# Voltage Stability Monitoring based on Adaptive Dynamic Mode Decomposition

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***Abstract*—This paper develops a new voltage stability monitoring method using dynamic mode decomposition (DMD) and its adaptive variance. First, state estimation (SE) is used to estimate the voltage in the system. Then, the measured voltages from the phasor measurement units (PMU) and estimations from SE are used as the inputs for DMD to predict the long-term voltage dynamic. Furthermore, to improve the prediction performance, the normal DMD is improved by adaptively changing the size of input samples based on the error in the training phase, named adaptive DMD (ADMD). The effectiveness of the proposed method is validated on the Nordic32 test system, which is recommended as the test system for voltage stability studies. Different contingency scenarios are used, and the results show that the proposed method is able to monitor the voltage stability after a disturbance (i.e., 4.3x10$^{-4}$ MAPE for a stable case and 0.0041 MAPE for an unstable case). Furthermore, the results from a scenario in which a real-world oscillation event is used show high accuracy in voltage stability monitoring of the proposed ADMD method.**

***Index Terms*—Adaptive DMD, dynamic mode decomposition, long-term voltage stability, voltage prediction.**

## I. Introduction

Voltage stability monitoring (VSM) is crucial for the reliable and secure operation of power systems [1]. Different instability indicators, such as the sensitivity of total reactive power generations to individual load reactive power or the maximum eigenvalue of the inverse Jacobian matrix, have been proposed in [2], [3] to assess voltage stability. However, a lack of measurements in real-time operation hinders the possibility of calculating the indicators mentioned above. To address that issue, the works in [4], [5] tried reconstructing unmonitored bus voltages in unobservable areas. In [4], a hybrid power flow model has been proposed which combines phasor measurement unit (PMU) and power flow equations to increase observable voltage. Furthermore, the authors in [5] proposed an early instability detection method based on a state estimator with limited PMUs. To further improve voltage instability detection, the concept of tracking network state has been proposed in [6], where the Kalman filter is used for voltage prediction. However, the method relied on the assumption of having a network model and complete system states, i.e., the voltage at unmeasured buses in real-time operation.

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With increasing available data from PMUs, machine learning has been recently considered a promising method to address the issue of requiring a full network model, as needed for the above methods. In [7], a massively parallel dynamic security assessment is proposed based decision tree (DT) method. Based on high-performance computing, massively contingency scenarios are generated; thus, the DT method is used for post-fault analysis and learning security rules, which are then applied for real-time operation. Similarly, authors in [8] presented a deep recurrent neural network for long-term voltage instability prediction. Approximately 140,000 training scenarios are created, including N-1 and N-1-1 contingencies for the training model. Those model-free methods show high prediction accuracy. However, the overall prediction performance highly depends on the set of training samples (i.e., the training set should cover all possible instability scenarios), which is limited for online applications (i.e., the model needs to be trained again when the system changes or new devices are added). Furthermore, the mentioned methods can only predict instability situations. This information is insufficient for the control tasks, where voltage evolution is required [9].

To tackle the mentioned limitations, we propose a new method based on dynamic model decomposition (DMD) for long-term voltage prediction. The DMD has been applied in the power system for global oscillation analysis [10], dynamic state reconstruction [11], and short-term load forecasting [12], to name a few. This paper combines the DMD with the static state estimation (SE) to predict long-term voltage evolution. Overall, the main contributions of this paper are as follows:

- The normal DMD is improved with an adaptive training sample input, named ADMD, which can enhance the prediction performance;
- The proposed method does not require any offline training or system model, thus reducing the computational burden and complexity when applied to large power systems;
- This work enhances the connection between the SE and the VSM, which can track and monitor long-term voltage dynamics after a disturbance.

The Nordic32 system and a real-world event in ISO New England are used to evaluate the proposed method. The numerical results showed that the proposed method could accurately predict long-term voltage evolution with high accuracy.

