Two measurement techniques to determine Higher Order Sinusoidal Input Describing Functions

Two measurement techniques to determine Higher Order Sinusoidal Input Describing Functions

P. Nuij, M. Steinbuch
Eindhoven University of Technology, Department of Mechanical Engineering
Den Dolech 2, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
E-mail: **p.w.j.m.nuij@tue.nl**

Abstract

For high precision motion systems, modelling and control design specifically oriented at friction effects is instrumental. The Sinusoidal Input Describing Function theory represents a solid mathematical framework for analysing non-linear system behaviour. This theory however limits the description of the non-linear system behaviour to an approximated linear relation between sinusoidal excitation and sinusoidal response. An extension to Higher Order Describing Functions can be realised by calculating the corresponding Fourier coefficients. The resulting Higher Order Sinusoidal Input Describing Functions (HOSIDFs) relate the magnitude and phase of the higher harmonics of the periodic system response to the magnitude and phase of a sinusoidal excitation. This paper describes two techniques to measure HOSIDFs. The first technique is FFT based. The second technique is based on IQ (=in phase/quadrature phase) demodulation. In a case study both techniques are used to measure the changes in dynamics due to friction as function of drive level in an electric motor.

1 Introduction

In the analysis and synthesis of dynamic systems, linearity is often a prerequisite. Frequency domain based concepts like the Frequency Response Function, Bode plot etc. describe linear system behaviour. If the system is non-linear, the Frequency Response Function describes a linearized version of the system behaviour in a working point or limited operating range. Every real life system is non-linear although the implications are not always noticeable in the operating range. As the required performance of mechanical systems increases, non-linear behaviour becomes of interest due to its adverse influence on system performance. In positioning systems, for example, friction can lead to limit cycling, which deteriorates positioning accuracy and increases wear and power consumption. Even in the simplest trajectory, the transition from the pre-sliding to the sliding regime causes changes in system dynamics due to a significant change in stiffness and damping in the friction contact, as has been demonstrated in [1], [2]. In ultra precision equipment like wafer-steppers and DVD mastering recorders the effect of friction is minimized by the application of gas bearings or fluid bearings. This results in rather complex, expensive mechanical constructions. In the majority of industrial positioning equipment the application of these ultra low friction bearings is too expensive. Hence, the negative effects of friction on the dynamics of the machine have to be taken into account. Often, servo controllers are used to reduce the position errors caused by friction. The complexity of these controllers varies from a basic PID action to sophisticated model based compensation schemes, combining advanced friction models with digital signal processing [3], [4]. Increasing demands on positioning performance call for a steady advance in the synthesis techniques of controllers. The influences of non-linear system behavior have to be taken into account. It is evident that one cannot do without reliable data both for validation of the sophisticated models as well as input for state dependent control actions like gain scheduling as put into practice in these advanced controllers. This requires practical but reliable measurement techniques, which are not limited to linear system behavior. In this work we would like to extend the well-known procedures from frequency response analysis for linear systems, towards a class of non-linear dynamical systems, with harmonic responses where amplitude dependent behavior is obvious. Some approaches have been addressing the describing function analysis [5], [6]. Although this theoretical framework allows for higher order analysis it ignores the influence of higher order components and in this way produces a linearized version of the non-linear system. In [7], [8] an extension of the single input sinusoidal describing functions to multiple inputs is proposed.

In this paper two measurement techniques are presented to determine the complex relations i.e. the magnitude and phase relations, between a fixed frequency sinusoid with variable amplitude and the individual components of the harmonic non-linear system response signal. Hereto we will define the notion of this framework in section 2. In section 3 we will introduce the measurement schemes and in section 4 experimental results of a motion system case study will be shown. Finally, the main results will be discussed in the form of conclusions in section 5.

2 Higher Order Sinusoidal Input Describing Function

2.1 Sinusoidal input describing function

Consider a stable, non-linear time invariant system with an odd non-linearity. Let $u(t) = \hat{a}\sin(\omega_0 t)$ be the input signal. The system response $y(t)$ is considered to be periodic with the fundamental frequency ω_0 of the input signal $u(t)$, i.e. we assume that the transient behaviour has died out. Response $y(t)$ can be written as a summation of harmonics of the input signal $u(t)$, each with an amplitude and phase, which can depend on the amplitude $\hat{a}$ and frequency ω_0 of the input signal, see figure 1.

