies an expected need for upward and/or downward reserve activation to cover the entire uncertainty range. We refer to this concept as the upward and downward reserve needs. Moreover, LRF2 creates a dispatch order that maximizes not only the energy and reserve procurement in DA (given the allowed deviation imposed by ρ in (2b)) but also minimizes the anticipated activation costs of the procured reserves. Since the reserve needs depend on the uncertainty ranges submitted by the accepted stochastic producers, the uncertainty bid format presented in LRF1 is expanded to also consider the upper bounds of the uncertainty ranges. In other words, in LRF2, the uncertainty range of a stochastic producer is composed of the submitted maximum negative ($\hat{m}_s^n$) and positive ($\hat{m}_s^p$) deviations from the most probable output (m_s^S), as illustrated in Fig. 3. In fact, $\hat{m}_s^n$ and $\hat{m}_s^p$ can be defined by the producer using different approaches, ranging from empirical observations to complex statistical analyses.

Moreover, in LRF2, conventional and stochastic producers also submit their anticipated upward and downward activation prices (p_g^{urG}, p_g^{drG}, p_s^{urS}, and p_s^{drS}). Note that the anticipated activation prices p_g^{urG}, p_g^{drG}, p_s^{urS}, and p_s^{drS} are defined and submitted by the producers, as part of their submitted bids, during the DA bidding process. In that regard, the anticipated upward (downward) activation prices correspond to the minimum (respectively maximum) price each producer is willing to receive (pay back) in exchange for the activation of upward (downward) reserves. Table 2, provides an overview of the bid formats of the market participants for each market formulation. The co-optimized market clearing LRF2 is introduced in Problem (3) as follows.

$$\min_{\Psi,\Xi} \sum_{s\in S}(-r_s^{dS} p_s^{drS} + r_s^{uS} p_s^{urS}) + \sum_{g\in \mathcal{G}}(-r_g^{dG} p_g^{drG} + r_g^{uG} p_g^{urG}), \tag{3a}$$

subject to:

$$\text{(2b)–(2c), (2f)–(2n)}, \tag{3b}$$

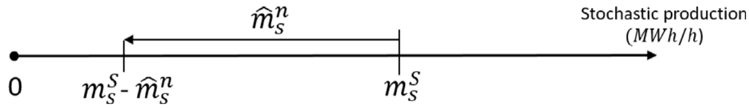

Fig. 1. Quantities in the uncertainty bids of stochastic producers participating in LRF1. Maximum anticipated negative deviation ($\hat{m}_s^n$) from their most probable production (m_s^S).

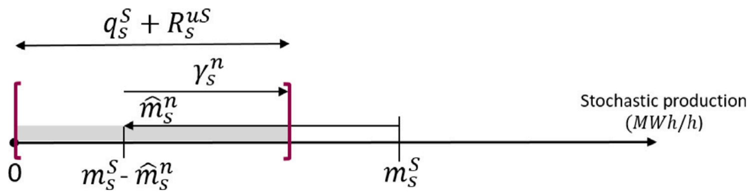

Fig. 2. Slack variable (γ_s), energy dispatch (q_s^S) and reserve (R_s^{uS}) bounds of stochastic units participating in LRF1.

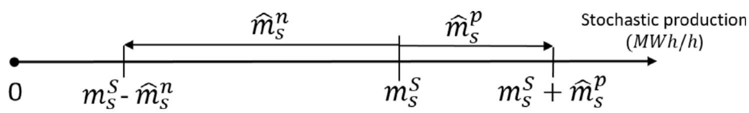

Fig. 3. Quantities in the uncertainty bids of stochastic producers participating in LRF2 and LRF3. Maximum anticipated negative ($\hat{m}_s^n$) and positive ($\hat{m}_s^p$) deviations from their most probable production (m_s^S).

$$q_s^S + R_s^{uS} - \gamma_s^n \leq m_s^S - \hat{m}_s^n \ \forall s \in \mathcal{S}, \tag{3c}$$

$$q_s^S + R_s^{uS} + \gamma_s^p = m_s^S + \hat{m}_s^p \ \forall s \in \mathcal{S}, \tag{3d}$$

$$\sum_{s\in\mathcal{S}} (r_s^{dS} - \gamma_s^p) + \sum_{g\in\mathcal{G}} r_g^{dG} = 0, \tag{3e}$$

$$\sum_{s\in\mathcal{S}} (r_s^{uS} - \gamma_s^n) + \sum_{g\in\mathcal{G}} r_g^{uG} = 0, \tag{3f}$$

$$r_s^{uS} \leq R_s^{uS} \ \forall s \in \mathcal{S}, \tag{3g}$$

$$r_s^{dS} \leq R_s^{dS} \ \forall s \in \mathcal{S}, \tag{3h}$$

$$r_g^{uG} \leq R_g^{uG} \ \forall g \in \mathcal{G}, \tag{3i}$$

$$r_g^{dG} \leq R_g^{dG} \ \forall g \in \mathcal{G}, \tag{3j}$$

$$\sum_{s\in\mathcal{S}} R_s^{dS} + \sum_{g\in\mathcal{G}} R_g^{dG} = \sum_{s\in\mathcal{S}} r_s^{dS} + \sum_{g\in\mathcal{G}} r_g^{dG}, \tag{3k}$$

$$\sum_{s\in\mathcal{S}} R_s^{uS} + \sum_{g\in\mathcal{G}} R_g^{uG} = \sum_{s\in\mathcal{S}} r_s^{uS} + \sum_{g\in\mathcal{G}} r_g^{uG}, \tag{3l}$$

$$q_d^D, q_s^S, q_g^G, R_s^{uS}, R_g^{uG}, R_s^{dS}, R_g^{dG}, r_s^{uS}, r_g^{uG}, r_s^{dS}, r_g^{dG}, \gamma_s^p, \gamma_s^n \geq 0 \quad \forall d \in \mathcal{D}, \forall s \in \mathcal{S}, \forall g \in \mathcal{G}, \tag{3m}$$

where Ξ is the set of decision variables $r_g^{dG}, r_g^{uG}, r_s^{dS}, r_s^{uS}$, corresponding to the anticipated activation of downward and upward reserves from conventional (G) and stochastic producers (S), and the slack variables γ_s^p and γ_s^n.

Due to the possibility that the output of a stochastic producer fall anywhere in the range $[m_s^S - \hat{m}_s^n, m_s^S + \hat{m}_s^p]$, the energy dispatch and reserve procurement from the stochastic producers creates an associated need for upward and downward reserves. These upward and downward reserve needs are captured by the slack variables γ_s^n and γ_s^p, respectively. In this respect, (3c) defines the level of compliance with the robustness goal (i.e., lower bound of the uncertainty range) by adjusting γ_s^n, setting in this manner the upward reserve needs from each stochastic producer. Similarly, the downward reserve needs (γ_s^p) are calculated in (3d) as the difference between the upper bound of the uncertainty range and the energy dispatch and reserve procurement ($q_s^S + R_s^{uS}$). The link between the variables and parameters in (3c) and (3d) is illustrated in Fig. 4.

With reference to Fig. 5, in a nominal market clearing (i.e., $\rho = 0$), the slack variables γ_s^p and γ_s^n would correspond to the positive and negative maximum deviations within the uncertainty range $\hat{m}_s^p$ and $\hat{m}_s^n$, respectively, as illustrated in Fig. 5a. As ρ increases and the market clearing becomes more robust — moving towards the lower bound of the uncertainty range, as in Fig. 5b, the needs for upward reserves decrease (γ_s^n) while the downward reserve needs increase (γ_s^p). As such, in the most robust market clearing, as illustrated in Fig. 5c, $\gamma_s^n \cong 0$ and $\gamma_s^p \cong \hat{m}_s^n + \hat{m}_s^p$. In other words, no upward reserves are required, while enough downward reserves are needed to cover for additional stochastic generation in the entire uncertainty range.

Constraints (3e) and (3f) guarantee that the anticipated reserve needs would be met by activating enough downward and upward reserves delivered by stochastic or conventional producers in RT. The cost of this anticipated activation of reserves is minimized by the objective function (3a). Since the problem is seen from the perspective of the MO (or TSO), the activation of upward reserves represents a cost for the system while the activation of downward reserves is included as revenues. Moreover, constraints (3g)–(3l) guarantee that enough reserves are procured (i.e., $R_g^{dG}, R_g^{uG}, R_s^{dS}, R_s^{uS}$) in advance to meet the anticipated activation of reserves (i.e., $r_g^{dG}, r_g^{uG}, r_s^{dS}, r_s^{uS}$). Lastly, the non-negativity constraints for all the decision variables are included in (3m).