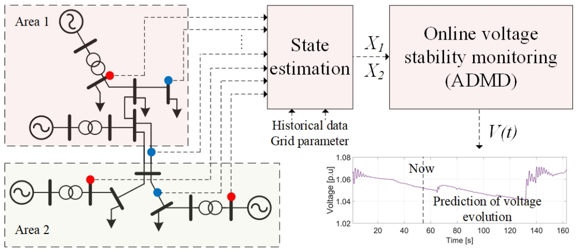


Fig. 1: The outline of the proposed approach.

## II. Proposed Method for VSM

This section presents the outline of our proposed VSM, which is presented in Fig. 1. It aims to capture long-term voltage dynamics after a disturbance. It is worth noting that the short-term dynamic is not tracked and is assumed to be stable. The underlying idea is summarized below:

- First, similar to the previous works [4], [5], unmeasured bus voltages are estimated using SE. The traditional weighted least square (WLS) is used for the SE model;
- Second, the system voltages (i.e., the estimated voltage from SE and the measured voltage from PMUs) are constructed to two sequential overlapping voltage profiles in the last seconds $X_1$ and $X_2$. Then, the ADMD method is used to learn the dynamic voltage of the underlying system, which is then used to predict the dynamic voltage evolution $V(t)$ in the future.

The details are explained in the following sections.

### A. Static State Estimation

The SE algorithm is a data processing given by a set of measurements to estimate the system state. The measurement function is as follows:

$$z = h(x) + \ e \tag{1}$$

where, $z$ is measurement vector, $h(x)$ is vector of function measurement of state vector $x$, and $e$ is measurement error vector. The measurement errors are assumed to be independent zero-mean Gaussian variables, and vector $\mathbf{R}$ is the covariance matrix of $e$. To solve Eq. (1), the widely used WLS is implemented, and the objective is to minimize the sum of the square of the residuals.

$$J\left(x\right) = \left[z - h(x)\right]^T \mathbf{R}^{-1}[z - h(x)] \tag{2}$$

Solving the first-order optimality condition of Eq. (2) equal zero will give the $x$ value, which satisfies the minimum value of objective function $J(x)$. It is given by:

$$g\left(x\right) = \frac{\partial J(x)}{\partial x} = -\mathbf{H}^T(x)\mathbf{R}^{-1}\left[z - h\left(x\right)\right] = 0 \tag{3}$$

where:

$$\mathbf{H}\left(x\right) = \frac{\partial h(x)}{\partial x} \tag{4}$$

The Gauss-Newton method is used to linearize the function, giving the result at the $k_{th}$ iterative as follows Eq. (5). The iterative continues until the given convergence criteria are satisfied.

$$x^k = x^{k-1} + \mathbf{G}\left(x^k\right)^{-1}\mathbf{H}\left(x^k\right)^T\mathbf{R}^{-1}\left[z - h\left(x\right)\right] \tag{5}$$

where $\mathbf{G}(x)$ is gain matrix.

$$\mathbf{G}\left(x^k\right) = \mathbf{H}(x^k)^T\mathbf{R}^{-1}\mathbf{H}(x^k) \tag{6}$$

### B. Dynamic Mode Decomposition based Prediction

*1) Dynamic Mode Decomposition:* Consider a discrete dynamical system sampled at every $\Delta t$ in time:

$$x_{k+1} = \mathbf{A}x_k, \tag{7}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$ is the dynamic matrix which is mapping the snapshot $x_k \in \mathbb{R}^n$ at time $k$ to the next snapshot $x_{k+1} \in \mathbb{R}^n$ at time $k+1$.

The solution to this system can be expressed as:

$$x_{k+1} = \sum_{j=1}^{n} \phi_j \lambda_j^k b_j, \tag{8}$$

where $\lambda_j$ and $\phi_j$ are eigenvalues and eigenvectors of dynamic matrix $\mathbf{A}$. $b_j$ is the initial magnitude of each eigenvector $\phi_j$.

