The role of data and priors in estimating climate sensitivity

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Abstract: In Bayesian theory, the data together with the prior produce a posterior. We show that it is also possible to follow the opposite route, that is, to use data and posterior information (both of which are observable) to reveal the prior (which is not observable). We then apply the theory to equilibrium climate sensitivity as reported by the Intergovernmental Panel on Climate Change in an attempt to get some insight into the prior beliefs of the IPCC scientists. It appears that the data contain much less information than one might think, due to the presence of correlation. We conclude that the prior in the fifth IPCC report was too low, and in the sixth report too high.

JEL Classification: C11, C52, C83, Q54.

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1 Introduction

When the radiation balance of the Earth is perturbed, the temperature will change. By how much is measured by the *equilibrium climate sensitivity* (ECS): the long-term temperature rise that is expected to result from a doubling of the atmospheric CO_2 concentration, usually relative to the pre-industrial level (around 1750). It is a prediction of the new global mean near-surface air temperature once the CO_2 concentration has stopped increasing and most of the feedbacks have had time to have their full effect. The ECS is an important diagnostic in climate modelling, but it cannot be measured directly and forms a large source of uncertainty. CO_2 levels rose from 280 parts per million (ppm) in the eighteenth century (IPCC, 2013, p. 100) to about 416 ppm by 2020, an increase of almost 50%. In the same period, the Earth's temperature rose by a little over one degree Celsius. The ECS will be our parameter of interest, and we shall denote this parameter by β.

In estimating β we rely exclusively on the Intergovernmental Panel on Climate Change (IPCC) reports. So far, six so-called Assessment Reports have appeared: the first in 1990, the sixth in 2021. In these reports we find estimates and precisions of studies on the ECS and IPCC's own estimates and precisions. Our interest is not so much in these estimates themselves but rather in the *process* that leads from the underlying studies to IPCC's estimates. Adopting a Bayesian framework, we shall think of the underlying studies as our data and of the published conclusions as our posterior. The question then arises whether we can reveal the IPCC's priors (which we do not observe) from these data and the posterior (which we do observe). This is indeed possible, and the purpose of this paper is to explain how this can be achieved, which challenges we encounter on the way, and what we learn from such an exercise.

The idea of reversing Bayesian thought and — rather than obtain a posterior from data and prior — recover the prior from data and posterior, was recently proposed by Ikefuji et al. (2023) and applied to inflation forecasting in the UK. Their application was carefully chosen to satisfy two requirements. First, that inflation forecasts can be credibly modeled using a symmetric distribution, such as normality. The assumption of symmetry and normality is, however, only plausible in a limited number of cases. We shall show that our theory goes through in the case of monotonic transformations of β, for which we shall take the log-transformation, which is not symmetric and particularly appropriate for our application.

Second, the data in Ikefuji et al. (2023) consisted of essentially *one* piece of information, i.e. the Phillips curve. But suppose we have two or more pieces of information as our data, which will typically be the case. How to then combine these pieces of information into one data distribution? If we combine too naively, then the resulting precision may be misleadingly high, leading to the theoretical impossibility that the posterior variance is larger (rather than smaller) than the data variance. To resolve this issue we use the recently developed theory in Magnus and Vasnev (2023), where the critical role of correlation was discussed and applied to Bank of Japan and European Central Bank forecasts.

The remainder of this paper is organized as follows. In Section 2 we analyze how to recover the prior from the data and the posterior in the case when there is only one parameter of interest, first within the framework of the normal distribution, then extending this framework to the lognormal distribution. In Section 3 we consider the conclusions of the fifth and sixth IPCC reports (our posteriors) concerning the ECS, and reformulate these conclusions using the lognormal distribution. In Section 4 we review the data sources that underly the IPCC conclusions, and find the appropriate lognormal distributions for each of the data sources. Next, we need to combine these data sources into one data source for each of the two reports. This is achieved using Magnus and Vasnev's (2023) approach through correlations, and we discuss and explain the procedure in Section 5. Without correlation the data variance is much too small, leading to data that are misleadingly precise. This, in fact, is one of the findings of the current paper: when data sources are (highly) correlated, as is often the case, they contain much less information than casual inspection might indicate. Now that we have the data and the posterior, we obtain and discuss the revealed prior in Section 6. The prior turns out to be important and the direction of the prior changes between the fifth and the sixth report. In Section 7 we study the relationship between the posterior in one period and the prior in the next period (dynamic consistency). Section 8 concludes.

2 From posterior to prior

Suppose we have data $y_1, \dots, y_n$, possibly correlated and heteroskedastic, generated from a normal distribution with common mean β, our parameter of interest. In other words, we have

$$y|\beta \sim \mathrm{N}(\beta\imath, \Omega), \tag{1}$$

where $y = (y_1, \dots, y_n)'$, $\imath$ is an $n \times 1$ vector of ones, and Ω is a positive definite $n \times n$ matrix. A frequentist would estimate the scalar parameter β

using the generalized least-squares estimator b_0 with variance σ_0^2:

$$b_0 = \frac{\imath'\Omega^{-1}y}{\imath'\Omega^{-1}\imath}, \qquad \sigma_0^2 = \frac{1}{\imath'\Omega^{-1}\imath}. \tag{2}$$

A Bayesian, on the other hand, would wish to take prior knowledge about β into account. If this prior information is given by $\beta \sim \mathrm{N}(b_1, \sigma_1^2)$, then the posterior distribution of β is $\beta|y \sim \mathrm{N}(b_2, \sigma_2^2)$ with posterior moments

$$b_2 = wb_1 + (1-w)b_0 = \frac{\sigma_0^2 b_1 + \sigma_1^2 b_0}{\sigma_0^2 + \sigma_1^2} \tag{3}$$

and

$$\sigma_2^2 = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2}\right)^{-1} = \frac{\sigma_0^2\sigma_1^2}{\sigma_0^2 + \sigma_1^2}, \tag{4}$$

where $w = \sigma_0^2/(\sigma_0^2 + \sigma_1^2) = \sigma_2^2/\sigma_1^2$. The expression (4) for the posterior variance implies the restrictions $\sigma_2^2 < \sigma_0^2$ and $\sigma_2^2 < \sigma_1^2$, and this makes sense: when two pieces of information (prior and data) are combined then the precision increases. These inequalities, in turn, imply that $0 < w < 1$. When the prior becomes uninformative, that is, when $\sigma_1^2 \to \infty$, then $b_2 \to b_0$ and $\sigma_2^2 \to \sigma_0^2$.