3.3. LRF3-combined reserve requirements

The two formulations LRF1 and LRF2 differ in the definition of the reserve requirements. In LRF1, reserve requirements are included as a deterministic fixed input to the optimization problem. In LRF2, reserve requirements are a variable by-product of the optimization problem and are calculated by looking at the anticipated uncertainty from (dispatched) stochastic producers. Both approaches bring particular advantages. In fact, fixed reserve requirements, captured in LRF1, allow meeting preset reliability requirements, while LRF2 allows the sizing of reserves based on the expected variability of VRES, and its allocation based on anticipated activation costs. However, since in LRF2 the reserve requirements depend entirely on the uncertainty bids submitted by the participants, the presence of strategic behaviour, or inherent forecast errors could represent a risk for the system's security of supply.

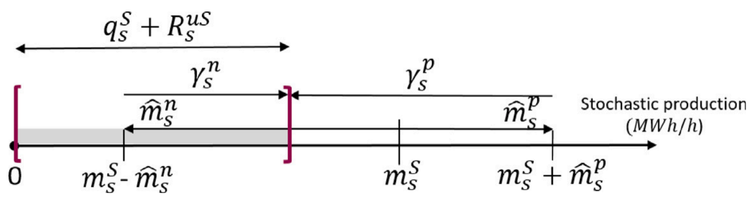

Fig. 4. Slack variables (γ_s^n and γ_s^p), energy dispatch (q_s^S), and reserve (R_s^{uS} and R_s^{dS}) bounds of stochastic units participating in LRF2 and LRF3.

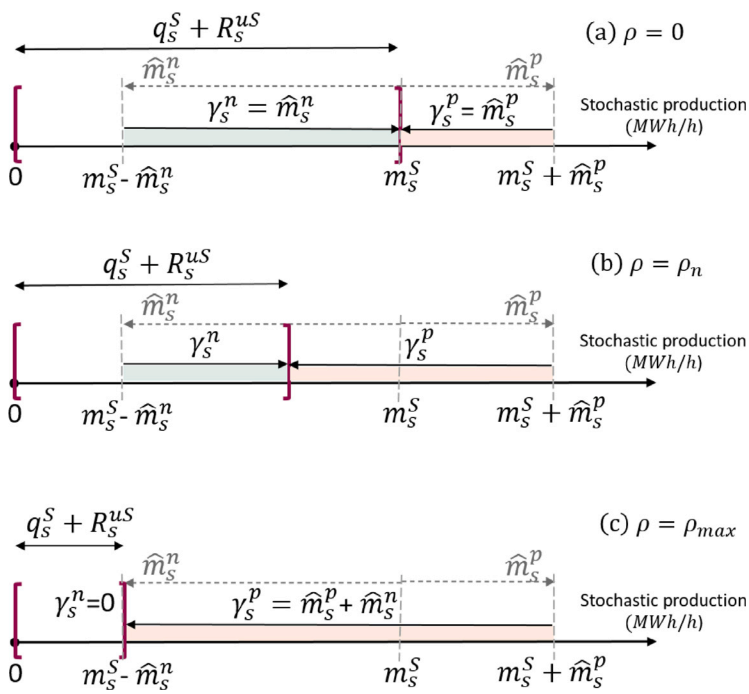

Fig. 5. Changes in variable reserve needs (γ_s^n and γ_s^p) under LRF2 and LRF3 for different levels of the conservatives parameter ρ. (a) deterministic solution $\rho = 0$, (b) light robust solution $\rho = \rho_n$, (c) most robust solution $\rho = \rho_{max}$.

Table 1
Main characteristics of the proposed formulations (N/A = not applicable).

	Reserve req.	Uncertainty range	Objective function	γ_s^n	γ_s^p	Anticipated activation costs
LRF1	Fixed	Negative deviations	Min. of slack variable	Slack on robustness goal	N/A	N/A
LRF2	Variable	Negative and positive deviations	Min. of anticipated activation costs	Upward reserve needs from player $s \in \mathcal{S}$	Downward reserve needs from player $s \in \mathcal{S}$	Minimized
LRF3	Fixed and variable	Negative and positive deviations	Min. of anticipated activation costs	Upward reserve needs from player $s \in \mathcal{S}$	Downward reserve needs from player $s \in \mathcal{S}$	Minimized

To capitalize on all the advantages of LRF1 and LRF2 and reduce the aforementioned risk, we present next a combined formulation in Problem (4), hereafter referred to as LRF3, that integrates fixed and variable reserve requirements:

$$\min_{\Psi,\Xi} \sum_{s\in\mathcal{S}}(-r_s^{dS} p_s^{drS} + r_s^{uS} p_s^{urS}) + \sum_{g\in\mathcal{G}}(-r_g^{dG} p_g^{drG} + r_g^{uG} p_g^{urG}), \tag{4a}$$

subject to:

$$\text{(2b)–(2c), (2f)–(2n)}, \tag{4b}$$

$$\text{(3c)–(3j), (3m)}, \tag{4c}$$

$$\sum_{s\in\mathcal{S}} R_s^{uS} + \sum_{g\in\mathcal{G}} R_g^{uG} = \sum_{s\in\mathcal{S}} r_s^{uS} + \sum_{g\in\mathcal{G}} r_g^{uG} + R^u_{fixed,req}, \tag{4d}$$

$$\sum_{s\in\mathcal{S}} R_s^{dS} + \sum_{g\in\mathcal{G}} R_g^{dG} = \sum_{s\in\mathcal{S}} r_s^{dS} + \sum_{g\in\mathcal{G}} r_g^{dG} + R^d_{fixed,req}. \tag{4e}$$

As observed, LRF3 is derived as a hybrid form of LRF1 and LRF2. Differently from LRF2, in LRF3 constraints (4d) and (4e) guarantee that enough reserves are procured to cover both: the fixed reserve requirements according to national regulations, as well as a VRES-dependent reserve component. As in LRF2, a complete uncertainty set (including both the upper and lower bounds of the set, as illustrated

Table 2
Summary of bid formats.

	Demand	Conventional producers	Stochastic producers
Nominal	m_d^D, p_d^D	$m_g^G, R_g^{uG,max}, R_g^{dG,max}, p_g^G, p_g^{dRG}, p_g^{uRG}$	$m_s^S, R_s^{uS,max}, R_s^{dS,max}, p_s^S, p_s^{dRS}, p_s^{uRS}$
LRF1	m_d^D, p_d^D	$m_g^G, R_g^{uG,max}, R_g^{dG,max}, p_g^G, p_g^{dRG}, p_g^{uRG}$	$m_s^S, R_s^{uS,max}, R_s^{dS,max}, p_s^S, p_s^{dRS}, p_s^{uRS}, \hat{m}_s^n$
LRF2	m_d^D, p_d^D	$m_g^G, R_g^{uG,max}, R_g^{dG,max}, p_g^G, p_g^{dRG}, p_g^{uRG}, p_g^{drG}, p_g^{urG}$	$m_s^S, R_s^{uS,max}, R_s^{dS,max}, p_s^S, p_s^{dRS}, p_s^{uRS}, \hat{m}_s^n, \hat{m}_s^p, p_s^{drS}, p_s^{urS}$
LRF3	m_d^D, p_d^D	$m_g^G, R_g^{uG,max}, R_g^{dG,max}, p_g^G, p_g^{dRG}, p_g^{uRG}, p_g^{drG}, p_g^{urG}$	$m_s^S, R_s^{uS,max}, R_s^{dS,max}, p_s^S, p_s^{dRS}, p_s^{uRS}, \hat{m}_s^n, \hat{m}_s^p, p_s^{drS}, p_s^{urS}$

by Fig. 3) is considered here for the bids of the stochastic producers. Table 1 provides a summary of the main characteristics of the proposed three formulations, while Table 2 lists the bid format requirements of each.

4. Pricing scheme

In the mathematical formulation of problems (2), (3), and (4), the objective functions (2a), (3a), and (4a) do not represent the SEW of the co-optimized markets. Hence, it is not possible to readily derive meaningful prices from the dual variables of the energy and reserve balance constraints, as is typically the case in uniform pay-as-clear remuneration schemes. As such, a different pricing method is proposed next to enable the extraction of dual variables to define the energy and reserve prices.

