Design of lightweight floor system with minimized vibration

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Abstract

During the last four decades floor systems used in housing and office-buildings in the Netherlands were mostly made of stone-like materials, and can be characterized as heavy. In recent years, in light of sustainable building methods, the trend is to reduce the use of materials and thus build lighter. Lightweight floor structures are however often found to be more susceptible to vibrations than heavier floor structures. The vibrations are caused by dynamic actions such as walking persons or vibrating machines such as a washing machine.

This paper focuses on a beam as a representation for a floor system, supported by hinges with variable rotational and translational springs. The influences of the parameters involved are described. An analytical approach is used which results in an new approximation formula that can be used in practice. The analytical results are compared to the results found in literature and from numerical calculations. Finally design recommendations are given for the design of lightweight floor systems.

1 Introduction

Traditionally floor systems used in housing and office-buildings in the Netherlands were made of stone-like materials. These floor systems, which can be characterized as heavy, normally posed little problems concerning vibrations. In recent years, in light of sustainable building methods, the trend is to reduce the use of materials and thus build lighter. Lightweight structures are however often found to be susceptible to vibrations. The vibrations are caused by dynamic actions such as walking persons or vibrating machines such as a washing machine. When one of the natural frequencies of the floor system, usually the first, is close to the frequency of excitation, problems can occur. Usually it is found that the higher the first natural frequency, the better the performance.

The vibration behavior of beams for several boundary conditions is well described in literature [2]. The cases discussed are mostly those with free or completely fixed end conditions. Hibbeler [1] discussed the case of a prismatic beam with rotational spring supports and presented a formula that could numerically be solved.

This paper discusses the case of a prismatic beam with rotational spring end supports and presents an approximation formula to find the first natural frequency. This allows for greater possibilities for analysis of the discussed case. Also a parametric study will be presented that will show the influence of relevant parameters on the natural frequency and recommendations for the design of lightweight floor systems will be given.

2 Analytical model

Medium and light weight floorsystems that have been developed in the past mostly have a principle direction for the loadbearing. This allows for a lightweight floor system to be regarded as a single span beam supported at both ends. In the engineering practice it is often assumed the supports are free hinges. However in most cases this is not true because the support is partly fixed as schematized in Figure 1.

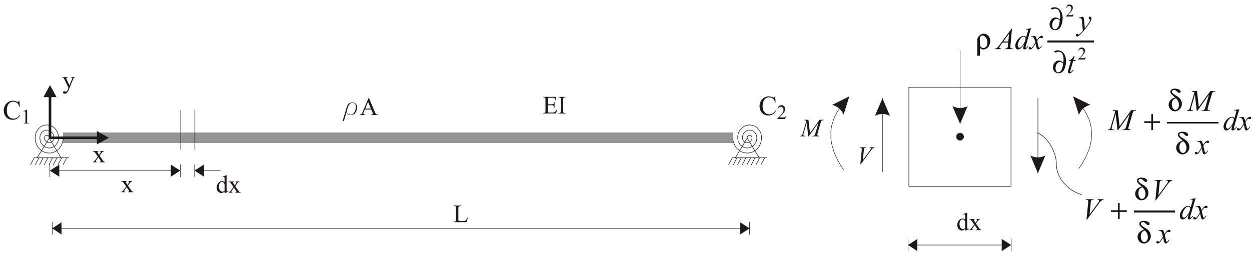

Figure 1, Scheme of structure

Where:

L	[m]	: Length of the beam between the supports
EI	[Nm^2]	: Bending stiffness of the beam
ρA	[kg/m’]	: Mass per unit length, acting as a distributed load
C_1, C_2	[Nm/rad]	: Rotational stiffness of left and right support respectively

2.1 Exact solution for the natural frequency

The exact solution for the natural frequency of the beam with rotational spring supports can be found by solving the differential equation (1) and application of the boundary conditions.

