

# Energy dissipation of a friction damper: experimental validation

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# Energy dissipation of a friction damper: experimental validation

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## Abstract

Friction is frequently seen as an unwanted phenomenon whose influence has to be either minimised or controlled. In this work one of the positive sides of friction is investigated: friction damping. The friction inherently present in a system can be positively used to increase the total damping or alternatively, a friction damper can be designed. Friction dampers can be a cheap and efficient way to reduce the vibration levels of a wide range of mechanical systems. In the present paper the conclusions of previous analytic and numerical results regarding friction damping are validated with results of laboratory experiments, where the energy dissipated through friction is measured. The test set-up consists of a mass sliding on parallel ball-bearings, where additional friction is created by a sledge attached to the mass, which is pre-stressed against a friction plate. No care has been taken to ensure pure dry (Coulomb) friction. Nevertheless, the measured energy dissipation is in good agreement with the theoretical results for Coulomb friction.

## 1 Introduction

Frictional forces arising from the relative motion of two contacting surfaces are a well-known source of energy dissipation [1, 2, 3]. Sometimes this is an unwanted effect of the design, but it can also be intentionally used to increase the damping of a certain system in a simple and cost-effective way. In [4] the energy dissipated through friction has been studied for a mass-spring-damper system subjected to frictional contact with a periodically oscillating base plane, Figure 1(a). In that work the existence and stability of a periodic solution has been proved and subsequently analytical expressions for the energy dissipated per cycle are derived. Expressions for the optimum friction force and maximum energy dissipation have also been obtained as a function of the system parameters. The friction force is modeled using the classical Coulomb friction model. In practice, this model is a too simplified representation of frictional interaction.

In later work [5] a numerical analysis has been presented for the same damper where the influence of the friction model on the predicted energy dissipation is studied. The stick-phase is modelled using continuous approximations (viscous damping, arctan function) and implementations of the Stribeck effect and viscous damping are considered for the slip-phase. It is shown that the use of continuous approximations to model the stick-phase (viscous damping, arctan function) has little influence on the predicted energy dissipation and that the exact behavior during the stick-phase will not significantly influence the prediction of the maximum energy dissipation and the optimal friction force. Furthermore the optimum friction force and maximum energy dissipation are relatively insensitive to the choice of friction model in the sliding region, as long as realistic values of the parameters are used.

In the present paper the conclusions of previous work are validated with results of laboratory experiments, where the energy dissipated through friction is measured. The test set-up consists of a mass sliding on parallel ball-bearings, where additional friction is created by a sledge attached to the mass, which is pre-stressed against a friction plate. No care has been taken to ensure pure dry (Coulomb) friction. Nevertheless, the

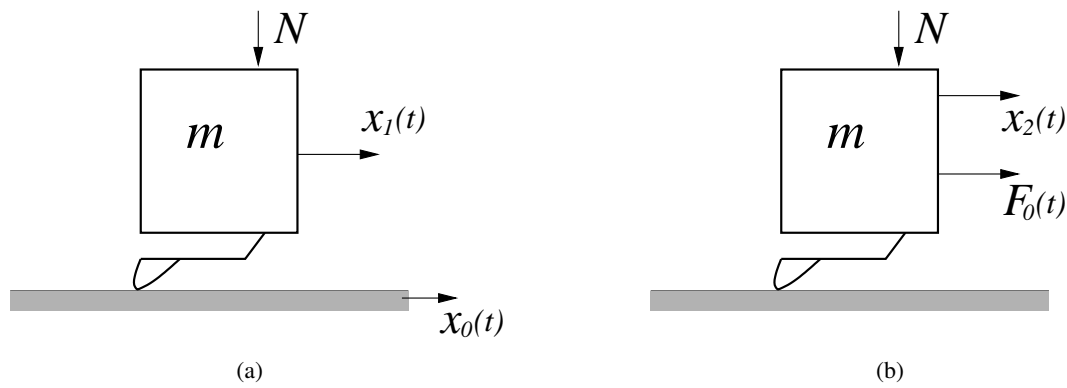


Figure 1: 1-DOF systems, (a) mass on an oscillating base and (b) mass driven by a periodic force.

measured energy dissipation is in good agreement with the theoretical results for Coulomb friction presented in [4].

This paper is organized as follows, in section 2 the main results of the theoretical analysis are summarized and extended to the case of a mass excited by a periodic force. The experimental set-up and the measurement procedure are described in section 3 and the experimentally determined energy dissipation is compared to the analytical results in section 4. In section 5 a thorough discussion of the findings from section 4 follows and finally, in section 6 the main conclusions are summarized.

## 2 Theoretical analysis of periodically forced systems with friction

### 2.1 Mass on an oscillating base

In the system of Figure 1(a) the base plane oscillates with a displacement  $x_0(t) = X_0 \sin(\omega_0(t))$ . If the classical Coulomb friction model as defined in (1) is used, closed-form expressions for the motion of mass  $m$  can be derived and consequently, the energy dissipated per cycle can be determined.

$$F_r = \begin{cases} F_d \text{sign}(v) & |v| > 0, \\ [-F_d, F_d] & |v| = 0 \end{cases} \quad (1)$$

where  $v$  is the relative velocity between mass and base.

Two non-dimensional parameters are defined: normalized friction force and normalized energy dissipation.

$$f_r = \frac{F_r}{mX_0\omega_0^2} \quad (2)$$

$$e_d = \frac{E_d}{mX_0^2\omega_0^2} \quad (3)$$

where  $F_r$  and  $E_d$  are the friction force and the energy dissipated per cycle.