The nominal problem (1) is taken as the starting point, which is then modified to derive the prices for the three proposed formulations. In this respect, three different modified nominal problems are derived as follows. For LRF1, the modified problem is made of problem (1) in which constraints (1c) and (1d) are replaced with constraints (2d), (2e) and (2o) while using the optimal generated value of γ_s^n from problem (2) (i.e., for $\gamma_s^n = \gamma_s^{n*}$). In the case of LRF2, constraints (1c) and (1d) are replaced with constraints (3c), (3d), (3g)–(3l) for $\gamma_s^n = \gamma_s^{n*}$, $\gamma_s^p = \gamma_s^{p*}$, $r_s^{uS} = r_s^{uS*}$, $r_g^{uG} = r_g^{uG*}$, $r_s^{dS} = r_s^{dS*}$, and $r_g^{dG} = r_g^{dG*}$. Lastly, when using LRF3, constraints (1c) and (1d) are replaced with constraints (3c), (3d), (3g)–(3l), (4d), and (4e), for $\gamma_s^n = \gamma_s^{n*}$, $\gamma_s^p = \gamma_s^{p*}$, $r_s^{uS} = r_s^{uS*}$, $r_g^{uG} = r_g^{uG*}$, $r_s^{dS} = r_s^{dS*}$, and $r_g^{dG} = r_g^{dG*}$.[3] By implementing the optimal indicated variables, obtained from the LR formulations in the modified problem, meaningful prices can be derived from the balance constraints, as shown next.

Indeed, incorporating the optimal LR slack variables from the LR formulation in the modified problem yields the modified problem to return the same optimal energy dispatch and reserve procurement variables as the corresponding LR formulation, hence, enabling the derivation of the prices. The equivalence between the decision variables is expressed in the following theorem. Note that the decision variables of the modified problem are identified with the subscript m.

Theorem 1. *If upward and downward reserves are expected to be activated in RT ($\sum_{s\in\mathcal{S}} r_s^{uS} + \sum_{g\in\mathcal{G}} r_g^{uG} > 0$ and $\sum_{s\in\mathcal{S}} r_s^{dS} + \sum_{g\in\mathcal{G}} r_g^{dG} > 0$) and at least one conventional producer is dispatched, then $q_{ms}^{S*} = q_s^{S*}$, $R_{ms}^{uS*} = R_s^{uS*}$, $R_{mg}^{uG*} = R_g^{uG*}$, $R_{ms}^{dS*} = R_s^{dS*}$, and $R_{mg}^{dG*} = R_g^{dG*}$ for all $s \in \mathcal{S}$ and $g \in \mathcal{G}$.*

Proof. The proof of Theorem 1 is provided in Appendix. □

[3] Note that $\gamma_s^{p*}, \gamma_s^{n*}, r_g^{uG*}, r_s^{uS*}, r_g^{dG*}$, and r_s^{dS*} correspond to the values of the decision variables of the LR problems at the optimal solution.

Given the equivalence of the energy dispatch and procured reserve quantities in both problems (i.e., solving the modified problem will return the same optimal decision variables as its corresponding LR formulation), the derivation of the prices from the modified formulation is possible.

A specific description of the process for each formulation is provided in Fig. 6. In this regard, the energy prices are derived from the dual variables (λ_{LRF1}, λ_{LRF2}, and λ_{LRF3}) of the energy balance constraint (1b) for each specific formulation. Moreover, the upward reserve prices are derived from the dual variables (λ_{LRF1}^u, λ_{LRF2}^u, and λ_{LRF3}^u) of the upward reserve balance constraints (2d), (3k), and (4d), while the prices for the downward reserves are obtained from the duals (λ_{LRF1}^d, λ_{LRF2}^d, and λ_{LRF3}^d) of the constraints (2e), (3l), and (4e). The general steps required to solve the formulations and derive the prices are illustrated in Fig. 7. This proposed method allows the derivation of the MCP (for energy and reserves) from the dual variables of the corresponding balance constraints, also known as shadow pricing. Shadow pricing has been traditionally used to calculate the prices in electricity markets [14].

5. Case study

To showcase the effects of the proposed formulations on the market clearing outcomes, the updated version of the IEEE RTS 24-bus system, proposed in [48], is considered. In this case study, a DA market with 12 conventional generation players, 17 demand players, and 6 wind farm producers with a capacity of 200 MW each is cleared for a given hour (this is sufficient since we do not consider inter-temporal dependencies through complex bids). The demand quantities (m_d^D), the prices (p_g^G) and quantities (m_g^G) for the energy bids, the maximum upward ($R_g^{uG,max}$) and downward ($R_g^{dG,max}$) reserve capacity, and the bid prices for the activation of upward (p_s^{urG}) and downward (p_s^{drG}) reserves of the conventional players are taken directly from [48]. The demand prices (p_d^D) were randomly chosen from a uniform distribution in the range [0, 50] €/MWh while the upward (p_g^{uRG}) and downward (p_g^{dRG}) reserve capacity prices of the conventional players were randomly chosen from uniform distributions in the range $[0.076p_g^G - 0.235p_g^G]$ and $[0.001p_g^G - 0.04p_g^G]$, respectively, to mirror pricing settings seen in practice[4]. Regarding the fixed upward and downward reserve requirements for LRF1 and LRF3, it is set equal to 154.33 MW which is calculated as the 7.019% of the total demand of the test system, to also reflect practical settings[5]. The input data corresponding to the bids from the conventional players and the demand are presented in Table 3 and 4, respectively.

Regarding the six stochastic players, historical wind power production data from the Belgium TSO Elia [52] is used to define the uncertainty bids. For each wind player, the selected historical data sets correspond to the aggregated forecast and the measured wind production in Belgium in a specific hour on a specific day, over ten years (from 2013 to 2022). The same hour (at 21:00 h), but a different day was chosen for each player (from the same six-day period corresponding to the windiest time of the year: 01–06 February). The most probable generation quantities (m_s^S) correspond to the mean of the measured capacity factors over the 10-year period. The maximum negative and positive deviations ($\hat{m}_s^n$ and $\hat{m}_s^p$) are obtained from the minimum and maximum difference between the forecasted and measured capacity

[4] According to the Belgium Commission for Electricity and Gas Regulation (CREG) [49], in 2015–2020, the average procurement cost for manual Frequency Restoration Reserves (mFRR) capacity ranged between 3.2 €/MW/h and 9.9 €/MW/h. These costs represented 7.6% and 23.51% of the average DA energy price in the same period, which was 42.1 €/MWh.

[5] The percentage 7.019% corresponds to the ratio between the average mFRR volume to be procured and the electricity demand in Belgium, (as approved by the CREG) [50,51].

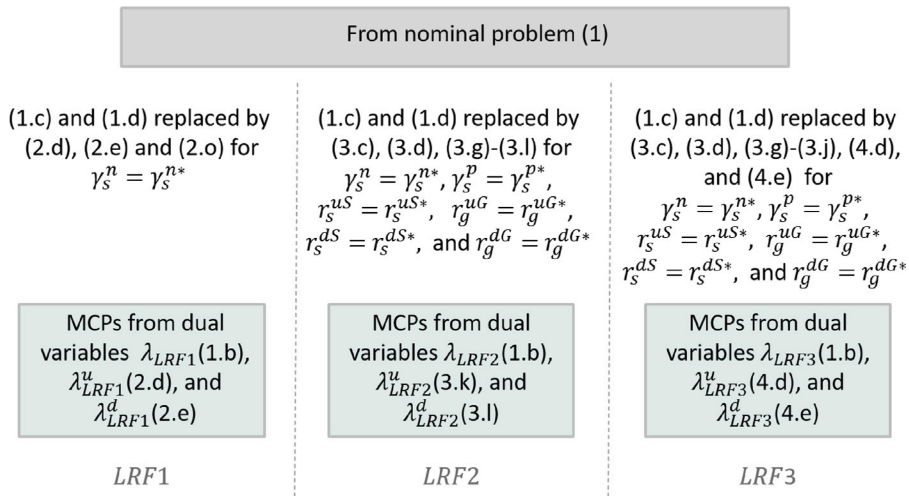

Fig. 6. Summary of the proposed mathematical methods for deriving the market clearing prices from formulations LRF1, LRF2, and LRF3.

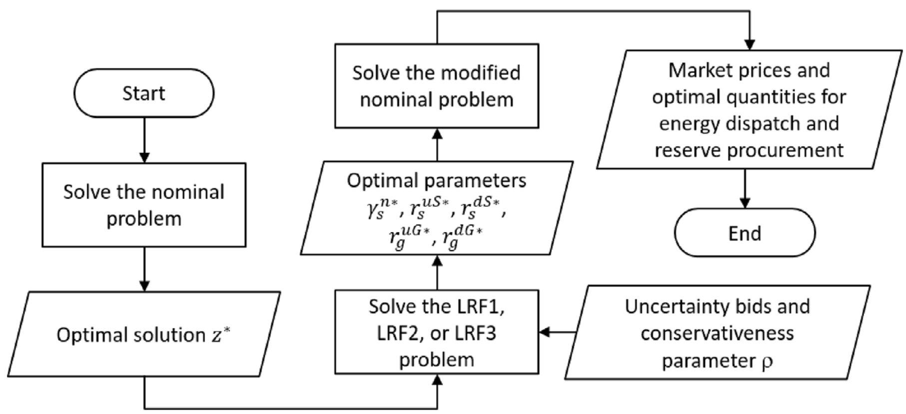

Fig. 7. General process to derive the market clearing prices in a light robust co-optimization model.