The differential equation [2] governing this structure is:

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}=0 \tag{1}$$

with y = deflection of the beam, t = time

The deflection of a beam in the first natural mode can be written as:

$$y(x,t)=Y(x)(E\cos\omega t+F\sin\omega t) \tag{2}$$

with Y(x) decribes the deflection of the beam as a function of x, ω = angular velocity in rad/s and E and F are constants. Introducing the parameter β as a measure for the frequency given by,

$$\beta^4=\omega^2(\rho A/EI) \tag{3}$$

β	[rad$^{1/2}$/m]	measure for the natural frequency
ω	[rad/s]	angular velocity
ρ	[kg/m^3]	mass
A	[m^2]	area of section
E	[N/m^2]	Young’s modulus of elasticity
I	[m^4]	moment of inertia.

equation (1) combined with equation (2) and (3) results in:

$$\frac{d^4Y}{dx^4} - \beta^4 Y = 0 \tag{4}$$

Equation (4) can be solved for β, by letting Y taking the form

$$Y(x) = A\sin\beta x + B\cos\beta x + C\sinh\beta x + D\cosh\beta x \tag{5}$$

The constants A, B, C and D can be determined by using the boundary conditions for the case under consideration. Hibbeler [Hibbeler, 1975] introduced two parameters, u_1 and u_2, to group the parameters that influence the boundary conditions.

$$u_1 = \frac{C_1 L}{EI}\text{rad}^{-1}, \quad u_2 = \frac{C_2 L}{EI}\text{rad}^{-1} \tag{6}$$

Using these, the boundary conditions can be written as follows.

At x = 0 deflection: $Y(0) = 0$

moment force: $C_1 Y'(0) = EIY''(0) \quad \Rightarrow \quad \frac{u_1}{L}Y'(0) = Y''(0)$ (7)

At x = L deflection: $Y(L) = 0$

moment force: $-C_2 Y'(L) = EIY''(L) \quad \Rightarrow \quad -\frac{u_2}{L}Y'(L) = Y''(L)$ (8)

Combining (5), (7) and (8) and substituting R=*βL*, an equation depending on only 3 variables is found with R [rad$^{1/2}$] as a measure for the natural frequency

$$\begin{aligned}&-u_1 R\sinh(R)\cos(R) + 2R^2\sinh(R)\sin(R) + u_1 R\cosh(R)\sin(R)\\&- u_1u_2\cosh(R)\cos(R) + u_2 R\cosh(R)\sin(R) + u_1u_2 - u_2 R\cos(R)\sinh(R) = 0\end{aligned} \tag{9}$$

Equation (9) has only one unknown variable, R, which has to be solved for. However this unknown is not explicit so it cannot be solved analytically. More than one value of R can be found for a combination of parameters, represented by u_1 and u_2, representing the different modes of vibration. Using numerical methods values for R can be found.

It should also be noted that Equation (9) is of a different form than found by Hibbeler, but it should result in the same values of R.

Equation (9) is graphically shown in Figure 2. It can be seen that for the same combination of values of u_1 and u_2 more values of R are valid. This of course is correct as these values describe different harmonics or higher natural frequencies.

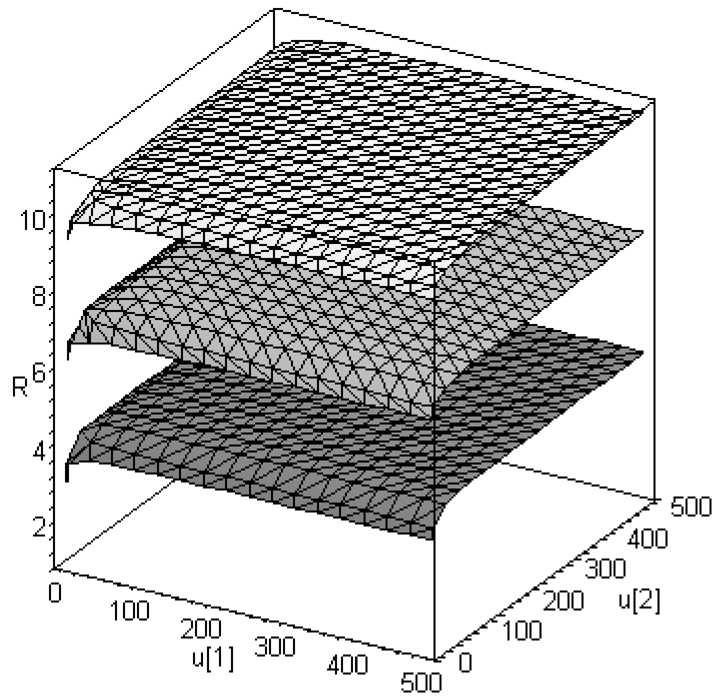

Figure 2, Graphical representation of equation (9)