Now consider the opposite situation where the data and the posterior are available but not the prior. Can we reveal the prior from the data and the posterior? Indeed we can, and in the special case of normality we obtain the prior moments as

$$b_1 = w^{-1}b_2 + (1-w^{-1})b_0 = \frac{\sigma_0^2 b_2 - \sigma_2^2 b_0}{\sigma_0^2 - \sigma_2^2} \tag{5}$$

and

$$\sigma_1^2 = \left(\frac{1}{\sigma_2^2} - \frac{1}{\sigma_0^2}\right)^{-1} = \frac{\sigma_0^2\sigma_2^2}{\sigma_0^2 - \sigma_2^2}, \tag{6}$$

where $w^{-1} = \sigma_1^2/\sigma_2^2 = \sigma_0^2/(\sigma_0^2 - \sigma_2^2)$. The prior mean is thus a 'weighted average' of b_0 and b_2, but w^{-1} does not lie between zero and one; in fact, $w^{-1} > 1$ and $1 - w^{-1} < 0$. Also, since variances are nonnegative, the expression for σ_1^2 implies an upper bound to the posterior variance, namely $\sigma_2^2 < \sigma_0^2$. This restriction does not play a role in the usual Bayesian framework where we go from data plus prior to posterior, because the underlying variances σ_0^2 and σ_1^2 are unrestricted (apart from being positive) and σ_2^2 will automatically satisfy the restriction. But it does play a role when we go from data plus posterior to prior, because now the restriction is not automatically satisfied. This has practical consequences as we shall see later. In summary, we can indeed reveal the prior given information on the data and the posterior.

This simple analysis can be generalized in various directions. First, we may have not one but several parameters of interest in which case our starting point is $y|\beta \sim \mathrm{N}(X\beta, \Omega)$ for some $n \times k$ matrix X; see Ikefuji et al. (2023). Second, the assumption of normality (implying symmetry) may be reasonable for some applications — as it was in Ikefuji et al. (2023) — but it is not in our's. So, a generalization of the framework is required. Recall the fundamental formula for completing the square,

$$\frac{(b_0 - \beta)^2}{\sigma_0^2} + \frac{(\beta - b_1)^2}{\sigma_1^2} = \frac{(\beta - b_2)^2}{\sigma_2^2} + \frac{(b_0 - b_1)^2}{\sigma_0^2 + \sigma_1^2}, \tag{7}$$

where b_2 and σ_2^2 are given in (3) and (4). Multiplying both sides by $-1/2$ and taking exponentials gives

$$f(b_0; \beta, \sigma_0^2)\, f(\beta; b_1, \sigma_1^2) = f(\beta; b_2, \sigma_2^2)\, f(b_0; b_1, \sigma_0^2 + \sigma_1^2), \tag{8}$$

where

$$f(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} \tag{9}$$

denotes the normal density. This, of course, is just Bayes' formula

$$f(b_0|\beta)\, f(\beta) = f(\beta|b_0)\, f(b_0), \tag{10}$$

and it shows that a normal likelihood plus a normal prior results in a normal posterior.

Now let $h(x)$ be a monotonic transformation of x, such as $h(x) = \log x$, and define the density

$$g(x; \mu, \sigma^2) = \frac{h'(x)}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{h(x)-\mu}{\sigma}\right)^2\right\}. \tag{11}$$

Then it follows from (7) that

$$f(b_0; h(\beta), \sigma_0^2)\, g(\beta; b_1, \sigma_1^2) = g(\beta; b_2, \sigma_2^2)\, f(b_0; b_1, \sigma_0^2 + \sigma_1^2). \tag{12}$$

In particular, letting $h(x) = \log x$ with $h'(x) = 1/x$ leads to the lognormal density $g(x)$ for $x > 0$. Hence, a normal likelihood plus a lognormal prior results in a lognormal posterior, so that prior and posterior remain conjugate distributions. Another way of arriving at this result is to realize that the lognormal distribution is not really a new distribution but rather a reparametrization. The parameter of interest remains β, but the analysis is performed on $\log\beta$ (more generally on $h(\beta)$).

In summary, if we have a likelihood $b_0|\beta \sim \mathrm{N}(\log\beta, \sigma_0^2)$ and a prior $\log\beta \sim \mathrm{N}(b_1, \sigma_1^2)$, then we obtain a posterior $\log\beta|b_0 \sim \mathrm{N}(b_2, \sigma_2^2)$. Reversing the process, if we have a likelihood $b_0|\beta \sim \mathrm{N}(\log\beta, \sigma_0^2)$ and a posterior $\log\beta|b_0 \sim \mathrm{N}(b_2, \sigma_2^2)$, then we obtain a prior $\log\beta \sim \mathrm{N}(b_1, \sigma_1^2)$, where b_1 and σ_1^2 are given by (5) and (6), respectively.

At this point we have two objects that can be regarded as 'data': the observations y and the estimate b_0. The observations y are what one would usually call data, but in our process the IPCC scientists consider b_0 as their data, add a prior b_1, and arrive at a posterior b_2. To avoid confusion and following Ikefuji et al. (2023), we shall not refer to b_0 as the data but as the *input*. In the absence of a prior, the IPCC scientists accept the input as their only tool: output = input, that is, $b_2 = b_0$. But if the scientists' prior plays a role (as of course it does), then output ≠ input and the difference is the prior, which is the object of our study.

3 The posterior distribution

We shall apply the theory of Section 2 to the estimation of equilibrium climate sensitivity, hereafter β, as presented by the IPCC in several reports. Their conclusions are our posterior, which we discuss in the current section. These conclusions are based on many scientific publications which serve as our inputs, and we shall discuss these in the next section. Below we review the IPCC conclusions and the posterior distributions derived from them, first for the fifth then for the sixth IPCC report.

3.1 The fifth IPCC report (posterior)

In the fifth report, more precisely the Working Group I contribution (IPCC, 2013), hereafter IPCC5, the authors state that 'no best estimate for equilibrium climate sensitivity can now be given because of a lack of agreement on values across assessed lines of evidence and studies' (IPCC5, p. 16, footnote). But later in the same report they do provide a confidence region (as opposed to a point estimate), as follows:

> '...ECS is likely in the range 1.5℃ to 4.5℃ with high confidence. ECS is positive, extremely unlikely less than 1℃ (high confidence), and very unlikely greater than 6℃ (medium confidence).' (IPCC5, pp. 83–84)

The IPCC also provides an interpretation of terms like 'extremely unlikely' and 'medium confidence' (IPCC5, p. 36), which differs slightly from the interpretation in the previous Assessment Report (IPCC, 2007, p. 22) by explicitly taking into account the degree of 'agreement' in the team about the evidence provided by each study.[1] Given this interpretation, IPCC5 concludes that

$$\begin{aligned}\Pr(1.5 < \text{ECS} < 4.5) &= 0.67,\\ \Pr(\text{ECS} < 1.0) &< 0.05, \text{ and}\\ \Pr(\text{ECS} > 6.0) &< 0.10.\end{aligned}$$

In addition (pp. 75 and 817), they summarize information of experiments by the Coupled Model Intercomparison Project Phase 5 (CMIP5) who report a range 2.1–4.7 for the ECS, without however stating the likelihood of this range.

Assuming the ECS β to be lognormally distributed, so that $\log\beta \sim \mathrm{N}(b_2, \sigma_2^2)$, we seek combinations (b_2, σ_2) such that the posterior probabilities closely match the probabilities in the IPCC5 report. There is no unique lognormal distribution that fits our inputs, but $b_2 = 1.07$ and $\sigma_2 = 0.53$ seems a reasonable approximation and is also in line with Hwang et al. (2013, Figure 4) where $b_2 = 1.071$ and $\sigma_2 = 0.527$.