Thus, the time-domain of $x(t)$ can be calculated using the eigenvalue and eigenvector of matrix $\mathbf{A}$:

$$x(t) = \sum_{j=1}^{n} \phi_j e^{\omega_j t} b_j = \mathbf{\Phi} e^{\mathbf{\Omega}_t}\mathbf{b}, \tag{9}$$

where $\mathbf{\Phi} \in \mathbb{C}^{\mathbf{n}\times\mathbf{n}}$ is the right eigenvector matrix of matrix $A$. $\mathbf{\Omega} = diag(\omega)$ is a diagonal matrix that contains eigenvalue $\omega_j$, where $\omega_j = ln(\lambda_j)/\Delta t$ showing the relationship between the discrete and continuous eigenvalue. $\mathbf{b} = \mathbf{\Phi}^\dagger x_1$ is a vector of coefficients $b_j$, where $\mathbf{\Phi}^\dagger$ denotes the Moore-Penrose pseudoinverse of $\mathbf{\Phi}$.

The DMD method can be used to identify eigenvector matrix $\mathbf{\Phi}$, eigenvalue matrix $\mathbf{\Lambda}$, and the coefficient matrix $\mathbf{b}$ for a set of the measurement values.

Having two data matrices $\mathbf{X_1} \in \mathbb{R}^{n\times(m-1)}$ and $\mathbf{X_2} \in \mathbb{R}^{n\times(m-1)}$ contain $n$ states which are collected at $m-1$ snapshots with constant time-interval $\Delta t$ as follows:

$$\mathbf{X_1} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_{m-1} \\ | & | & & | \end{bmatrix}, \tag{10}$$

$$\mathbf{X_2} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \dots & \mathbf{x}_{m} \\ | & | & & | \end{bmatrix}, \tag{11}$$

where the matrix $\mathbf{X_2}$ is time-shifted of the matrix $\mathbf{X_1}$. Then, the Eq. (7) can also be written as follows:

$$\mathbf{X_2} = \mathbf{A}\mathbf{X_1}. \tag{12}$$

The matrix $\mathbf{A}$ can be defined by the following:

$$\mathbf{A} = \mathbf{X_2}(\mathbf{X_1})^\dagger. \tag{13}$$

Normally, the matrix $\mathbf{X_1}$ and $\mathbf{X_2}$ are large-size matrices; thus, the singular value decomposition (SVD) is applied to improve computationally. Hence, the SVD of $\mathbf{X_1}$ is:

$$\mathbf{X_1} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^* \approx \mathbf{U_r}\mathbf{\Sigma_r}\mathbf{V_r^*}, \tag{14}$$

where $\mathbf{U} \in \mathbb{R}^{n\times n}$, $\mathbf{\Sigma} \in \mathbb{R}^{n\times(m-1)}$, and $\mathbf{V}^* \in \mathbb{R}^{(m-1)\times(m-1)}$ ($*$ denotes the complex conjugate transpose). For the matrix reduction, a rank $r$ will be chosen, which keeps the most dominant modes in the SVD; see [13] for choosing $r$ methods. Thus, $\mathbf{U_r} \in \mathbb{R}^{n\times r}$, $\mathbf{\Sigma_r} \in \mathbb{R}^{r\times r}$, and $\mathbf{V_r}^* \in \mathbb{R}^{r\times(m-1)}$ contain the first $r$ singular values.

Substituting Eq. (14) into Eq. (13), the matrix $\mathbf{A}$ can be expressed as follows:

$$\mathbf{A} \approx \mathbf{X_2}(\mathbf{U_r}\mathbf{\Sigma_r}\mathbf{V_r^*})^{\dagger} = \mathbf{X_2}(\mathbf{V_r}\mathbf{\Sigma_r^{-1}}\mathbf{U_r^*}), \tag{15}$$

where $\mathbf{U_r}$ and $\mathbf{V_r}$ are unitary matrices (i.e., $\mathbf{U_r} * \mathbf{U_r} = \mathbf{I}$ and $\mathbf{V_r} * \mathbf{V_r} = \mathbf{I}$). Then, the reduced-order dynamic matrix $\mathbf{A_r} \in \mathbb{R}^{r\times r}$ can be calculated by projecting $\mathbf{A}$ onto $\mathbf{U_r}$ basis:

$$\mathbf{A_r} = \mathbf{U_r^*}\mathbf{A}\mathbf{U_r} = \mathbf{U_r^*}\mathbf{X_2}\mathbf{V_r}\mathbf{\Sigma_r^{-1}}. \tag{16}$$