Table 3
Energy and reserve bids from conventional producers.

g	p_g^G (€/MWh)	m_g^G (MWh)	$R_g^{uG,max}$ (MW)	p_g^{uRG} (€/MW)	$R_g^{dG,max}$ (MW)	p_g^{dRG} (€/MW)	p_g^{urG} (€/MWh)	p_g^{drG} (€/MWh)
g1	13.32	152	40	1.38	40	0.28	15	11
g2	13.32	152	40	2.37	40	0.33	15	11
g3	20.70	350	70	2.51	70	0.31	24	16
g4	20.93	591	180	2.88	180	0.30	25	17
g5	26.11	60	60	3.78	60	0.96	28	23
g6	10.52	155	30	1.10	30	0.32	16	7
g7	10.52	155	30	2.05	30	0.36	16	7
g8	6.02	400	0	0.00	0	0.00	0	0
g9	5.47	400	0	0.00	0	0.00	0	0
g10	0.00	300	0	0.00	0	0.00	0	0
g11	10.52	310	60	2.48	60	0.28	14	8
g12	10.52	350	40	1.28	40	0.20	16	8

factors observed in the data set. Note that the producers may also use other empirical and statistical methods to define their maximum negative and positive deviations. For example, in the case the stochastic generation is considered to follow a normal distribution, $\hat{m}_s^n$ and $\hat{m}_s^p$ can then be set equal to -2σ and $+2\sigma$ to include around 95% of the historical values. As such, the corresponding uncertainty bids from the stochastic players are shown in Table 5. To align with the original case study and for simplicity, it is assumed that no reserves are offered by the stochastic producers.

To study the effect of the proposed formulations, the energy and reserve markets were jointly cleared using LRF1, LRF2, and LRF3 for different values of the conservativeness parameter ρ. The results of the simulations are presented in the following three subsections. In Section 5.1, three aspects are discussed from the system perspective: (i) the evolution of the SEW; (ii) the anticipated activation costs; and (iii) the aggregated dispatch and procured reserves. Section 5.2 discusses individual results for stochastic and conventional producers. Final remarks are included in Section 5.3.

5.1. System results

Fig. 8 illustrates the evolution of the SEW in the DA under the proposed formulations. The bars correspond to the SEW from the energy dispatch (marked on the left-hand vertical axis as 'SEW energy'),

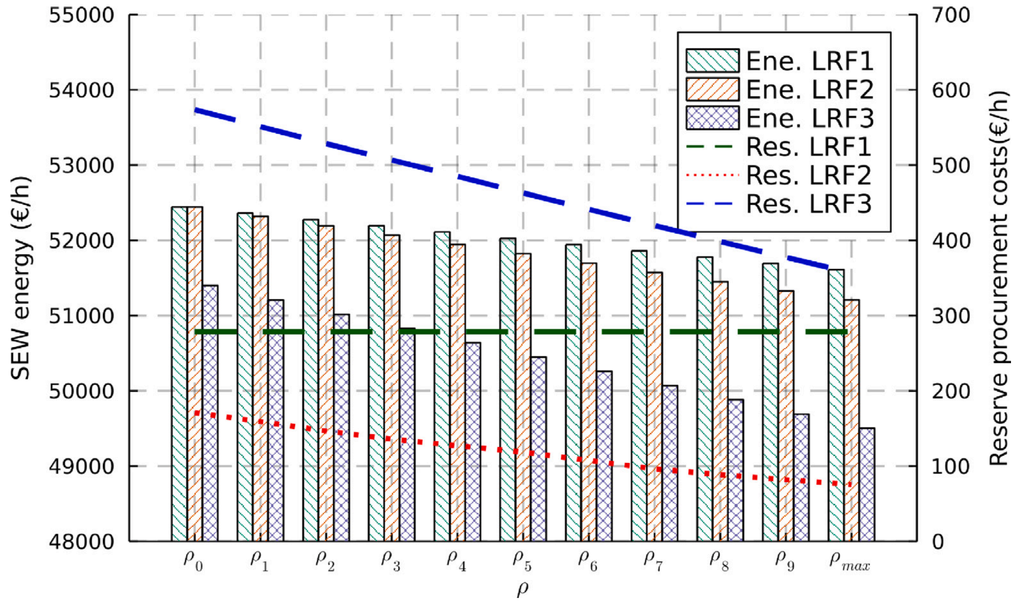

Fig. 8. Comparison of the socio-economic welfare (SEW) from the energy dispatch and reserve procurement under LRF1, LRF2, and LRF3 for different levels of the conservativeness parameter ρ.

Table 4
Energy bids from demand players.

d	p_d^D (€/MWh)	m_d^D (MWh)	d	p_d^D (€/MWh)	m_d^D (MWh)
d1	15	83.55	d10	3	149.51
d2	13	74.75	d11	24	204.48
d3	44	138.52	d12	11	149.518
d4	16	57.16	d13	30	244.06
d5	30	54.97	d14	36	76.95
d6	33	105.54	d15	45	257.26
d7	23	96.74	d16	26	140.72
d8	40	131.92	d17	40	98.94
d9	29	134.12			

Table 5
Uncertainty bids from stochastic producers.

s	p_s^S (€/MWh)	m_s^S (MWh)	$\hat{m}_s^n$ (MWh)	$\hat{m}_s^p$ (MWh)
s1	1	120.05	11.21	13.90
s2	1.5	103.08	24.11	14.46
s3	2	78.84	19.21	76.01
s4	3	115.68	29.65	26.56
s5	2.5	120.05	11.49	2.43
s6	3.5	121.52	4.98	23.67

while the lines represent the costs incurred to procure upward and downward reserve capacity (marked on the right-hand side vertical axis as ‘reserve procurement costs’). The combination of the SEW from the energy dispatch minus the reserve procurement costs (also called ‘DA SEW’ in the following) corresponds to the absolute value of the left side of (2b). ρ_0 corresponds to the nominal case in which the robustness goal is entirely relaxed, resulting in the nominal DA SEW (1a). To obtain more robust market clearings, deviations from these nominal solutions are allowed by increasing ρ. When ρ reaches its maximum value, the maximum deviation from the nominal solution is permitted, resulting in the most robust solution. Overall, the effect of the conservativeness parameter in the ‘SEW energy’ and optimal dispatch coincide with the expected performance of a market under LR optimization – the SEW energy decreases as ρ increases and the generation from stochastic producers is constrained – confirming the results in [40].

Comparing the proposed formulations, the lowest SEW from the energy dispatch is obtained when the market is cleared using LRF3. For example, when $\rho = \rho_{max}$ is chosen, LRF3 results in a ‘SEW energy’ of €49,504 while LRF1 and LRF2 result in a ‘SEW energy’ of €51,610 and €51,209, respectively. As expected, LRF3 procures more reserves than the other two formulations (83.4% and 119.8% more than LRF1 and LRF2, respectively), increasing the reserve procurement costs and dispatching more expensive units whose downward reserves are required. When LRF1 is used, the costs associated with the procurement of reserves are constant (€278.4), due to the imposed fixed reserve requirements (154.33 MW for upward and 154.33 MW for downward reserves), contrary to the decreasing trend seen in the reserve procurement costs of LRF2 and LRF3. Notably, reserve procurement costs are the lowest in LRF2 with a maximum value of €171 and a minimum of €75.5 (since LRF2 procures fewer reserves than the other formulations) and ‘SEW energy’ remains relatively close to what is found in LRF1. Still, ‘SEW energy’ is higher in LRF1 due to the fact that it only maximizes the ‘DA SEW’ and does not consider the activation costs, which have an effect on the dispatch and, therefore on the ‘SEW energy’.

The anticipated activation costs also provide an insightful perspective. Fig. 9 presents the activation costs of reserves that would be paid (positive) or received (negative) by the TSO if all the procured reserves had been activated in RT. Since LRF2 and LRF3 minimize the activation costs while meeting the robustness goal, their anticipated activation costs are significantly lower when compared with LRF1 (€1,234.6), which only aims at minimizing the level of relaxation of the robustness goal, represented by the slack variable γ_s^n. Comparing LRF2 and LRF3, it is observed that LRF3 results in lower activation costs than LRF2 (61% lower costs when $\rho = \rho_{max}$) since LRF3 anticipates the activation of more downward reserves (to cover the fixed and variable reserve needs), which are represented as revenue for the operator.