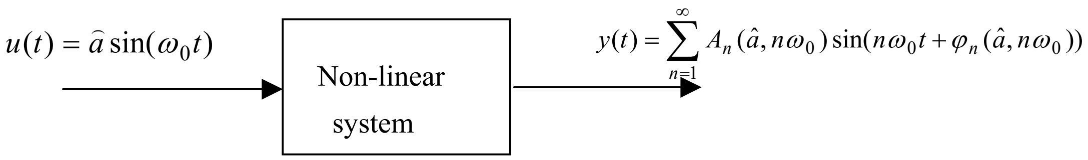

Figure 1: General sinusoidal input-output relation

The describing function H(â,ω) of the system is the complex ratio of the fundamental component $\tilde{y}(t)$ of the system response and the input sinusoid $u(t)$, see figure 2.

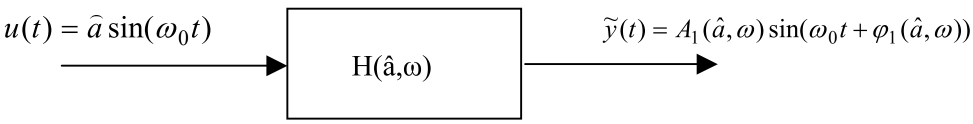

Figure 2: Describing function representation

The describing function H(â,ω) is defined as [9]:

$$H(\hat{a},\omega) = \frac{A_1(\hat{a},\omega)e^{j(\omega_0 t + \varphi_1(\hat{a},\omega))}}{\hat{a}e^{j\omega_0 t}} = \frac{1}{\hat{a}}(b_1 + ja_1) \tag{1}$$

The Fourier coefficients a_1 and b_1 are calculated as in (2), (3) with $T_0 = \frac{1}{\omega_0}$.

$$a_1 = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} y(t)\cos(\omega_0 t)dt \tag{2}$$

$$b_1 = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} y(t)\sin(\omega_0 t)dt \tag{3}$$

2.2 Higher order sinusoidal input describing function

The describing function as defined above can be interpreted as the first order representation of a more global describing function $H_n(\hat{a},\omega)$, see figure 3. This function can be defined as the complex ratio of the n^{th} harmonic component in the output signal to a virtual n^{th} harmonic signal derived from the excitation signal (4). This virtual harmonic has equal amplitude as the fundamental sinusoid and zero phase.

$$H_n(\hat{a},\omega)=\frac{A_n(\hat{a},\omega)e^{j(n\omega_0 t+\varphi_n(\hat{a},\omega))}}{\hat{a}e^{jn\omega_0 t}}=\frac{A_n(\hat{a},\omega)e^{j\varphi_n(\hat{a},\omega)}}{\hat{a}}=\frac{1}{\hat{a}}(b_n+ja_n) \tag{4}$$

In this paper $H_n(\hat{a},\omega)$ will be referred to as the Higher Order Sinusoidal Input Describing Function (HOSIDF).

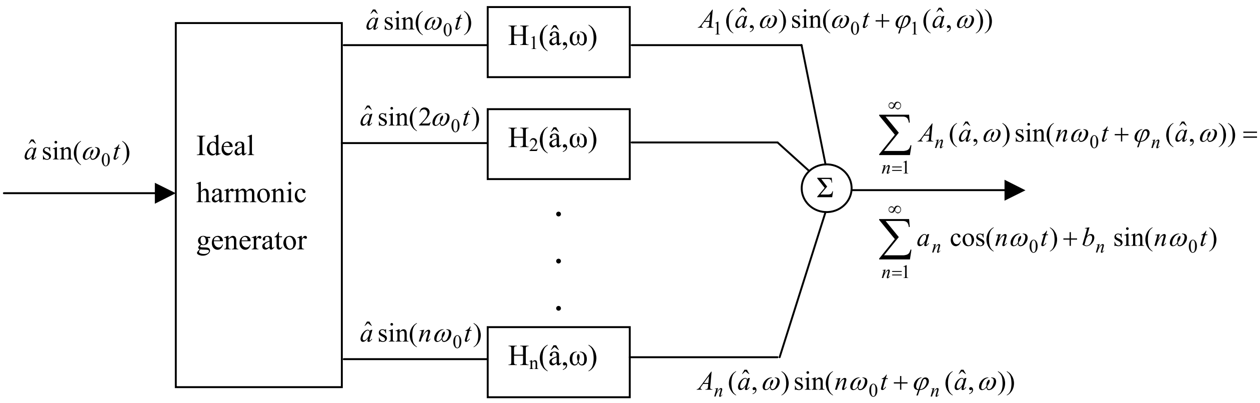

Figure 3: Higher order sinusoidal input describing function representation

3 Measurement techniques for HOSIDFs

As stated in (4), determining HOSIDFs requires a method to measure the complex ratio of two bandpass filtered signals. Two methods are investigated. The first method employs the FFT techniques to determine auto and cross-spectral information and operates upon blocks of data. The second method uses IQ demodulation and is sample based [10], [11].