Applying the eigendecomposition to the reduced-order matrix $\mathbf{A_r}$, we have:

$$\mathbf{A_r}\mathbf{W} = \mathbf{W}\mathbf{\Lambda}, \tag{17}$$

where the eigenvectors are the columns of matrix $\mathbf{W} \in \mathbb{C}^{r\times r}$, and the eigenvalues are the diagonal entries of matrix $\mathbf{\Lambda} \in \mathbb{C}^{r\times r}$. In the DMD, the relationship between matrices $\mathbf{A_r}$ and $\mathbf{A}$ is proved in [14], showing that the eigenvalues of the two matrices are equivalent, and the eigenvectors are related via a linear transformation. Thus, the eigenvectors $\mathbf{\Phi} \in \mathbb{C}^{n\times r}$ of the original dynamic matrix $\mathbf{A}$ can be recovered by:

$$\mathbf{\Phi} = \mathbf{U_r}\mathbf{W}. \tag{18}$$

Furthermore, the vector of coefficients $\mathbf{b} \in \mathbb{C}^{r\times 1}$ is computed:

$$\mathbf{b} = \mathbf{\Phi}^{\dagger} x_1. \tag{19}$$

Finally, the time-domain response of $x(t)$ in Eq. (9) can be predicted using eigenvector and coefficients value from Eq. (18) and Eq. (19).

*2) DMD with Hankel Matrix:* In this section, the idea of the Hankel matrix is used to improve the learning of the DMD method. In case of fewer input measurement states or recorded snapshots, the data matrix $\mathbf{X}$ is shift-stacking to increase the dimension of the measurement matrix, which helps to capture the dynamic relationship between different time steps [14]. The Hankel matrix is constructed as follows:

$$\mathcal{H} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_{m-(s-1)} \\ \mathbf{x}_2 & \mathbf{x}_3 & \dots & \mathbf{x}_{m-(s-2)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_s & \mathbf{x}_{s+1} & \dots & \mathbf{x}_m \end{bmatrix}, \tag{20}$$

where $\mathcal{H} \in \mathbb{R}^{(n*s)\times(m-s+1)}$, and $s$ is the number of stacking time. Then, the Hankel matrix is separated into two overlapping matrices:

$$\mathbf{X_1}^{\mathcal{H}} = \mathcal{H}(:, 1 : m - s), \text{and} \tag{21}$$

$$\mathbf{X_2}^{\mathcal{H}} = \mathcal{H}(:, 2 : m - s + 1). \tag{22}$$

This two matrices are used to replace $\mathbf{X_1}, \mathbf{X_2}$ in Eq. 12.

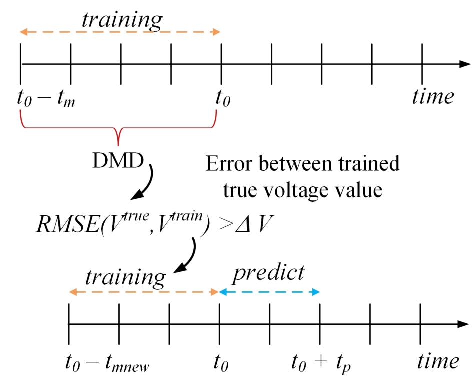


Fig. 2: The proposed adaptive dynamic mode decomposition.