The dissipated energy per period,  $E_d$ , can be calculated according to the following expression:

$$E_d = \int_0^T -F_r(\dot{x}_0(t) - \dot{x}_1(t)) dt \quad (4)$$

where  $T$  is the period length of the periodic solution. An expression for the energy dissipation as a function of the friction force has been found in [4] and is plotted in figure 2. It is also shown that the normalized

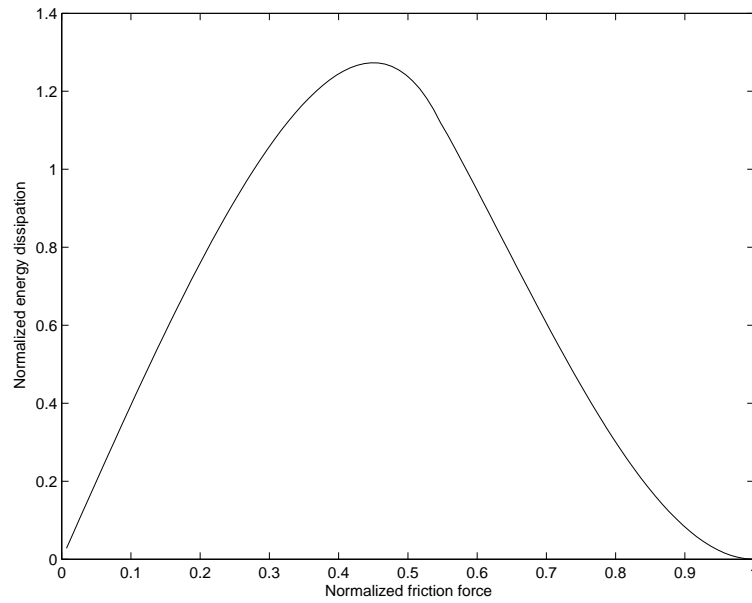


Figure 2: Normalized energy dissipation vs. normalized friction force.

optimum friction force and maximum energy dissipation are:

$$f_r|_{max} = \frac{\sqrt{2}}{\pi} \quad (5)$$

$$e_d|_{max} = \frac{4}{\pi} \quad (6)$$

The system in Figure 1(a) is difficult to realize in an experimental set-up, whereas the system in Figure 1(b) is relatively straight-forward to build. In the next section it will be shown that these two systems are equivalent and therefore, the normalized energy dissipation as plotted in Figure 2 is also valid for the system in Figure 1(b).

## 2.2 Mass driven by a periodic force

The equations of motion for the mass on an oscillating base shown in Figure 1(a) can be written as

$$m\ddot{x}_1(t) = m\ddot{x}_0(t) \quad (\text{stick}) \quad (7)$$

$$m\ddot{x}_1(t) = -F_r \text{sign}(\dot{x}_1(t) - \dot{x}_0(t)) \quad (\text{slip}). \quad (8)$$

Similarly, the equations of motion for the mass excited by a periodic force (Figure 1(b)) are

$$m\ddot{x}_2(t) = 0 \quad (\text{stick}) \quad (9)$$

$$m\ddot{x}_2(t) = F_0(t) - F_r \text{sign}(\dot{x}_2(t)) \quad (\text{slip}). \quad (10)$$

Comparing both sets of equations it can be concluded that the systems are equivalent if

$$\dot{x}_2(t) = \dot{x}_1(t) - \dot{x}_0(t) \quad (11)$$

$$F_0(t) = -m\ddot{x}_0(t). \quad (12)$$

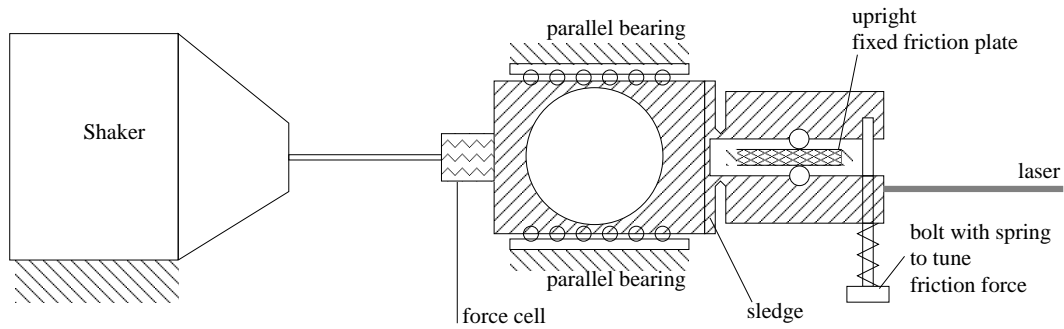


Figure 3: Top-view of the experimental setup with the new friction addition.

Consequently the normalized friction force and energy dissipation are redefined for the mass excited by a periodic force:

$$f_r = \frac{F_r}{F_0} \quad (13)$$

$$e_d = \frac{m\omega_0^2 E_d}{F_0^2} \quad (14)$$

where  $F_0$  is the amplitude of the periodic force ( $F_0(t) = F_0 \sin(\omega_0 t)$ ). These equations imply that the optimum friction force is independent of the mass and the excitation frequency and only depends on the amplitude of the excitation. The corresponding maximum normalized energy dissipation is then fixed and independent of the mass and excitation frequency and amplitude.

The dissipated energy for the mass excited by a periodic force can be obtained from (4), considering the equivalence given in (11). Therefore, with the normalized friction force and energy dissipation defined as in (13, 14), the energy dissipation as a function of the friction force for a mass excited by a periodic force is shown in Figure 2. Furthermore, the optimum normalized friction force and the maximum energy dissipation are also the same as for the mass on an oscillating base and given by (5, 6).

Since the two systems are equivalent, in the following the results of the analytical model for the mass on an oscillating base will be validated with experiments performed for a mass excited by a periodic force.