3.1 FFT method

In this measurement technique both the input signal $u(t)$ and output signal $y(t)$, see figure 1, are Fourier transformed. The data block length is chosen equal to p times the period T_0 of the excitation signal. This assures that all the power of the excitation signal is concentrated in line p. The power of the response signal is fully concentrated in frequency lines n*p so leakage is absent. Let us consider the calculation of the k^{th} order HOSIDF, see figure 4.

The frequency line p with value a_p+jb_p represents the input signal. The Fourier coefficients a_p and b_p are calculated as:

$$a_p = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} y(t)\cos(p\omega_0 t)dt \tag{5}$$

$$b_p = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} y(t)\sin(p\omega_0 t)dt \tag{6}$$

The k[th] component of the output signal is contained in frequency line k*p of its spectrum and has the complex value $a_{kp}+jb_{kp}$. Its Fourier coefficients are calculated as in (5), (6), with p=kp.

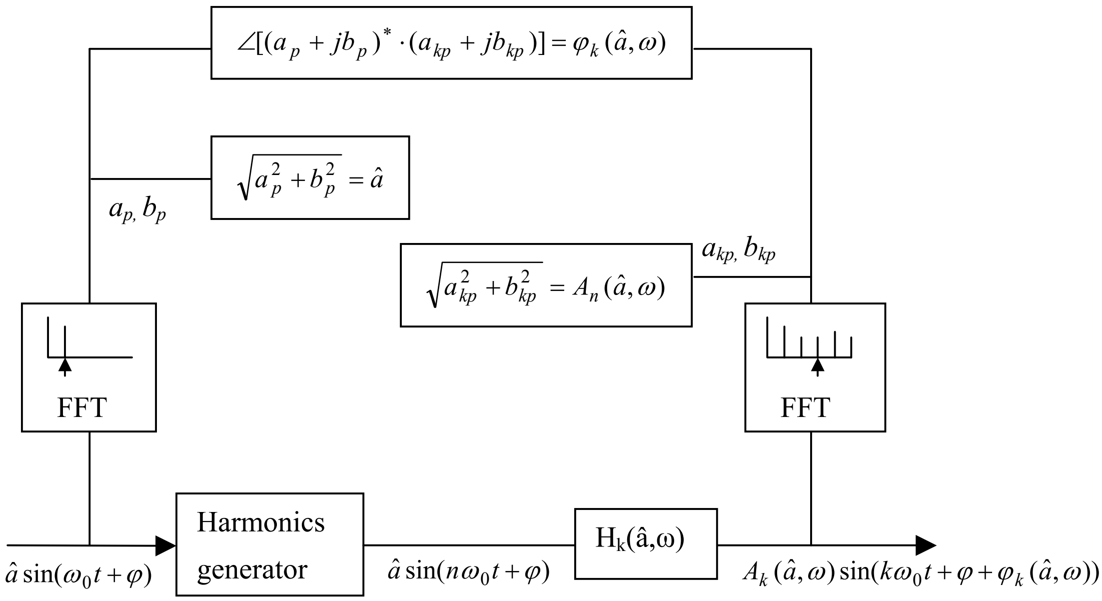

Figure 4. Determination of k[th] order HOSIDF using FFT techniques

The amplitudes $\hat{a}$ of the excitation signal and $A_k(\hat{a},\omega)$ of the k[th] harmonic component in the output signal are calculated from the autospectra. The phase $\varphi_k(\hat{a},\omega)$ of the k[th] order describing function is the phase difference between the real k[th] harmonic component of the output signal and the virtual k[th] harmonic derived from the input signal. In linear system theory the system phase at frequency ω is the phase of the measured cross-spectrum from the input and output signals at frequency ω. In this non-linear situation however we calculate a cross spectrum component between two FFT lines with different frequency.

Let a_p+jb_p be the value of FFT line p of the input signal with frequency ω_0 and let $a_{kp}+jb_{kp}$ be the value of FFT line kp of the output signal with frequency $k\omega_0$, then:

$$a_p + jb_p = e^{j\varphi} \tag{7}$$

$$a_{kp} + jb_{kp} = e^{j(\varphi+\varphi_k(\hat{a},\omega))} \tag{8}$$

$$(a_p + jb_p)^*(a_{kp} + jb_{kp}) = e^{j\varphi_k(\hat{a},\omega)} \tag{9}$$

So the phase of the cross spectrum between the two FFT lines containing the excitation signal and its k[th] harmonic component of the output signal equals the phase of the k[th] order describing function.