Finally, since robustness is achieved by moving towards the lower bound of the uncertainty ranges (i.e., robustness goals in (2o) and (3c)), when ρ increases, the dispatch of the accepted stochastic units is constrained. This is illustrated in Fig. 10 for all three formulations. The reduction in stochastic dispatch is covered by conventional producers, which increase their production as a more robust solution is chosen by the MO through the choice of ρ. Fig. 10 also depicts the aggregated procurement of upward and downward reserves for different robustness levels. In the case of LRF2, at $\rho = 0$, enough upward and downward reserves are procured to maintain the balance given any realization within the aggregated negative (−100,65 MW) or positive

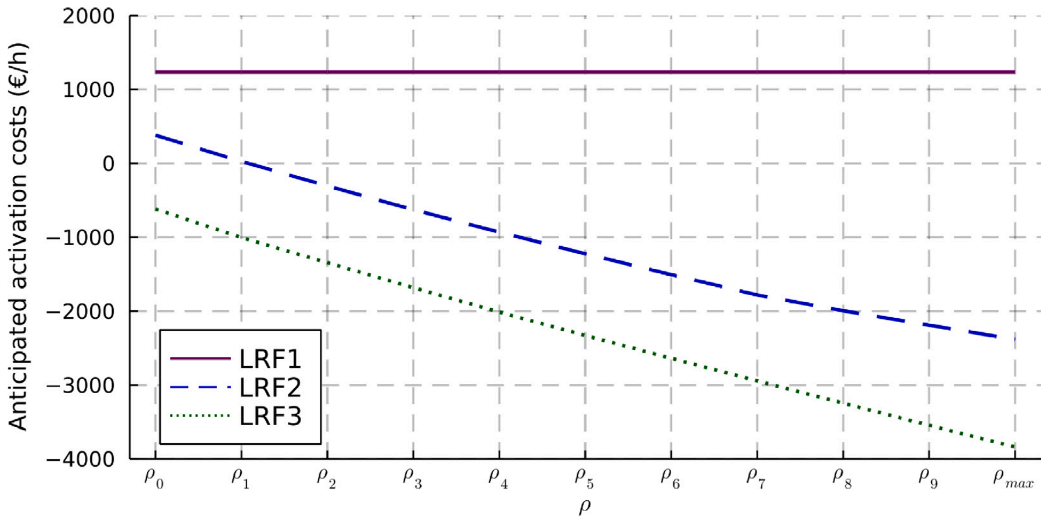

Fig. 9. Comparison of the anticipated activation costs obtained under LRF1, LRF2, and LRF3 for different levels of the conservativeness parameter ρ.

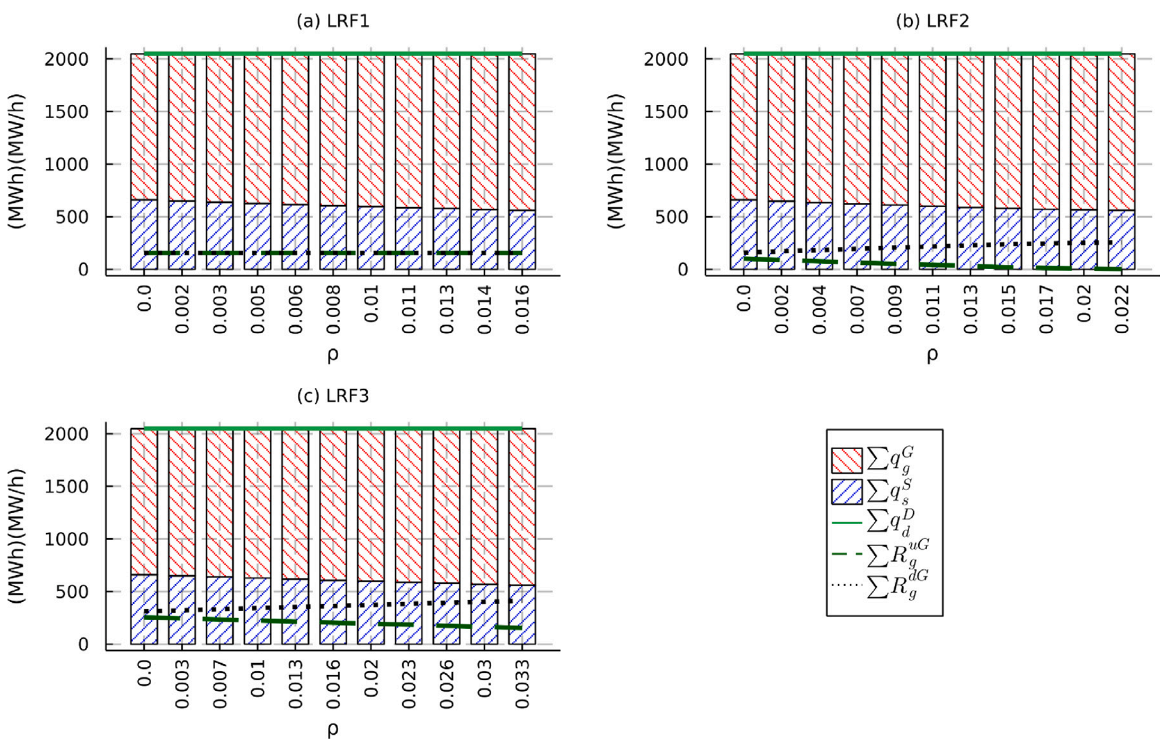

Fig. 10. Aggregated energy dispatch (q_s^S, q_g^G) and procured upward (R_g^{uG}) and downward (R_g^{dG}) reserves for different levels of the conservativeness parameter (ρ) under LRF1, LRF2 and LRF3.

(+157.03 MW) uncertainty ranges, respectively. As the co-optimized market clearing moves toward a more robust solution (by increasing ρ), fewer upward reserves and more downward reserves are needed. Therefore, at $\rho = \rho_{max}$, no upward reserves are procured while 257.6 MW of downward reserves are purchased. A similar trend is observed in LRF3, in which, in addition to the variable reserve needs, extra reserve capacity is procured to cover the fixed reserve requirements. In contrast, when only fixed reserve requirements are imposed (as in LRF1), no change in the procurement of reserves is seen regardless of variations in ρ.

5.2. Individual participant results

A detailed view of some of the results of LRF2 for the stochastic producers is presented in Figs. 11 and 12 - similar trends in terms of dispatched quantities and reserve needs are observed for the two other formulations. Fig. 11 shows the reduction of the optimal dispatch levels of each stochastic unit as greater values for ρ are chosen, where the dispatch of s_6 (the most expensive stochastic player with a bid of €3.5/MWh) and s_1 (the cheapest with a bid of €1/MWh) are, respectively, the first and last to reach the lower bound of their uncertainty

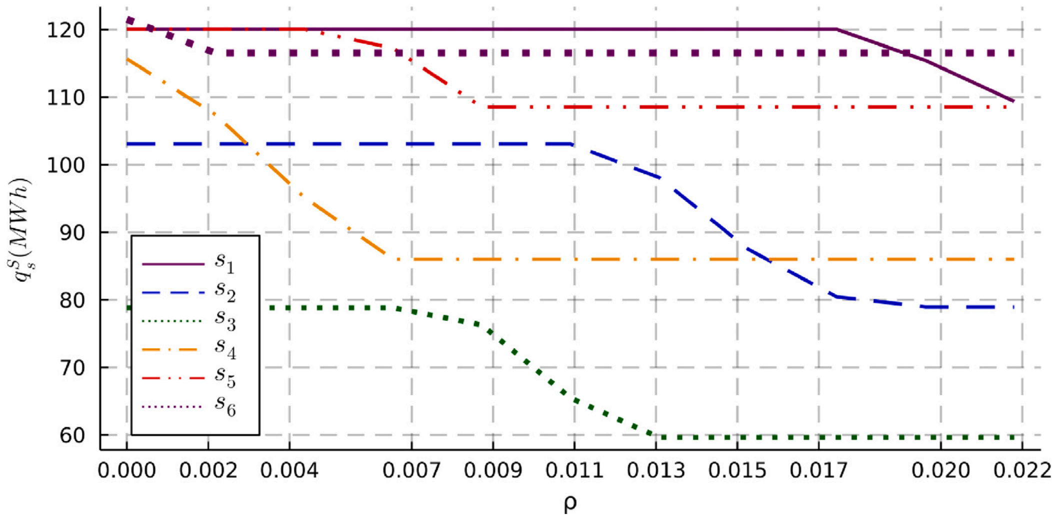

Fig. 11. Dispatched quantities of stochastic producers (q_s^S) participating in LRF2 for different levels of the conservativeness parameter ρ.