### 3 Description of the experiments

#### 3.1 Description of the experimental setup and measurement procedure

The experimental investigation of energy dissipation of dry friction has been done at a setup where a shaker forces a sledge back and forth. In Figure 3 a top-view sketch of the setup is depicted. As friction lip a steel part is used which is stiff in the translating direction and elastic in the direction perpendicular to the translation. A bearing ball is attached to the part and used as friction tip. The bearing balls slide along a polished silicium-carbonate plate which is fixed to the underground. The normal force between the bearing ball and the plate is adjustable by turning the bolt which pushes a spring. In this setup the sledge behaves as a free mass in a frequency range from approximately 10 to 120 Hz.

A laser-doppler velocimeter is used to acquire the velocity and the displacement. The force cell measures the force of the shaker on the sledge and at the sledge is an acceleration sensor attached. The data-acquisition and the control of the excitation force is done with a Siglab interface. To excite the sledge periodically by the shaker, a sinusoidal voltage signal is sent from the Siglab interface to a current amplifier. The current amplifier transforms the voltage signal to a current signal that is sent to the electromechanical shaker.

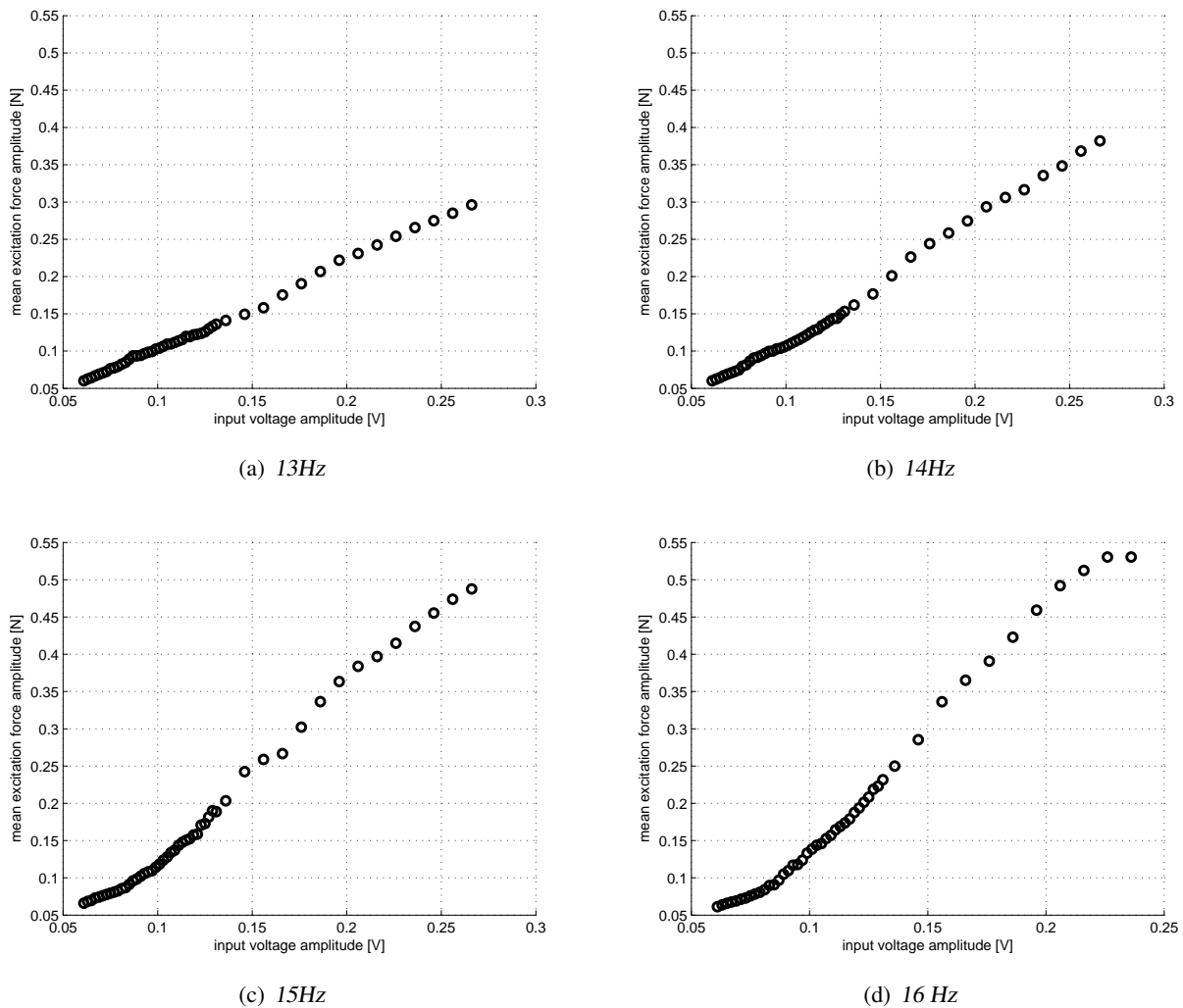


Figure 4: Force amplitude against input voltage amplitude, (a) 13Hz, (b) 14Hz, (c) 15Hz and (d) 16Hz.

The normal force is kept constant during a measurement series and the type and amount of friction in the set-up is completely unknown. During an experiment a sinusoidal input signal is used to let the shaker excite the sledge with a periodic force. At a defined excitation frequency  $\omega_0$ , the voltage amplitude,  $V_0$  of the sinusoid is increased or decreased in small steps. The variation of normalized friction force ( $f_r = F_r/F_0$ ) is therefore achieved by varying the excitation force,  $F_0$  of the shaker on the sledge. The range of the excitation amplitude levels used in the experiments is sufficiently large to ensure that the transition from stick-slip to continuous sliding and the maximum normalized energy dissipation can be observed.