The selected posterior distribution, plotted in Figure 1 (bold line, in red), satisfies

$$\begin{aligned}\Pr(1.5 < \beta < 4.5) &= 0.69 \text{ (about 67\%)},\\ \Pr(0 < \beta < 1.0) &= 0.02 \text{ (less than 5\%), and}\\ \Pr(\beta > 6.0) &= 0.09 \text{ (less than 10\%)},\end{aligned}$$

hence close to the IPCC5 conclusions. In addition, the interquartile range is $\Pr(2.04 < \beta < 4.17) = 50\%$ and there is a 1% probability of $\beta > 10.0$. The skewness of the distribution is well illustrated by the fact that the mode and median of β are quite different: the mode is $e^{b_2 - \sigma_2^2} = 2.20$, while the median is $e^{b_2} = 2.92$.

3.2 The sixth IPCC report (posterior)

In the sixth report, hereafter IPCC6, the authors write:

[1]In fact, the interpretation of these terms is far from easy. Kause et al. (2022) studied how IPCC experts from different disciplines interpret the recommended uncertainty language, and they found that physical science experts were more familiar with the IPCC instructions than other experts, and followed it more often; that experts' confidence levels increased more with perceptions of evidence than with agreement; and that experts' estimated probability intervals for climate variables were wider when likelihood terms were presented with 'medium confidence' rather than with 'high confidence' and when seen in context of IPCC sentences rather than out of context.

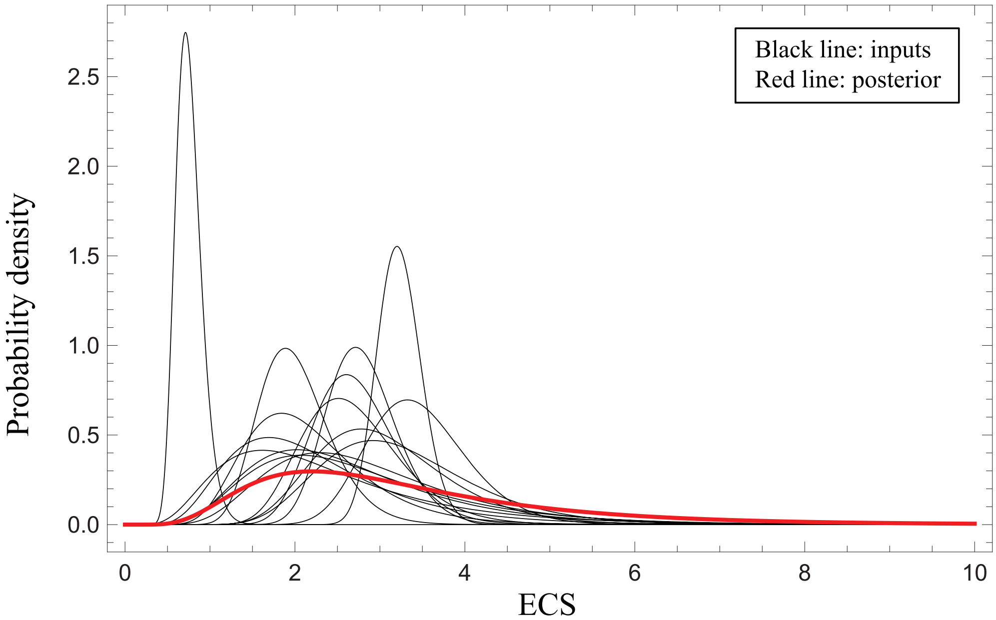

Figure 1: Posterior distribution (in red) of ECS in the fifth report and the fifteen studies, lognormal distributions

> '...Based on multiple lines of evidence, the very likely range of equilibrium climate sensitivity is between 2℃ (high confidence) and 5℃ (medium confidence). The (...) best estimate is 3℃ with a likely range of 2.5℃ to 4℃ (high confidence).'
>
> (IPCC6, A.4.4, p. 11)

Later (p. 926) they add that it is 'virtually certain that ECS is larger than 1.5℃.'

The 'multiple lines of evidence' are: the understanding of climate processes, the instrumental record, paleoclimates, and model-based emergent constraints (IPCC6, p. 11, footnote). The definition of 'likely', 'very likely', and 'virtually certain' is the same as in the fifth report (IPCC6, p. 4, footnote 4), and hence we obtain the following posterior probabilities:

$$\Pr(2.0 < \text{ECS} < 5.0) = 0.90,$$
$$\Pr(2.5 < \text{ECS} < 4.0) = 0.67, \text{ and}$$
$$\Pr(\text{ECS} > 1.5) = 0.99,$$

where we interpret the 'best estimate' of 3.0℃ as the median of the ECS.

The posterior in the sixth report differs markedly from the posterior in the fifth report, not so much in the mean (which increases a little) but rather in the variance (which is much lower). The reduction in uncertainty is caused, not by a single breakthrough or discovery, but by combining evidence from many different sources and a better understanding of their strengths and weaknesses (IPCC6, p. 1024, FAQ 7.3; Sherwood et al., 2020).

Assuming again that the ECS β follows a lognormal distribution, so that $\log \beta \sim \mathrm{N}(b_2, \sigma_2^2)$, we seek combinations (b_2, σ_2) such that the posterior probabilities closely match the above probabilities. If $\mathrm{median}(\beta) = 3.0$ then $b_2 = \log 3 = 1.099$ and

$$\begin{aligned} \Pr(2.0 < \beta < 5.0) = 0.90 &\implies \sigma_2 = 0.27, \\ \Pr(2.5 < \beta < 4.0) = 0.67 &\implies \sigma_2 = 0.24, \text{ and} \\ \Pr(\beta > 1.5) = 0.99 &\implies \sigma_2 = 0.30. \end{aligned}$$