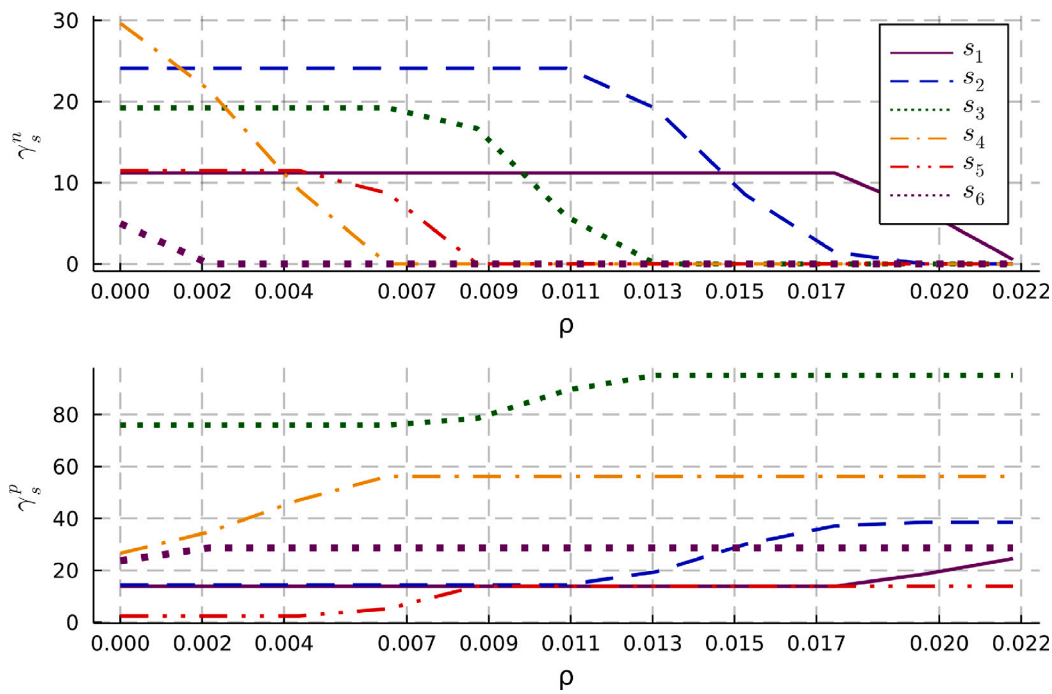

Fig. 12. Upward reserve (above) and downward reserve (below) needs as derived from the bids of the stochastic producers participating in LRF2 for different levels of the conservativeness parameter ρ.

ranges. A similar trend is shown in Fig. 12, where the slack variables γ_s^n and γ_s^p, which in LRF2 represent the needs for upward and downward reserves based on each uncertainty range, are presented. At the nominal realization, when the robustness goal is relaxed, the slack variables γ_s^n and γ_s^p correspond to the negative and positive deviations of each stochastic producer ($\hat{m}_s^n$ and $\hat{m}_s^p$), respectively. As greater levels of ρ are chosen, γ_s^n constraints the dispatch q_s^S towards the lower bound of the uncertainty range, limiting the need for upward reserves and increasing γ_s^p and, therefore, the need for downward reserves. When the MO chooses the most robust market clearing setting, no need for upward reserves is imposed ($\sum \gamma_s^n$), since all the stochastic players are constrained to their lower bound. On the contrary, the needs for downward reserves are equivalent to the sum of the entire uncertainty range of each stochastic producer ($\gamma_s^n = \hat{m}_s^n + \hat{m}_s^p$), to account for additional stochastic generation at RT.

As for the conventional generators, differences across the three proposed formulations are more pronounced. The energy dispatch and the procurement of reserve capacity for each conventional producer participating in the co-optimized market under LRF1 are depicted in Fig. 13(a). Since the market is co-optimized, the optimal solution corresponds to the best output for both the energy and the reserve procurement markets. As expected, the cheapest players (g_8, g_9 and g_{10} with bids of €6.02/MWh, €5.47/MWh, and €0/MWh, respectively), which did not offer reserve services, are dispatched at 100% of their capacity. Players that offered upward reserve services and energy at very competitive prices are partially dispatched to set aside some capacity in case upward reserves are needed, such as g_6 and g_7. On the contrary, players offering cheap downward capacity, such as g_{12} with a downward reserve price bid of €0.2/MW, are dispatched and preferred, over other players with lower energy prices, in case downward reserves are needed. As higher values of the conservativeness parameters are

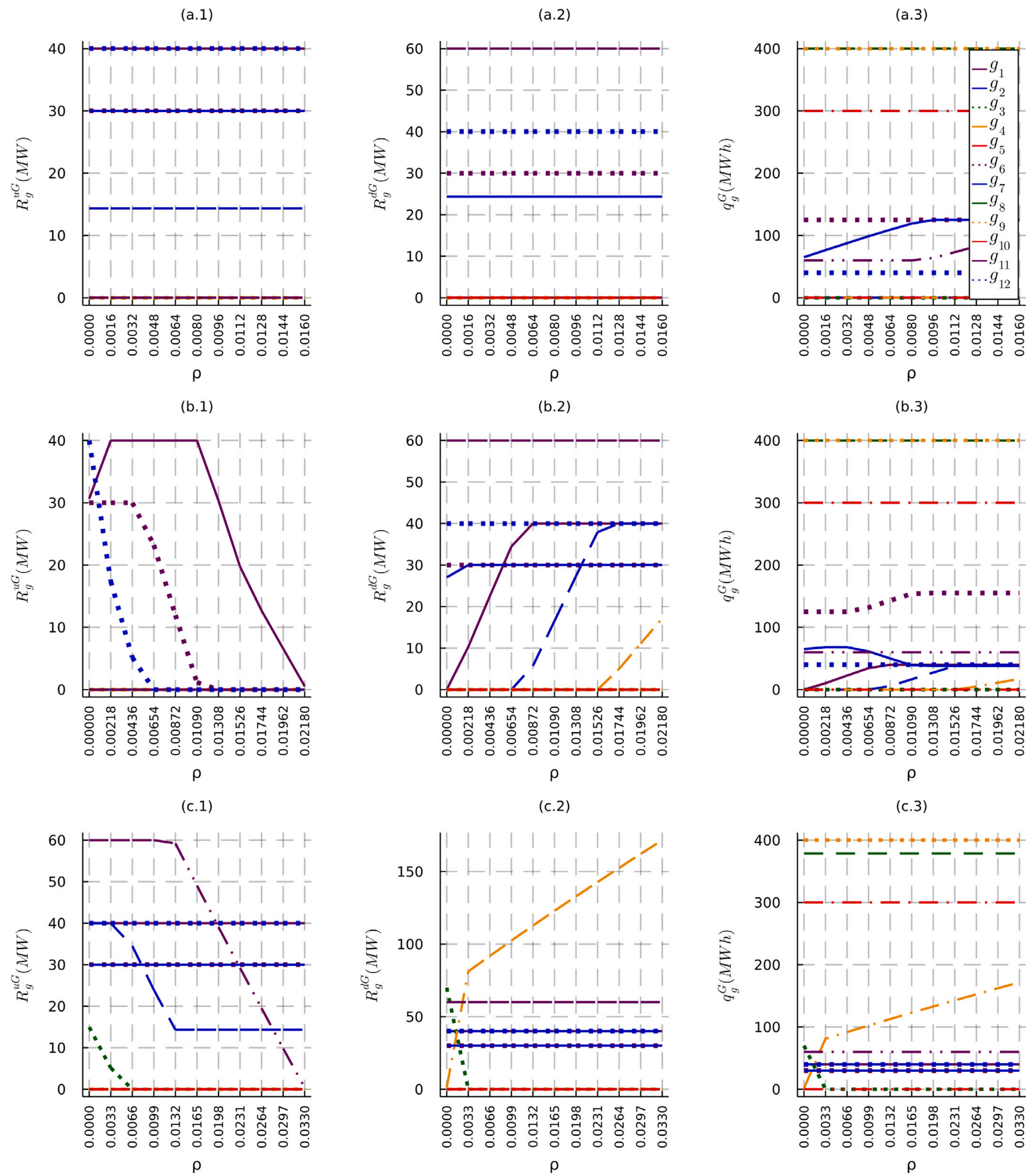

Fig. 13. Dispatched quantities (q_g^G) and procured upward (R_g^{uG}) and downward (R_g^{dG}) reserves for conventional producers under LRF1 (a), LRF2 (b), and LRF3 (c) for different levels of the conservativeness parameter ρ.

chosen, the production of players g_7 and g_{11} increase to cover the reduction of stochastic production. Overall, the outcomes are rather stable for changes in ρ.