2.2 Approximation function

In this paper the focus is primarily on the first natural frequency. In practice the first natural frequency is the most important for analyzing floor systems, as this will generally be in the range of the excitation frequency that is below 7 Hz. Higher order frequencies are of less importance. Though the exact solution of (9) will result in accurate values for the first natural frequency, it is not adequate for practical use as one has to use numerical techniques to find a solution. A solution that results in a formula that gives the first natural frequency explicitly is desired. As stated above this cannot be achieved analytically, but it proves possible to find a very accurate approximation function where R is indeed explicit. The approximation function for R, called $\tilde{R}$ will be taken in the form of

$$\tilde{R}_{(u_1,u_2)} = \frac{S_1(u_1+u_2)+S_2(u_1u_2)+S_5}{S_3(u_1+u_2)+S_4(u_1u_2)+1} \tag{10}$$

This function results in generally the same type of graph as with the exact formula. This function has an adequate ability to fit the curve of Figure 2 by choosing adequate values for the five constants, S_1 to S_5. These constants can be found by examining the limit cases as will be shown below. In this approximation function five constants have to be determined. As switching the left and the right side of the structure, resulting in swapping u_1 and u_2, should yield the same value for $\tilde{R}$ the constants for u_1 and u_2 of the same order have to be equal and thus are grouped together.

The first constant to be solved is S_5 and can be solved by examining the limit case where $u_1=u_2=0$. This reduces (10) to $\tilde{R}_{(u_1=u_2=0)} = S_5$. Numerically solving (9) for $u_1=u_2=0$, results in the value $S_5=\pi$, which is obvious as this represents the case of an unrestrained beam. The quotient S_1 / S_3 can be found by examining the limit state where u_1 =0 and $u_2 \to \infty$, which represent a beam fixed at one end and freely supported at the other end. Solving (9) numerically for this case, (10) reduces to

$$\tilde{R}_{(u_1=0;u_2\to\infty)} = \frac{S_1}{S_3} = 3.92660 \tag{11}$$

Expressing S_1 as a function of S_3 and assign u_1=0 we can rewrite (10) as follows:

$$\tilde{R}_{(u_1=0,u_2)} = \frac{3.92660 S_3(u_2)+\pi}{S_3(u_2)+1} \tag{12}$$

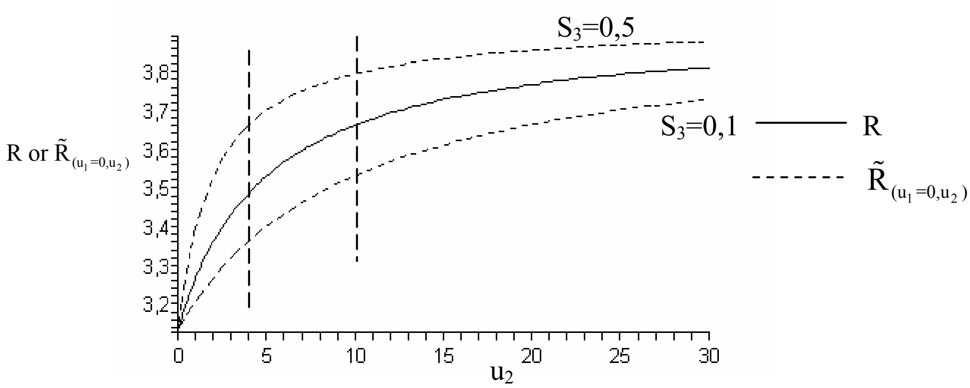

Figure 3, R and $\tilde{R}$ for different values of S_3

Equation (12) will be correct for the extreme values of u_2 =0 and $u_2\to\infty$, combined with u_1 =0 regardless of the value chosen for S_3, but has also to be optimal for all values of u_2 in between. This can be done by choosing strategically a value for u_2 where equations (9) and (12) have to be equal. The graphs of these to equations, using different values for S_3 are shown in Figure 3. The value chosen for u_2 to calculate S_3 is a value where the difference, ΔR, between (9) and (12) is the largest. The range for suitable values of u_2 is shown in Figure 3 by the dashed vertical lines. The value for u_2 of 7 proves to result in the best approximation. After numerically solving (9) for u_2 = 7 we obtain,

$$\tilde{R}_{(u_1=0;u_2=7)} = \frac{3.92660 S_3(7)+\pi}{S_3(7)+1} = 3.59933 \quad \Rightarrow \quad S_3 = 0.19981,\ S_1 = 0.78457 \tag{13}$$