*3) Adaptive Dynamic Mode Decomposition:* As discussed in the previous section, the DMD's idea is to learn the underlying dynamic system based on a set of measurements in time. The dynamic matrix $\mathbf{A}$ in Eq. (7) shows the linear mapping between the snapshot $x_k$ and $x_{k+1}$. Furthermore, the dynamic matrix $\mathbf{A}$ is changing in time, corresponding to the natural or forced responses. Thus, the dynamic matrix $\mathbf{A}$ needs to capture the right current dynamic response of the system. Therefore, this work proposed an adaptive DMD to improve the learning phase of DMD, which result in higher prediction performance. The proposed ADMD method is in Fig. 2 is explained as follows:

- At the time step $t_0$, $m$ voltage snapshots from $t_0 - t_m$ to $t_0$ are collected as the input for the DMD. After that, the voltage, $V^{train}$ calculated using the Eq. (9), is compared with $V^{true}$ by the RMSE indicator;
- If the RMSE is larger than a predefined $\Delta V$, the training samples will be shorten, i.e., $mnew = m - \Delta m$. This means that more recent data will be used to learn the dynamic matrix $\mathbf{A}$.

The above concept is repeated (moving) in time. Thus, the dynamic voltage evolution is predicted with higher accuracy.

## III. Simulation Results and Discussion

In this section, different case studies have been investigated to evaluate the proposed methods' performance. The Nordic32 test system is used to validate the proposed method in the first and second cases. This system was specifically developed for voltage stability analysis as the IEEE PES Task Force recommended a "Test system for voltage stability analysis and security assessment" [15]. Then, in the third case, a real-world voltage oscillation event is used to see how the method performs with a real-world event.

### A. Testing with The Nordic32 System

This case study presents the implementation of the proposed method on the Nordic32 test system. This system is recommended for voltage stability studies [15], [16], which has 20 machines and 52 buses. Each generator includes its dynamic models. Loads are modeled as an exponential model with the constant current for the active power and constant admittance for the reactive power. It is worth mentioning that the induction motors are not modeled in this study as their electromagnetic transient can be neglected in long-term voltage stability analysis. Transformers are equipped with a load tap changer (LTC), with a delay of 30 seconds for the first tap action (i.e., a shorter delay on the following tap actions, different for different transformers). The setup of measurement location is based on the work in [5] (i.e., 7-PMU for complex voltage measurements, 72 power injection measurements, 20 zero injection measurements, see Ref. [5] for more details).

*1) Case 1- Stable voltage:* In this case study, an identical generator with the generator g16 is added to the same bus (i.e., bus 4051), which increases the total active power injection at bus 4051 to 1200 MW. Then, a three-phase fault on line 4032-4044 is applied. To isolate the fault, this line is open (after 0.1 s). After the short-term period (i.e., 0.25 s), the voltage of 52 buses is collected with the total samples of $m = 500$, which is used for ADMD. The threshold for training phase $\Delta V$ is selected as 0.001, and the $\Delta m = 50$ is considered for the ADMD. Once the eigenvector and coefficients value from Eq. (18) and Eq. (19) are obtained, the time-domain voltages evolution in the next 30 samples are predicted by Eq. (9). In Fig. 3, voltage evolution at bus 4062 is predicted using DMD and ADMD methods. In general, both methods accurately predict the voltage. However, at $t = 32$ seconds, where the LTC is activated, the normal DMD failed to predict the voltage. The corresponding percentage error of the two methods compared with the true value is plotted in the bottom sub-figure of Fig. 3. It clearly shows the performance of the proposed ADMD in this stable case after the fault.

*2) Case 2- Unstable voltage:* Unlike the previous case, this second case study considers a voltage unstable after the same fault as case 1. One generator with a capacity of 600 MW is disconnected from bus 4051. After the fault, the system is short-term stable. However, due to the effect of LTCs and OELs, the system gets unstable in long-term (the detailed explanation can be seen from [15]). Similarly, the voltage evolution at bus 4062 is shown in Fig. 4. It can be seen the sub-figure in the middle, at $t = 65$ seconds, the DMD failed to predict the voltage due to the control response of OELs or LTCs. However, the proposed ADMD with adaptive training

