WORKING PAPER

Global Value Chains and International Risk Sharing

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Abstract

Unlike final-goods trade, intermediate-input trade through Global Value Chains (GVCs) creates supply-side linkages across borders. We show that, even when GVCs themselves are efficient, they have welfare implications because these linkages affect countries' ability to share risks in incomplete financial markets. This novel interaction between trade and finance arises from a distinct channel of cross-border transmission with GVCs, the marginalcost effect. Productivity shocks, by moving relative prices, impact the marginal cost of production both at home and abroad, and therefore, in equilibrium, their relative supply. When international financial markets are incomplete, these supply-side linkages will affect household wealth in both countries, and, in turn, the degree of international risk sharing. The direction and strength of this effect varies with the trade elasticity and the degree of GVC integration, with non-monotonic effects leading to 'fragmentation traps' in which small increases in GVC integration reduce risk sharing, while large increases would improve it. We show that in the quantitatively relevant case, GVCs reduce cross-border misalignments, and so endogenously support international risk sharing.

Keywords

Acknowledgements

1 Introduction

Global Value Chains (GVCs) are now a prominent feature of the international macroeconomic landscape, and recent large and synchronised shocks to supply chains have brought them to the forefront of macroeconomic policy debates (World Bank, 2020; D'Aguanno, Davies, Dogan, Freeman, Lloyd, Reinhardt, Sajedi, and Zymek, 2021; International Monetary Fund, 2022). In contrast to trade in final goods, trade in intermediate inputs, through GVCs, creates supply-side linkages across borders, giving rise to a distinct channel of cross-border transmission. In this paper, we put this channel front and centre, asking: how do GVCs influence the transmission of shocks and what does this imply for the degree of international risk sharing?

We document a novel link between international trade linkages and financial-market integration. The ability of economies to hedge their consumption against idiosyncratic shocks by sharing business-cycle risk with each other is a key theme in the international macroeconomics literature, due to its implications for macroeconomic dynamics and social welfare. We show that GVC integration affects how countries share risks when international financial markets are incomplete, and therefore impacts welfare even if trade itself is efficient. This occurs because trade in GVCs allows productivity shocks in one country to impact production in another, by influencing the supply and marginal costs of intermediate inputs for both domestic and foreign producers. When international financial markets are incomplete, these supply-side linkages will affect household wealth in both countries, and, in turn, the degree of international risk sharing.

We ground our analysis in a workhorse two-country, two-good New Open-Economy Macro (NOEM) model with 'roundabout' technologies that take intermediate goods as inputs into production. This class of models is widely used in academic and policy analysis, and our general specification nests a range of empirically-relevant cases. Within the model, households in each country consume domestic- and foreign-produced final goods, giving rise to trade in final goods. Domestic production combines household labour supply with both domestic- and foreign-produced intermediate inputs, giving rise to GVCs within the model. Firms' output is subject to risk from country-specific productivity shocks, generating incentives for households to borrow and save through, potentially incomplete, international financial markets, with implications for aggregate macroeconomic outcomes.

In response to a positive productivity shock, a vast body of empirical work indicates that all countries benefit, with positive macroeconomic transmission and co-movement across borders (see, e.g., Corsetti, Dedola, and Leduc, 2014). When risk sharing across borders is complete, this is the efficient outcome: a greater abundance of goods from one country following a positive productivity shock allows domestic and foreign consumers alike to consume a greater quantity, albeit through a counterfactual decline in the relative price due to a '*substitution effect*'.

However, the literature has shown that without trade in intermediate goods—a case nested within our model—incomplete international risk sharing can make it challenging to generate the positive transmission and co-movement seen in the data. In addition to the substitution effect described above, these price changes have a '*wealth effect*' when financial markets are incomplete: lower prices push down on current income for imperfectly-insured domestic households. Corsetti,

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Dedola, and Leduc (2008) show that when domestic and imported final goods are sufficiently complementary, these wealth effects can help to reconcile evidence on the correlation between relative consumption and relative prices (Backus and Smith, 1993; Kollmann, 1995). This can happen in one of two ways: either the negative wealth effect dominates such that the decline in the relative price immiserises Home consumption; or, to support higher Home consumption, relative prices rise and generate positive Home wealth effects.

In our model allowing for intermediate-goods trade, productivity shocks work through an additional channel: the '*marginal-cost effect*'. Movements in relative prices affect the marginal cost of intermediate inputs both at home and abroad, to which producers respond endogenously by adjusting output. We show that, all else equal, a decline in the relative price of the domestic good lowers Home output relative to Foreign. This effect offers a novel supply-side channel through which macroeconomic shocks can be transmitted across borders.

To illustrate how this effect influences risk sharing, we analytically compare a model with extreme financial market incompleteness—with financial autarky across borders—to a benchmark with full international risk sharing—with complete markets due to the presence of a full set of Arrow-Debreu securities. Following Corsetti, Dedola, and Leduc (2010, 2023) and Viani (2011), we define a welfare-relevant wedge, the '*wealth gap*', which characterises deviations from full risk sharing period-by-period. This wedge captures cross-country relative-consumption misalignments as well as relative-price misalignments, relative to the complete markets benchmark. A positive (negative) wealth gap implies that domestic consumption is inefficiently high (low) relative to foreign, after adjusting for purchasing power.

Remarkably, we show that, even when GVC trade is efficient, the degree of GVC integration has welfare implications because of its influence on the wealth gap, and so the degree of international risk sharing. The sign and magnitude of this relationship depends on a number of factors, including the degree of openness and trade elasticities.

When domestic and foreign intermediate goods are highly substitutable (i.e., the elasticity of intermediate-goods trade is high), the predominant effect is substitution, which raises relative-consumption misalignments. Following a productivity increase at Home, Foreign firms will flock towards the more abundant and, in equilibrium, cheaper Home input. This enables Foreign firms to produce more, which will, in turn, support even greater Home supply. Since equilibrium relative-price movements are small, the marginal-cost effect is dampened and the marginal impact of integration is small. Nonetheless, as relative Home supply rises by more in the presence of GVCs, substitution acts to amplify the relative-consumption misalignments. With consumers unable to smooth consumption intertemporally, this will lead to Home relative consumption being inefficiently high compared to the complete-markets benchmark. Therefore, the wealth gap increases in the degree of GVC integration.

In contrast, when domestic and foreign intermediate goods are highly complementary in production (i.e., the elasticity of intermediate-goods trade is low), GVC integration has ambiguous implications for misalignments. In cases where the negative wealth effect dominates, GVC integration works to mitigate misalignments. Following a positive Home productivity shock, higher

complementarity leads to larger relative-price movements compared to substitutability, and so the marginal-cost effect in the Foreign economy is stronger, and Foreign output can even rise by more than Home output in response to the Home productivity shock. In equilibrium, this means that higher GVC integration implies a smaller fall in relative prices—as relative supply absorbs some of the adjustment—and a weaker negative wealth effect in the Home economy. For a given elasticity, this reduces the adjustment in relative consumption, and so the magnitude of the wealth gap is decreasing in the degree of GVC integration. On the other hand, in cases where relative-price responses switch sign to generate positive domestic wealth effects, the opposite is true: all else equal the marginal-cost effect at Home is amplified, leading to a larger rise in Home output and further reinforcing the positive wealth effect. This amplifies the relative-consumption misalignments and so the wealth gap is larger. In other words, in this case, GVC integration reduces risk sharing.

In fact, we show that with sufficient GVC integration, this latter equilibrium is ruled out altogether. In the absence of GVCs, with high complementarity, a rise the relative price is needed to ensure that the demand for domestic goods rises to meet the increased supply. However, with GVCs, the marginal-cost effect induces an endogenous adjustment in relative supply, requiring less adjustment in demand, and thereby supporting the equilibrium with declining prices.

These different types of equilibria mean that, for sufficient complementarity, the impact of GVC integration on risk sharing is non-monotonic. At low levels of GVC integration, for a given trade elasticity, the marginal impact of higher integration may be to worsen risk sharing, even while a sufficiently large increase in integration can improve it. In this sense, countries may find themselves in a '*fragmentation trap*': even though large increases in integration can be beneficial, small increases in integration, which may be required in transition, can be costly. On the other hand, highly integrated economies operate in an '*integration oasis*', where reductions in GVC integration can significantly reduce risk sharing, disincentivising policies that could fragment trade, whereas further integration continues to yield increased risk sharing.

To assess the quantitative relevance of our analytical results, we carry out two complementary exercises. First, we use country-level input-output tables to calibrate the key parameters related to trade openness in both final and intermediate goods. We remain agnostic as to the exact calibration of final- and intermediate-goods trade elasticities, given the wide range of empirical estimates for these parameters. For different combinations of intermediate- and final-goods trade elasticities, we calculate whether risk sharing increases or decreases as we move between the calibrations corresponding to the 'Most Integrated', 'Average' and 'Least Integrated' countries in our sample. Across a range of exercises, we find that more GVC integration supports international risk sharing for quantitatively relevant values of the intermediate- and final-goods elasticities.

Second, we carry out a macro-level empirical analysis, in which we construct measures of both GVC integration and the wealth gap for advanced economies. The findings here validate the conclusion from the quantitative exploration of our model, namely that GVC integration is associated with increased risk sharing. Controlling for country size, overall trade openness and

other measures of integration, we show that, on average, greater GVC integration is associated with significantly more financial-market risk sharing.

Overall, our analysis highlights an important interaction between intermediate-goods trade and international financial markets, building on a growing literature assessing the nexus of trade and finance (e.g., Lloyd and Marin, 2023). Even with a simple frictionless model of trade in intermediates, GVCs can have non-trivial implications for the transmission of shocks across borders and, in turn, international risk sharing.

Related Literature. The contribution of intermediate goods trade to the increase in international trade seen since the 1980s has motivated a vast body of research into the aggregate consequences of GVCs, recently surveyed in Antràs and Chor (2022). Within this literature, particular emphasis has been placed on their role in shaping business cycles and cross-country co-movement (Burstein, Kurz, and Tesar, 2008; Johnson, 2014; Huo, Levchenko, and Pandalai-Nayar, 2019). Our paper takes a novel perspective, considering the implications of GVCs for risk sharing in international financial markets, and emphasising a specific channel, the marginal-cost effect, that is distinct to GVCs.

Our work also contributes to the broader debate that has been brought to the fore by post-Covid supply-chain shortages (see, e.g., Baldwin and Freeman, 2020a,b), fuelling pre-existing political fears around globalisation (Antràs, 2021)—in particular, a debate around the 'risks' from GVCs (D'Aguanno et al., 2021; Baldwin and Freeman, 2022). A particular concern is that, while GVCs have been shown to boost productivity, this could come at the expense of increased macroeconomic volatility (Caselli, Koren, Lisicky, and Tenreyro, 2020). But volatility in and of itself is not necessarily inefficient or detrimental to welfare, especially when there are no frictions in the trade linkages themselves (D'Aguanno et al., 2021). In this paper, we focus on the interaction of GVCs with a welfare-relevant inefficiency, namely incomplete financial markets, uncovering a novel mechanism through which GVCs have welfare implications by allowing risk to be shared across borders.

Related to our paper, Bai and Zhang (2012) show that de-jure financial integration, in the form of financial liberalisation, is not always correlated with de-facto risk sharing across borders, highlighting the role of implicit barriers to capital flows. In a similar vein, we show that even when de-jure financial integration is limited, de-facto risk sharing can be high due to the presence of implicit risk sharing via GVCs.

The mechanisms explored in this paper are also related to the recent literature on 'Keynesian supply shocks', which explores how changes in relative prices induced by supply shocks can affect demand through wealth effects (Cesa-Bianchi and Ferrero, 2021; Guerrieri, Lorenzoni, Straub, and Werning, 2022). While that literature explores this mechanism in multi-sector closed economies with uninsured households, we focus on the same mechanism in open economies with incomplete international risk sharing. In this sense, our approach is similar in spirit to that of Cole and Obstfeld (1991), Kraay and Ventura (2002) and Corsetti et al. (2008), who study risk sharing in international economies in models without GVCs.

Finally, this paper relates to the recent literature on financial integration and international

risk sharing.[1] Empirically, Cimadomo, Ciminelli, Furtuna, and Giuliodori (2020) and Ferrari and Rogantini Picco (2023) look at how risk sharing has evolved over time in the euro area, and decompose this into private and public channels of risk sharing. On the theoretical side, Cho, Kim, and Kim (2023) look at the wealth gap in the presence of inefficient shocks, showing that under certain conditions welfare under complete markets can be lower than financial autarky, and Hamano (2022) looks at a model with heterogeneous firms, finding that the self-selection of firms into export markets affects the degree of risk sharing. Relatedly, Esposito (2022) looks at how firms can diversify their production location across countries, so as to insure against demand uncertainty.

Outline. The rest of this paper is structured as follows: Section 2 lays out the theoretical model of trade in intermediate inputs. Section 3 presents our main analytical results, highlighting the marginal-cost effect and the consequences of GVC integration for international risk sharing in the face of relative productivity shocks. Section 4 explores the quantitative relevance of the analytical results, presenting results from the calibrated version of the model, as well as the assessment of the model predictions in the macro data. Section 5 concludes.

2 The Model

We begin by setting up a simple two-country real NOEM model, with GVCs. For simplicity and analytical tractability, we assume that the two countries, Home and Foreign, are symmetric. Consumers enjoy goods produced in both countries, such that there is trade in final goods. Firms in each country produce these goods using a combination of domestically supplied labour and both domestic and imported inputs, such that there is trade in intermediate goods, capturing a role for GVCs.

The following subsections describe the Home economy in more detail: the structure of the Foreign economy is analogous. Where necessary, the conventional $*$ denotes Foreign variables or parameters, and the subscripts H and F denote variables or parameters related to goods produced in the Home and Foreign economy respectively.

2.1 Households

Intertemporal Allocation. The representative Home household maximises lifetime utility from aggregate consumption, C_t. Their additively separable utility function is defined as:

$$U_t = \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\frac{C_{t+\tau}^{1-\sigma} - 1}{1 - \sigma} \right] \tag{1}$$

where $\mathbb{E}_t$ is the expectations operator conditional on information at time t, $\beta \in (0, 1)$ is the discount factor, and $\sigma > 0$ is the coefficient of relative risk aversion.

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[1]Of course this literature builds on important earlier work, but here we deliberately highlight the recent papers on this topic.

Let P_t denote the price of a unit of aggregate consumption, W_t the wage per unit of labour and L the inelastic labour supply of the household.[2] The intertemporal budget constraint of the representative household is: $\sum_{\tau=0}^{\infty} (P_{t+\tau}C_{t+\tau} - W_{t+\tau}L) \leq 0$. The per-period budget constraint, and hence the intertemporal optimality conditions, will depend on the financial assets available to the households, which will be determined by the nature and degree of cross-border financial linkages. We describe this in Section 2.4.