It can be shown that S_2 and S_4 can be found by examining the limit case were $u_1 = u_2 \to \infty$. The resulting approximation function is:

$$\tilde{R} = \frac{0.78457(u_1+u_2)+0.15976(u_1u_2)+\pi}{0.19981(u_1+u_2)+0.03377(u_1u_2)+1} \tag{14}$$

2.3 Validation of approximation function

The derived approximation function (14) has a deviation compared to the exact solution, given implicitly by equation (9). In this section we will show the distribution of this deviation, ΔR, and express it in percent. ΔR at given values of u_1 and u_2 is defined by:

$$\Delta R = \frac{R^2(u_1,u_2) - \tilde{R}^2(u_1,u_2)}{\tilde{R}^2(u_1,u_2)} \bullet 100\% \tag{15}$$

We can now use the solution found by (9) for R and $\tilde{R}$ given by the approximation function (14) with equation (15). This results in an equation depending on only three variables, being ΔR, u_1 and u_2. This equation is plotted in Figure 4. It can be seen from this figure that the maximum error is about 0.07% and this occurs only for small values of u_1 and u_2 between 0 and 10. For larger values the error reduces to 0%, which of course should be the case as we determined the constants S_1 through S_5 by using the limit cases. It can also be seen from the error distribution that the values chosen to determine constants S_1 and S_3 were correct as the maximum positive error equals the maximum negative one.

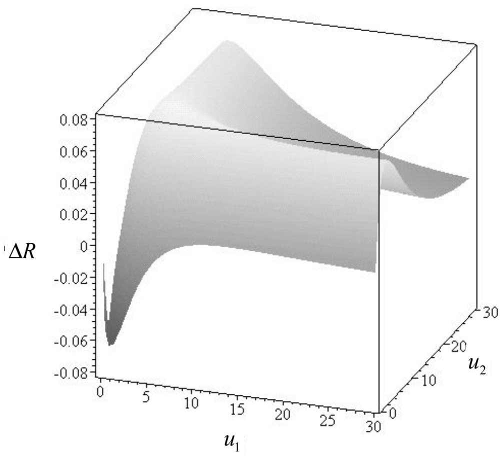

Figure 4, Distribution of deviation according to (15) for approximation formula (14)

3 Results

The first eigen frequency can be found combining equations (3), (14) and $f_e = \omega/(2\pi)$:

$$f_e = \frac{\tilde{R}^2_{(L,EI,C_1,C_2)}}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \quad \text{[Hz]} \tag{16}$$

$\tilde{R}$ depends on 4 parameters that can be described by u_1 and u_2. For the purpose of comparison of the result with the available results in literature the material and geometrical properties are chosen in such a way as to obtain the same values for u_1 and u_2 as used by Hibbeler.

For further validation a simple numeric model has been made in the finite element program Ansys.

Table 1, Validation of approximation formula (Extreme combinations, C_1=0…∞, C_2 = 0 or ∞).

							Zegers	Hibbeler	Ansys
ρA	EI	L	C_1	C_2	u_1	u_2	f_e	f_e	f_e
400	7,00E+06	8	0	0	0	0	3,247	3,248	3,247
400	7,00E+06	8	875000	0	1	0	3,523	3,524	3,525
400	7,00E+06	8	8750000	0	10	0	4,418	4,419	4,418
400	7,00E+06	8	87500000	0	100	0	4,976	4,975	4,976
400	7,00E+06	8	∞	0	∞	0	5,072	5,076	5,071
400	7,00E+06	8	∞	∞	∞	∞	7,363	7,360	7,358

Table 2, Validation of approximation formula(Intermediate combination of C_1 and C_2 values).

							Zegers	Hibbeler	Ansys
ρA	EI	L	C_1	C_2	u_1	u_2	f_e	f_e	f_e
400	7,00E+06	8	875000	875000	1	1	3,798	3,129	3,8
400	7,00E+06	8	875000	8750000	1	10	4,704	7,463	4,702
400	7,00E+06	8	875000	87500000	1	100	5,277	5,484	5,275
400	7,00E+06	8	8750000	8750000	10	10	5,685	8,967	5,681
400	7,00E+06	8	87500000	87500000	100	100	7,088	6,588	7,087
400	7,00E+06	8	8750000	∞	10	∞	6,461	6,456	6,457
400	7,00E+06	8	87500000	∞	100	∞	7,223	7,221	7,222

As can be seen from table 1 and 2 the results obtained from the approximation formula, (9), and the results from Ansys correspond very well. The maximum difference that can be found is 0,004 Hz. Also for most values equation (9) corresponds very well with the results found by Hibbeler. Only in the case of u_1 and u_2 being small, i.e. less then 100, and both being greater than zero, a significant difference can be found. Further analysis of the results shows that Hibbeler's function has an asymptote at certain values. This influences the results significantly. The approximation formula presented in this paper doesn't show this behaviour and can be considered valid for the whole frequency range.