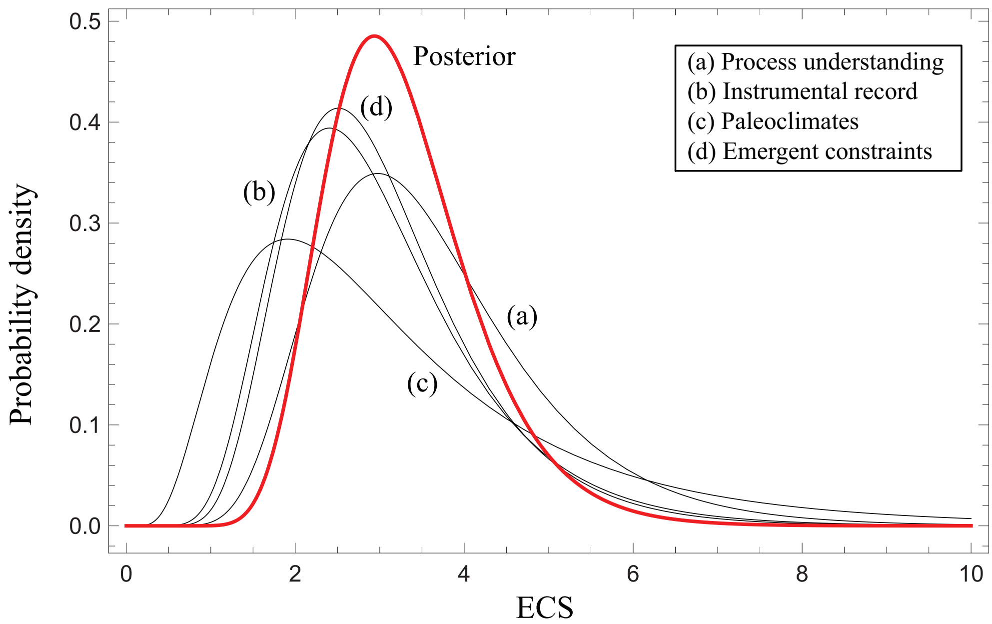

Figure 2: Posterior distribution (in red) of ECS in the sixth report and the four inputs, lognormal distributions

Thus, a reasonable approximation is provided by $b_2 = 1.15$ and $\sigma_2 = 0.27$. The selected posterior distribution, plotted in Figure 2 (bold line, in red), satisfies

$$\begin{aligned} \Pr(2.0 < \beta < 5.0) &= 0.91 \text{ (about 90\%)}, \\ \Pr(2.5 < \beta < 4.0) &= 0.62 \text{ (about 67\%)}, \text{ and} \\ \Pr(\beta > 1.5) &= 0.997 \text{ (about 99\%)}, \end{aligned}$$

and we find median(β) = 3.16 and mode(β) = 2.94, hence close to the IPCC6 conclusions. In addition, the interquartile range is $\Pr(2.63 < \beta < 3.79) = 50\%$, much tighter than in the IPCC5 report, and there is a 1% probability of $\beta > 5.9$.

4 The distribution of the inputs

In addition to the posterior we need information on the inputs. These inputs are not 'raw' data but outcomes of scientific publications, each with its own merits and viewpoints. In the fifth report we have fifteen inputs, one of which we shall consider an outlier and disregard; in the sixth report we have four. We discuss these inputs and the resulting distributions below.

4.1 The fifth IPCC report (inputs)

Our inputs in the IPCC5 report consist initially of $m = 15$ studies. Fourteen of these studies are contained in Figure 10.20 (p. 925) and Box 12.2 (p. 1110) of the report, and one study (Huber et al., 2011) is included in Figure 2 of Knutti et al. (2017) and referred to in various places of IPCC5.

The ith study produces a range (l_i, u_i) (lower and upper bound) with an associated probability p_i (typically 90% or 95%) for the ECS β, our parameter of interest. Given the range (l_i, u_i) and the associated probability $\Pr(l_i < \beta_i < u_i) = p_i$ we can identify the parameters of the associated lognormal distributions, as follows. Since $\log \beta_i \sim \mathrm{N}(b_{0i}, \sigma_{0i}^2)$ in the ith study, we have

$$\Pr\left(\frac{\log l_i - b_{0i}}{\sigma_{0i}} < z_i < \frac{\log u_i - b_{0i}}{\sigma_{0i}}\right) = p_i, \qquad z_i \sim \mathrm{N}(0, 1), \tag{13}$$

and hence

$$b_{0i} = \log(l_i u_i)^{1/2}, \qquad \sigma_{0i} = \frac{\log(u_i/l_i)^{1/2}}{q_i}, \qquad \Phi(q_i) = \frac{p_i + 1}{2}, \tag{14}$$

where Φ denotes the c.d.f. of the standard-normal distribution. In this way we end up with fifteen inputs b_{0i} with associated standard deviations σ_{0i}; see Table 1.

The IPCC5 report distinguishes between an instrumental period and a palaeoclimatic period. The instrumental period is the short period in the Earth's long history where direct instrumental records on climate are available, while the palaeoclimatic period is the long period preceding such records. Of our fifteen studies some only use data from the instrumental period (studies 1, 3, 5–9, 12, 13), some only from the palaeoclimatic period

Table 1: Inputs and lognormal approximations, fifth report

	Study		Bounds		Lognormal approx.	
		p_i	l_i	u_i	b_{0i}	σ_{0i}
1	Lindzen and Choi, 2011	95%	0.5	1.1	−0.30	0.20
2	Schmittner et al., 2011	90%	1.4	2.8	0.68	0.21
3	Aldrin et al., 2012	90%	1.2	3.5	0.72	0.33
4	Hargreaves et al., 2012	90%	1.0	4.2	0.72	0.44
5	Lewis, 2013	90%	2.0	3.6	0.99	0.18
6	Bender et al., 2010	95%	1.7	4.1	0.97	0.22
7	Otto et al., 2013	90%	0.9	5.0	0.75	0.52
8	Schwartz, 2012	90%	1.2	4.9	0.89	0.43
9	Lin et al., 2010	90%	2.8	3.7	1.17	0.08
10	Libardoni and Forest, 2011	90%	1.2	5.3	0.93	0.45
11	Köhler et al., 2010	90%	1.4	5.2	0.99	0.40
12	Olson et al., 2012	95%	1.8	4.9	1.09	0.26
13	Huber et al. 2011	67%	2.9	4.0	1.23	0.17
14	Holden et al., 2010	90%	2.0	5.0	1.15	0.28
15	Palaeosens, 2012	95%	1.1	7.0	1.02	0.47

(2, 4, 11, 14, 15), and some combine different lines of evidence (3, 10, 12). Studies 3 and 12 appear in both the groups instrumental and combination. This highlights the first of several problems: comparability.

The second problem is that the first study (Lindzen and Choi, 2011) has a big impact and should be considered an outlier. The IPCC raises doubts about the reliability of this study (IPCC5, pp. 923–924), but it has not removed the study from their report. Not only is the revealed value of b_{0i} much lower than in the other studies, but the effect is much strengthened by the fact that the reported precision is high.

To gain further insight, Figure 1 presents the fifteen lognormal curves corresponding to the fifteen studies. From the figure several things become clear. First, that the first study (with the highest peak) is an outlier, and that we should therefore disregard this study from our analysis, thus ending up with $m = 14$ studies. Second, that what the inputs tell us is ambiguous and uncertain. The modes range from 0.71 (1.62 if we exclude the first study) to 3.32, and the medians from 0.74 (1.97 if we exclude the first study) to 3.42.

4.2 The sixth IPCC report (inputs)

In earlier IPCC reports, the assessment of ECS relied on either CO_2-doubling experiments using global atmospheric models coupled with mixed-layer ocean models or on standardized CO_2-quadrupling experiments using fully coupled ocean-atmosphere models or Earth system models. In the sixth report, the assessment of ECS is based on multiple lines of evidence, with Earth system models representing only one of several sources of information (IPCC6, p. 993). These 'multiple lines of evidence' are divided in four classes: process understanding, the instrumental record, paleoclimates, and emergent constraints.