Intratemporal Allocation. The aggregate consumption bundle of Home households consists of domestically-produced and imported final goods. The Home basket has the following constant elasticity of substitution (CES) form à la Armington (1969):

$$C_t \equiv \left[a_H^{\frac{1}{\phi_C}} c_{H,t}^{\frac{\phi_C-1}{\phi_C}} + a_F^{\frac{1}{\phi_C}} c_{F,t}^{\frac{\phi_C-1}{\phi_C}} \right]^{\frac{\phi_C}{\phi_C-1}} \tag{2}$$

where $a_H \in (0,1)$ and $a_F = (1 - a_H)$ are the weights on H and F goods, $c_{H,t}$ and $c_{F,t}$ are the quantities consumed at time t, and $\phi_C > 0$ is the elasticity of substitution between them.[3]

The intratemporal problem of the representative Home household at time t involves minimising total expenditure, taking as given the level of aggregate consumption, C_t, and the prices of H and F goods—$P_{H,t}$ and $P_{F,t}$, respectively. This yields the demand functions:

$$c_{H,t} = a_H \left(\frac{P_{H,t}}{P_t} \right)^{-\phi_C} C_t \tag{3}$$

$$c_{F,t} = a_F \left(\frac{P_{F,t}}{P_t} \right)^{-\phi_C} C_t \tag{4}$$

and the definition of the consumer price index (CPI), P_t:

$$P_t = \left[a_H P_{H,t}^{1-\phi_C} + a_F P_{F,t}^{1-\phi_C} \right]^{\frac{1}{1-\phi_C}} \tag{5}$$

It is useful re-write these equations in terms of the Home country's terms of trade, $\mathcal{T}_t \equiv P_{F,t}/P_{H,t}$—i.e., the price of imports relative to domestic goods. Re-arranging (5) to write relative prices—$P_{H,t}/P_t$ and $P_{F,t}/P_t$—as a function of the terms of trade, and plugging the

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[2]Throughout our analytical exposition, we assume inelastic labour supply to abstract from wealth effects on labour supply that would otherwise generate counterfactually negative cross-country macroeconomic co-movement. With this assumption, in the absence of intermediate inputs our model nests an endowment model. We assess the robustness of our quantitative model results to endogenous labour supply in Section 4.

[3]Lisack, Lloyd, and Sajedi (2022) show that the parameters of the Armington aggregator capture the sufficient statistics for first-order demand-side dynamics in a general class of open-economy macroeconomics models with homothetic preferences, including this one.

resulting expressions into (3) and (4) yields the following expressions for final-goods demand:

$$c_{H,t} = a_H \left(a_H + a_F \mathcal{T}_t^{1-\phi_C} \right)^{\frac{\phi_C}{1-\phi_C}} C_t \tag{6}$$

$$c_{F,t} = a_F \mathcal{T}_t^{-\phi_C} \left(a_H + a_F \mathcal{T}_t^{1-\phi_C} \right)^{\frac{\phi_C}{1-\phi_C}} C_t \tag{7}$$

The Foreign consumption bundle, CPI and relative demand function are analogously defined.

2.2 Production

In each economy, perfectly competitive final-goods firms produce a country-specific good using a 'roundabout' production function. Home firms combine domestic labour with a bundle of intermediate goods, X_t, to produce gross output, $Y_{H,t}$, according to a Cobb-Douglas technology:

$$Y_{H,t} = A_t L^{1-\alpha} X_t^{\alpha} \tag{8}$$

where $\alpha \in [0,1)$ is the share of intermediates in gross output, and A_t is the exogenous country-specific total factor productivity (TFP).

Firms take the price of their goods, $P_{H,t}$, as well as the wage, W_t, and price of the intermediate bundle, $P_{X,t}$, as given. Maximising profits, their optimal demand schedules for labour and intermediate goods are as follows:

$$W_t = (1-\alpha) P_{H,t} \frac{Y_{H,t}}{L} \tag{9}$$

$$P_{X,t} = \alpha P_{H,t} \frac{Y_{H,t}}{X_t} \tag{10}$$

Trade in Intermediate Inputs. Home firms' intermediate-inputs bundle is an aggregate of domestic inputs, $x_{H,t}$, and imported inputs, $x_{F,t}$, similar to the final consumption bundle (2):

$$X_t \equiv \left[b_H^{\frac{1}{\phi_X}} x_{H,t}^{\frac{\phi_X-1}{\phi_X}} + b_F^{\frac{1}{\phi_X}} x_{F,t}^{\frac{\phi_X-1}{\phi_X}} \right]^{\frac{\phi_X}{\phi_X-1}} \tag{11}$$

where $b_H \in (0,1)$ and $b_F = (1-b_H)$ are the weights on H and F inputs, $x_{H,t}$ and $x_{F,t}$ are the quantities of inputs used at time t, and $\phi_X > 0$ is the elasticity of substitution between them.

As with the consumption bundle, the cost minimisation leads to the following definition for the intermediate-inputs price index:

$$P_{X,t} = \left[b_H P_{H,t}^{1-\phi_X} + b_F P_{F,t}^{1-\phi_X} \right]^{\frac{1}{1-\phi_X}} \tag{12}$$

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and the relative demand functions:

$$x_{H,t} = b_H \left(b_H + b_F \mathcal{T}_t^{1-\phi_X} \right)^{\frac{\phi_X}{1-\phi_X}} X_t \tag{13}$$

$$x_{F,t} = b_F \mathcal{T}_t^{-\phi_X} \left(b_H + b_F \mathcal{T}_t^{1-\phi_X} \right)^{\frac{\phi_X}{1-\phi_X}} X_t \tag{14}$$

2.3 Goods Market Clearing

In equilibrium, goods markets must clear. The gross outputs of H and F goods must satisfy the world demand for both consumer goods and intermediate inputs:

$$Y_{H,t} = c_{H,t} + c_{H,t}^* + x_{H,t} + x_{H,t}^* \tag{15}$$

$$Y_{F,t}^* = c_{F,t} + c_{F,t}^* + x_{F,t} + x_{F,t}^* \tag{16}$$

2.4 Financial Market Linkages

In addition, global financial markets must clear too. To assess the role of international financial markets for GVCs, we consider two alternative international financial market structures: Complete and Incomplete Markets.

With Complete Markets (CM), households can exchange a full set of state-contingent securities across borders. The intertemporal optimisation of households in each country will be given by the familiar Euler equations. In this case, with symmetry, there will be full risk sharing across borders and, since there are no frictions in the model (including in global supply chains), the resulting allocation will be first best from the perspective of a global planner. We obtain the equilibrium risk-sharing condition by equalising the Euler equations in each country:

$$\left(\frac{C_t}{C_t^*} \right)^\sigma \frac{1}{\mathcal{Q}_t} = 1 \tag{17}$$

where $\mathcal{Q}_t \equiv P_t^*/P_t$ is the real exchange rate.

In contrast, Incomplete Markets introduce inefficiency, limiting the extent to which households can smooth consumption intertemporally and share risks across borders. For our analytical results, we focus on a specific and extreme form of financial-market incompleteness: Financial Autarky (FA) in which no cross-border trade in financial assets is allowed, meaning that there is no risk sharing through financial markets. Under this assumption, trade must be balanced in each period:

$$P_{F,t} \left(c_{F,t} + x_{F,t} \right) = P_{H,t} \left(c_{H,t}^* + x_{H,t}^* \right) \tag{18}$$

Although this represents a somewhat stark assumption, the intuition from this case carries over to more general incomplete-market settings.[4]

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[4]For instance, in a bond economy, where international trade in financial assets is limited to one non-state-contingent bond that pays a known interest rate in the next period, there will be limited risk-sharing through financial markets. We assess the robustness of our quantitative model findings to this alternative financial-market structure in Section 4.

Incomplete-market settings will give rise to deviations from the perfect risk-sharing condition, equation (17). Following Corsetti et al. (2010), we can define a 'wealth gap' measure, which acts as a summary statistic quantifying departures from perfect international risk sharing:

$$\mathcal{W}_t \equiv \left(\frac{C_t}{C_t^*}\right)^\sigma \frac{1}{\mathcal{Q}_t} \tag{19}$$

Under CM, equation (17) implies that $\mathcal{W}_t^{CM} = 1$ for all t. With incomplete markets the magnitude of the wealth gap relative to the CM benchmark—i.e., $|\mathcal{W}_t - \mathcal{W}_t^{CM}|$—will represent the deviation from perfect risk sharing in international financial markets. Moreover, equation (19) clarifies that the gap is comprised of two components: one pertaining to misalignments in relative consumption across countries, and another linked to misalignments in relative prices.

Under FA, the wedge can be interpreted as the shadow value of income in the Foreign economy relative to Home.[5] As such, $\mathcal{W}_t > 1$ can be interpreted as showing that the marginal utility of an extra unit of income in the Foreign country exceeds that of the Home country. For given relative prices, this implies that relative consumption in the Foreign country will be inefficiently low. Alternatively, given relative consumption, the relative price of Home consumption is inefficiently high (i.e. the real exchange rate is over-appreciated).

3   Analytical Results

In this section, we present our main results, first highlighting how the supply-side linkages coming from GVCs give rise to a 'marginal-cost effect', and then analysing the links between the wealth gap and GVC integration. We present these results analytically by deriving the log-linear equilibrium equations, where we denote by $\hat{x}_t$ the deviation of any variable, x_t, from its value in the deterministic symmetric steady state.

3.1   Marginal-Cost Effect

In our model, the gross output of each country, $Y_{H,t}$ and $Y_{F,t}^*$, is endogenous due to the presence of intermediate inputs in the production function. To see this, we can plug the first-order condition for intermediates, equation (10), and the definition of the aggregate intermediate-input price, equation (12), into the production function:

$$Y_{H,t} = A_t^{1/(1-\alpha)} L(\alpha)^{\frac{\alpha}{1-\alpha}} \left[b_H + b_F \mathcal{T}_t^{1-\phi_X}\right]^{-\frac{1}{1-\phi_X}\frac{\alpha}{1-\alpha}} \tag{20}$$

and the Foreign equivalents:

$$Y_{F,t}^* = A_t^{*\,1/(1-\alpha)} L^*(\alpha)^{\frac{\alpha}{1-\alpha}} \left[b_H + b_F \mathcal{T}_t^{-(1-\phi_X)}\right]^{-\frac{1}{1-\phi_X}\frac{\alpha}{1-\alpha}} \tag{21}$$

If $\alpha = 0$, meaning that there are no intermediate inputs in the production function, our

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[5]This can be shown formally because under FA the wealth gap is equal to the ratio of the Foreign marginal-utility from current income relative to the Home equivalent.

model nests an endowment economy, with the endowments equal to $A_t L$ and $A_t^* L^*$.[6] When $\alpha > 0$, however, gross output will depend on the endogenous choice of intermediate inputs in production. The higher is α, the higher the share of intermediate inputs in overall production and, therefore, the higher the degree of GVC integration.[7]

To see how this impacts the equilibrium of the model, take the ratio of equation (20) and (21) and log-linearise to get the expression for relative supply:

$$\widehat{Y}_{H,t} - \widehat{Y}_{F,t} = \frac{1}{1-\alpha}\left(\widehat{A}_t - \widehat{A}_t^*\right) - 2b_F \frac{\alpha}{1-\alpha}\widehat{\mathcal{T}}_t. \tag{22}$$

This expression highlights the role of the *marginal-cost effect* in this model. With GVCs in the model, relative supply is now not only a function of relative productivity but also the terms of trade.[8] In other words, the terms of trade also represents a component of the firms' marginal costs, leading to a new channel of transmission through supply: a shock that moves the terms of trade will also affect relative supply.

Specifically, relative Home supply falls as the terms of trade increases. This is because, for Home firms, the terms of trade represents the cost of one of their inputs to production, relative to the price of their output. So, as the relative price of the imported intermediate input increases relative to the domestic good—i.e., if the terms of trade rises—this increases the cost of the intermediate inputs relative to the price of the domestic firm's output, and the firm's optimal response is to decrease domestic production. The reverse occurs for Foreign firms, with a rise in the terms of trade leading them to optimally increase their demand for Home inputs, and therefore produce more. Overall, this means that a rise in the terms of trade pushes down on relative Home output.

3.2 GVC Integration and Risk Sharing

We now consider how this marginal-cost effect impacts the equilibrium dynamics of the model, and in particular how this affects risk sharing under different degrees of GVC integration, as captured by α.

We first use the definition of CPI, equation (5) and its Foreign counterpart, to derive an equilibrium relationship between the real exchange rate, $\mathcal{Q}_t$, and the terms of trade, $\mathcal{T}_t$:

$$\mathcal{Q}_t^{1-\phi_C} = \frac{a_F + a_H \mathcal{T}_t^{1-\phi_C}}{a_H + a_F \mathcal{T}_t^{1-\phi_C}} \quad \Rightarrow \quad \widehat{\mathcal{Q}}_t = (2a_H - 1)\widehat{\mathcal{T}}_t \tag{23}$$

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[6] From this we can also see that L and L^* can be interpreted as either labour inputs or total domestic value added, and adding a fixed capital stock would not change these results.

[7] More accurately, b_H and α together determine the degree of GVC integration, with the former specifically determining the imported content of total intermediates. We focus on α as the key parameter, in particular because $\alpha = 0$ exactly nests the endowment model with trade in final goods, which is a commonly used and well understood benchmark. For many of our results, $b_H = 1$ will have the same effect. Note that, under the roundabout production structure, lowering b_H and α both effectively imply replacing domestic value added with foreign value added.

[8] Note that α also affects the size of the direct impact of the relative productivity shock on relative output. For a given shock to TFP, the presence of intermediate inputs in the production function will amplify the effect on gross output. We abstract from this effect in our discussion.

This relationship tells us that as long as there is home bias in the consumption bundle, $a_H > 0.5$, then the terms of trade and the real exchange rate will co-move positively. For the rest of the paper, we will focus our analysis on the terms of trade when discussing international relative prices, but all the results will hold for the real exchange rate as well, as per equation (23).

We can plug this into the log-linearised definition of the wealth gap, equation (19):

$$\widehat{\mathcal{W}}_t = \sigma \left(\widehat{C}_t - \widehat{C}_t^* \right) - \widehat{\mathcal{Q}}_t = \sigma \left(\widehat{C}_t - \widehat{C}_t^* \right) - (2a_H - 1) \widehat{\mathcal{T}}_t \tag{24}$$

Since the wealth gap remains fixed under complete markets, i.e., $\widehat{\mathcal{W}}_t^{CM} = 0$, we can interpret these fluctuations in the wealth gap around its steady state as deviations from perfect risk sharing. As such, the two terms on the right-hand of this expression can be interpreted as the relative-consumption and relative-price misalignments, respectively.

To see how this wedge fluctuates in response to shocks under incomplete markets, therefore, we consider the equilibrium dynamics of each component in turn.

3.2.1 Terms of Trade

We first focus on the terms of trade, summarising its determination in equilibrium with the following lemma.