3.2 IQ demodulation method

An alternative to the FFT method is the IQ demodulation method [10], [11]. In this method n IQ demodulators decompose the system response signal into its n components, see figure 5.

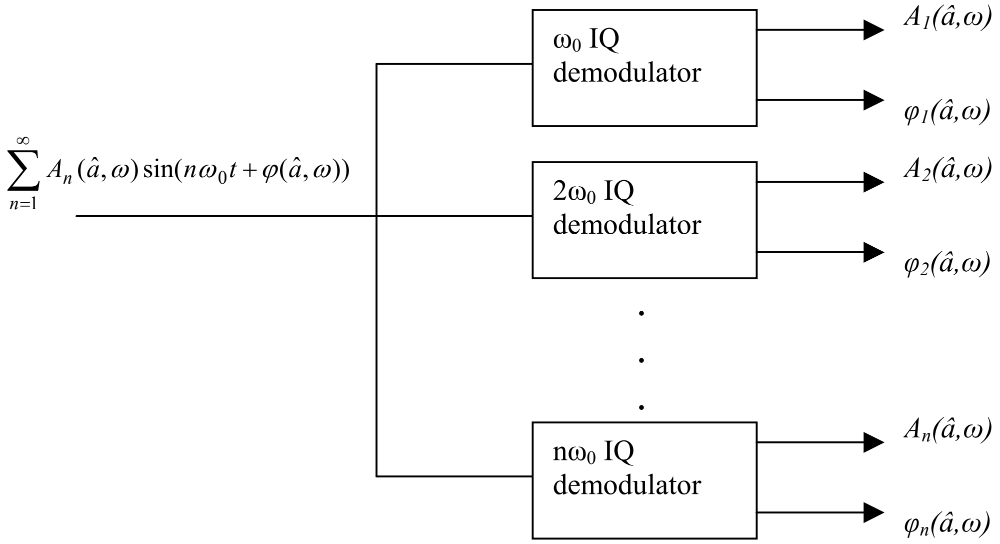

Figure 5: Signal decomposition using IQ demodulation

In figure 6 the IQ demodulator for the kth harmonic of the output signal is explained in more detail. The signal is multiplied with sin($k\omega_0$) and cos($k\omega_0$) in two separate branches. These multiplications result in the generation of two new signals, each consisting of the sum and difference frequencies of the original signal and the oscillator signals (10), (11). These new signals are 90 deg apart.

$$2\sin(k\omega_0 t)A_k(\hat{a},\omega)\sin(k\omega_0 t+\varphi_k(\hat{a},\omega)) = A_k(\hat{a},\omega)\{\cos(\varphi_k(\hat{a},\omega)-\cos(2k\omega_0 t+\varphi_k(\hat{a},\omega))\} \tag{10}$$

$$2\cos(k\omega_0 t)A_k(\hat{a},\omega)\sin(k\omega_0 t+\varphi_k(\hat{a},\omega)) = A_k(\hat{a},\omega)\{\sin(\varphi_k(\hat{a},\omega)+\sin(2k\omega_0 t+\varphi_k(\hat{a},\omega))\} \tag{11}$$

After low-pass filtering the remaining signals representing the kth harmonic are $A_k(\hat{a},\omega)\sin(\varphi_k(\hat{a},\omega))$ called the I-signal (= in phase) component and $A_k(\hat{a},\omega)\cos(\varphi_k(\hat{a},\omega))$ called the Q signal (= quadrature) component. From the I and Q components $A_k(\hat{a},\omega)$ and $\varphi_k(\hat{a},\omega)$ are computed.