Low excitation frequencies are preferred, because at low excitation frequencies the excitation force amplitude is less sensitive to changes in the input voltage amplitude level. This is shown in Figure 4, where the mean excitation force amplitude over 25 cycles is plotted as a function of the input voltage amplitude for 4 different excitation frequencies. It is clear that the slope of the lines increases as the excitation frequency increases. This is caused by the dynamic behavior of the electromagnetic shaker [6]. As a consequence of this effect, small input voltage variations lead to large excitation force variations at higher frequencies and the stick-slip phase cannot be measured, since the resolution of the measurement system is too coarse to allow for small enough input voltage variations. Therefore the excitation frequency is kept low in order to be able to identify the stick-slip phase and the transition from stick-slip to continuous sliding.

Measurements are performed at excitation frequencies of 13, 14, 15, 16 and in one measurement also at 17 and 18 Hertz. A series always starts at 13 Hertz increased til the highest frequency. At each excitation

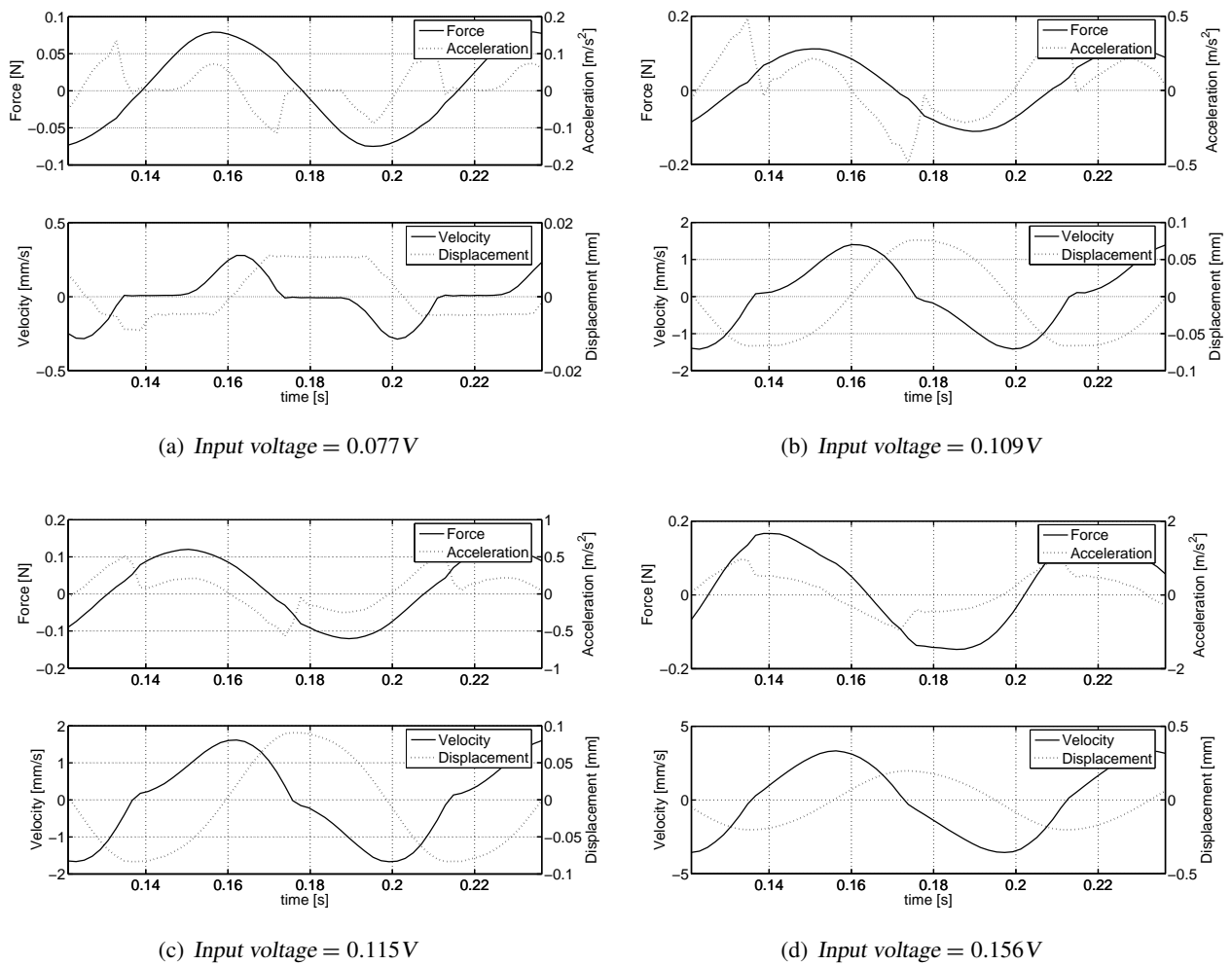


Figure 5: Time plots of force and acceleration, velocity and displacement for different input amplitude voltage level at a frequency of 13 Hz.

frequency the excitation amplitude is first increased and then decreased with small steps. At each excitation voltage amplitude level the actual excitation force on the sledge and the acceleration, velocity and displacement of the sledge are measured for 25 excitation periods. This together is called a measurement series. After one series wear has affected the plate and bearing balls too much to be used again and plate and the bearing balls are changed for a new series of measurements. In total four measurement series have been completed to check the repeatability and robustness of the results.

An impression of the measured signals is given in Figure 5, where the measured force, acceleration, velocity and displacement are plotted for an excitation frequency of 13 Hz and four different input voltage amplitudes. A similar behavior is observed at other excitation frequencies.