3.1 Practical results

A range of values of practical relevance for the parameters involved has been determined. Graphs can be made showing the influence on the first natural frequency of two parameters while choosing a constant value for the other parameters. A total of five parameters define the system.

Table 3, Validation of approximation formula.

Parameter	unit	description
L	[m]	Length of the beam between the supports
EI	[Nm^2]	Bending stiffness of the beam
ρA	[kg/m']	Mass per unit length
C1, C2	[Nm/rad]	Rotational stiffness of left and right support respectively

For this parameter study for each parameter a range of values is determined that are of interest in building practice. The focus of this research is on office and housing buildings. The range of the parameters is characteristic for these types of buildings. Each of these ranges is discussed below.

3.2 Parameter L

The parameter L describes the length of the single span of the floor system. Typical spans in buildings are ranging from a minimal of 5 meters to a maximum of 14 meters.

3.3 Parameter ρA

This parameter describes the mass per unit length. Traditionally wooden floors are the lightest (ca 50 kg/m^2) in normal practice while concrete slab floors are the heaviest, traditionally up to a thickness of 300 mm which corresponds with 750 kg/m^2. Depending on the use of additional materials this can be increased by roughly 250 kg/m^2. Considering a beam with a width of one meter this results in a minimum value of ρA of 50 kg/m' up to 1000 kg/m'.

3.4 Parameter EI

This parameter describes the stiffness of the floor system. The required stiffness depends on the requirements for maximal deflection allowed for the floor. The weight of the floor system combined with the live load determines the loading of the system, defined as q. The live load can vary between 175 kg/m^2 up to 400 kg/m^2 for the regarded buildings. The criterium for the maximum deflection u_{max} = 0.004 * L.

The deflection under a distributed load is given by:

$$u_{\max} = \frac{5}{384}\frac{qL^4}{EI} < 0.004 * L \tag{17}$$

Solving for EI results in:

$$EI > \frac{5qL^3}{384 * 0.004} = 3.255qL^3 \tag{18}$$

The distributed load, q, which is a combination of the weight and the live load ranges from 225 kg/m^2 up to 1400 kg/m^2, or 2.25 – 14 kN/m^2. Combined with the already defined range for parameter L, this results in the range for the stiffness of 0.9 kNm^2 to 12500 kNm^2

3.5 Parameter C_1 and C_2

These parameters describe the amount of rotation stiffness of the supports. The extremes for these parameters are unconstrained or fixed. The unconstrained condition corresponds to a value of 0 while the fixed condition corresponds with a value of ∞. The completely fixed condition will not be practical to achieve, and further examination of the influence of this parameter learns that a maximum value for C = 10^8 Nm/rad gives nearly the same results as a completely fixed support.

3.6 Graphical results and discussion

In Figure 5 the effect every combination of two parameters has on the first natural frequency is shown. For each graph the other parameters are taken constant and the value is the average of the range chosen in the previous paragraphs for that parameter. This results in the center of the graph being at the same level for each graph, so the relative effect of the parameters can be examined.