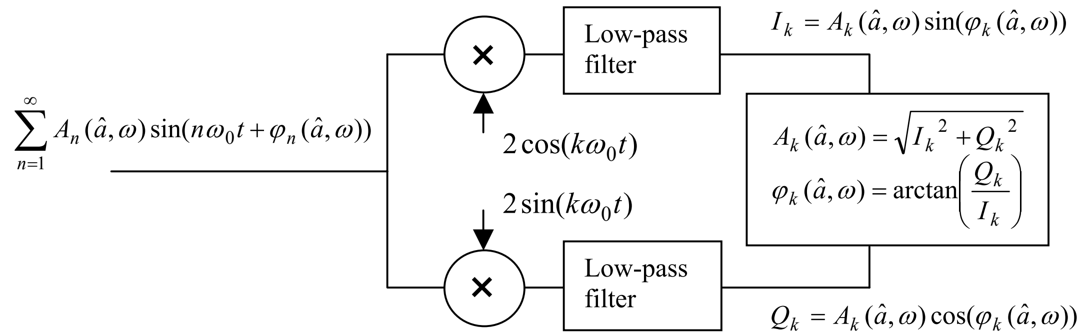

Figure 6: Determination of kth order HOSIDF using IQ demodulation

4 Case study

4.1 Description of the system

In this case study the HOSIDFs of a real mechanical system with friction are measured. The test object is a system, which consists of a 20 W electric DC collector motor with encoder. The motor is powered by a voltage-to-current converter see figure 7. The input to the system, i.e. the motor current I_m, is measured using a current probe with a sensitivity of 2 A/V. The response signal is angular velocity ω_{out}. For small rotations this angular velocity can be measured with a dual fibre laser vibrometer as the linear velocity difference between two points spaced 180° on the circumference of the shaft divided by the spacing of the points. The resulting sensitivity is 0.588 rad/s/V. A block diagram of the measurement set-up is given in figure 8. J_1 and J_2 represent the inertias of the motor and the encoder. T is the driving torque and C the stiffness of the motor shaft. As in the LuGre model [13] the friction influence is modelled with σ_0 the bristle stiffness, σ_1 the bristle damping and σ_2, the viscous damping. The combination of the bristle stiffness σ_0 and the inertias J_1 and J_2 will cause a friction-induced resonance [1], [2], [4] which frequency will depend upon the excitation level.

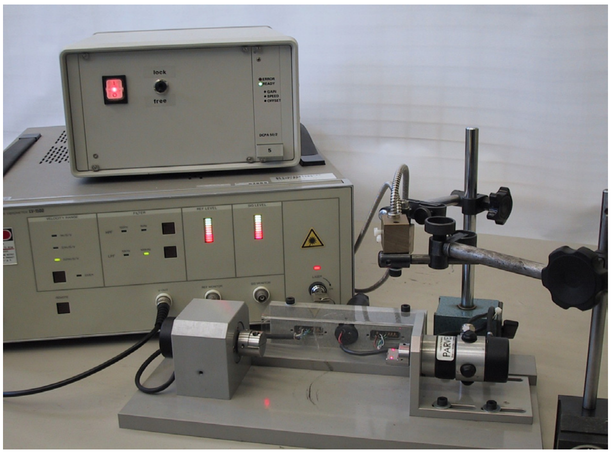

Figure 7: Case study on a small DC motor

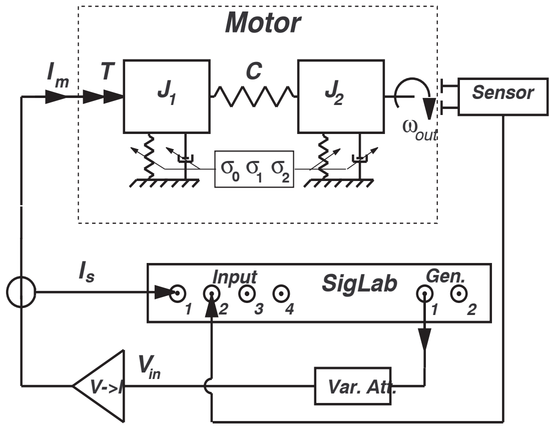

Figure 8: Block diagram of measurement set-up

4.2 Measurement of the FRF using white noise excitation

Using a SigLab 20-42 dynamic signal analyser [13] with 90 dB aliasing protection the H_1 frequency response function ω_{out}/I_m was measured with $\Delta f = 0.313$ Hz and Hanning weighting in a frequency range of 0 Hz to 1 kHz. The excitation signal was band-limited random noise in an equal frequency range. The crest factor of the noise was 3; the (very low) excitation levels were 1.5 mA_{RMS}, 6 mA_{RMS} and 36 mA_{RMS}, in order to be able to measure the non-linear phenomenon. This situation occurs frequently in accurate point-to-point motion tasks. Figure 9 shows the results after 20 averages per measurement. In the plots two resonances are visible. The resonance at approximately 650 Hz is caused by the finite stiffness C in combination with the bristle stiffness σ_0 (see figure 8). The amplitude dependent friction induced resonance is visible and varies between 540 Hz and 200 Hz. Its damping varies too as can be seen from the differences in phase gradients. The fact that the system behaviour depends upon the amplitude of the excitation signal is reflected in the coherence plot.