In Figure 5(a) the system is clearly in stick-slip phase. At a higher excitation force, the system passes from stick-slip phase into continuous sliding for an input voltage between 0.109V and 0.115V. At an input voltage of 0.109V stick-slip is seen in Figure 5(b) where the velocity signal is sticking at zero and at the same time the acceleration is zero as well, while at an input voltage of 0.115V, depicted in Figure 5(c), a non-zero acceleration signal is measured at the time instant where the velocity is zero which indicates that the system is in continuous sliding. It can also be seen that higher input voltage amplitudes result in a larger phase shift between force and velocity. It is due to this phase-shift that frictional energy dissipation occurs. The phase-shift between velocity and force at maximum energy dissipation, in Figure 5(d), is about 0.01 second which is about one eighth of the period. This value is in good agreement with the theoretical value for the free

mass model found in [4].

The force of the shaker to the sledge is not a perfect sine wave. This is due to the fact that the exerted force is altered by the velocity and friction forces on the sledge. This deviation from the sine wave is not problematic for the current study as long as the force is periodic. Moreover, the amplitude of the force is non-linear with the amplitude of the input voltage control signal. This is not considered a problem since the actual excitation force is measured.

### 3.2 Calculation of normalized energy dissipation and the normalized friction force

In the measured frequency range the experimental setup can be modeled as a free mass excited by a periodic force as in Figure 1(b). The friction force can be calculated from

$$F_r(t) = F_0(t) - m a(t) \quad (15)$$

where  $F_0(t)$  is the measured force on the sledge,  $m$  is the mass and  $a(t)$  the measured acceleration of the sledge. The dissipated energy is calculated with the following expression.

$$E_d = \frac{\Delta t}{n} \sum_{k=1}^{n \cdot N} F_r(t_k) v(t_k) \quad (16)$$

where  $N$  is the number of time samples in one period,  $n$  is number of periods measured,  $v(t)$  the velocity of the sledge and  $\Delta t$  is the sampling time. This energy dissipation can be normalized by applying 14, where the amplitude of the excitation force is calculated as follows:

$$F_0 = \frac{1}{2n} \sum_{i=1}^n \max(F_0(t_k)) - \min(F_0(t_k)) \quad \text{with } k = (i-1)N + 1, \dots, iN \quad (17)$$

To obtain the normalized friction force,  $f_r = F_r/F_0$ , the expression for the normalized friction force at the threshold of stick-slip to continuous sliding obtained in [4] has been applied:

$$f_r |_{threshold} = \sqrt{\frac{1}{1 + \frac{\pi^2}{4}}} = 0.5370 \quad (18)$$

This equation implies that, for a given friction force, the level of excitation force needed to bring the system in continuous sliding is independent of the mass and the excitation frequency. The normalized friction force has been determined based on the excitation force at the threshold of stick-slip to continuous sliding  $F_0(threshold)$ . The threshold is determined by comparing the acceleration and velocity signals. Stick is observed if the acceleration signal crosses zero when the velocity is zero. The accuracy is limited by the increments used for the excitation amplitudes and the sample frequency. The transition from stick-slip to continuous sliding occurs often at the same excitation force for all periods. The friction force can then be estimated from  $f_r |_{threshold} \times F_0(threshold)$  and the normalized friction force becomes:

$$f_r = \frac{f_r |_{threshold} \times F_0(threshold)}{F_0} \quad (19)$$

In the next section the experimentally determined normalized energy dissipation from equations (14) and (16) is plotted as a function of the normalized friction force obtained from (19) and compared to the analytical result from [4] (see Figure 2).