Lemma 1 (Terms-of-Trade Determination) *In the FA equilibrium, the terms of trade is pinned down by (exogenous) differences in productivity across country:*

$$\widehat{\mathcal{T}}_t = \frac{1}{(1-\alpha)\left(D_\alpha + 2b_F \frac{\alpha}{1-\alpha}\right)} \left(\widehat{A}_t - \widehat{A}_t^* \right), \tag{25}$$

where:

$$D_\alpha \equiv \frac{a_F(1-\alpha)\left[1 - 2a_H(1 - \phi_C)\right] + b_F \alpha \left[1 - 2b_H(1 - \phi_X)\right]}{a_F(1-\alpha) + b_F \alpha}. \tag{26}$$

Proof: Inserting the individual demand functions into the trade-balance condition, equation (18), we can derive an expression for relative demand as a function of the terms of trade:

$$\widehat{Y}_{H,t} - \widehat{Y}_{F,t} = D_\alpha \widehat{\mathcal{T}}_t, \tag{27}$$

with D_α defined as in equation (26).

In equilibrium, relative demand equals relative supply, meaning that we can equate equations (27) and (22):

$$D_\alpha \widehat{\mathcal{T}}_t = \frac{1}{1-\alpha} \left(\widehat{A}_t - \widehat{A}_t^* \right) - 2b_F \frac{\alpha}{1-\alpha} \widehat{\mathcal{T}}_t.$$

Rearranging this yields equation (25). See Appendix A for full derivations. □

It is a known result in the NOEM literature that the terms of trade is pinned down in

equilibrium by relative productivities. Lemma 1 shows that this continues to hold in the model with roundabout production and trade in intermediates.

Equation (25) also shows that the sign of the response of the terms of trade to a (relative) productivity shock is ambiguous. In particular:

$$\text{sign}\left(\frac{\partial \widehat{\mathcal{T}}_t}{\partial \widehat{A}_{R,t}}\right) = \text{sign}\left(D_\alpha + 2b_F \frac{\alpha}{1-\alpha}\right)$$

where $\widehat{A}_{R,t} \equiv (\widehat{A}_t - \widehat{A}_t^*)$.

When $\alpha = 0$, when the model simplifies to the baseline endowment model with no GVCs, this simplifies to the sign of $(1 - 2a_H(1 - \phi_C))$. This leads to the familiar result explored in Corsetti et al. (2008): there exists a threshold value of elasticity of substitution, $\widetilde{\phi}_{TOT} \equiv 1 - 1/2a_H$, below which the terms of trade appreciates in response to a domestic productivity increase, and above which the terms of trade depreciates.

To understand why, recall first that a fall in the relative price of the domestic good induces a rise in demand through substitution towards the cheaper domestic good. However, with incomplete markets, this price adjustment induces a negative wealth effect for domestic households. Following a positive domestic productivity shock, markets clear with a terms of trade depreciation so long as the substitution effect is sufficiently large. When the trade elasticity is below the threshold value, the strength of the substitution effect is small relative to the wealth effect, and market clearing requires an appreciation of the terms of trade, inducing a rise in demand for the domestic good through a positive wealth effect for domestic households.

We show how this result extends to the cases where $\alpha > 0$, when there is a role for GVCs, with the following proposition.

Proposition 1 (GVC Integration and the Terms of Trade) *When the home bias in the CES aggregator for the consumption goods and intermediate inputs are the same, i.e., $b_H = a_H$, the sign of the response of the terms of trade to a relative productivity shock depends on whether the weighted-average trade elasticity, defined as:*

$$\Phi_\alpha = (1-\alpha)\phi_C + \alpha\phi_X, \tag{28}$$

is above or below the threshold:

$$\widetilde{\phi}_{TOT}(\alpha) = 1 - \frac{1}{2a_H} - \frac{a_F}{a_H}\frac{\alpha}{1-\alpha}. \tag{29}$$

Proof: Inserting $b_H = a_H$ into equation (25), the equilibrium expression for the terms of trade simplifies to:

$$\widehat{\mathcal{T}}_t = \frac{1}{(1-\alpha)\left(1 - 2a_H(1 - \Phi_\alpha) + 2a_F\frac{\alpha}{1-\alpha}\right)}\left(\widehat{A}_t - \widehat{A}_t^*\right), \tag{30}$$

so that:

$$\text{sign}\left(\frac{\partial \widehat{\mathcal{T}}_t}{\partial \widehat{A}_{R,t}}\right) = \text{sign}\left(1 - 2a_H(1 - \Phi_\alpha) + 2a_F \frac{\alpha}{1 - \alpha}\right).$$

Since the expression on the right-hand side is a continuous and monotonic function of Φ_α, we can define the threshold value as the value of the elasticity that sets that expression to zero:

$$1 - 2a_H(1 - \widetilde{\phi}_{TOT}(\alpha)) + 2a_F \frac{\alpha}{1 - \alpha} = 0 \quad \Rightarrow \quad \widetilde{\phi}_{TOT}(\alpha) = 1 - \frac{1}{2a_H} - \frac{a_F}{a_H}\frac{\alpha}{1 - \alpha}.$$

$\square$

This proposition shows that, for this case with $b_H = a_H$, given all other parameters, the terms of trade will depreciate in response to the positive Home productivity shock when $\Phi_\alpha > \widetilde{\phi}_{TOT}(\alpha)$, and it will appreciate if $\Phi_\alpha < \widetilde{\phi}_{TOT}(\alpha)$. Since the sign depends on the denominator of the coefficient, there is an asymptote at $\Phi_\alpha = \widetilde{\phi}_{TOT}(\alpha)$.

There are two ways in which the degree of GVC integration, α, enters in this result. First, and most evidently, through the weights in the weighted-average trade elasticity, Φ_α. If the trade elasticities for consumption goods and intermediate inputs are different, then relative demand is determined by their weighted average. So long as the home bias is the same, however, this weighted average elasticity is what matters for the equilibrium relative price.

If we assume that the trade elasticity for consumption goods and intermediate inputs are also the same (i.e., $\Phi_\alpha = \phi_X = \phi_C$), then the demand for intermediate inputs and consumption goods responds in the same way to changes in the relative price. This means that the presence of GVCs does not affect the demand side of the equilibrium.[9]

However, even in this case, there is a second, and important, role for GVCs—namely through the threshold $\widetilde{\phi}_{TOT}(\alpha)$. Specifically, the threshold is a decreasing function of α: as intermediates play an increased role in the production function, such that α rises, the threshold below which the terms of trade appreciates declines.

This is shown graphically in Figure 1, which depicts the impact response of the terms of trade to a positive Home productivity shock, against different values of the common trade elasticity, $\phi_C = \phi_X = \phi$, plotted for different values of α. The orange line shows the model with $\alpha = 0$, illustrating the result from Corsetti et al. (2008). At $\phi = \widetilde{\phi}_{TOT}$, there is an asymptote below which the terms of trade appreciates and above which it depreciates. As we introduce GVCs, in the purple line with $\alpha = 0.25$, the asymptote shifts left, shrinking the region in which the terms of trade appreciates.

The green line further illustrates that for a sufficiently high degree of GVCs, this region can disappear entirely. To explore this, note that $\widetilde{\phi}_{TOT}(\alpha)$ is a continuous and monotonic function

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[9]To see this, note that inserting $b_H = a_H$ and $\phi_X = \phi_C = \phi$ into the equation for relative demand, (27), removes α from that equation:

$$\widehat{Y}_{H,t} - \widehat{Y}_{F,t} = (1 - 2a_H(1 - \phi))\widehat{\mathcal{T}}_t.$$

Figure 1: Impact Response of Terms of Trade to Home Productivity Shock
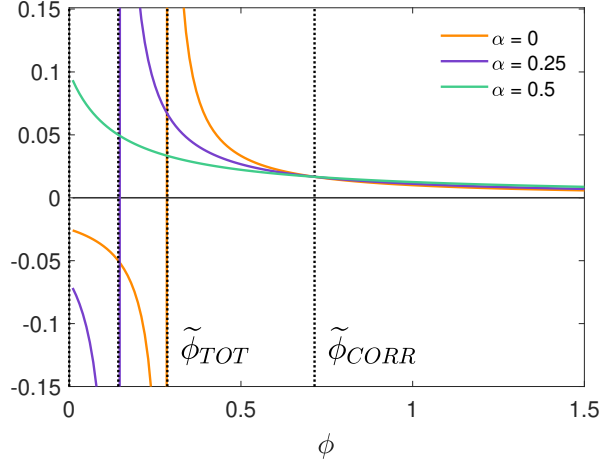
of α, which means that we can define a threshold, $\widetilde{\alpha}$, such that $\widetilde{\phi}_{TOT}(\widetilde{\alpha}) = 0$:

$$0 = 1 - \frac{1}{2a_H} - \frac{a_F}{a_H}\frac{\widetilde{\alpha}}{1 - \widetilde{\alpha}} \quad \Rightarrow \quad \widetilde{\alpha} = 1 - 2a_F$$

For all $\alpha \geq \widetilde{\alpha}$, like the green line in Figure 1, there are no feasible values of the elasticity that can lead to an appreciation of the terms of trade in response to a positive Home productivity shock.

Why does this happen? Recall that this special case, with the CES parameters for trade in consumption goods and intermediate goods equalised, means that the presence of GVCs does not affect the demand side of the equilibrium. Instead, the impact of the presence of GVCs comes through the effect on inputs costs and relative supply—i.e., the marginal-cost effect.

Putting the parameter restrictions into equation (22) yields:

$$\widehat{Y}_{H,t} - \widehat{Y}_{F,t} = \frac{1}{1 - \alpha}\left(\widehat{A}_t - \widehat{A}_t^*\right) - 2a_F\frac{\alpha}{1 - \alpha}\widehat{\mathcal{T}}_t.$$

As discussed above, with GVCs in the model, relative supply is now a decreasing function of the terms of trade. Indeed, for a sufficiently large depreciation of the terms of trade, supply in the Home economy can fall relative to the Foreign economy in response to a positive Home productivity shock. This is illustrated in Figure 2, which depicts the impact response of the relative output to a Home productivity shock, against different values of the trade elasticity, ϕ, plotted for different values of α. The orange line, corresponding to the model with $\alpha = 0$, is flat, showing the exogenous rise in relative endowment in the no-GVC model. With GVCs, however, we see that the response of relative output to the same exogenous shock in (relative) productivity depends on both ϕ and α.

Specifically, the figure shows that when $\phi < \widetilde{\phi}_{TOT}$, which was the threshold below which the terms of trade appreciated in the model without GVCs, relative output falls, for all $\alpha > 0$.

Figure 2: Impact Response of Relative Output to Home Productivity Shock
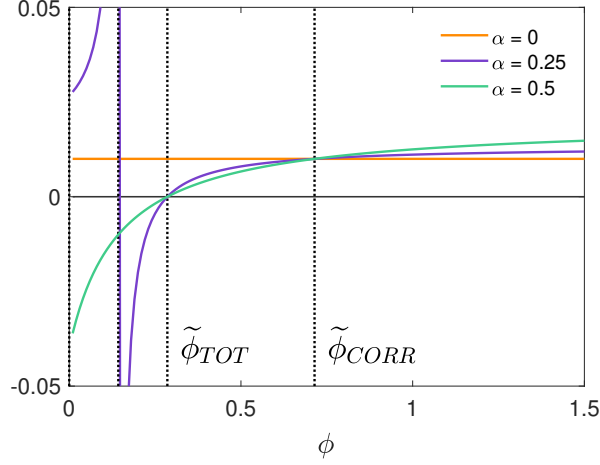
Hence, in this region, where previously the price adjustment had to reverse in order to increase relative demand to clear the market, now, in the presence of GVCs, it is the relative supply that adjusts. This sustains the equilibrium with a terms of trade depreciation for a wider range of the parameter ϕ. When $\alpha \geq \widetilde{\alpha}$, in the green line, relative output declines in response to the shock for all $\phi < \widetilde{\phi}_{TOT}$. As noted before, in this case, the terms of trade continues to depreciate, clearing the market by reducing relative demand for the Home good.

3.2.2 Relative Consumption

Having discussed the response of terms of trade to relative productivity shocks, we now look at the other component of the risk-sharing wedge, namely relative consumption. The following lemma summarises how this is determined.

Lemma 2 (Relative-Consumption Determination) *In the FA equilibrium, relative consumption is pinned down by the terms of trade according to:*

$$\widehat{C}_t - \widehat{C}_t^* = (D_\alpha - 2a_F)\,\widehat{\mathcal{T}}_t \tag{31}$$

with D_α defined as in equation (26).

Proof: We use the trade-balance and market-clearing conditions to derive the following relationship between aggregate gross output and aggregate consumption in each country:

$$\widehat{C}_t = \widehat{Y}_{H,t} + \widehat{P}_{H,t} - \widehat{P}_t,$$
$$\widehat{C}_t^* = \widehat{Y}_{F,t} + \widehat{P}_{F,t} - \widehat{P}_t^*.$$

Taking the difference, plugging in relative demand from equation (27), and using the relationship between the real exchange rate and the terms of trade in equation (23), we arrive at

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equation (31). Full derivations are in Appendix B. $\square$

The lemma shows that the sign of the comovement between the relative consumption and the terms of trade—the 'Backus-Smith-Kollmann correlation' (Backus and Smith, 1993; Kollmann, 1995)—is ambiguous, and depends on the sign of $(D_\alpha - 2a_F)$.

In the baseline without GVCs, imposing $\alpha = 0$:

$$D_\alpha - 2a_F = 2a_H \phi - 1.$$

As shown in Corsetti et al. (2008), the sign of this correlation depends on whether the elasticity is above or below a threshold value, $\widetilde{\phi}_{CORR} = 1/2a_H$. When $\phi > \widetilde{\phi}_{CORR}$, while the terms of trade depreciates (rises), relative consumption also rises. When $\phi < \widetilde{\phi}_{TOT} < \widetilde{\phi}_{CORR}$, the terms of trade appreciates (falls) and relative consumption rises. Finally, when $\widetilde{\phi}_{TOT} < \phi < \widetilde{\phi}_{CORR}$, while the terms of trade depreciates, relative consumption falls. Corsetti et al. (2008) refer to this middle region, in which the increase in Home relative productivity leads to a decline in their relative consumption, as a region of 'immiserising growth'.

Again, we show how this result extends to the cases where $\alpha > 0$, when there is a role for GVCs, by considering the following proposition.

Proposition 2 (GVC Integration and Relative Consumption) *When the home bias in the CES aggregator for the consumption goods and intermediate inputs are the same, i.e., $b_H = a_H$, the correlation between relative consumption and terms of trade depends on whether the weighted-average trade elasticity, defined as in equation (28), is above or below the threshold:*

$$\widetilde{\phi}_{CORR} = \frac{1}{2a_H}. \tag{32}$$

Proof: Inserting $b_H = a_H$ into equation (31), this simplifies to:

$$\widehat{C}_t - \widehat{C}_t^* = (2a_H \Phi_\alpha - 1)\, \widehat{\mathcal{T}}_t \tag{33}$$

so that:

$$\text{sign}\left(\frac{\partial \widehat{C}_{R,t}}{\partial \widehat{A}_{R,t}}\right) = \text{sign}\,(2a_H \Phi_\alpha - 1)$$

where $\widehat{C}_{R,t} \equiv (\widehat{C}_t - \widehat{C}_t^*)$.