Each class is analyzed separately and is based on a number of underlying studies (IPCC6, Section 7.5, pp. 992–1011). Our input can thus be viewed as a two-step procedure: first from underlying studies to a distribution of β in each of the four classes, then from these four class distributions to the final input distribution. To estimate the input distribution in two steps would, in principle, be possible, but it would require a two-level aggregation theory where at the moment a one-level aggregation theory is already a major challenge as we shall see in Section 5.

Table 2: Probabilities for the four inputs, sixth report

			Probability		
		median	0.67	0.90	0.95
(a)	Process understanding	3.4	2.5–5.1	2.1–7.7	
(b)	Instrumental record	2.5–3.5	> 2.2	> 1.8	> 1.6
(c)	Paleoclimates	3.3–3.4	< 4.5	> 1.5	< 8.0
(d)	Emergent constraints	2.4–3.3		1.5–5.0	

Hence, we shall take the $m = 4$ intermediate conclusions of the IPCC as our inputs. These conclusions are presented in Table 2, which summarizes Table 7.13 in IPCC6. The table contains the median and probabilities of 67% (likely), 90% (very likely), and 95% (extremely likely) attached to various ranges in each class.[2] In contrast to the IPCC5 report, there seems to be broad agreement across multiple lines of evidence.

Table 3: Lognormal approximations based on the four inputs, sixth report

		b_{0i}	σ_{0i}	mode	Quantiles 5%	50%	95%
(a)	Process understanding	1.22	0.36	2.98	1.87	3.39	6.12
(b)	Instrumental record	1.03	0.39	2.41	1.47	2.80	5.32
(c)	Paleoclimates	1.20	0.61	2.29	1.22	3.32	9.06
(d)	Emergent constraints	1.05	0.36	2.51	1.58	2.86	5.17

If we assume, as in Section 4.1, that $\log \beta_i \sim \mathrm{N}(b_{0i}, \sigma_{0i}^2)$ for the ith input, then we can approximate the values of b_{0i} and σ_{0i} from the given probabilistic ranges in Table 2. These approximations are presented in Table 3 together with the implied mode, and 5%, 50% (the median), and 95% quantiles. The four implied distributions will serve as our inputs.

Table 4: Probabilities of the four inputs based on the lognormal approximations, sixth report

	Target = 0.67	Target = 0.90	Target = 0.95
(a)	$\Pr(2.5 < \beta < 5.1) = 0.67$	$\Pr(2.1 < \beta < 7.7) = 0.90$	
(b)	$\Pr(\beta > 2.2) = 0.73$	$\Pr(\beta > 1.8) = 0.87$	$\Pr(\beta > 1.6) = 0.92$
(c)	$\Pr(\beta < 4.5) = 0.69$	$\Pr(\beta > 1.5) = 0.90$	$\Pr(\beta < 8.0) = 0.93$
(d)		$\Pr(1.5 < \beta < 5.0) = 0.90$	

The precision of our approximations is verified in Table 4, where we calculate the implied probabilities for the ranges presented in Table 2. The

[2]The IPCC writes about the 'best' or the 'central' estimate, and they also occasionally mention the median but never the mode of a distribution. For this reason we interpret the 'best' estimate as the median rather than as the mode, but some ambiguity remains.

probabilities in Table 4 should be close to those in Table 2, and they are. Figure 2 presents the four lognormal curves corresponding to the four lines of evidence. The distributions are closer to each other and less ambiguous than the distributions in Figure 1. The modes range from 2.29 to 2.98 (rather than from 1.62 to 3.32 in IPCC5) and the medians range from 2.80 to 3.39 (rather than from 1.97 to 3.42 in IPCC5).

5 Combining the inputs

Combining information from various sources into a single piece of information is a nontrivial exercise. One needs to know the nature of the sources, how relevant and reliable they are, and if and to what extent they are correlated. We shall concentrate on the effect of correlation, first in general, then specifically for our application.

In the general case, we consider a sequence of correlated random variables $x = (x_1, x_2, \ldots, x_m)'$ with common mean μ, so that $\mathrm{E}(x) = \mu\,\imath$ and $\mathrm{var}(x) = \sigma^2 V$, where $\imath$ denotes the vector of ones and V is positive definite. Without loss of generality we normalize V by imposing the restriction $\mathrm{tr}(V) = m$. The generalized least-squares estimators of μ and σ^2 are

$$\hat{\mu} = \frac{\imath' V^{-1} x}{\imath' V^{-1} \imath}, \qquad \hat{\sigma}^2 = \frac{(\imath' V^{-1} \imath)(x' V^{-1} x) - (\imath' V^{-1} x)^2}{m\, \imath' V^{-1} \imath}, \tag{15}$$

while $\tau^2 = \mathrm{var}(\hat{\mu})$ is estimated by

$$\hat{\tau}^2 = \frac{\hat{\sigma}^2}{\imath' V^{-1} \imath} = \frac{(\imath' V^{-1} \imath)(x' V^{-1} x) - (\imath' V^{-1} x)^2}{m\, (\imath' V^{-1} \imath)^2}. \tag{16}$$

Important is the difference between σ^2 and τ^2. The parameter σ^2 is the variance of the process, while τ^2 measures the precision of the estimator $\hat{\mu}$. As the number of observations increases we are able to estimate μ more and more precisely (as measured by τ^2), but the variance of the process (measured by σ^2) is not affected by the number of observations. The behavior of the two estimators $\hat{\sigma}^2$ and $\hat{\tau}^2$ as a function of V is also very different. We are interested in $\hat{\sigma}^2$, the estimator of $\mathrm{tr}(\mathrm{var}(x))/m$, the average variance of the x_i.

When the x_i are uncorrelated then $\hat{\sigma}^2$ is bounded and finite, but when the x_i are correlated then this is no longer the case. To see why, let us write $V^{-1} = |V|^{-1} C_V$, where C_V denotes the cofactor matrix of V. In the expression for $\hat{\mu}$ the determinant $|V|$ cancels as it appears in both the numerator and the denominator, but in the expression for $\hat{\sigma}^2$ it does not cancel and we are left with one factor $|V|$ in the denominator. If $|V|$ goes to zero then the ratio goes to ∞. This heuristic argument was recently made precise in Magnus and Vasnev (2023) and applied to forecast combinations of GDP and CPI by the Bank of Japan and the European Central Bank. They conclude that ignoring possible correlation in the observations can lead to estimates of σ^2 which are (much) too small.

We shall adopt the ideas in Magnus and Vasnev (2023) to aggregate the $m = 14$ inputs in IPCC5 and the $m = 4$ inputs in IPCC6. Our task is to find an estimator $b_0 \sim \mathrm{N}(\log\beta, \sigma_0^2)$ based on the fourteen inputs $b_{0i} \sim \mathrm{N}(\log\beta, \sigma_{0i}^2)$ in Section 4.1 and, similarly, an estimator b_0 based on the four inputs b_{0i} in Section 4.2. Thus we need to find two lognormal distributions which can be thought of as reasonable representations of the two sets of inputs. These distributions must have a larger variance than the red curves in Figures 1 and 2, because theory prescribes that the posterior variance is smaller than the input variance (and also than the prior variance).