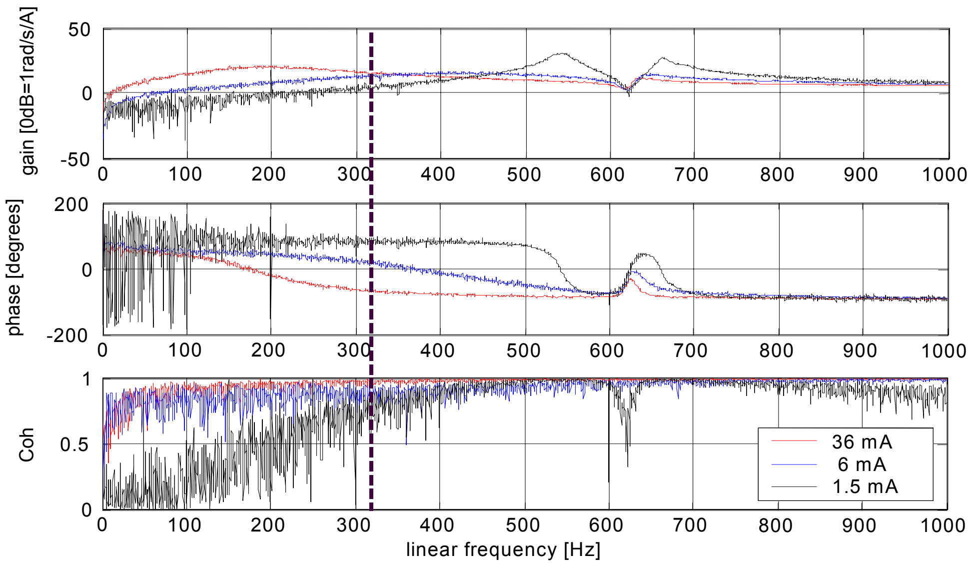

Figure 9 H_1 estimate of FRF

4.3 Measurements of the HOSIDFs

To further investigate this non-linear behaviour, the HOSIDFs were determined using the measurement techniques described in 3.1 and 3.2. The frequency of the generator signal was chosen 320 Hz to excite the system both above and below its friction induced resonance frequency depending upon the instantaneous amplitude of the excitation signal, see the vertical dashed line in figure 9. Other considerations are that 320 Hz is not a multiple of the 50 Hz mains frequency and that the signal can be generated with an integer number of 12,8 kHz samples per period, being one of the sampling frequencies of the SigLab 20-42 dynamic signal analyser. Figure 10 shows the generator signal, the input current signal and the system response.

The main parameters used for the FFT method are a block-size of 1600 samples and a sampling frequency of 12,8 kHz so Δf = 8 Hz, hamming window, no overlap processing. For the IQ method the low-pass filters are 8^{th} order Butterworth with 4 Hz cut-off frequency.