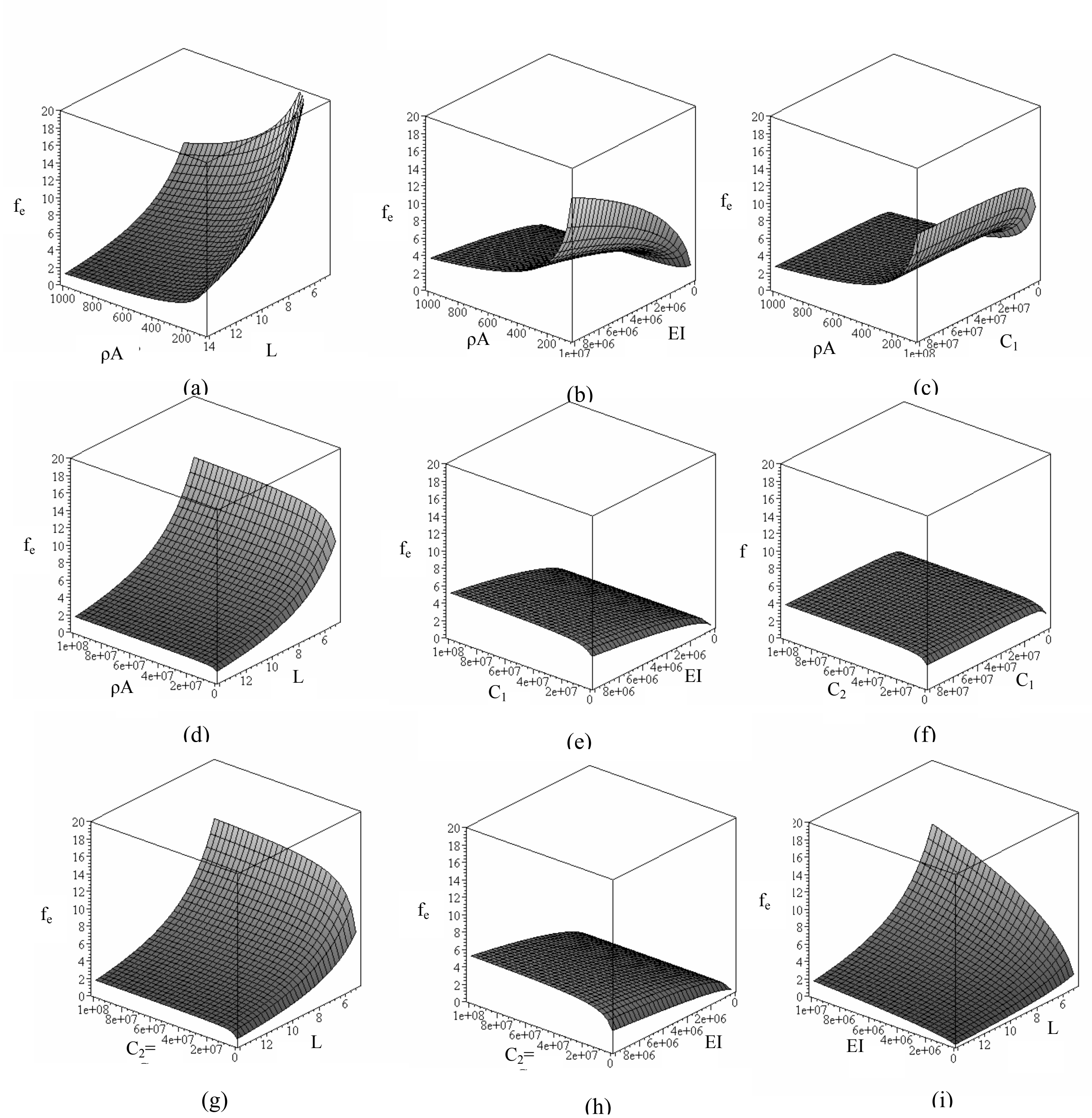

Figure 5, Natural frequency, f_e, depending on two parameters with the others constant

In the Figure 5 some interesting points can be found. The first point to be discussed is graph (f). In this graph the effect of the spring stiffness of the supports on the natural frequency is shown. This graphs shows that already for relatively small values for the spring stiffnesses the natural frequency becomes constant. You only have to design a support with a small stiffness to reduce the first natural frequency considerably. This effect is even stronger on lightweight floor systems. The second point to be mentioned is related to the first point and is shown in graph (g). For smaller spans the absolute effect on the natural frequency of the spring stiffness is greater than for larger spans. Thirdly it is mentioned that the beam stiffness has a larger effect on a beam on lower masses than on higher masses as can be seen in graph (b). Lastly it should be mentioned that the length of the span has the biggest influence on the natural frequency. But if you keep this parameter constant, the mass has the second greatest impact.

4 Conclusions

- An accurate approximation function for the first natural frequency of a beam with rotational spring supports has been derived
- Lightweight floor systems benefit more from rotational spring stiffness at the supports than heavier floors.
- The majority of the possible increase of the natural frequency, due to rotational springs, can be obtained already by designing supports with relatively small rotational spring stiffness.
- Lightweight floors benefit more from higher stiffness than heavier floors.

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