As before, because the right-hand side is a continuous and monotonic function of Φ_α, we can define the threshold as the value that sets the expression to zero:

$$2a_H \widetilde{\phi}_{CORR} - 1 \quad \Rightarrow \quad \widetilde{\phi}_{CORR} = \frac{1}{2a_H}$$

$\square$

Importantly, this proposition shows that the threshold, $\widetilde{\phi}_{CORR}$, has the same value and interpretation as the $\alpha = 0$ case.

Figure 3: Impact Response of Relative Consumption to Home Productivity Shock
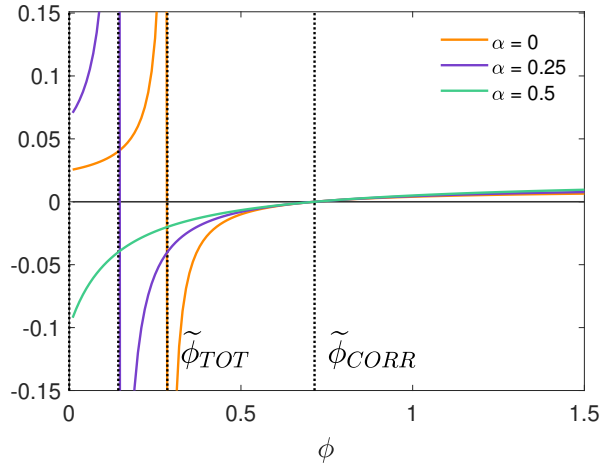
Note: Positive shock to A_t with persistence 0.8, with $b_H = a_H = 0.7$.

This result is shown graphically in Figure 3, where we again focus on the further simplification with $\Phi_\alpha = \phi_X = \phi_C$. For values of the trade elasticity above $\widetilde{\phi}_{CORR}$, relative consumption rises for all α. Below $\widetilde{\phi}_{CORR}$, relative consumption follows the same pattern as seen in the terms of trade in Figure 1: there is an asymptote at $\widetilde{\phi}_{TOT}(\alpha)$, at which point the response of relative consumption switches sign.

With this, we see that shifting the threshold $\widetilde{\phi}_{TOT}(\alpha)$ to the left implies an expansion of the immiserising-growth region. As $\alpha \geq \widetilde{\alpha}$, shown by the green line, where before we saw that the terms of trade depreciated for all values of ϕ, we now see that the response of relative consumption is negative for all $\phi < \widetilde{\phi}_{CORR}$. This is again due to the endogenous decline in relative output in this region, as shown in Figure 2.

3.2.3 Wealth Gap and International Risk Sharing

Having shown the equilibrium dynamics of its two components, we can now bring these results together to investigate how the wealth gap, $\widehat{\mathcal{W}}_t$, responds to productivity shocks, and how the size of this response depends on the degree of GVC integration.

Recall from equation (23) that fluctuations in $\widehat{\mathcal{W}}_t$, away from the constant perfect risk-sharing benchmark, comprise of relative-consumption and relative-price misalignments. When the relative consumption of the Home households is inefficiently high, the wealth gap rises above the benchmark, while when the terms of trade is inefficiently high (ie, depreciated), the wealth gap falls below the benchmark. According to Proposition 2, when $\Phi_\alpha < \widetilde{\phi}_{CORR}$, relative consumption and the terms of trade comove negatively, which means that the two components of the wealth gap work in the same direction. On the other hand, when $\Phi_\alpha > \widetilde{\phi}_{CORR}$, the relative consumption and the terms of trade comove positively, meaning that they have offsetting effects on the wealth gap, and the net effect will depend on their relative size. This observation leads to the following lemma, which extends the well-known result from Cole and Obstfeld (1991) to

our model with GVCs.[10]

Lemma 3 (Cole-Obstfeld with GVCs)   *When $b_H = a_H$, perfect international risk-sharing can be attained under Financial Autarky when the weighted-average trade elasticity, Φ_α, is equal to $\widetilde{\phi}_{CO}$, defined as:*

$$\widetilde{\phi}_{CO} \equiv \frac{1}{2a_H} + \frac{2a_H - 1}{\sigma 2a_H}$$

Proof: From equation (23), take the derivative of $\widehat{\mathcal{W}}_t$ with respect to the relative productivity shock, $\widehat{A}_{R,t} \equiv (\widehat{A}_t - \widehat{A}_t^*)$:

$$\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}} = \sigma \frac{\partial \widehat{C}_{R,t}}{\partial \widehat{\mathcal{T}}_t} \frac{\partial \widehat{\mathcal{T}}_t}{\partial \widehat{A}_{R,t}} - (2a_H - 1) \frac{\partial \widehat{\mathcal{T}}_t}{\partial \widehat{A}_{R,t}} = \left(\sigma \frac{\partial \widehat{C}_{R,t}}{\partial \widehat{\mathcal{T}}_t} - (2a_H - 1) \right) \frac{\partial \widehat{\mathcal{T}}_t}{\partial \widehat{A}_{R,t}} \qquad (34)$$

where we have used the notation introduced earlier, $\widehat{C}_{R,t} \equiv (\widehat{C}_t - \widehat{C}_t^*)$.

The two derivatives in the final expression can be read from equations (30) and (33), with the parameter restriction $b_H = a_H$:

$$\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}} = \frac{\sigma(2a_H \Phi_\alpha - 1) - (2a_H - 1)}{(1 - \alpha)\left(1 - 2a_H(1 - \Phi_\alpha) + 2a_F \frac{\alpha}{1-\alpha} \right)} \qquad (35)$$

The response of the wealth gap is zero when the numerator is equal to zero, i.e. when:

$$\sigma(2a_H \Phi_\alpha - 1) - (2a_H - 1) = 0 \Rightarrow \Phi_\alpha = \widetilde{\phi}_{CO} \equiv \frac{1}{2a_H} + \frac{2a_H - 1}{\sigma 2a_H}$$

Full derivations are in Appendix C.   □

At this precise value of the weighted-average trade elasticity, the relative-price and relative-consumption misalignments under FA exactly offset, and therefore there are no inefficient fluctuations in the wealth gap. This can only occur when the two components move in opposite directions, which is reflected in the fact that $\widetilde{\phi}_{CO} > \widetilde{\phi}_{CORR}$.[11]

Away from this knife-edge case, the wealth gap under FA will deviate from the perfect risk-sharing benchmark, as shown graphically in Panel A of Figure 4 for the case in which $\phi_C = \phi_X = \phi$. The $\phi = \widetilde{\phi}_{CO}$ line is where the wealth gap crosses the x-axis for all α. Instead when $\phi > \widetilde{\phi}_{CO}$, the relative-consumption misalignments dominate and the wealth gap rises: Home households are not able to smooth their positive productivity shock and have inefficiently high consumption relative to perfect risk sharing. When $\widetilde{\phi}_{CORR} < \phi < \widetilde{\phi}_{CO}$, the larger deterioration in the terms-of-trade means that the relative-price misalignments dominate and the wealth gap falls with the Home productivity shock. Finally, when $\phi < \widetilde{\phi}_{CORR}$, the wealth gap is negative in the immiserising-growth region, due to both the decline in relative consumption and the

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[10]Corsetti and Pesenti (2001) similarly show that the Cole and Obstfeld (1991) result extends to production economies, focusing on the adjustment of factor prices that offset relative-marginal-cost distortions.
    [11]To see this, note that $\widetilde{\phi}_{CO} \equiv \frac{1}{2a_H} + \frac{2a_H - 1}{\sigma 2a_H} = \widetilde{\phi}_{CORR} + \frac{2a_H - 1}{\sigma 2a_H} > \widetilde{\phi}_{CORR}$ whenever $a_H > 0.5$.

Figure 4: Impact Response of Risk-sharing Wedge to Home Productivity Shock

Note: Positive shock to A_t with persistence 0.8, with $b_H = a_H = 0.7$ and $\sigma = 2$.
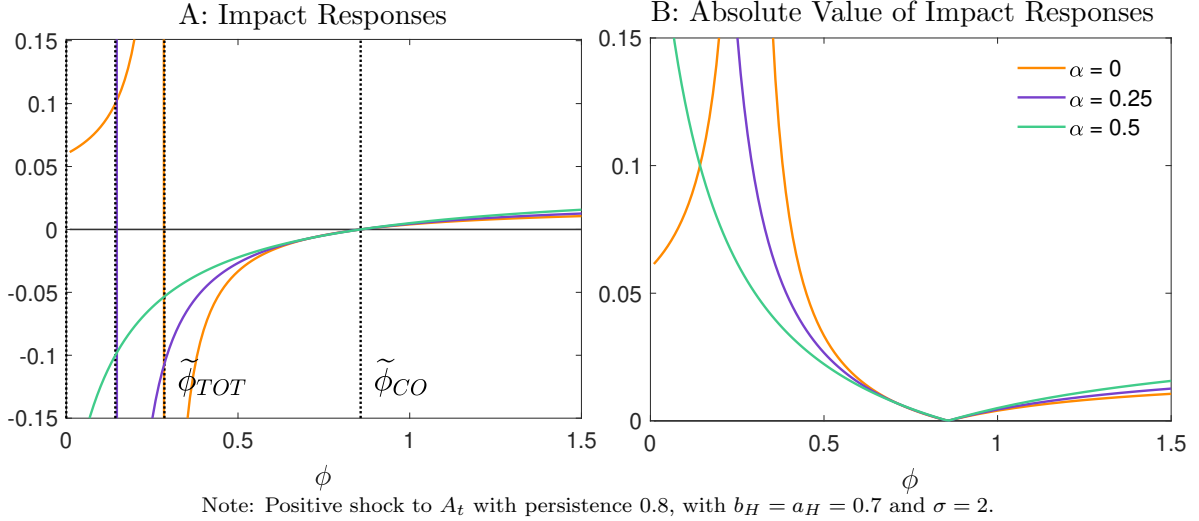
deterioration in the terms of trade, and, conversely, positive when $\phi < \widetilde{\phi}_{TOT}(\alpha)$ where we saw that the terms of trade improves and relative consumption rises. As before, when α is sufficiently high, shown in the green line in the figure, that latter region disappears.

Panel B of Figure 4 instead shows the *absolute value* of these responses, which captures the size of the deviation from the perfect risk-sharing benchmark, irrespective of whether the wealth gap is inefficiently high or inefficiently low. As well as showing that these deviations are small for higher values of ϕ, this figure also shows that the impact of GVC integration on the degree of risk sharing is ambiguous at low values of ϕ.

To explore this formally, we look at how the absolute value of the derivative in equation (35) varies with the degree of GVC integration, α. To facilitate this, it is useful to recall that, for any function $f(x)$:

$$\frac{\partial |f(x)|}{\partial x} = \text{sign}(f(x)) \ \frac{\partial f(x)}{\partial x} \tag{36}$$

where sign(.) is the sign function, which is equal to 1 when its argument is positive and -1 when it is negative.[12] Applying this to the response of the wealth gap yields important results about the impact of GVC integration on risk sharing, summarised in the following propositions.

Proposition 3 (GVC Integration and Risk Sharing) *When $b_H = a_H$ and $\phi_X = \phi_C = \phi$, the marginal impact of increased GVC integration is to reduce risk sharing whenever the wealth gap is positive, and increase risk sharing whenever the wealth gap is negative, so long as either $\phi < \widetilde{\phi}_{CORR}$ or $\phi > \widetilde{\phi}_{CO}$.*

Proof: Imposing the parameter restriction $\phi_C = \phi_X = \phi$ on equation (35), and taking the

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[12]Intuitively, the derivative of the absolute value is equal to the derivative of the function wherever the function is positive, and minus the derivative wherever the function is negative.

derivative with respect to α, we can show that:[13]

$$\frac{\partial}{\partial \alpha}\left(\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}}\right) = \frac{\sigma(2a_H\phi - 1) - (2a_H - 1)}{\left[(1-\alpha)\left(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right)\right]^2}(2a_H\phi - 1)$$

The sign of this derivative depends on the elasticity, ϕ. The denominator of the fraction is unambiguously positive, while the numerator of the fraction is positive if and only if $\phi > \widetilde{\phi}_{CO}$, and the term $(2a_H\phi - 1)$ is positive if and only if $\phi > \widetilde{\phi}_{CORR}$. Using the fact that $\widetilde{\phi}_{CO} > \widetilde{\phi}_{CORR}$, this implies that when $\phi > \widetilde{\phi}_{CO}$, both terms are positive, and when $\phi < \widetilde{\phi}_{CORR}$ both terms are negative, and in both cases the overall sign of the derivative is positive.

In these two cases, therefore, equation (36) shows that the sign of the derivative of the *absolute value* of the wealth-gap response—which captures how the degree of risk sharing responds to GVC integration—is the same as the sign of the wealth-gap response. Whenever the response of the wealth gap to the productivity shock is positive, the size of the response increases with α, such that more GVC integration decreases risk sharing. Whenever the response of the wealth gap is negative, the size of the response decreases with α, such that more GVC integration increases risk sharing.[14]

Appendix D provides a formal derivation of this result. □

This result shows that GVC integration can work to either amplify or mitigate relative-consumption and terms-of-trade misalignments across borders, depending on the trade elasticity. As we saw previously, the wealth gap falls in response to a productivity shock when Home and Foreign inputs are sufficiently complementary. Here, the marginal-cost effect in the Foreign economy is larger following the positive Home productivity shock, such that Foreign output can rise by more than Home output. This leads to a smaller depreciation of the terms of trade, implying more muted negative wealth effects in the Home economy, and, in turn, leading to a smaller adjustment of relative consumption. In other words, with sufficient complementarity, GVC integration acts to mitigate both relative-consumption and relative-price misalignments, and so reduces the deviation of the wealth gap from the perfect risk-sharing benchmark.
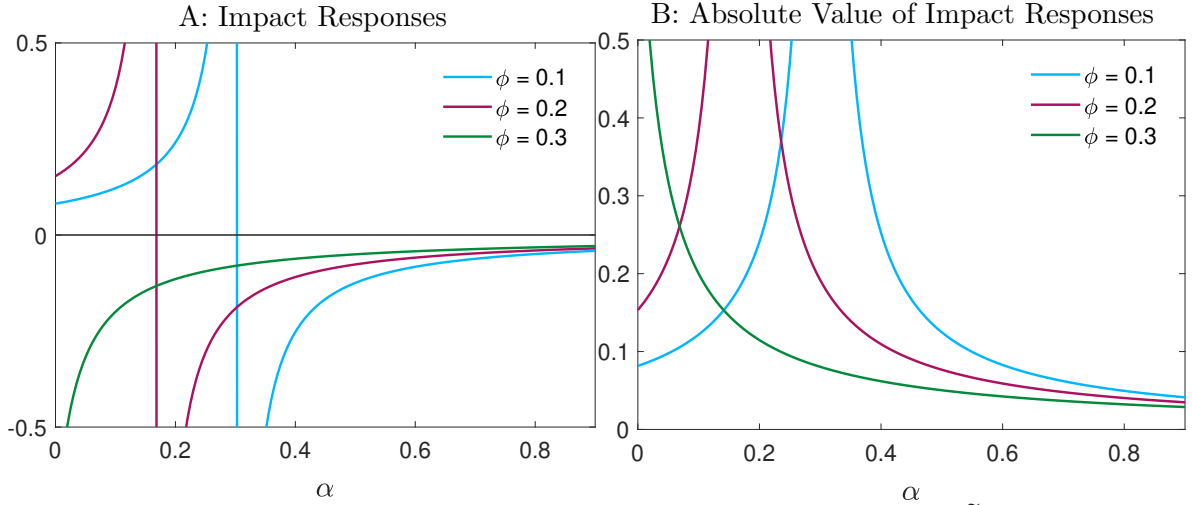
Furthermore, with sufficient complementarity, the impact of GVC integration on the degree of risk sharing can be non-monotonic. Proposition 3 refers to regions in which the wealth gap rises or falls in response to the the productivity shock. In fact, these regions can themselves be a function of α, through the threshold elasticity $\widetilde{\phi}_{TOT}(\alpha)$. The implication of this is described formally in the following proposition.