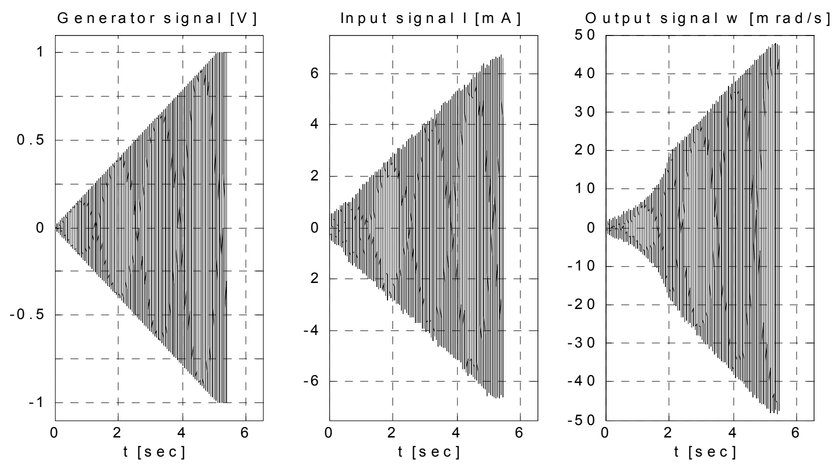

Figure 10 Generator signal, input and output measurement signals

4.4 Results

Figure 11 shows the amplitude dependency of the HOSIDFs measured at the fixed frequency of 320 Hz. The blue (solid) line shows the results from the IQ method, the (red) dots indicate the measurements from the block based FFT method. In the left column the magnitude plots are presented for the odd order Sinusoidal Input Describing Functions (SIDF). The even orders are all zero because of the odd characteristic of the non-linearity. The right hand column gives the corresponding phase relations. In the magnitude plot of the first order SIDF we can distinguish three regions. From 0 to approximately 0.5 mA the system gain is excitation independent. Between 0.5 mA and approximately 2.5 mA a strong excitation level dependency is visible. Above 2.5 mA the gain is independent of the excitation level at a stable 18 dB but the system remains non-linear as can be concluded from the plots of the higher order SIDFs. The gain of the third order FRF decreases initially until it reaches a minimum at an excitation of 0.5 mA. This is due to the low signal to noise ratios in this region resulting in large uncertainties in the calculations. For increasing excitation its magnitude increases and reaches a maximum of –8 dB at approximately 2.5 mA. Above that excitation level the gain decreases again slightly. The same pattern is visible for the fifth order SIDF, however its maximum of –15 dB is reached at an excitation level of 4.5 mA.

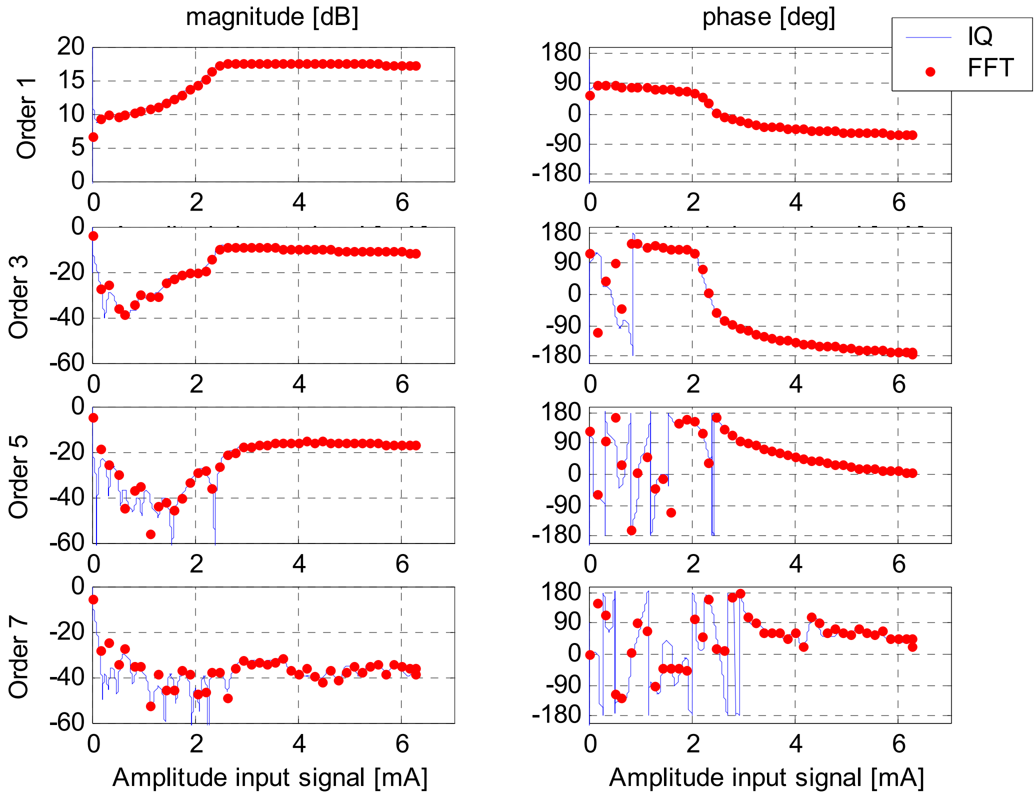

Figure 11: HOSIDFs measured with IQ and FFT method

In figure 11 first row, right column the phase relation of the first order FRF must not be mistaken for the phase graph of the bode-plot of a standard linear second order system relating input torque and output angular velocity, since the x-axis does not indicate frequency but input signal magnitude. This plot however does contain information about the resonance frequency of the system as function of excitation level. In the phase plot of figure 9 we see for low excitation levels at 320 Hz a phase of approximately 90 deg and a resonance frequency above 320 Hz. For high excitation levels the phase is dropped to –90 deg, and the resonance frequency is shifted below 320 Hz. At the actual resonance frequency the phase will be 0 deg. These results match with the first order phase plot in figure 11. From this phase plot the excitation level required for the system to resonate at 320 Hz could be determined to be approximate 2.5 mA.