Let V_0 denote the diagonal matrix containing the diagonal elements of V. Then, $P = V_0^{-1/2} V V_0^{-1/2}$ is the correlation matrix associated with V. In the absence of correlation we have $P = I_m$ and we obtain $b_0 = 1.07$ (fifth report), $b_0 = 1.11$ (sixth report), and

$$\sigma_0 = 0.27 < 0.53 = \sigma_2 \text{ (fifth report)}$$
$$\sigma_0 = 0.10 < 0.27 = \sigma_2 \text{ (sixth report)}.$$

These input variances violate the restriction $\sigma_0 > \sigma_2$ and are therefore too small. But the assumption of zero correlation is unrealistic anyway. We expect the inputs to be correlated in our case, probably highly correlated.[3] To take possible correlation into account we shall assume equicorrelation, so that the correlation between each pair of inputs is the same, say ρ. The correlation matrix P then takes the form

$$P = \begin{pmatrix} 1 & \rho & \rho & \dots & \rho \\ \rho & 1 & \rho & \dots & \rho \\ \vdots & \vdots & \vdots & & \vdots \\ \rho & \rho & \rho & \dots & 1 \end{pmatrix} \qquad (0 \leq \rho < 1). \tag{17}$$

Since P depends on only one parameter, we can now investigate the effect of correlation on our input moments.

[3] IPCC6 (p. 1006) is also concerned with the question on how to combine the inputs, but with a view to *narrow* the range of ECS values. See Annan and Hargreaves (2006), Stevens et al. (2016), and Sherwood et al. (2020) for a Bayesian perspective.

Table 5: Combined inputs, fifth and sixth reports

	Fifth report			Sixth report		
ρ	b_0	σ_0	τ	b_0	σ_0	τ
0.00	1.07	0.27	0.04	1.11	0.10	0.04
0.50	1.20	0.32	0.06	1.10	0.13	0.09
0.80	1.22	0.49	0.06	1.07	0.21	0.14
0.90	1.22	0.68	0.06	1.05	0.30	0.17
0.95	1.23	0.96	0.06	1.03	0.41	0.19
0.99	1.23	1.98	0.06	1.01	0.89	0.21
1.00	1.23	∞	0.06	1.00	∞	0.22

The input moments are presented in Table 5, where we let the correlation ρ increase from 0 to 1.[4] The restriction $\sigma_0 > \sigma_2$ is only satisfied when $\rho > 0.83$ (fifth report) and $\rho > 0.88$ (sixth report), confirming that the estimation results in the underlying inputs are highly correlated. When there is no correlation, then b_0 increases from 1.07 to 1.11 between the fifth and sixth report. But when we take correlation into account, then b_0 decreases from 1.23 to 1.03 (at $\rho = 0.95$). Since the posterior b_2 increases from 1.07 to 1.15 it will need a strong prior to achieve this change of direction.

While b_0 is quite stable for $\rho \geq 0.9$, this is not the case for σ_0. The precision in the input distribution depends heavily on the value of ρ, that is, on the degree in which the inputs are correlated with each other. In the limit, when $\rho \to 1$ and $\sigma_0 \to \infty$, the inputs provide *no* additional information and hence the posterior is equal to the prior. We conclude that the inputs provide less information (σ_0 is larger) in the presence of correlation, and given our assumptions we can quantify how much less. We do not know what the value of ρ is and hence, when we reveal the priors, we shall work with three scenarios: $\rho = 0.90$, 0.95, and 0.99.

6 The revealed prior

Given the moments (b_2, σ_2) from the posterior and (b_0, σ_0) from the input, we can now reveal the implied prior moments. Recall from Section 2 that, given a posterior $\log \beta | b_0 \sim \mathrm{N}(b_2, \sigma_2^2)$ and an input $b_0|\beta \sim \mathrm{N}(\log \beta, \sigma_0^2)$, the prior

[4]The results are obtained from (15) and (16) for $\rho < 1$. For $\rho = 1$ the matrix P is singular, but we can calculate the limiting values from the theory developed in Magnus and Vasnev (2023).

distribution is $\log \beta \sim \mathrm{N}(b_1, \sigma_1^2)$, where b_1 and σ_1^2 are given by (5) and (6), respectively:

$$b_1 = \frac{\sigma_0^2 b_2 - \sigma_2^2 b_0}{\sigma_0^2 - \sigma_2^2} = \frac{b_2 - (1-w) b_0}{w} \tag{18}$$

and

$$\sigma_1^2 = \frac{\sigma_0^2 \sigma_2^2}{\sigma_0^2 - \sigma_2^2} = \frac{\sigma_2^2}{w}, \tag{19}$$

where $w = 1 - \sigma_2^2/\sigma_0^2$, as defined in Section 2. The prior mean b_1 thus depends only on the posterior mean b_2 (which we set at 1.07 and 1.15 respectively), the input mean b_0 (which is quite stable at approximately 1.23 and 1.03 respectively), and on the weight w (which is not stable and depends heavily on the assumed correlation). In contrast, the prior standard deviation σ_1 depends only on the posterior standard deviation σ_2 (which we set at 0.53 and 0.27 respectively) and on the weight w.

Table 6: Input, prior, and posterior

	ρ	Fifth report			Sixth report		
		b_0	σ_0	w	b_0	σ_0	w
	0.90	1.22	0.68	0.39	1.05	0.30	0.16
Input	0.95	1.23	0.96	0.69	1.03	0.41	0.58
	0.99	1.23	1.98	0.93	1.01	0.89	0.91
	1.00	1.23	∞	1.00	1.00	∞	1.00
		b_1	σ_1		b_1	σ_1	
	0.90	0.83	0.85		1.66	0.67	
Prior	0.95	1.00	0.64		1.24	0.36	
	0.99	1.06	0.55		1.16	0.28	
	1.00	1.07	0.53		1.15	0.27	
		b_2	σ_2		b_2	σ_2	
Posterior		1.07	0.53		1.15	0.27	

Table 6 shows the revealed prior means and standard deviations for both reports in the range $0.9 \leq \rho \leq 1.0$, together with the corresponding moments for the input and the posterior. We observe that there is essentially no difference in the prior at $\rho = 0.99$ and $\rho = 1.00$. In the latter case, the input variance is ∞ and hence prior = posterior and the inputs (the underlying academic studies) play no role at all. Although there are situations where the posterior is completely determined by the prior (and hence the data, however convincing, are disregarded), we do not believe this is a credible description of the process by which the IPCC scientists reach their conclusions. On the other hand, for $\rho < 0.83$ (fifth report) and $\rho < 0.88$ (sixth report) the weight w becomes negative and the theoretical constraint $\sigma_2 < \sigma_0$ is violated. Let us therefore concentrate on the two cases $\rho = 0.90$ and $\rho = 0.95$.