More dynamic results are observed when LRF2 is used. The main difference in the results is caused first by the variable reserve needs of the system. As illustrated in Fig. 13(b), the procurement of upward reserves follows a downward trend as more robust results are chosen, contrary to the case of downward reserves, in which an increasing trend is observed. Second, LRF2 not only considers the maximization of the DA SEW (i.e., energy and reserve procurement), as in LRF1, but also aims at minimizing the anticipated activation costs in RT. As expected, as more downward capacity is required, additional players, able to provide downward services, are dispatched. The selected players – in this case g_1, g_2, and g_4 – are dispatched not only due to their downward reserve capacity bid (p_g^{dRG} equal to €0.25/MW, €0.33/MW, and €0.30/MW, respectively) but also due to the offered price for the activation of downward reserves (p_g^{drG} equal to €11/MWh, €11/MWh and €17/MWh). Similarly, as less upward reserve capacity is needed, the capacity procured by the selected players decreases. The order in which the upward capacity procurement decreases depends on the bid price for reserve procurement and activation of each producer. For example, the provision of upward capacity from player g_{12} is the first to decrease, as bid prices of g_{12} (p_g^{uRG} and p_g^{urG}) are the least efficient among the players providing upward services.

The dynamic of the results obtained for different values of ρ in LRF3 is in between that observed for LRF1 and LRF2, as shown in Fig. 13(c). The results differ from the results of LRF2 due to the fact that additional fixed reserve capacity is required. Additional units are required to provide upward capacity, especially if a nominal market clearing is chosen. As less upward capacity is needed, the required capacity from the more expensive units ($p_g^{uRG} + p_g^{urG}$), among those considered initially, decreases. In order to meet the additional downward requirements, the dispatch is modified. For example, the dispatch of cheap units g_6, g_7, and g_8 decreases, when compared to the results of LRF2 and LRF1, as additional generation (in this case from player g_4 whose energy bid price is €20.93/MWh) is required to provide the extra downward capacity. Player g_4 is then increasingly dispatched as greater values of ρ are chosen to guarantee that increasing downward reserve needs are met.

5.3. Discussion

The case study results clearly show the impact of the conservativeness parameter on the dispatch of the stochastic producers and the associated system reserve needs. In practice, the choice of ρ provides a trade-off between the 'SEW energy' (which is at its greatest value when more VRES are dispatched, i.e. $\rho = 0$) and the reserve procurement and activation costs (which are at their lowest values when the dispatch from the stochastic producers is limited to the lower bound of the uncertainty ranges, i.e. $\rho = \rho_{max}$). As more robust solutions are chosen, the dispatch from stochastic producers is constrained (within their uncertainty ranges) and, in LRF2 and LRF3, the estimation of the reserve needs changes, as illustrated in Figs. 10 and 12. Moreover, when the reserve needs are based on the uncertainty range of the stochastic producers (as in LRF2 and LRF3), there is an effect of ρ not only in the total procured reserves, but also in the economic allocation of these reserves across conventional producers and, correspondingly, on their energy dispatch (Figs. 13(b)–13(c)). When LRF1 is used, the reserve procurement quantities and allocation remain constant regardless of the value of ρ (Fig. 13(a)).

The main difference between LRF2 and LRF3 is illustrated by the system indicators discussed in Section 5.1: when LRF3 is used, more reserves are scheduled than in LRF2, and expensive producers (willing to provide more reserve capacity) are dispatched, resulting in a lower 'SEW energy' and higher reserve procurement cost when compared with LRF2. For example, when $\rho = \rho_{max}$ is chosen, LRF2 results in a DA SEW which is 3.44% greater than in LRF3. This is mainly due to the dispatch in LRF3 of more expensive units, such as g_4, needed to cover additional downward reserve requirements.

Finally, when comparing LRF2 with LRF1, system results also suggest that there is a comparative benefit in using LRF2. Even though LRF1 performs slightly better than LRF2 in terms of 'SEW energy' (with a maximum difference of 0.78%), LRF2 performs considerably better than LRF1 in terms of reserve procurement costs (between 27.1% and 61.4% lower) and anticipated activation costs (with costs that are 69.1% and 293% lower). Instead, LRF3 achieves even lower anticipated activation costs , with costs 61% and 263% lower than LRF2, but it also presents the lowest 'SEW energy' of the three formulations.

6. Conclusions and future work

This work has proposed to co-optimize the energy dispatch and reserve procurement in a joint DA market setting under uncertainty using a novel LR market clearing formulation, which allows the market operator to choose the robustness level of the co-optimized solution via a conservativeness parameter. Three different formulations were proposed to specify the reserve requirements: LRF1 captured the traditional situation in which fixed pre-defined reserve requirements are considered. LRF2, on the other hand, proposed to define the reserve requirements as variable by-products of the co-optimization problem. These reserve requirements reflect the system upward and downward reserve needs, estimated based on the uncertainty stemming from the VRES participating in the market, which are captured using their submitted uncertainty bids. Finally, LRF3 introduced a combined formulation, including fixed and variable reserve requirements. To capture the uncertainty from the stochastic producers, different uncertainty bid formats, that allow these producers to define their uncertainty ranges, have been proposed. A pricing mechanism to derive energy and reserve prices from the dual variables of the proposed LR formulations was also introduced.

The effects of the proposed formulations and the conservativeness parameter on the total SEW, the dispatched quantities, and the procured reserves were showcased through a case study. The numerical results have illustrated the relative economic benefit of endogenizing the uncertainty of the stochastic producers in the definition of the reserve requirements (LRF2), compared to the case in which fixed predefined reserve requirements were considered (LRF1 and, to a lesser extent, LRF3). However, it is essential to highlight that the sizing of the reserves in LRF2 depends on the submitted uncertainty bids, which can be subject to strategic manipulation and therefore represent a risk for the system security of supply. This risk can be attenuated through the use of the LRF3 formulation, which combines a fixed reserve requirement with a variable requirement that depends on the uncertainty stemming from VRES. As the volume of fixed requirement can be adjusted, this can be a parameter tunable by the TSO, given its own security of supply analyses and the recommendations of the energy regulatory entity.

This work sets the stage for a number of promising future research directions, such as the analysis of the impact of the proposed formulation on the total system imbalance costs. This would enable the identification and investigation of the further potential economic benefits that the co-optimization of energy and reserves under uncertainty can deliver. In addition, a game-theoretic analysis of the participants' behaviour under this co-optimization setting can complement the current analysis by investigating the participants' bidding behaviours (e.g., submitted maximum negative and positive deviations) and their effects on potential profits and system performance. Finally, considering inter-temporal constraints and the link of the uncertainty bids within different time periods is another future research direction. The inclusion of inter-temporal constraints would allow, e.g., the assessment of the compatibility of the proposed method with current market setups where, e.g., block multi-temporal bids are allowed.

CRediT authorship contribution statement

Lina Silva-Rodriguez: Conceptualization, Investigation, Software, Writing – original draft. **Anibal Sanjab:** Conceptualization, Validation, Writing – review & editing. **Elena Fumagalli:** Conceptualization, Writing – review & editing. **Madeleine Gibescu:** Conceptualization, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Appendix. Proof of pricing mechanism

This section presents the mathematical proof of Theorem 1, required for the introduced pricing mechanism in Section 4. The proof process introduces four different Lemmas, whose proofs are also provided. The proof of LRF2 is presented as an example, as a similar mechanism can be derived for either LRF1 or LRF3, due to their similar mathematical structure.

The Karush–Kuhn–Tucker (KKT) conditions of the LRF2 formulation (problem (3)) required in the proof correspond to (A.1)–(A.3). Similarly, the required KKT conditions of the modified nominal problem of the LRF2 formulation (as described in Section 4) correspond to (A.4). Note that the decision variables of the modified problem are identified with the subscript m.

$$\frac{\partial \mathcal{L}}{\partial \gamma_s^n} = -\lambda_2 - \mu_{10}^s - \mu_{27}^s = 0 \ \forall s \in \mathcal{S}, \tag{A.1}$$

$$0 \leq -q_s^S - R_s^{uS} + m_s^S - \hat{m}_s^n \ + \gamma_s^n \perp {\mu_{10}}^s \geq 0 \ \forall \ s \in \mathcal{S}, \tag{A.2}$$

$$0 \leq \gamma_s^n \perp {\mu_{27}}^s \geq 0 \ \forall \ s \in \mathcal{S}, \tag{A.3}$$

$$0 \leq -q_{ms}^S - R_{ms}^{uS} + m_s^S - \hat{m}_s^n \ + \gamma_s^{n*} \perp {\mu_{m9}}^s \geq 0 \ \forall \ s \in \mathcal{S}. \tag{A.4}$$

Theorem 1. *If upward and downward reserves are expected to be activated in RT ($\sum_{s\in\mathcal{S}} r_s^{uS} + \sum_{g\in\mathcal{G}} r_g^{uG} > 0$ and $\sum_{s\in\mathcal{S}} r_s^{dS} + \sum_{g\in\mathcal{G}} r_g^{dG} > 0$) and at least one conventional producer is dispatched, then $q_{ms}^{S*} = q_s^{S*}$, $R_{ms}^{uS*} = R_s^{uS*}$, $R_{mg}^{uG*} = R_g^{uG*}$, $R_{ms}^{dS*} = R_s^{dS*}$, and $R_{mg}^{dG*} = R_g^{dG*}$ for all $s \in \mathcal{S}$ and $g \in \mathcal{G}$.*

Proof. Without loss of generality, let us consider stochastic producer i (i.e. $s \triangleq i$) to be the cheapest stochastic producer. For the derivation of the proof of Theorem 1, we first begin by proving the following four lemmas.