4.5 Discussion

An important difference between the FFT method and the IQ method is the way of processing data. The FFT method is block oriented and generates results every 0.125 sec where the IQ method is sample based so the time between two measurements equals the sample period of 1/12800 sec. This results in less measurement points for the FFT method in the corresponding HOSIDF plots as can be seen in figure 11. Apart from this difference the results of the two methods are substantially equivalent. The visible differences which are mainly concentrated in the range of low excitation levels are probably due to low signal to noise ratios.

5 Conclusion and further research

An extension of the theory of Describing Functions was presented. Higher Order Sinusoidal Input Describing Functions (HOSIDF) can be defined as the excitation amplitude dependent gain and phase relations between a virtual set of harmonics of the excitation signal of a non-linear system and the corresponding real harmonics in the output signal. The theory developed assumes the non-linear system to respond harmonically at a sinusoidal excitation. To measure the HOSIDF two measurement methods were described. The first method is FFT based and uses auto and cross spectrum information to determine the system gain and phase of the HOSIDF as function of the input signal amplitude and frequency. The second measurement technique uses IQ demodulation techniques and is sample based. A case study was presented describing the application of the two measurement techniques. In this case study the device under test was a small current fed DC electric motor. Due to its construction this motor exhibits a significant amount of friction. Initial FRF measurements with various levels of random noise excitation display a significant input level dependency of the system dynamics. The friction-induced stiffness causes one resonance to vary significantly in frequency and damping. With both measurement techniques the HOSIDFs have been successfully identified. The results clearly show non-linear system behaviour as function of excitation level.

Since the HOSIDF is not only a function of amplitude but also of excitation frequency many single frequency measurements have to be done in order to quantify its frequency dependency. Future work will comprise merging the multi-sine excitation techniques [14] with the HOSIDF technique described in this paper. Implementation of this combined knowledge in hardware like FPGAs will hopefully result in the construction of a new, valuable and practical measurement tool.

References

[1] W. Symens, F. Al-Bender, J. Swevers, H. Van Brussel, *Harmonic analysis of a mass subject to hysteretic friction: experimental validation,* in P. Sas editor, *Proceedings of The 2002 International Seminar on Modal Analysis, Leuven, Belgium,2002 September,* Leuven (2002), pp. 1229-1240.

[2] P. Nuij, *Measurement technique to determine modal parameters of friction induced resonance,* in P. Sas editor, *Proceedings of The 2002 International Seminar on Modal Analysis, Leuven, Belgium, 2002 September,* Leuven (2002), pp. 479-484.

[3] B. Armstrong-Hélouvry, P. Dupont, C. Canudas de Wit, *A survey of Models, Analysis Tools and Compensation Methods for the Control of Machines with Friction*, Automatica, Vol. 30 No. 7, (1994), pp.1083-1138.

[4] R. Hensen, *Controlled Mechanical Systems with friction.* Eindhoven University of Technology, The Netherlands Ph.D.Thesis. (2002)

[5] A. Gelb, W.E. Vander Velde, *Multiple input describing functions and non-linear system design.* New York: McGraw Hill, (1968).

[6] J. Taylor, *Electrical Engineering Encyclopedia* Describing Functions. New York, John Wiley&Sons, Inc. (1999).

[7] J. Verspecht, *Describing Functions Can Better Model Hard Nonlinearities In The Frequency Domain Than The Volterra Theory*, Annex to Doctoral Dissertation, Vrije Universiteit Brussel, (1995).

[8] J Verspecht, D. Schreurs, A. Barel and B. Nauwelaers, *Black Box Modelling of Hard Nonlinear Behavior in the Frequency Domain. Conference Record of the IEEE Microwave Theory and Techniques Symposium,* (1996), pp. 1735-1738.

[9] J. Slotine, Weiping Li, *Applied Nonlinear Control,* Prentice-Hall International, (1991)

[10] C.M. Rader *A simple method for sampling in-phase and quadrature components,* IEEE Transactions on Aerospace and Electronic Systems, AES-Vol 20, no 6, (1984), pp. 821-824.

[11] R.I. Mitchell *Creating Complex Signal Samples From a Band-Limited Real Signal.* IEEE Transactions on Aerospace and Electronic Systems, AES-Vol 25 no 3, (1989), pp. 425-427.

[12] Canudas de Wit C,. Olsson H., Åström K.J., and Lischinsky P., *A New Model for Control of Systems with Friction*, IEEE Transactions on Automatic Control, 40, (1995), pp. 419-425,

[13] Spectral Dynamics Inc., *SigLab 20-42 dynamic signal analyser system, User Guide*

[14] R. Pintelon, J. Schoukens, *System Identification: A Frequency Domain Approach*, John Wiley&Sons, Inc., (2001)