Proposition 4 (Fragmentation Traps and Integration Oases) *When $b_H = a_H$ and $\phi_X = \phi_C = \phi < \widetilde{\phi}_{TOT}$, GVC integration can reduce risk sharing when starting from a low level*

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[13]Note that the restriction that $\phi_X = \phi_C$ is imposed in order to avoid including the derivative of Φ_α with respect to α. In other words, we can interpret Proposition 3 without this restriction, with the additional assumption that we abstract from the impact of increased GVC integration on the weighted-average elasticity.

[14]In the (small) remaining region, when $\widetilde{\phi}_{CORR} < \phi < \widetilde{\phi}_{CO}$, both the response of the wealth gap and its derivative with respect to α are negative, implying that higher GVC integration leads less risk sharing.

Figure 5: Impact Response of Risk-sharing Wedge to Home Productivity Shock with Respect to Openness

A: Impact Responses   B: Absolute Value of Impact Responses

Note: Positive shock to A_t with persistence 0.8, with $b_H = a_H = 0.7$, and $\sigma = 2$. Here, $\widetilde{\phi}_{TOT} = 0.29$.

of integration, leading to 'fragmentation traps'; while further GVC integration, starting from a higher level, can increase risk sharing, leading to 'integration oases'.

Proof: Recall that $\widetilde{\phi}_{TOT}(\alpha)$ is a decreasing function of α, with $\widetilde{\phi}_{TOT}(\alpha) \in [0, \widetilde{\phi}_{TOT}]$ for $\alpha \in [0, 1)$, where the upper limit is the threshold in the no-GVC case. A given $\phi < \widetilde{\phi}_{TOT}$, therefore, will move from satisfying $\phi < \widetilde{\phi}_{TOT}(\alpha)$ for low α to satisfying $\phi > \widetilde{\phi}_{TOT}(\alpha)$ as α increases.

As we have seen, when $\phi < \widetilde{\phi}_{TOT}(\alpha)$ the wealth gap rises in response to the productivity shock. Applying Proposition 4, this means that the marginal impact of more GVC integration is to reduce risk sharing when α is low. Conversely, when $\phi > \widetilde{\phi}_{TOT}(\alpha)$ the wealth gap falls in response to the productivity shock. Again applying Proposition 4, this means that the marginal impact of more GVC integration is to improve risk sharing when α is high.

Appendix D provides a formal derivation of this result. □

This result is illustrated graphically in Figure 5, which plots the response of the wealth gap, this time against α along the x-axis, for three different values of ϕ. Both panels show that for $\phi > \widetilde{\phi}_{TOT}$, shown in the green line, the size of the wealth-gap response unambiguously falls as integration increases. However, in the purple and blue lines, where $\phi < \widetilde{\phi}_{TOT}$, this effect is non-monotonic. For low levels of α, the economy is in a region where the wealth gap is positive and the marginal impact of higher GVC integration is to reduce risk sharing—the economy is stuck in a Fragmentation Trap. However, if GVC integration is raised enough, the economy moves to a region where the wealth gap is negative. As highlighted by Panel B, which plots the absolute value of the wealth-gap response, this means that the impact of further increases in GVC integration becomes to increase risk sharing—the economy moves towards the Integration Oasis.

This highlights how situations can arise where, even though large increases in integration can

be beneficial for risk sharing, small increases in integration, which may be required in transition, can be costly. This non-monotonic relationship could thereby act to disincentivise policies to integrate into GVCs, even though they could ultimately improve risk sharing.

4   Quantitative Relevance

In this section, we consider the quantitative relevance of our analytical results, to assess the directional influence of GVC integration on international risk sharing in practice. We first do so by calibrating our theoretical model using country-level data on final- and intermediate-goods trade. We explore the implications of trade integration for the degree of risk sharing under this calibration using a variety of experiments, including for alternative parameterisations of final- and intermediate-goods trade elasticities. We complement this model-based analysis with reduced-form evidence using macroeconomic data in which we construct measures of the wealth gap for a panel of advanced economies. Despite their differences, both approaches indicate that, on balance, higher GVC integration is generally associated with greater international risk sharing, not less.

4.1   Calibrated Model

The analytical results presented so far have relied on tight restrictions on parameter values. In particular, we have used the restriction $a_H = b_H$ to simplify the expressions, and varied the parameter α to capture the degree of GVC integration. If we remove this restriction, the dynamics of the terms of trade and relative consumption are pinned down by equations (25) and (31), with D_α defined in (26). In other words, the sign and size of the responses of the wealth gap to productivity shocks, as well as the impact of GVC integration, depend on all five key parameters: α, a_H, b_H, ϕ_C and ϕ_X.

In order to explore the implications of the model in this more general form, we turn to data to pin down the appropriate values of the first three of these parameters, using variation in the cross-section of countries to compare different degrees of GVC integration. We continue to explore the parameter space with different values of the trade elasticities, ϕ_C and ϕ_X, as these are famously difficult to estimate and there is a wide range of estimates that have been found in the literature (Feenstra, Luck, Obstfeld, and Russ, 2018; Freeman, Larch, Theodorakopoulos, and Yotov, 2021).

We calibrate three parameters using input-output data, defining α as one minus the share of value added in total gross output, a_H as the domestic share of consumption and b_H as the domestic share of intermediate inputs.

We use the World Input-Output Tables (WIOT) data to do this. This data is available for 28 advanced economies, at annual frequency over the period 2000-2014.[15] We calculate each share on average over time for each country in the sample, to smooth out the impact of

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[15]The full set of countries are: AUS, AUT, BEL, CAN, CHE, CZE, DEU, DNK, ESP, EST, FIN, FRA, GBR, GRC, IRL, ITA, JPN, KOR, LTU, LUX, LVA, NLD, NOR, PRT, SVK, SVN, SWE, USA.

relative prices. Table 1 shows the resulting parameter values. On average across these countries, intermediate inputs account for 53% of gross output, and there is significant home bias in both final consumption and intermediate inputs, 83 and 87% respectively.

Table 1: Calibrated Parameter Values

	Average	Most Integrated	Least Integrated
α	53%	67%	44%
a_H	83%	60%	95%
b_H	87%	62%	96%

Note: Average refers to the unweighted mean over the 28 countries. Most Integrated refers to Luxembourg, which has the highest α and lowest a_H and b_H. Least Integrated refers to the US, which has the lowest α, and highest a_H and b_H.
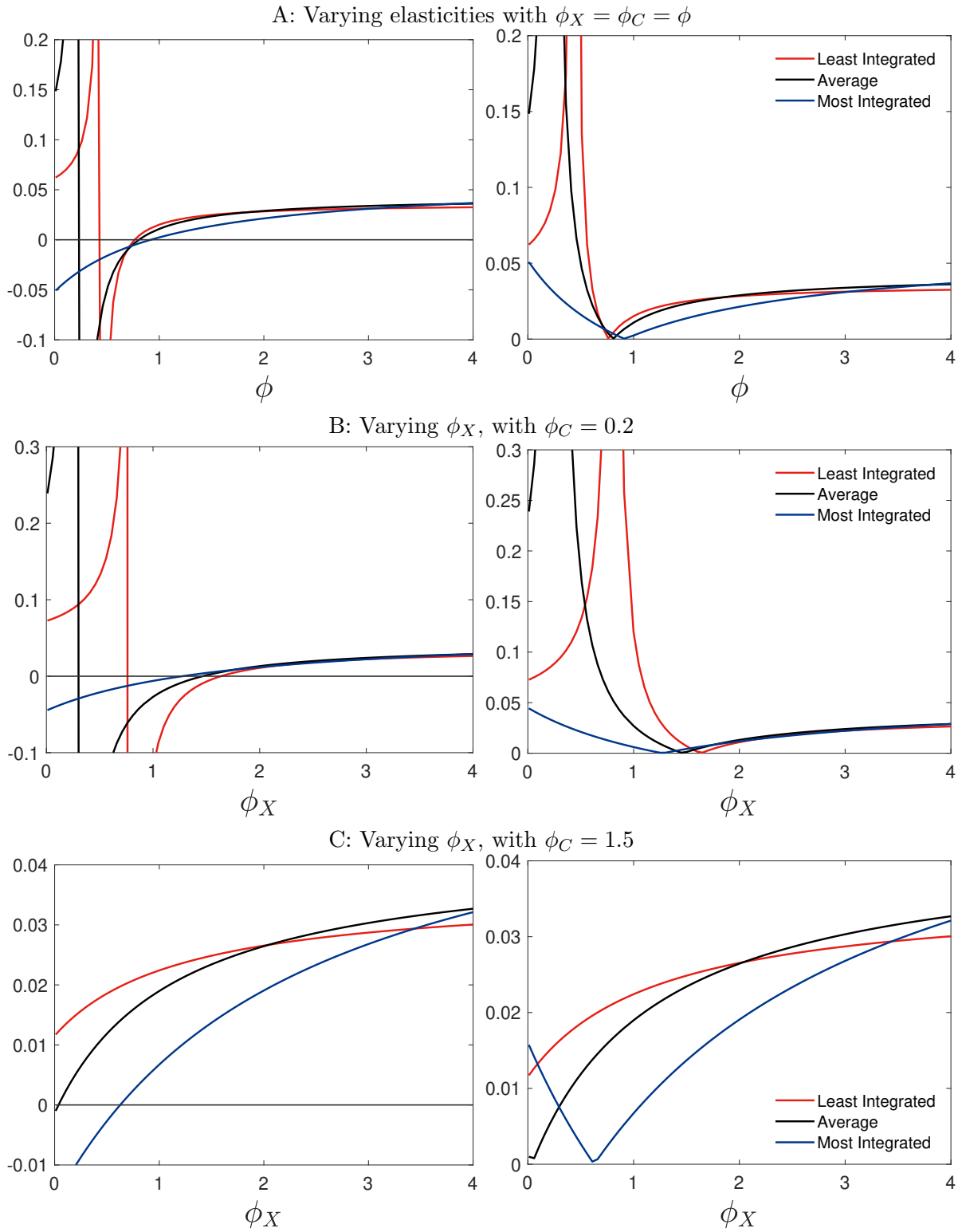
We can also use the cross-section of our data to get a plausible range for each parameter. In line with expectations, across our sample of countries, Luxembourg is the 'Most Integrated' and the US is the 'Least Integrated'. Interestingly, this is consistent across all three parameters: Luxembourg has the highest intermediate-inputs share and the lowest home bias, and the US has the lowest intermediate-inputs share and the highest home bias. This means that they are the most open or closed in both final-goods and intermediate-inputs trade, and that openness to trade is related to a lower value-added share in gross output.

Results. Figure 6 shows the response of the risk-sharing wedge to a Home productivity shock under these three calibrations. As in Section 3, we consider both the impact response and its absolute value, in the left- and right-hand side respectively. Here, we further consider different combinations of final- and intermediate-goods trade elasticities.

Panel A of the figure shows the results for $\phi_X = \phi_C = \phi$. We see a similar pattern to that observed in Figure 4. For the 'Least Integrated' calibration, the impact response of the wealth gap is positive to the left of an asymptote, and negative to the right, in a region where the relative-consumption and terms-of-trade misalignments are moving together. As the elasticity increases, where the two misalignments now work in opposite directions, the relative-consumption misalignments dominate and raise the wealth gap. As we increase trade integration with the 'Average' calibration, this asymptote shifts left, and in the 'Most Integrated' calibration there is no longer an asymptote. Comparing the relative size of the responses with the absolute values on the right-hand chart, we see that the response is smallest for the Most Integrated calibration for most of the parameter space. Interestingly, unlike in the case where we imposed $a_H = b_H$ and compared across different α, this now continues to be true for higher values of ϕ, including after the point where the wealth gap response is zero.

Panels B and C of the figure show the same responses, now for different values of ϕ_X while keeping ϕ_C fixed. Panel B sets $\phi_C = 0.2$. We can see that the lines are effectively stretched to the right along the horizontal axis, echoing the results that we saw analytically in terms of the weighted-average trade elasticity: given the high degree of complementarity in final goods, there needs to be more substitutability in the intermediate inputs to achieve a similar aggregate effect. Conversely, in Panel C, which sets $\phi_C = 1.5$, there is no longer a region with the asymptotes for

Figure 6: Risk-sharing Wedge Responses to Home Productivity Shock
Impact Response (Left) and Absolute Value (Right)

A: Varying elasticities with $\phi_X = \phi_C = \phi$

B: Varying ϕ_X, with $\phi_C = 0.2$

C: Varying ϕ_X, with $\phi_C = 1.5$

Note: Average, Most Integrated and Least Integrated are defined as in Table 1.