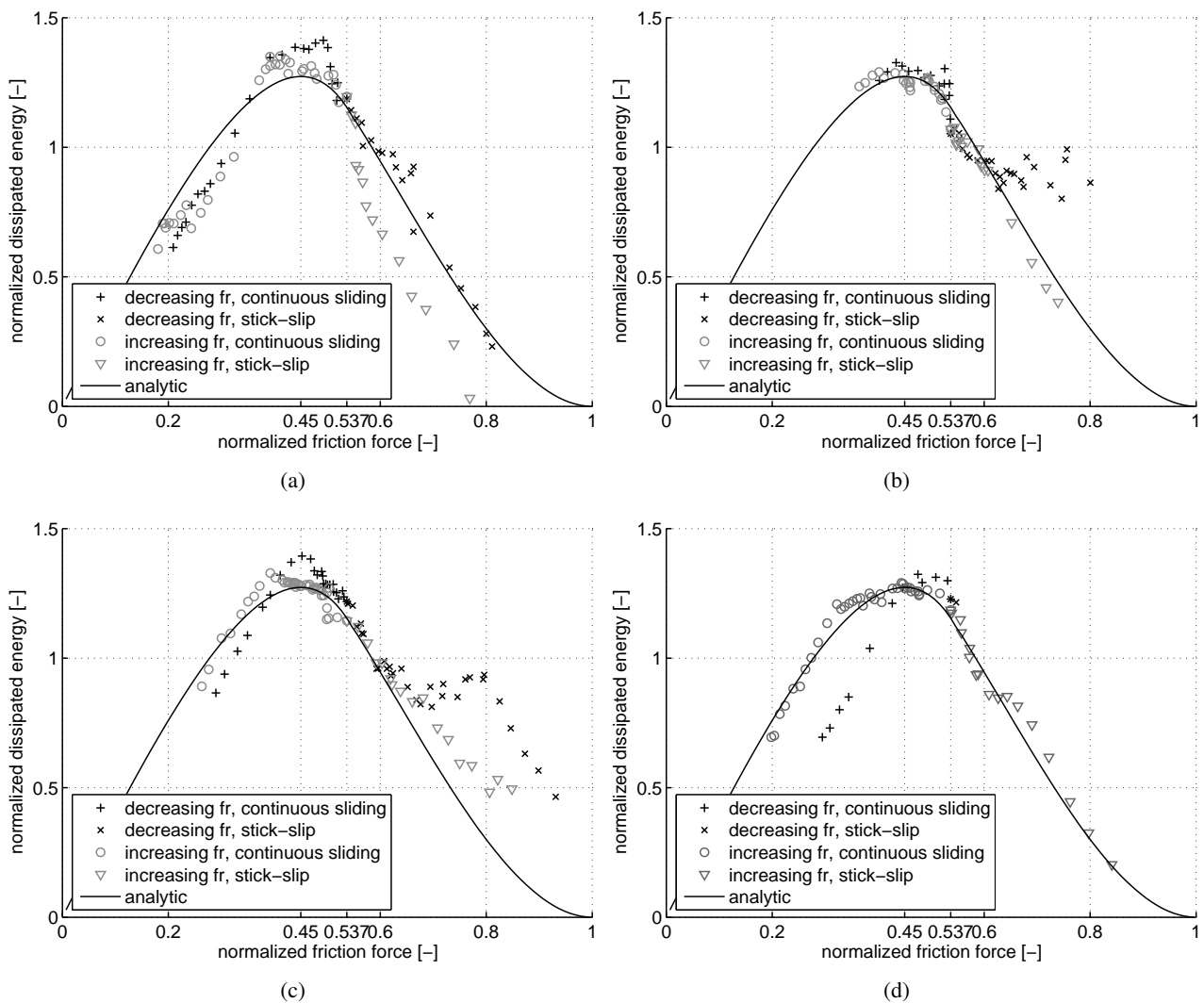


Figure 6: Normalized energy dissipation against normalized friction force at 13 Hz for 4 different measurement series, (a)A, (b)B, (c)C, (d)D

## 4 Comparison of calculated and measured energy dissipation

It has already been mentioned that four measurement series have been completed and that, for each measurement series several different excitation frequencies have been measured. The experimentally determined energy dissipation for the excitation frequencies of 13 Hz and 16 Hz is compared to the analytical curve in Figures 6 and 7 respectively. The results for the four measurement series are shown. In every plot a distinction is made between decreasing and increasing normalized friction force and between the stick-slip and continuous sliding regions. Please note that a decreasing normalized friction force corresponds to an increasing excitation force and vice-versa. The continuous solid line corresponds to the theoretically determined normalized energy dissipation from [4].

In general a very good agreement is found between the experimentally determined friction force and the theoretical results. The experimental results follow the theoretical curve nicely in most cases and the results are repeatable. Deviations are seen mostly in the stick-slip region (higher values of  $f_r$ ), where the experimentally determined normalized energy dissipation is often higher than the predicted value. This can be explained by the difficulty to perform accurate measurements in this region and possibly by viscous effects as will be shown later. An exception to this behavior is found in Figure 7(a), where the experimentally determined energy dissipation suddenly drops to values below the theoretical curve in the stick-slip region. This