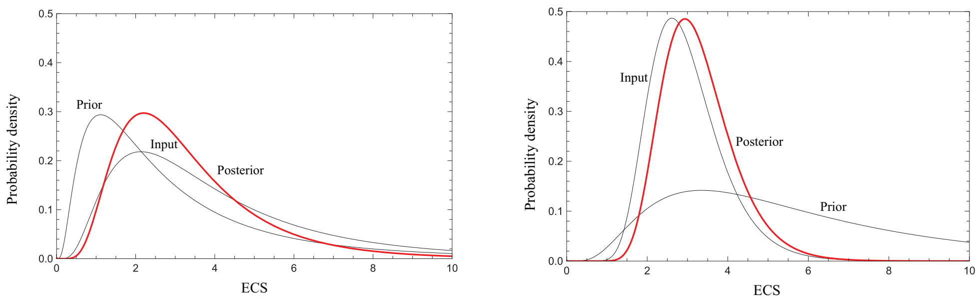

Figure 3: Input, prior, and posterior for $\rho = 0.90$, fifth report (left) and sixth report (right)

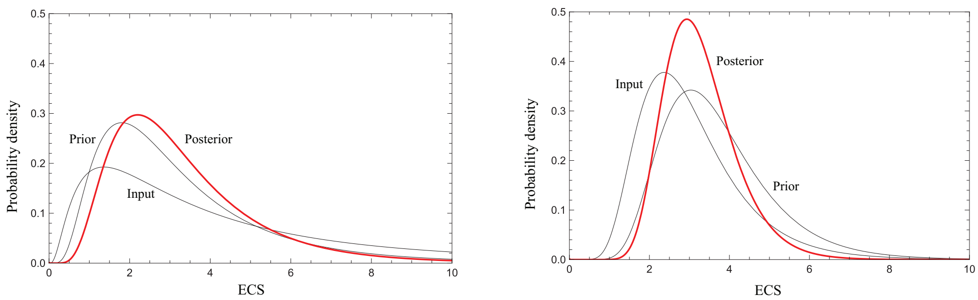

Figure 4: Input, prior, and posterior for $\rho = 0.95$, fifth report (left) and sixth report (right)

The table confirms what we already know from the theory, namely that when ρ increases then w and σ_0 also increase, while σ_1 decreases. This reflects the fact that more correlation in the inputs reduces the precision of the combined input and makes the prior more important.

The prior mean b_1 is much larger in the sixth than in the fifth report and the prior standard deviation σ_1 is much smaller. Specifically, b_1 increased from 0.83 to 1.66 ($\rho = 0.90$) and from 1.00 to 1.24 ($\rho = 0.95$), implying an increase in the prior median of β from 2.30 to 5.27 ($\rho = 0.90$) and from 2.72 to 3.45 ($\rho = 0.95$). At the same time, the standard deviation σ_1 decreased from 0.85 to 0.67 ($\rho = 0.90$) and from 0.64 to 0.36 ($\rho = 0.95$). Hence, the

IPCC scientists changed their prior views between the fifth and the sixth report, and they held these views with more conviction.

It is remarkable that $b_1 < b_2$ in the fifth report, while $b_1 > b_2$ in the sixth report. This means that in the fifth report the IPCC scientists used their prior views to tune the input (= data) information down, arriving at median(β) = 2.92 in the posterior while median(β) = 2.72 (at $\rho = 0.95$) in the prior. In contrast, in the sixth report the prior views are tuned up arriving at median(β) = 3.16 in the posterior while median(β) = 3.45 (at $\rho = 0.95$) in the prior.

In Figures 3 and 4 we draw the lognormal distributions of β at $\rho = 0.90$ and $\rho = 0.95$, respectively. The figures illustrate that as ρ increases the prior becomes more important, and that the prior distribution shifts to the right and becomes more concentrated (lower variance) when we compare the fifth and sixth reports.

7 Dynamic consistency

One would expect that the posterior in one period serves as the prior in the next period, at least approximately. When this happens, the decision maker is rational and the process is said to be 'dynamically consistent.' In our case, dynamic consistency requires that the prior in the sixth report is equal to the posterior in the fifth report. But is it?

The posterior in the fifth report is $b_2 = 1.07$ ($\sigma_2 = 0.53$), while the prior in the sixth report is $b_1 = 1.24$ ($\sigma_1 = 0.36$) at $\rho = 0.95$. Hence, the median of the posterior in the fifth report is 2.92 (the mode is 2.20) and the median of the prior in the sixth report is 3.45 (the mode is 3.04). These are large differences, both in terms of location (median, mode) and in terms of precision.

One possible explanation of this large difference is that there is indeed a lack of dynamic consistency. One can easily imagine a situation where the decision maker remains too loyal to their original prior, which one may call 'prior stubbornness' (bunching). This stubbornness may be politically motivated and it could continue until some bound has been reached (a tipping point) after which the prior is adjusted and moves to a new level.

But there is also another possible explanation. There is a gap of eight years between the IPCC5 and IPCC6 reports. During this period the IPCC experts were exposed to climate news (floods, fires, rising temperatures), they attended conferences, they read newspapers, etc. Their beliefs were therefore influenced and updated before they started working on IPCC6. In other words, an adjustment to their priors occurred.

To analyze this possibility we perform a thought experiment. In the relationship between input, prior, and posterior we have so far encountered input+prior = posterior (the standard Bayesian method) and also posterior− input = prior (our perspective to reveal the prior). We now rewrite our equations based on posterior − prior = input, assuming that the decision maker behaves dynamically consistent. That is, we create a hypothetical input by considering the IPCC5 posterior as our prior and the IPCC6 prior as our posterior. To move from this new prior to the new posterior we need a new input, which is constructed from

$$b_0 = \frac{\sigma_1^2 b_2 - \sigma_2^2 b_1}{\sigma_1^2 - \sigma_2^2}, \qquad \sigma_0^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 - \sigma_2^2}, \tag{20}$$

using (3) and (4) or, alternatively, (5) and (6). The revealed hypothetical input thus represents the input that would be required to bridge the gap between the posterior from IPCC5 and the prior from IPCC6.

Table 7: Consistency gap and hypothetical input at $\rho = 0.95$

Source	Experiment	b	σ	median	mode
IPCC5					
Input		1.23	0.96	3.41	1.36
Prior		1.00	0.64	2.72	1.81
Posterior	Prior	1.07	0.53	2.92	2.20
IPCC6					
Input		1.03	0.41	2.80	2.36
Prior	Posterior	1.24	0.36	3.45	3.04
Posterior		1.15	0.27	3.16	2.94
Gap	Input	1.37	0.48	3.95	3.14

In Table 7 we copy the posterior in the fifth report and the prior in the sixth report (at $\rho = 0.95$) from Table 6, and then use (20) to calculate the input that would be required to bridge the gap and bring about dynamic consistency. In this experiment the IPCC5 posterior thus plays the role of prior and the IPCC6 prior plays the role of posterior.