Figure 7: Regions of Elasticity Parameter Space where GVC Integration Improves Risk Sharing
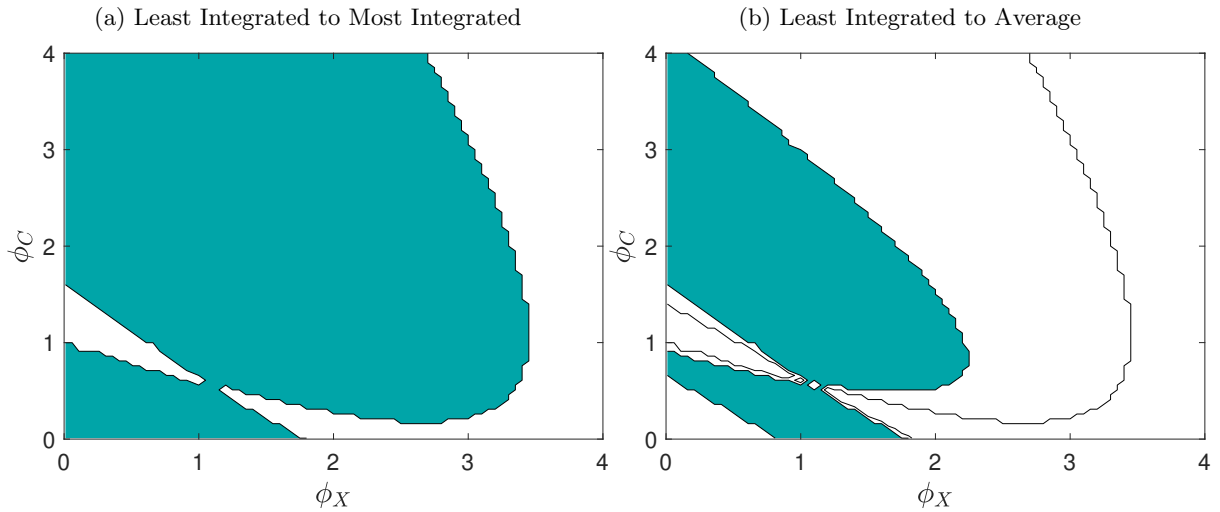
(a) Least Integrated to Most Integrated

(b) Least Integrated to Average

Note: Shaded area denotes combinations of final- and intermediate-goods trade elasticities, ϕ_C and ϕ_X, for which an increase in GVC integration increases international risk sharing. Increased international risk sharing is defined as occurring when the theoretical variance of the wealth gap is lower for a more open calibration of trade integration vis-à-vis a more closed one. Theoretical variances are calculated accounting for independent Home and Foreign productivity shocks, with persistence $\rho = 0.8$. Average, Most Integrated and Least Integrated calibrations for trade integration are defined as in Table 1.

any of the calibrations, showing that even a high degree of complementarity in the intermediate inputs cannot replicate that effect when there is substitutability in final goods. In this case, the Most Integrated calibration yields the smallest response of the wealth gap for most values of ϕ_X, again highlighting that increased openness can be associated with a greater degree of risk sharing. However, there are wide ranges of the ϕ_X parameter space in which the link between GVC integration and risk sharing appears to be non-monotonic. For values of ϕ_X from around 2 to 3.5, the ordering of absolute impact responses is consistent with the fragmentation-trap logic described in Section 3.

4.2 Extensions and Robustness

To assess the extent to which these results generalise, we move away from looking at the impact response to a Home productivity shock, and instead calculate the theoretical variance of the wealth gap in the face of independent Home and Foreign productivity shocks. For this exercise, we explore the parameter space for the final- and intermediate-goods trade elasticities, ϕ_C and ϕ_X, more fully. For each combination of ϕ_C and ϕ_X, we calculate whether the variance of the wealth gap increases or decreases when comparing different degrees of trade integration. When comparing a more closed economy to a more open one, a *decrease* in the variance of the wealth gap represents outcomes that are closer to the complete-markets benchmark and, therefore, higher welfare.

Figure 7 plots the region where this difference is positive in green, and where it is negative in white. The results are most stark in Panel (a), which compares outcomes for the Most Integrated economy with those of the Least Integrated. Strikingly, our results suggest that

greater GVC integration improves risk sharing for most combinations of elasticities. Indeed, for intermediate elasticities, ϕ_X, below around 3.5, this is true for essentially any value of the final-goods elasticity, ϕ_C. Standard values for elasticities used in the international macroeconomics literature typically lie around 1.5, suggesting that, for increases in GVC integration to be associated with improved risk sharing, intermediate inputs need not be highly complementary, and can indeed be substitutable in an absolute sense.

Panel (b) shows the same results comparing the Least Integrated calibration to the Average calibration. A very similar pattern holds, though for a smaller region of the parameter space. Still, without a significant degree of substitutablity in both final- and intermediate-goods trade, there is a greater degree of risk sharing with more integration. This area is a subset of that coloured in Panel (a). So for values of ϕ_C and ϕ_X in the green area in Panel (a) that does not intersect with the green area in Panel (b), a large increase (from Least Integrated to Most Integrated) in GVC integration is associated with improved risk sharing, while a small increase (from Least Integrated to Average) is associated with reduced risk sharing. This suggests scope for 'fragmentation traps' and 'integration sweet spots' to arise in reasonable values of the parameter space.

Finally, we summarise the robustness of these findings to extending the model in various dimensions. Full results from our robustness analysis are presented in Appendix E.

Alternative Financial Market Incompleteness. Building on our analytical results, our benchmark results come from a model with extreme financial market incompleteness: Financial Autarky. We verify that our quantitative findings are robust to a model setting in which financial markets remain incomplete, but households have access to non-state-contingent one-period international bonds, which are in zero net supply across countries in each period, to smooth consumption partially.[16]

Endogenous Labour Supply. In our benchmark setting, labour is supplied exogenously to abstract from wealth effects via labour supply that can reduce cross-country co-movement in incomplete-market multi-country models. We verify the robustness of our quantitative findings to endogenous labour supply. To do this, we assume that households have period-by-period labour disutility $L_t^{1+1/\psi}/(1+1/\psi)$, where ψ is the inverse Frisch elasticity of labour supply—which we calibrate to a standard value, $\psi = 0.5$, in our simulations.

Alternative Production Function. Our benchmark results use a Cobb-Douglas production function between labour and intermediate inputs. To account for the possibility of there being non-unitary elasticity of substitution between factor inputs, we also verify the robustness of our quantitative model results to the use of a Constant Elasticity of Substitution production function.

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[16]Following Bodenstein (2011), we additionally include Uzawa-style discounting in this model variant to ensure that (i) bond positions are stationary and (ii) the equilibrium is unique.

4.3 Macro-Level Evidence

To provide a model-free test for the links between GVC integration and openness, we turn to a macro-level empirical exercise. To do this, we construct empirical measures of GVC integration and financial-market risk sharing for the same 28 economies, and the same 2000-2014 sample period, used to calibrate the model.[17]

GVC Integration. As our benchmark measure, we use the share of imported intermediate inputs in gross output. This measure is consistent with our model, where it can be obtained by combining firms' optimal demand for intermediate inputs, equation (10) and the optimal demand for imported intermediate inputs, equation (14):

$$\frac{x_{F,t}}{Y_{H,t}} = \alpha b_F \left(\frac{P_{F,t}}{P_{X,t}} \right)^{-\phi_X} \frac{P_{H,t}}{P_{X,t}}$$

If, on average, relative price movements are equal to one, then the mean of this GVC integration measure will capture integration due to αb_F as discussed in our analytical results.

Wealth Gap. To complement this, we construct measures of the wealth gap by combining country-level data for real final consumption expenditure and consumer price indices (CPI) from the OECD with nominal exchange rate data (measuring currency per US dollar) from the BIS. To match the periodicity of our GVC measures, our baseline data is annual and covers the period, although we carry out a robustness check to ensure that our results are robust to wealth-gap measures constructed using quarterly data for the same 2000-2014 period.

We construct relative consumption for each country by assuming that 'foreign' consumption for each is a PPP-weighted average of the remaining countries. We use IMF World Economic Outlook data to construct the weights. Real exchange rates are given by the ratio of CPI across countries, converted into US dollar terms using the nominal exchange rate data, where, again, 'foreign' prices are PPP-weighted averages of the remaining countries in each period.

With relative consumption and real exchange rate data, we then construct empirical measures of the wealth gap at country level, using the definition in equation (24).

Empirical Association. Consistent with the theoretical predictions outlined in Section 4.1, Figure 8 plots the cross-sectional association between GVC integration and risk sharing. It compares variance of the wealth gap and the average degree of GVC integration, at country level, over the period 2000-2014. To construct the wealth gap, we assume that $\sigma = 1$ in the baseline.

The figure illustrates that the degree of GVC linkages, shown on the horizontal axis, are (weakly) negatively associated with the variance of the wealth gap, on the vertical axis. This

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[17]While measures of the wealth gap can be constructed for a longer sample, the time span of this exercise is constrained by the availability of information on imported intermediate goods usage at country level in the WIOT data, which we use to construct measures of GVC integration.

Figure 8: Bivariate Association Between GVC Integration and Variance of Wealth Gap Across Countries
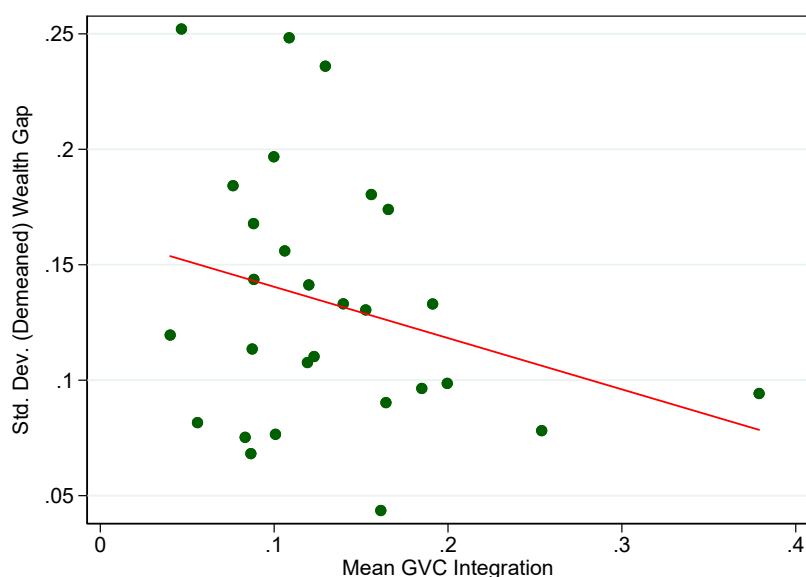
Note: Average country-level GVC linkages measured using the gross ouput share of imported intermediate inputs for the period 2000-2014 using the data from WIOT. Country-level wealth gap measured using HP-filtered relative consumption and CPI data from OECD and nominal exchange rate data from BIS, with foreign quantities PPP-weighted; variance calculated over the period 2000-2014.

Table 2: Regressions for Association Between GVC Integration and Variance of Wealth Gap Across Countries

| | DEP. VAR.: Variance of Wealth Gap by Country $\text{var}_t(\mathcal{W}_{i,t})$ | | | | |
	$\sigma = 1$				$\sigma = 2$
$\overline{GVC}_i$	-0.222	-0.418*	-1.712**	-1.766**	-2.959**
	(0.151)	(0.229)	(0.722)	(0.714)	(1.054)
$\overline{VA}_i$		-0.340	1.299	1.525*	3.001**
		(0.302)	(0.768)	(0.779)	(1.149)
$\overline{Int}_i$			1.834**	2.069**	3.457***
			(0.758)	(0.771)	(1.138)
$\overline{TrOpen}_i$			0.356*	0.339	0.565*
			(0.204)	(0.201)	(0.297)
$\overline{RelGDP}_i$			0.003	0.003	-0.009
			(0.009)	(0.009)	(0.014)
$\mathbb{1}_i^{EU}$				0.028	0.089**
				(0.022)	(0.033)
Observations	28	28	28	28	28
R^2	0.077	0.122	0.326	0.373	0.448

Note: Average country-level GVC linkages measured using the gross ouput share of imported intermediate inputs for the period 2000-2014 using the data from WIOT. Country-level wealth gap measured using HP-filtered relative consumption and CPI data from OECD and nominal exchange rate data from BIS, with foreign quantities PPP-weighted; variance calculated over the period 2000-2014.

suggests that the magnitude of deviations from perfect risk sharing is decreasing in the degree of GVC integration.

To complement the bivariate association shown in Figure 8, Table 2 reports coefficient estimates from cross-sectional regressions of the by-country variance of the wealth gap on the average country-level GVC linkages ($\overline{GVC}_i$) and other control variables. These control variables are intended to capture other differences across countries, to help refine the implications from Figure 8. In this instance, the addition of additional control variables—including the average value-added share ($\overline{VA}_i$, a proxy for $1 - \alpha$), the average intermediate goods share ($\overline{Int}_i$, a proxy for α), average trade openness ($\overline{TrOpen}_i$), average relative GDP ($\overline{RelGDP}_i$, both proxies of relative country size), and an indicator variable for euro area countries in the sample ($\mathbb{1}_i^{EU}$)—increases the significance attributed to the association between the variance of the risk-sharing wedge and the degree of GVC integration. In columns (3) and (4), with all controls included, our estimates indicate that higher GVC integration is, all else equal, associated with less volatility in the wealth gap, with estimates significant at the 5% level.

Overall, therefore, our empirical analysis supports the implications of our model calibration. Both suggest that higher GVC integration is associated with smaller deviations from perfect risk sharing—i.e., increased financial-market risk sharing.

5   Conclusion

In this paper, we have documented an important interaction between intermediate goods trade and international financial. We have showed that, even when GVCs are frictionless, intermediate-goods-trade linkages can affect countries' ability to share risks in incomplete international financial markets. This occurs because GVCs allow productivity shocks in one country to affect production in another, by influencing the supply of intermediate inputs for production.

The model that we use—a two-country, two-good NOEM model with roundabout production—is somewhat stylised to allow us to derive analytical solutions. However, this class of models is widely used in academic and policy analysis, and our general specification nests a range of empirically-relevant cases. And so our results have important implications for contemporary macroeconomic policy debates around the benefits and risks associated with GVC integration (D'Aguanno et al., 2021; International Monetary Fund, 2022).

In general, we show that the relationship between GVC integration and risk sharing is ambiguous in theory. However, our results—from both a calibrated theoretical model and macro-level evidence—highlight that the most empirically relevant case is one in which GVC integration can serve to reduce misalignments and, in effect, 'complete' otherwise incomplete international financial markets most when domestic and foreign intermediate inputs.

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Appendices

A Proof of Lemma 1

We will start by using the Trade Balance equation, which holds in every period under Financial Autarky:

$$\mathcal{T}_t \left(C_{F,t} + X_{F,t} \right) = C^*_{H,t} + X^*_{H,t} \tag{A.1}$$

We proceed by plugging in the optimal demand functions for each component, written as a function of terms of trade $\mathcal{T}_t$ and gross output of the Home and Foreign country.