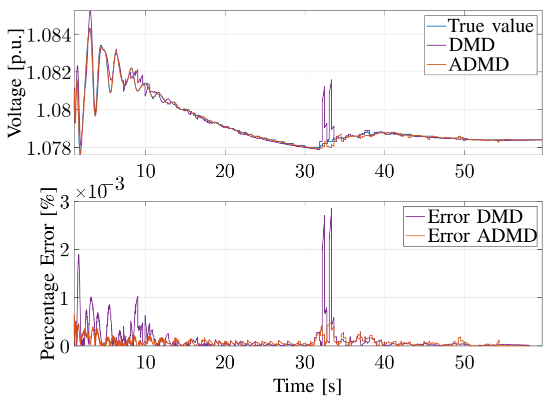


Fig. 3: The voltage evolution prediction using normal DMD and ADMD methods.

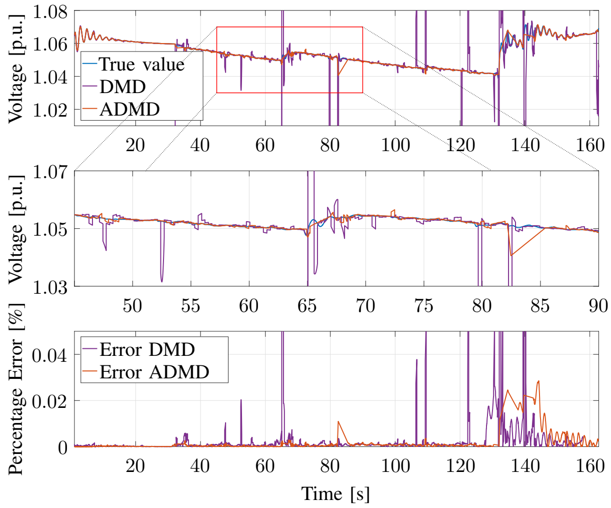


Fig. 4: The voltage evolution prediction using normal DMD and ADMD methods.

samples can achieve better results with a maximum percentage error of around 0.03%, which can be seen from the bottom sub-figure. It again confirms the performance of the ADMD in VSM, even with an unstable voltage situation.

### *B. A Real-world Event in ISO New England*

In this section, a real oscillation event in ISO New England is used to test our method. This is a disturbance in a large generator on July 20, 2017. The detail of the event and recorded data can be found in [17].

*1) Case 3- Voltage oscillation in ISO New England:* There are 3 minutes of recorded data from PMUs. In this case, other grid information (i.e., grid topology and grid parameters are unknown). Thus, the measurement voltage data from PMUs are used directly to the ADMD model without SE. In total, 18 voltage states are used. Other settings are the same as in the previous cases, i.e., $m = 500$, $\Delta V = 0.001$, and $\Delta m = 50$. The voltage at one of the buses is predicted as shown in Fig. 5. It can be seen that at the time $t = 50$ seconds, both methods cannot predict the right voltage evolution due to an unknown force in the system. However, the ADMD obtained very good accuracy, with a maximum error of around 0.01%.

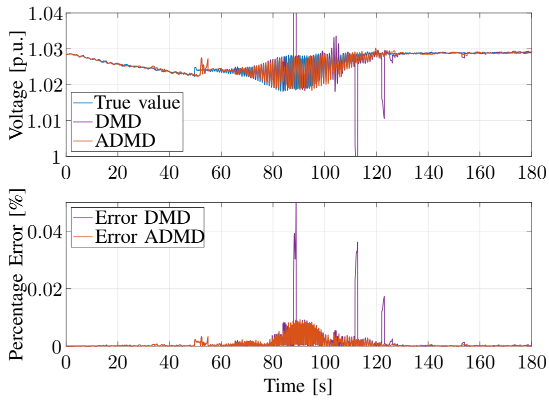


Fig. 5: The voltage evolution prediction using normal DMD and ADMD methods.