When we compare the hypothetical input with the actual input in the sixth report (also displayed in Table 7), we see that the hypothetical input distribution is located to the right of the actual input distribution. In fact, both the median and the mode of the hypothetical input are much larger than of the actual input: for the median we have $3.95 \gg 2.80$ and for the mode $3.14 \gg 2.36$.

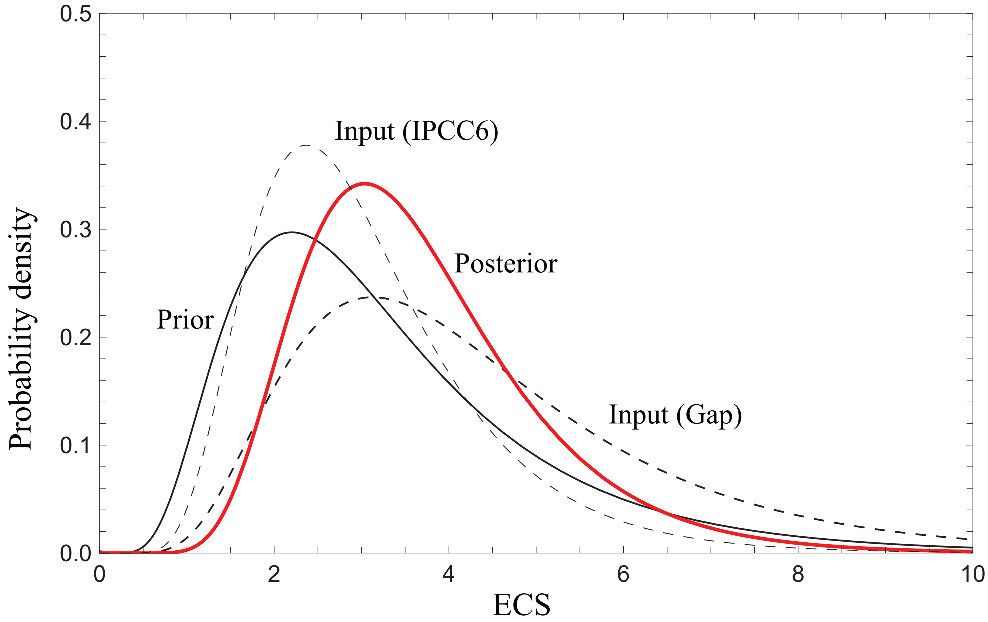

Figure 5: The consistency gap at $\rho = 0.95$

The gap is illustrated in Figure 5 where we display the prior and the posterior of our experiment, and the implied input which we wish to compare with the IPCC6 input (also displayed). The figure confirms what we found in the table, namely that the gap is too large to explain the dynamic inconsistency. One can imagine that IPCC6 scientists felt the pressure caused by the increase of climate exposure in the media. There has been political pressure and an abundance of qualitative information — information that was not reflected in the underlying scientific reports underlying IPCC6. But the consistency gap is too large too be credibly explained by political and media pressure alone.

If the gap experiment does not explain the dynamic inconsistency, then what does? Perhaps the IPCC6 scientists were not fully confident with the conclusions from IPCC5, because they now had access to multiple lines of evidence which were not available in IPCC5. They would then not necessarily accept the IPCC5 posterior as their prior. Another possibility is prior stubbornness, as mentioned earlier. Interestingly, it seems that the opposite has occurred here. We know from the previous section that the prior in IPCC5 is smaller than the posterior,

$$\text{prior} < \text{posterior} < \text{input} \qquad \text{(IPCC5)},$$

that is, scientific evidence (the input) indicated a higher value of the ECS than the IPCC scientists' prior views.[5] But in IPCC6 the situation is reversed:

$$\text{input} < \text{posterior} < \text{prior} \qquad \text{(IPCC6)}.$$

The inconsistency is apparently caused, at least in part, by overcompensation. The prior in IPCC5 was too low, so the IPCC scientists adjusted their prior upward in IPCC6, perhaps too much.

8 Concluding remarks

Let us translate our findings back to the language of the IPCC. As discussed in Section 3.1, IPCC5 concludes that ECS is likely in the range 1.5℃ to 4.5℃, that it is positive, extremely unlikely less than 1.0℃, and very unlikely greater than 6.0℃. This reflects the posterior distribution as presented by the IPCC itself. In Section 5 we deduce the combined input and approximate its distribution, from which we find (at $\rho = 0.95$) that

> ECS is likely in the range 1.4℃ to 8.6℃, is positive, extremely unlikely less than 0.7℃, and very unlikely greater than 11.6℃.

Then, in Section 6, we present the prior which tells us that

> ECS is likely in the range 1.5℃ to 5.0℃, is positive, extremely unlikely less than 1.0℃, and very unlikely greater than 6.2℃.

In the same way, IPCC6 concludes (our posterior, Section 3.2) that the very likely range of ECS is between 2.0℃ and 5.0℃, that the best estimate is 3.0℃ with a likely range of 2.5℃ to 4.0℃, and that it is virtually certain that ECS is larger than 1.5℃. From the distribution corresponding to the combined input we find that

> the very likely range of ECS is between 1.4℃ and 5.5℃, the best estimate is 2.8℃ with a likely range of 1.9℃ to 4.2℃, and it is virtually certain that ECS is larger than 1.1℃.

Then, combining posterior and input, the induced prior tells us that

[5] The inequality holds for b and the median, but not for the mode, where the inequality is reversed.

> the very likely range of ECS is between 1.9℃ and 6.2℃, the best estimate is 3.4℃ with a likely range of 2.4℃ to 4.9℃, and it is virtually certain that ECS is larger than 1.5℃.

These verbal summaries correspond exactly to the estimates obtained in the previous sections, and they confirm that the prior plays an important role in the IPCC decision process when drawing conclusions from the data (our inputs). In the fifth report the prior is more optimistic than the input with a prior median of 2.7 versus an input median of 3.4, while in the sixth report the prior is more pessimistic with a prior median of 3.5 versus an input median of 2.8. The ECS is a critical parameter in the climate debate but it remains difficult to estimate, and it seems that the scientific evidence (the inputs) and the prior views held by the IPCC scientists are not well synchronized.

In addition to an in-depth analysis of the estimation process of the ECS in the fifth and sixth reports of the IPCC, the current paper makes two theoretical contributions. First, we show how the prior can be revealed from the input and the posterior, also when the underlying distributions are skewed in which case the normal distribution is unsuitable. Second, we show how inputs can be combined, and the critical role of correlation in this aggregation process. The underlying studies (the inputs) are naturally correlated and if we ignore the correlation we overestimate the precision of our combined estimator: under correlation the data contain much less information than a naive investigator might assume. These two theoretical contributions will also be relevant in other applications where priors can be revealed.

Priors as discussed in the current paper are the human filter between data and conclusions. Such a filter is present in any scientific project. When the data and the conclusions are publicly available, which is often the case, then our methodology makes it possible to infer the underlying prior, which should make scientific and policy reports more transparent.

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