Optimal Home consumption and intermediate demand for Foreign produced goods are:

$$
\begin{aligned}
C_{F,t} &= a_F \left(\frac{P_{F,t}}{P_t} \right)^{-\phi_C} C_t \\
&= a_F \left(\frac{P_{H,t}}{P_t} \frac{P_{F,t}}{P_{H,t}} \right)^{-\phi_C} (1-\alpha) \frac{P_{H,t}}{P_t} Y_{H,t} \\
&= \frac{a_F \mathcal{T}_t^{-\phi_C}}{a_H + a_F \mathcal{T}_t^{1-\phi_C}} (1-\alpha) Y_{H,t}
\end{aligned}
$$

$$
\begin{aligned}
X_{F,t} &= b_F \left(\frac{P_{F,t}}{P_{X,t}} \right)^{-\phi_X} X_t \\
&= b_F \left(\frac{P_{H,t}}{P_{X,t}} \frac{P_{F,t}}{P_{H,t}} \right)^{-\phi_X} \alpha \frac{P_{H,t}}{P_{X,t}} Y_{H,t} \\
&= \frac{b_F \mathcal{T}_t^{-\phi_X}}{b_H + b_F \mathcal{T}_t^{1-\phi_X}} \alpha Y_{H,t}
\end{aligned}
$$

These yield:

$$
C_{F,t} + X_{F,t} = \left[\frac{a_F \mathcal{T}_t^{-\phi_C}}{a_H + a_F \mathcal{T}_t^{1-\phi_C}} (1-\alpha) + \frac{b_F \mathcal{T}_t^{-\phi_X}}{b_H + b_F \mathcal{T}_t^{1-\phi_X}} \alpha \right] Y_{H,t} \tag{A.2}
$$

Similarly, Foreign consumption and intermediates demand for Home produced goods, can be expressed as:

$$
\begin{aligned}
C^*_{H,t} &= a_F \left(\frac{P_{H,t}}{P^*_t} \right)^{-\phi_C} C^*_t \\
&= a_F \left(\frac{P_{H,t}}{P^*_t} \right)^{-\phi_C} (1-\alpha) \frac{P_{F,t}}{P^*_t} Y_{F,t} \\
&= a_F \left(\frac{P_{H,t}}{P^*_t} \right)^{-\phi_C} (1-\alpha) \frac{P_{H,t}}{P^*_t} \mathcal{T}_t Y_{F,t} \\
&= \frac{a_F \mathcal{T}_t}{a_F + a_H \mathcal{T}_t^{1-\phi_C}} (1-\alpha) Y_{F,t}
\end{aligned}
$$

$$X^*_{H,t} = b_F \left(\frac{P_{H,t}}{P^*_{X,t}}\right)^{-\phi_X} X^*_t$$

$$= b_F \left(\frac{P_{H,t}}{P^*_{X,t}}\right)^{-\phi_X} \alpha \frac{P_{F,t}}{P^*_{X,t}} Y_{F,t}$$

$$= b_F \left(\frac{P_{H,t}}{P^*_{X,t}}\right)^{-\phi_X} \alpha \frac{P_{H,t}}{P^*_{X,t}} \mathcal{T}_t Y_{F,t}$$

$$= b_F \left(\frac{P_{F,t}}{P^*_{X,t}}\right)^{1-\phi_X} \mathcal{T}_t^{\phi_X} \alpha Y_{F,t}$$

$$= \frac{b_F \mathcal{T}_t}{b_F + b_H \mathcal{T}_t^{1-\phi_X}} \alpha Y_{F,t}$$

These yield:

$$C^*_{H,t} + X^*_{H,t} = \left[\frac{a_F \mathcal{T}_t}{a_F + a_H \mathcal{T}_t^{1-\phi_C}}(1-\alpha) + \frac{b_F \mathcal{T}_t}{b_F + b_H \mathcal{T}_t^{1-\phi_X}} \alpha\right] Y_{F,t} \tag{A.3}$$

Plugging these in to the trade balance equation (A.1) gives us the equilibrium relationship between relative demand and the terms of trade, which we can then log-linearising around the symmetric, deterministic steady state, obtaining:

$$\widehat{Y}_{R,t} = \frac{a_F(1-\alpha)\left[1 - 2a_H(1-\phi_C)\right] + b_F\alpha\left[1 - 2b_H(1-\phi_X)\right]}{a_F(1-\alpha) + b_F\alpha} \widehat{\mathcal{T}}_t \tag{A.4}$$

where $\widehat{Y}_{R,t} = \widehat{Y}_{H,t} - \widehat{Y}_{F,t}$.

We will next derive the relative supply as a function of terms of trade using the production side of the economy. As discussed, we can re-write the production function as:

$$Y_{H,t} = A_t^{1/(1-\alpha)} L(\alpha)^{\frac{\alpha}{1-\alpha}} \left[b_H + b_F \mathcal{T}_t^{1-\phi_X}\right]^{-\frac{1}{1-\phi_X} \frac{\alpha}{1-\alpha}}$$

$$Y_{F,t} = (A^*_t)^{1/(1-\alpha)} L^*(\alpha)^{\frac{\alpha}{1-\alpha}} \left[b_H + b_F \mathcal{T}_t^{\phi_X-1}\right]^{-\frac{1}{1-\phi_X} \frac{\alpha}{1-\alpha}}$$

Taking the ratio and log-linearising yields:

$$\widehat{Y}_{R,t} = \frac{1}{1-\alpha}\widehat{A}_{R,t} - 2b_F\frac{\alpha}{1-\alpha}\widehat{\mathcal{T}}_t \tag{A.5}$$

where $\widehat{A}_{R,t} = \widehat{A}_{H,t} - \widehat{A}_{F,t}$.

Combining the log-linearised relative demand, equation (A.4) and relative supply, equation (A.5), together:

$$\frac{a_F\alpha\left[1 - 2a_H(1-\phi_C)\right] + b_F(1-\alpha)\left[1 - 2b_H(1-\phi_X)\right]}{a_F\alpha + b_F(1-\alpha)}\widehat{\mathcal{T}}_t = \frac{1}{\alpha}\widehat{A}_{R,t} - 2b_F\frac{1-\alpha}{\alpha}\widehat{\mathcal{T}}_t$$

We define:

$$D_\alpha \equiv \frac{a_F(1-\alpha)\left[1-2a_H(1-\phi_C)\right] + b_F\alpha\left[1-2b_H(1-\phi_X)\right]}{a_F(1-\alpha)+b_F\alpha}$$

so that:

$$D_\alpha\widehat{\mathcal{T}_t} = \frac{1}{1-\alpha}\widehat{A}_{R,t} - 2b_F\frac{\alpha}{1-\alpha}\widehat{\mathcal{T}_t}$$

$$\widehat{\mathcal{T}_t} = \frac{1}{(1-\alpha)\left(D_\alpha + 2b_F\frac{\alpha}{1-\alpha}\right)}\widehat{A}_{R,t} \tag{A.6}$$

B   Proof of Lemma 2

We start by using the Trade Balance condition as in the previous proof.

$$P_{H,t}C^*_{H,t} + P_{H,t}X^*_{H,t} = P_{F,t}C_{F,t} + P_{F,t}X_{F,t}$$

Combining this with the market clearing condition for Home (gross) output:

$$
\begin{aligned}
P_{H,t}Y_{H,t} &= P_{H,t}C_{H,t} + P_{H,t}X_{H,t} + P_{H,t}C^*_{H,t} + P_{H,t}X^*_{H,t} \\
&= P_{H,t}C_{H,t} + P_{H,t}X_{H,t} + P_{F,t}C_{F,t} + P_{F,t}X_{F,t} \\
&= P_tC_t + P_{X,t}X_t
\end{aligned}
$$

From Cobb-Douglas production function, we obtain:

$$P_{X,t}X_t = \alpha P_{H,t}Y_{H,t} \tag{B.1}$$

which also implies:

$$P_tC_t = (1-\alpha)P_{H,t}Y_{H,t} \tag{B.2}$$

Equivalently for the Foreign economy:

$$P^*_{X,t}X^*_t = \alpha P_{F,t}Y_{F,t} \tag{B.3}$$

$$P^*_tC^*_t = (1-\alpha)P_{F,t}Y_{F,t} \tag{B.4}$$

By taking the ratio of equations (B.2) and (B.4), we can obtain a relationship between relative consumption and relative output. In log-linearised form, this can be expressed as:

$$\widehat{RER}_t + \widehat{Y}_{H,t} - \widehat{Y}_{F,t} = \widehat{\mathcal{T}_t} + \widehat{C}_t - \widehat{C}^*_t \tag{B.5}$$

Recall the relationship between RER and $\mathcal{T}$:

$$\widehat{RER}_t = (2a_H - 1)\widehat{\mathcal{T}_t}$$

Plug this along with equation (A.4) into equation (B.5):

$$(2a_H - 1)\widehat{\mathcal{T}}_t + \frac{a_F(1-\alpha)\left[1 - 2a_H(1 - \phi_C)\right] + b_F\alpha\left[1 - 2b_H(1 - \phi_X)\right]}{a_F(1-\alpha) + b_F\alpha}\widehat{\mathcal{T}}_t = \widehat{\mathcal{T}}_t + \widehat{C}_t - \widehat{C}_t^*$$

and rearrange:

$$\widehat{C}_t - \widehat{C}_t^* = \left\{2a_H - 1 + \frac{a_F(1-\alpha)\left[1 - 2a_H(1 - \phi_C)\right] + b_F\alpha\left[1 - 2b_H(1 - \phi_X)\right]}{a_F(1-\alpha) + b_F\alpha} - 1\right\}\widehat{\mathcal{T}}_t$$

$$= \left\{\frac{a_F(1-\alpha)\left[1 - 2a_H(1 - \phi_C)\right] + b_F\alpha\left[1 - 2b_H(1 - \phi_X)\right]}{a_F(1-\alpha) + b_F\alpha} - 2a_F\right\}\widehat{\mathcal{T}}_t$$

$$\widehat{C}_{R,t} = [D_\alpha - 2a_F]\,\widehat{\mathcal{T}}_t \tag{B.6}$$

where $\widehat{C}_{R,t} = \widehat{C}_t - \widehat{C}_t^*$.

C Proof of Lemma 3

When $b_H = a_H$, the composite parameter, D_α, simplifies to:

$$D_\alpha = \frac{a_F(1-\alpha)\left[1 - 2a_H(1 - \phi_C)\right] + b_F\alpha\left[1 - 2b_H(1 - \phi_X)\right]}{a_F(1-\alpha) + b_F\alpha}$$

$$= (1-\alpha)\left[1 - 2a_H(1 - \phi_C)\right] + \alpha\left[1 - 2a_H(1 - \phi_X)\right]$$

$$= 1 - 2a_H[1 - (1-\alpha)\phi_C - \alpha\phi_X]$$

$$= 1 - 2a_H(1 - \Phi_\alpha)$$

where:

$$\Phi_\alpha = (1-\alpha)\phi_C + \alpha\phi_X$$

The relationship between the terms of trade and the relative productivity, equation (A.6) becomes:

$$\widehat{\mathcal{T}}_t = \frac{1}{(1-\alpha)\left(1 - 2a_H(1 - \Phi_\alpha) + 2a_F\frac{\alpha}{1-\alpha}\right)}\widehat{A}_{R,t} \tag{C.1}$$

and the relationship between the relative consumption and the terms of trade, (B.6) becomes:

$$\widehat{C}_{R,t} = (2a_H\Phi_\alpha - 1)\,\widehat{\mathcal{T}}_t \tag{C.2}$$

We can express relative consumption as a function of relative productivity by plugging in equation (C.1) into (C.2):

$$\widehat{C}_{R,t} = \frac{(2a_H\Phi_\alpha - 1)}{(1-\alpha)\left(1 - 2a_H(1 - \Phi_\alpha) + 2a_F\frac{\alpha}{1-\alpha}\right)}\widehat{A}_{R,t} \tag{C.3}$$

We can now take the derivative of the log-linearised definition of the wealth gap, with respect

to relative productivity:

$$\widehat{\mathcal{W}}_t = \sigma \widehat{C}_{R,t} - (2a_H - 1)\widehat{\mathcal{T}}_t$$

$$\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}} = \sigma \frac{\partial \widehat{C}_{R,t}}{\partial \widehat{A}_{R,t}} - (2a_H - 1)\frac{\partial \widehat{\mathcal{T}}_t}{\partial \widehat{A}_{R,t}}$$

$$\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}} = \sigma \frac{(2a_H\Phi_\alpha - 1)}{(1-\alpha)\left(1 - 2a_H(1-\Phi_\alpha) + 2a_F\frac{\alpha}{1-\alpha}\right)} - (2a_H - 1)\frac{1}{(1-\alpha)\left(1 - 2a_H(1-\Phi_\alpha) + 2a_F\frac{\alpha}{1-\alpha}\right)}$$

$$\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}} = \frac{\sigma(2a_H\Phi_\alpha - 1) - (2a_H - 1)}{(1-\alpha)\left(1 - 2a_H(1-\Phi_\alpha) + 2a_F\frac{\alpha}{1-\alpha}\right)} \tag{C.4}$$

This derivative is equal to zero if and only if the numerator is equal to zero, in other words if:

$$\sigma(2a_H\Phi_\alpha - 1) = (2a_H - 1)$$
$$2a_H\Phi_\alpha = 1 + \frac{2a_H - 1}{\sigma}$$
$$\Phi_\alpha = \frac{1}{2a_H} + \frac{2a_H - 1}{\sigma 2a_H}$$

Furthermore, notice that the numerator of the derivative is positive if the weighted-average elasticity is above this value, and negative otherwise. From Proposition 1, we also know that the denominator is positive when the weighted-average elasticity is above the threshold $\widetilde{\phi}_{TOT}(\alpha)$, and negative otherwise. This means that:

$$\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}} \begin{cases} \geq 0 & \phi \geq \widetilde{\phi}_{CO} \\ < 0 & \widetilde{\phi}_{TOT}(\alpha) < \phi < \widetilde{\phi}_{CO} \\ > 0 & \phi < \widetilde{\phi}_{TOT}(\alpha) \end{cases}$$

D Proof of Propositions 3 and 4

Start from the expression for the response of the wealth gap, equation (C.4), and impose the restriction $\phi_C = \phi_x = \phi$:

$$\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}} \equiv f(\alpha) = \frac{\sigma(2a_H\phi - 1) - (2a_H - 1)}{(1-\alpha)\left(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right)}$$

Taking the derivative with respect to α:

$$\frac{\partial f(\alpha)}{\partial \alpha} = - \frac{\sigma(2a_H\phi - 1) - (2a_H - 1)}{\left[(1-\alpha)\left(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right)\right]^2}$$
$$\times \frac{\partial.}{\partial \alpha}\left((1-\alpha)\left(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right)\right)$$

$$\frac{\partial.}{\partial \alpha}\left((1-\alpha)(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha})\right)$$
$$= -(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}) + (1-\alpha)(2a_F\left(\frac{1}{(1-\alpha)^2}\right))$$
$$= -1 + 2a_H(1-\phi) - 2a_F\frac{\alpha}{1-\alpha} + 2a_F\frac{1}{(1-\alpha)}$$
$$= -1 + 2a_H(1-\phi) + 2a_F$$
$$= -(2a_H\phi - 1),$$

hence:

$$\frac{\partial f(\alpha)}{\partial \alpha} = \frac{\sigma(2a_H\phi - 1) - (2a_H - 1)}{\left[(1-\alpha)\left(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right)\right]^2}(2a_H\phi - 1).$$

We are interested in the sign of the derivative of the absolute value of the wealth-gap response. Recall that, for any function $g(x)$:

$$\frac{\partial |g(x)|}{\partial x} = \text{sgn}(g(x))\,\frac{\partial g(x)}{\partial x}$$

hence:

$$\frac{\partial (|f(\alpha)|)}{\partial \alpha} = \text{sgn}(f(\alpha))\frac{\partial f(\alpha)}{\partial \alpha}$$
$$= \frac{f(\alpha)}{|f(\alpha)|}\frac{\partial f(\alpha)}{\partial \alpha}$$

where we have used the formal definition of the sign function, $\text{sgn}(x) \equiv x/|x|$.