### C. Accuracy Indices

To evaluate the overall accuracy of long-term voltage prediction for the whole system, two indicators have been considered:

$$\text{MAPE} = \frac{1}{M}\sum_{k=1}^{M}\frac{1}{N}\sum_{i=1}^{N}\frac{|V_{ik}^{pred} - V_{ik}^{true}|}{V_{ik}^{true}}, \tag{23}$$

$$\text{RMSE} = \frac{1}{M}\sum_{k=1}^{M}\sqrt{\sum_{i=1}^{N}\frac{(V_{ik}^{pred} - V_{ik}^{true})^2}{N}}, \tag{24}$$

where $V_{ik}^{pred}$ is the predicted voltage value of bus $i$ at snapshot $k$ either by the normal DMD or our proposed adaptive DMD, $V_{ik}^{true}$ is the voltage value of bus $i$ at snapshot $k$ from time domain simulation (i.e., with the Nordic32 test system) or real collected data (i.e., with ISO New England). $M$ is number of snapshot and $N$ is the number of predicted voltage buses (i.e., $N = 52$ for the Nordic32 test system, and $N = 18$ for the ISO New England).

The values of mean absolute percentage error (MAPE) and root mean square error (RMSE) indices are given in Table I. It can be seen that both methods obtained good prediction results in case 1 (i.e., the stable system). Furthermore, the proposed ADMD method shows an improved prediction result compared with the normal DMD. In case 2 of the long-term voltage instability, the ADMD method is around 6 and 5 times better than normal DMD with the MAPE and RMSE indicators, respectively. Similarly, the ADMD shows a better prediction result for the real event in ISO New England.

Furthermore, the eigenvalue $\mathbf{\Lambda}$ obtained from the dynamic matrix $\mathbf{A_r}$ in Eq. 17 are shown in Fig. 6. This corresponds to case 2, the unstable voltage case of the Nordic32 system. Following the voltage at bus 4041, the eigenvalue of dynamic matrix $\mathbf{A_r}$ corresponding to areas 1, 2, and 3 are plotted in the unit circle. It clearly shows the moving of eigenvalues from inside the unit circle (in area 1) to the near unit circle (in area 2) and outside of the circle (in area 3). It confirms that the dynamic matrix $\mathbf{A_r}$ has the right property of system response.

TABLE I: Comparison of DMD and ADMD methods

| **Indicator** | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| MAPE-DMD | $5.8 \times 10^{-4}$ | 0.0248 | 0.0007 |
| MAPE-ADMD | $4.3 \times 10^{-4}$ | 0.0041 | 0.0004 |
| RMSE-DMD | $3.9 \times 10^{-4}$ | 0.0285 | 0.1909 |
| RMSE-ADMD | $1.9 \times 10^{-4}$ | 0.0051 | 0.1062 |

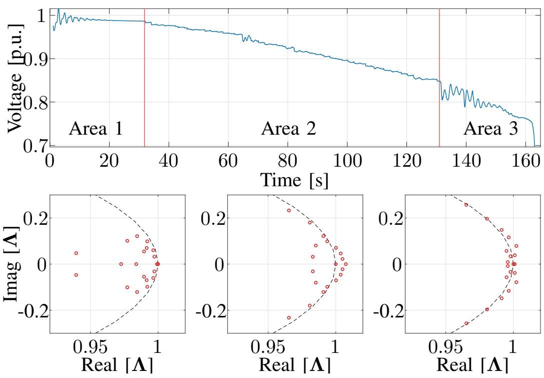


Fig. 6: Eigenvalue tracking along with voltage evolution. The top sub-figure is the voltage at bus 4041 in case 2. The bottom sub-figures from left to right are the eigenvalue plots of areas 1, 2, and 3, respectively.

## IV. Conclusions

A method for voltage stability monitoring based on adaptive dynamic mode decomposition (ADMD) has been proposed in this paper. The ADMD is used to learn the dynamic response of voltages. Then, the voltage evolution is predicted using the learned dynamic model. The simulation results of different scenarios with the Nordic32 system and a real-world oscillation event verify the high-accuracy performance of the proposed method in dynamic voltage prediction. Unlike the conventional DMD method, the proposed ADMD with adaptive training samples is able to predict both stable and unstable evolutions of the voltage, as well as the voltage dynamic as a result of the control response of OELs or LTCs.

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