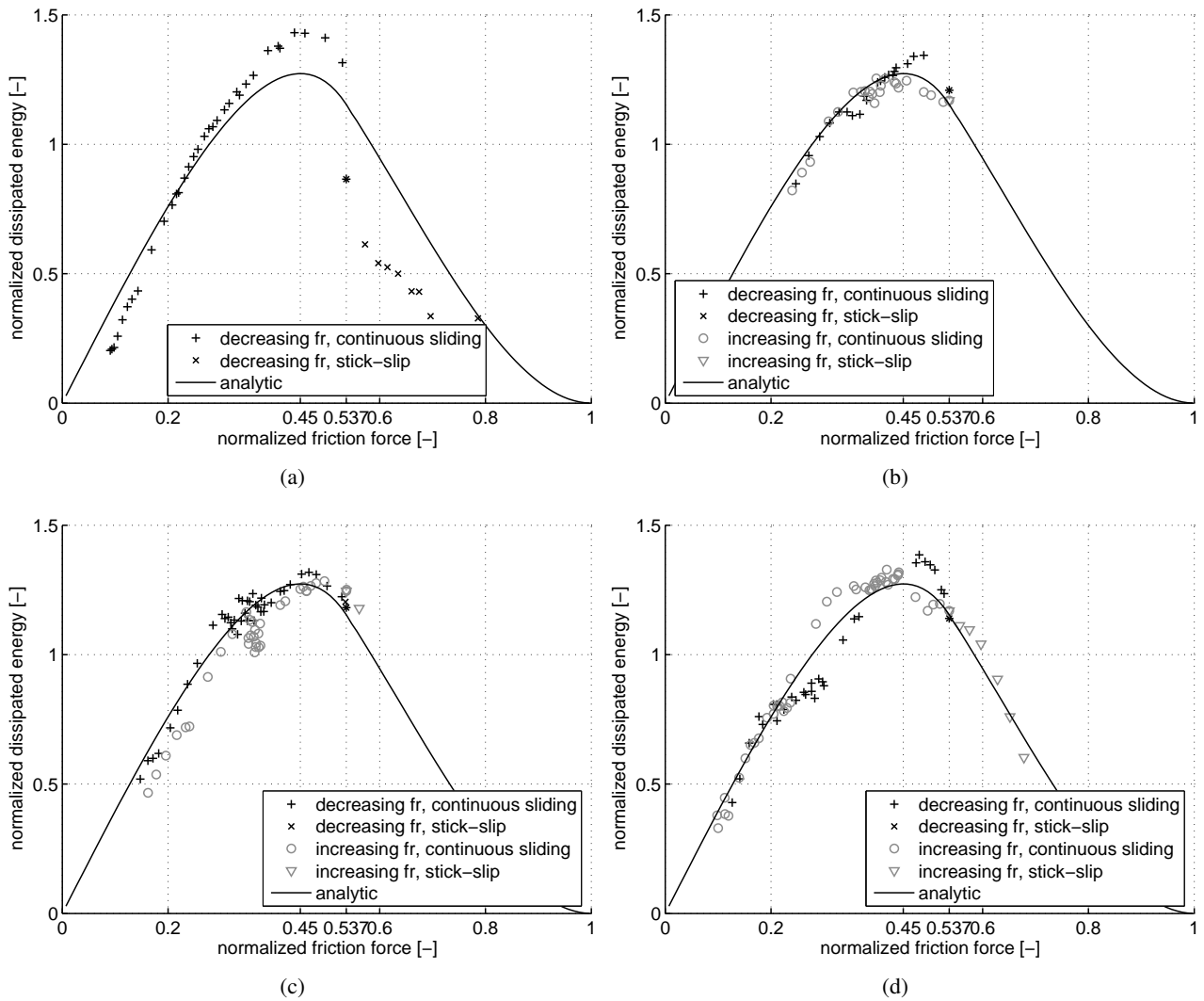


Figure 7: Normalized energy dissipation against normalized friction force at 16 Hz for 4 different measurement series, (a)A, (b)B, (c)C, (d)D

might be due to a difference between the static and dynamic friction force in the set-up, as shown in [4].

In the continuous sliding region (lower values of  $f_r$ ) the experimentally determined values show a good agreement with the theory except for the result in Figure 6(d) with increasing excitation force. In this case, the measured values are well below the predicted normalized energy dissipation. A close look reveals that the optimum normalized friction force is also higher than the theoretical value in this case. This together with the fact that it was not possible to measure the stick-slip region in this case lead to the conclusion that the discrepancies might be due to an error in determining  $F_0(threshold)$  in (19) and, therefore, to an error in the scaling of the normalized friction force.

When looking at the results in Figures 6 and 7 one should bear in mind that before each measurement series the friction plate and bearings are replaced and that the normal force is manually adjusted by fastening the bolt shown in Figure 3. No care has been taken to achieve a given normal force, which means that the actual friction force can vary from one measurement series to the next and this is probably the case. Therefore, the fact that the measured maximum energy dissipation does not significantly change for the different measurement series leads to the conclusion that the maximum normalized energy dissipation does not depend on the actual value of the friction force, as predicted in (6).

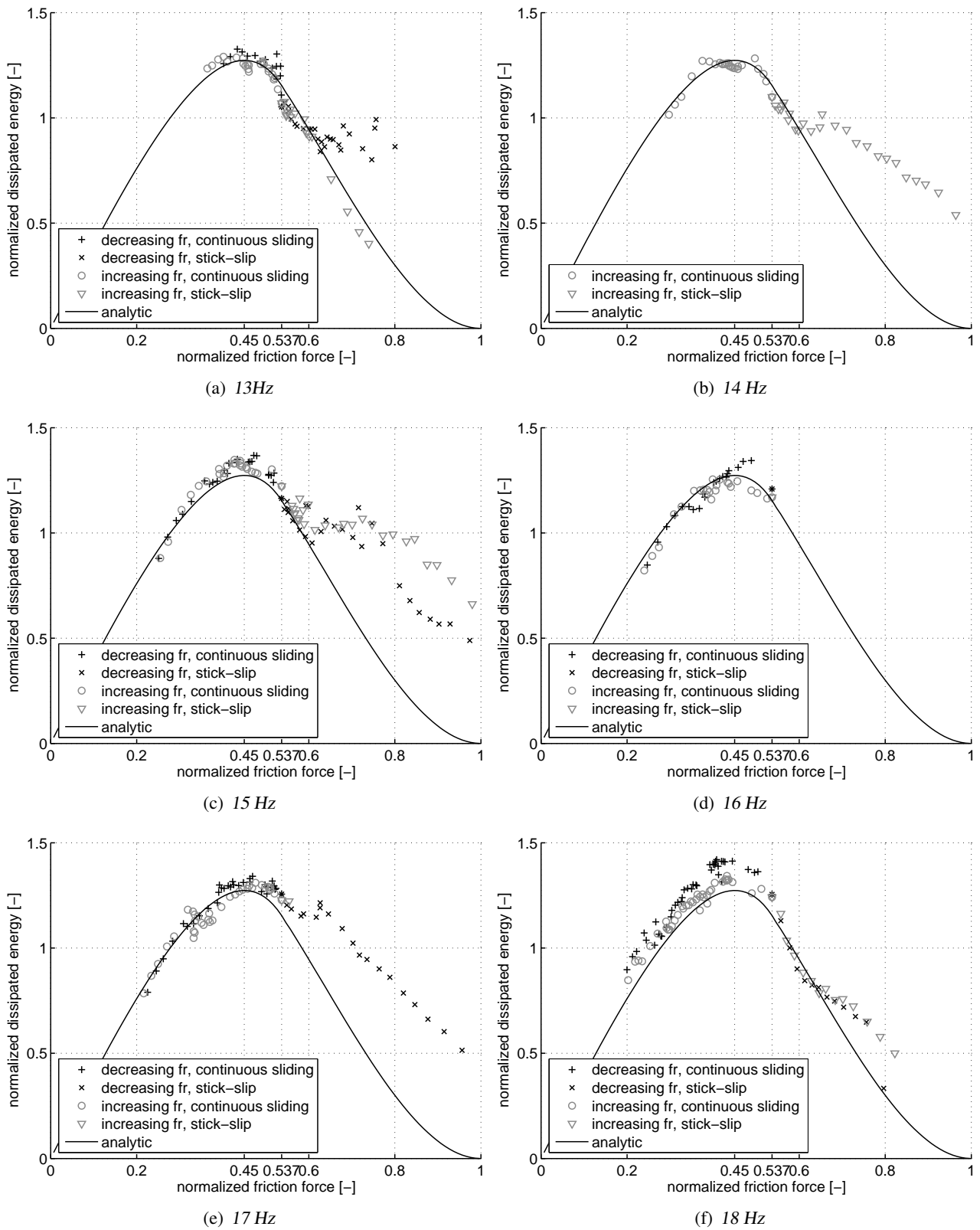


Figure 8: Normalized energy dissipation against normalized friction force for measurement series B, (a) 13Hz, (b) 14Hz, (c) 15Hz and (d) 16Hz.