Lemma 1. *If $\sum_{s\in\mathcal{S}} r_s^{uS} + \sum_{g\in\mathcal{G}} r_g^{uG} > 0$, then (3c) is binding and at least one stochastic producer (i.e., i) is dispatched ($q_i^S + R_i^{uS} > 0$).*

Proof. If $\sum_{s\in\mathcal{S}} r_s^{uS} + \sum_{g\in\mathcal{G}} r_g^{uG} > 0$, then $\sum_{s\in\mathcal{S}} \gamma_s^n > 0$, see (3f). If $\sum_{s\in\mathcal{S}} \gamma_s^n > 0$, then at least one stochastic producer has an associated $\gamma_s^n > 0$, namely, $\gamma_i^n > 0$. By complementary slackness, if $\gamma_i^n > 0$, then ${\mu_{27}}^s = 0$, see (A.3). If ${\mu_{27}}^s = 0$, then ${\mu_{10}}^s = -\lambda_2$, see (A.1). By definition, we know that ${\mu_{10}}^s \leq 0$. Therefore, $\lambda_2 \geq 0$. However, from (3a), we know that if there is a marginal increment on the right side of (3f), the objective function (3a) increases resulting on an increase on the activation costs. Therefore, $\lambda_2 > 0$. If $\lambda_2 > 0$, then ${\mu_{10}}^s < 0$. By complementary slackness (A.2), if ${\mu_{10}}^s < 0$, then (3c) is binding:

$$q_s^S + R_s^{uS} - \gamma_s^n = m_s^S - \hat{m}_s^n. \tag{A.5}$$

By definition, we know that $m_i^S - \hat{m}_i^S > 0$. Therefore, if $\gamma_i^n > 0$, then $q_i^S + R_i^{uS} > 0$. This captures the case of stochastic producers, as their bids are typically cheaper than the bids of conventional generators. □

Lemma 2. *If $\sum_{s\in\mathcal{S}} r_s^{uS} + \sum_{g\in\mathcal{G}} r_g^{uG} > 0$ and $\sum_{s\in\mathcal{S}} r_s^{dS} + \sum_{g\in\mathcal{G}} r_g^{dG} > 0$, then $R_s^{uS} = r_s^{uS}$, $R_s^{dS} = r_s^{dS}$, $R_g^{uG} = r_g^{uG}$, and $R_g^{dG} = r_g^{dG}$.*

Proof. If $\sum_{s\in\mathcal{S}} r_s^{uS} + \sum_{g\in\mathcal{G}} r_g^{uG} > 0$ and $\sum_{s\in\mathcal{S}} r_s^{dS} + \sum_{g\in\mathcal{G}} r_g^{dG} > 0$, then $\sum_{s\in\mathcal{S}} R_s^{uS} + \sum_{g\in\mathcal{G}} R_g^{uG} > 0$ and $\sum_{s\in\mathcal{S}} R_s^{dS} + \sum_{g\in\mathcal{G}} R_g^{dG} > 0$, see (3k) and (3l). Constraints (3h)–(3j) guarantee that at least enough reserve capacity is procured per player to cover the anticipated activation of reserves in RT. Since (3l) and (3m) ensure that the total quantity of procured reserves equals the anticipated activation quantities, then constraints (3h)–(3j) are always binding:

$$r_s^{uS} = R_s^{uS} \ \forall s \in \mathcal{S}, \tag{A.6}$$

$$r_s^{dS} = R_s^{dS} \ \forall s \in \mathcal{S}, \tag{A.7}$$

$$r_g^{uG} = R_g^{uG} \ \forall g \in \mathcal{G}, \tag{A.8}$$

$$r_g^{dG} = R_g^{dG} \ \forall g \in \mathcal{G}. \quad \square \tag{A.9}$$

Lemma 3. *If $\sum_{s\in\mathcal{S}} r_s^{uS*} + \sum_{g\in\mathcal{G}} r_g^{uG*} > 0$, then (3c) (in the modified problem) is binding.*

Proof. As in Lemma 1, we know that if $\sum_{s\in\mathcal{S}} r_s^{uS} + \sum_{g\in\mathcal{G}} r_g^{uG} > 0$, then $\sum_{s\in\mathcal{S}} \gamma_s^n > 0$. If $\sum_{s\in\mathcal{S}} \gamma_s^n > 0$, then at least one stochastic producer has an associated $\gamma_s^n > 0$, namely, $\gamma_i^n > 0$. Since all the decision variables $r_s^{uS}, r_g^{uG}, r_s^{dS}, r_g^{dG}$, and $\gamma_s^n > 0$ are passed to the modified problem as parameters, the same logic applies. Looking at duality theory and the effect of (3c) in the objective function (1a) of the modified problem, we can deduct the following: since $m_i^S - \hat{m}_i^n + \gamma_i^n$ is always positive, if there is a marginal increment on the right side of (3c), $q_{ms}^S + R_{ms}^{uS}$ increases. If there is an increase in $q_{ms}^S + R_{ms}^{uS}$, then the total costs that are minimized by the objective function increase (i.e., the total SEW decreases), then ${\mu_{m9}}^s < 0$. By complementary slackness (A.4), if ${\mu_{m9}}^s < 0$, then (3c) is binding:

$$q_{ms}^S + R_{ms}^{uS} = m_s^S - \hat{m}_s^n \ + \gamma_s^{n*}, \quad \square \tag{A.10}$$

Lemma 4. *If $\sum_{s\in\mathcal{S}} r_s^{uS*} + \sum_{g\in\mathcal{G}} r_g^{uG*} > 0$ and $\sum_{s\in\mathcal{S}} r_s^{dS*} + \sum_{g\in\mathcal{G}} r_g^{dG*} > 0$, then $R_{ms}^{uS} = r_s^{uS*}$, $R_{ms}^{dS} = r_s^{dS*}$, $R_{mg}^{uG} = r_g^{uG*}$, and $R_{mg}^{dG} = r_g^{dG*}$.*

Proof. If $\sum_{s\in\mathcal{S}} r_s^{uS*} + \sum_{g\in\mathcal{G}} r_g^{uG*} > 0$ and $\sum_{s\in\mathcal{S}} r_s^{dS*} + \sum_{g\in\mathcal{G}} r_g^{dG*} > 0$, then $\sum_{s\in\mathcal{S}} R_{ms}^{uS} + \sum_{g\in\mathcal{G}} R_{mg}^{uG} > 0$ and $\sum_{s\in\mathcal{S}} R_{ms}^{dS} + \sum_{g\in\mathcal{G}} R_{mg}^{dG} > 0$, see (3k) and (3l) in the modified problem. Constraints (3h)–(3j) (in the modified problem) guarantee that at least enough reserve capacity is procured per player to cover the anticipated activation of reserves in RT. Since (3l) and (3m) ensure that the total quantity of procured reserves equals the anticipated activation quantities, then constraints (3h)–(3j), in the modified problem, are always binding:

$$r_s^{uS*} = R_{ms}^{uS} \ \forall s \in \mathcal{S}, \tag{A.11}$$

$$r_s^{dS*} = R_{ms}^{dS} \ \forall s \in \mathcal{S}, \tag{A.12}$$

$$r_g^{uG*} = R_{mg}^{uG} \ \forall g \in \mathcal{G}, \tag{A.13}$$

$$r_g^{dG*} = R_{mg}^{dG} \ \forall g \in \mathcal{G}. \quad \square \tag{A.14}$$

Given the results of Lemmas 2 and 4, the equivalence between the procured reserve quantities in the two formulations can be readily shown. Indeed, since the decision variables from LRF2 ($r_s^{uS}, r_g^{uG}, r_s^{dS}$, and r_g^{dG}) are passed to the modified problem as parameters $r_s^{uS*}, r_g^{uG*}, r_s^{dS*}$, and r_g^{dG*} then $R_{ms}^{uS} = R_s^{uS*}$, $R_{mg}^{uG*} = R_g^{uG*}$, $R_{ms}^{dS*} = R_s^{dS*}$, and $R_{mg}^{dG*} = R_g^{dG*}$ for all $s \in \mathcal{S}$ and $g \in \mathcal{G}$.

Similarly, if ($R_{ms}^{uS} = R_s^{uS*}$) and given the results of Lemmas 1 and 3, the equivalence between the energy dispatch quantities for the stochastic producers can be readily derived. Solving (A.10) for γ_s^n and substituting it in (A.5), it follows that $q_{ms}^{S*} = q_s^{S*}$, completing the proof of Theorem 1

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