Comparing Figure 6 to Figure 7 leads to the conclusion that the excitation frequency has no influence on the normalized energy dissipation, as was expected from theory. This can be more clearly seen in Figure 8 where the experimentally determined normalized energy dissipation is plotted for 6 different excitation frequencies within one measurement series. Again a distinction is made between decreasing and increasing friction force and the stick-slip and continuous sliding regions. The theoretical normalized energy dissipation is given by the solid line as before. It can be concluded that the normalized energy dissipation as a function of the normalized friction force is roughly independent of the excitation frequency. Furthermore, the difference between the experimentally determined maximum normalized energy dissipation and the value predicted from (6) is less than 10% in all cases and the corresponding optimum normalized friction force is approximately equal to the predicted value.

Finally, the results in Figure 8 show that the experimentally determined normalized energy dissipation in the stick-slip region is significantly larger than the theoretical curve for most excitation frequencies. The possible reasons for this discrepancy will be discussed in the next section.

## 5 Discussion

The experimental results of the previous section show a very good agreement with the results from the analytical model presented in [4]. This is a remarkable result, since the theoretical results have been obtained for the most simple form of friction model, namely Coulomb friction, and the type and amount of friction present in the set-up are unknown. This result validates the conclusions derived in [5], where numerical experiments for several different friction models are reported and it is shown that the normalized energy dissipation versus normalized friction force is relatively insensitive to the choice of friction model. In particular, the optimum value of the friction force and the maximum normalized energy dissipation seem to be rather robust, since approximately the same maximum energy dissipation and optimum friction force is found in all experiments (4 measurement series, 6 different excitation frequencies). It should be noted that the friction plate and the bearing balls are replaced after each measurement series, which means that variations in the friction force can be expected.

The main discrepancies between experiments and theory are found in the stick-slip region (higher values of  $f_r$ ). This could be due to the difficulties to perform accurate and repeatable measurements in this region, but it is more likely that the larger energy dissipation measured is due to the presence of viscous damping in the system. Possibly the lubrication oil in the linear bearings introduces a small amount of viscous damping. In Figure 9 the calculated normalized energy dissipation for a system with viscous damping in the stick phase is shown for increasing values of the damping parameter. The highest curve corresponds to an unrealistically high value of the damping parameter [5].

It is clear that the effect of viscous damping is to increase the energy dissipation in the stick-slip region. Therefore the higher normalized energy dissipation values found in the experiments can partly be explained by viscous damping. However, viscous damping does not explain the fact the normalized energy dissipation seems to stay at a constant value in the region  $0.6 < f_r < 0.8$ . No satisfactory explanation has been found yet for this discrepancy between theory and experiments.

## 6 Conclusions

In the present paper the energy dissipation through friction for a mass excited by a periodic force has been experimentally determined and compared to theoretical results from a model with Coulomb friction. The measured energy dissipation is in excellent agreement with the energy dissipation predicted by the model. This is a remarkable result, since the theoretical results have been obtained for the most simple form of friction model, namely Coulomb friction, and the type and amount of friction present in the set-up are unknown. Furthermore, the measured maximum energy dissipation and the corresponding optimum friction

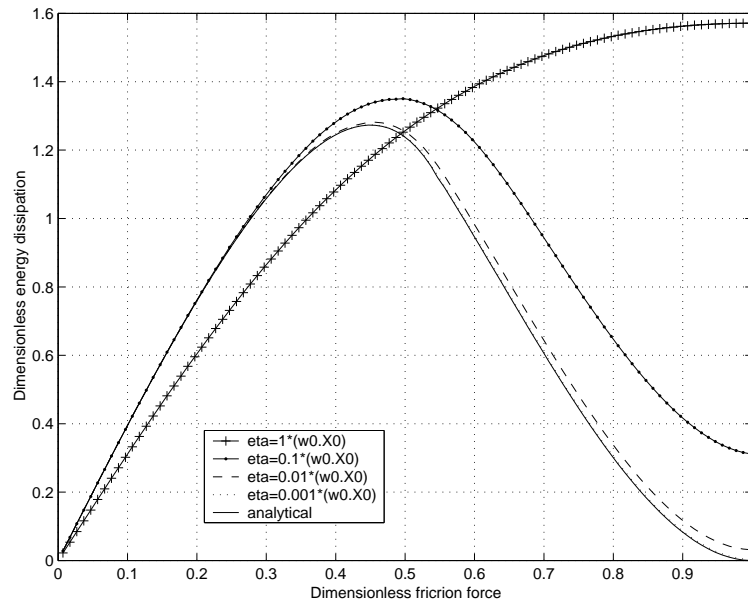


Figure 9: Normalized energy dissipation vs normalized friction force with viscous damping [5].

force agree within 10% with the theoretical prediction. This result is particularly interesting because no care has been taken to ensure predetermined friction characteristics and variations in friction force between each measurement series are very likely. Therefore the maximum energy dissipation and optimum friction force are relatively robust properties of the system, as expected from the theoretical analysis.

It can be concluded that models to predict energy dissipation through friction using Coulomb friction give a good estimation of the maximum energy dissipation and optimal friction force.

## Acknowledgements

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