Substituting in the definitions:

$$
\begin{aligned}
\frac{\partial\left(|f(\alpha)|\right)}{\partial \alpha} &= \frac{1}{|f(\alpha)|} \frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}} \frac{\partial}{\partial \alpha}\left(\frac{\partial \widehat{\mathcal{W}}_t}{\partial \widehat{A}_{R,t}}\right) \\
&= \frac{1}{|f(\alpha)|} \frac{\sigma(2a_H\phi - 1) - (2a_H - 1)}{(1-\alpha)\left(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right)} \\
&\qquad \times \frac{\sigma(2a_H\phi - 1) - (2a_H - 1)}{\left[(1-\alpha)\left(1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right)\right]^2}(2a_H\phi - 1) \\
&= \underbrace{\frac{[\sigma(2a_H\phi - 1) - (2a_H - 1)]^2}{|f(\alpha)| \cdot (1-\alpha)^3}}_{\equiv A} \underbrace{\frac{(2a_H\phi - 1)}{\left[1 - 2a_H(1-\phi) + 2a_F\frac{\alpha}{1-\alpha}\right]^3}}_{\equiv B}
\end{aligned}
$$

Since $\alpha < 1$, the term A is always positive, and so the sign of this derivative depends on the sign of the term B, which in turn depends on the relative sign of its numerator and denominator. As in Proposition 2, we know that the numerator is weakly positive when:

$$
\phi \geq \frac{1}{2a_H} \equiv \widetilde{\phi}_{CORR},
$$

and as in Proposition 1, we know that the denominator is strictly positive when:

$$
\phi > 1 - \frac{1}{2a_H} - \frac{a_F}{a_H}\frac{\alpha}{1-\alpha} \equiv \widetilde{\phi}_{TOT}(\alpha).
$$

Since $\widetilde{\phi}_{CORR} > \widetilde{\phi}_{TOT}(\alpha)$, we can conclude that:

$$
\frac{\partial\left(|f(\alpha)|\right)}{\partial \alpha} \begin{cases} \geq 0 & \phi \geq \widetilde{\phi}_{CORR} \\ < 0 & \widetilde{\phi}_{TOT}(\alpha) < \phi < \widetilde{\phi}_{CORR} \\ > 0 & \phi < \widetilde{\phi}_{TOT}(\alpha) \end{cases} \tag{D.1}
$$

Proposition 3. We know from the proof of Lemma 3 that the wealth-gap response is positive when either $\phi > \widetilde{\phi}_{CO}$ or $\phi < \widetilde{\phi}_{TOT}(\alpha)$. Since $\widetilde{\phi}_{CO} > \widetilde{\phi}_{CORR}$, this means that the derivative of the absolute value of the wealth-gap response, as described in (D.1), is positive in all regions that the response itself is positive. For $\widetilde{\phi}_{TOT}(\alpha) < \phi < \widetilde{\phi}_{CORR}$, the sign of the response is negative, and its derivative is also negative. Finally, when $\widetilde{\phi}_{CORR} < \phi < \widetilde{\phi}_{CO}$, instead, the wealth-gap response is negative, and its derivative is positive.

Proposition 4. Equation (D.1) shows that the impact of GVC integration on risk sharing depends on the trade elasticity relative to the threshold $\widetilde{\phi}_{TOT}(\alpha)$, which is itself a function of GVC integration. In particular, this threshold is a decreasing function of α, with $\widetilde{\phi}_{TOT}(\alpha) \in [0, \widetilde{\phi}_{TOT}]$ for $\alpha \in [0, 1)$, where the upper limit is the threshold in the no-GVC case.

This means that for a given $\phi < \widetilde{\phi}_{TOT}$, increasing α will move the equilibrium from the

case where $\phi < \widetilde{\phi}_{TOT}(\alpha)$ to the case where $\phi > \widetilde{\phi}_{TOT}(\alpha)$. According to equation (D.1), this means that for low values of α, the derivative of the absolute value of the wealth-gap response will be positive—such that higher GVC integration reduces risk sharing at the margin—but for higher values of α the derivative of the absolute value of the wealth-gap response will be negative—such that higher GVC integration increases risk sharing at the margin.

Figure D.1 presents the logic underpinning the proof to Proposition 4 visually, plotting derivative of the absolute value of the wealth-gap response against α, for different values of ϕ.

For $\phi = 1.5$, $\phi > \widetilde{\phi}_{CORR}$. In line with the exposition in the above proof, $|f(\alpha)|$ is strictly increase in α: higher integration is associated with less risk sharing for all α. The converse is true for $\phi = 0.5$, when $\widetilde{\phi}_{TOT}(\alpha) < \phi < \widetilde{\phi}_{CORR}$, such that the function is strictly decreasing in α. Higher integration is associated with increased risk sharing here.

For $\phi = 0.1$, $\phi < \widetilde{\phi}_{TOT}$, and, as a result, the function is neither strictly increasing, nor decreasing, with respect to GVC integration. At low values of integration, the derivative is positive, and a marginal increase in α is associated with less risk sharing. However, past a certain point, the equilibrium switches and the derivative becomes negative, such that a marginal increase in α is associated with more risk sharing.

This means that, starting from a low level of integration, while a marginal increase in integration reduces risk sharing, a large increase would improve it.
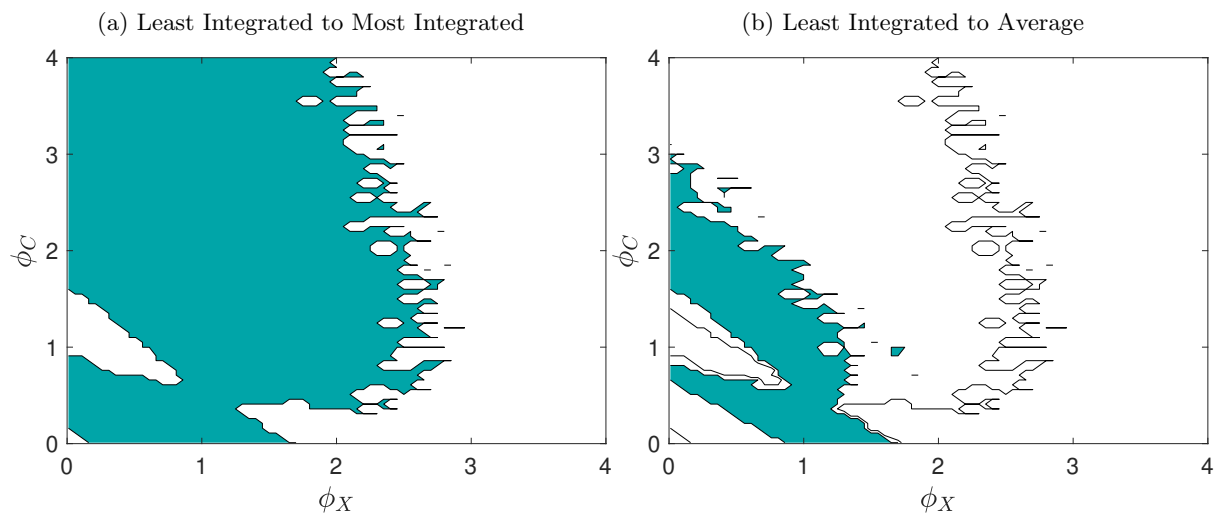
Figure D.1: Sign of Marginal Increase in Integration for Risk Sharing $|f(\alpha)|'$
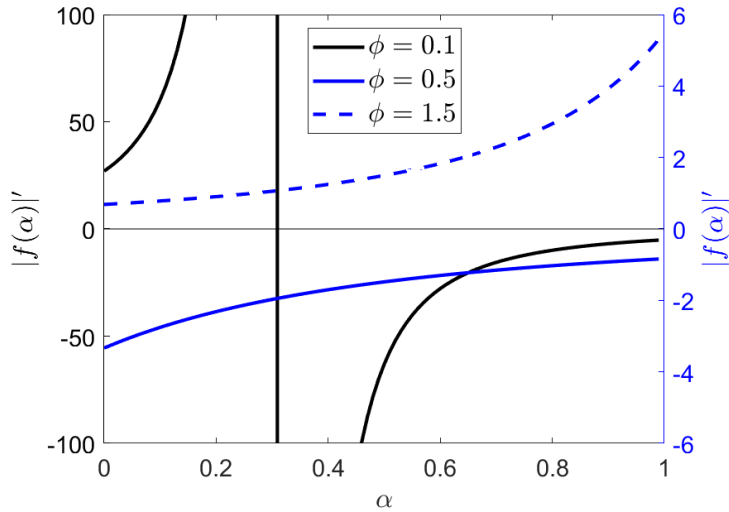
Note: Positive shock to A_t with persistence 0.8, with $b_H = a_H = 0.7$, $\sigma = 2$ and $\beta = 0.99$.

E   Robustness of Quantitative Model Results

Figure E.1: Bond-Economy Model: Regions of Elasticity Parameter Space where GVC Integration Improves Risk Sharing
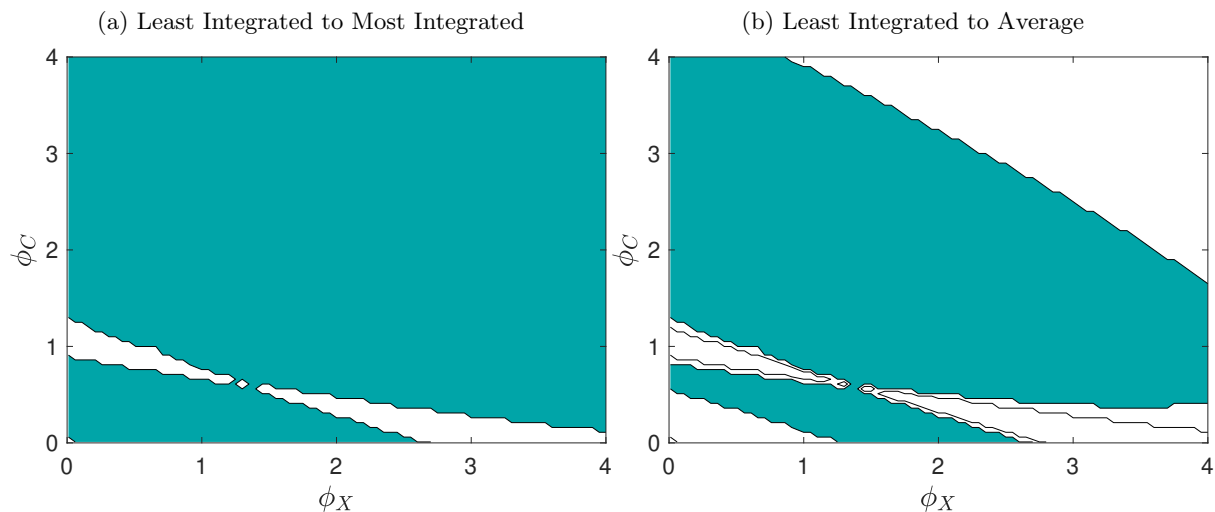
(a) Least Integrated to Most Integrated   (b) Least Integrated to Average

Note: Shaded area denotes combinations of final- and intermediate-goods trade elasticities, ϕ_C and ϕ_X, for which an increase in GVC integration increases international risk sharing. Increased international risk sharing is defined as occurring when the theoretical variance of the wealth gap is lower for a more open calibration of trade integration *vis-à-vis* a more closed one. Theoretical variances are calculated accounting for independent Home and Foreign productivity shocks, with persistence $\rho = 0.8$. Average, Most Integrated and Least Integrated calibrations for trade integration are defined as in Table 1.

Figure E.2: Model with Endogenous Labour Supply: Regions of Elasticity Parameter Space where GVC Integration Improves Risk Sharing
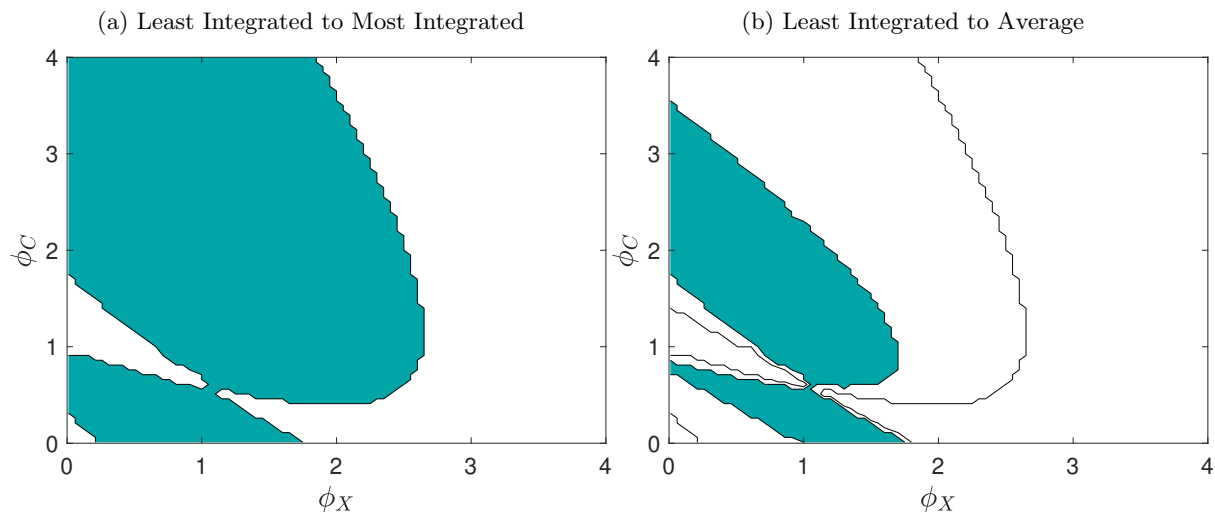
(a) Least Integrated to Most Integrated

(b) Least Integrated to Average

Note: Shaded area denotes combinations of final- and intermediate-goods trade elasticities, ϕ_C and ϕ_X, for which an increase in GVC integration increases international risk sharing. Increased international risk sharing is defined as occurring when the theoretical variance of the wealth gap is lower for a more open calibration of trade integration *vis-à-vis* a more closed one. Theoretical variances are calculated accounting for independent Home and Foreign productivity shocks, with persistence $\rho = 0.8$. Average, Most Integrated and Least Integrated calibrations for trade integration are defined as in Table 1.

Figure E.3: CES Production Function: Regions of Elasticity Parameter Space where GVC Integration Improves Risk Sharing

(a) Least Integrated to Most Integrated

(b) Least Integrated to Average

Note: Shaded area denotes combinations of final- and intermediate-goods trade elasticities, ϕ_C and ϕ_X, for which an increase in GVC integration increases international risk sharing. Increased international risk sharing is defined as occurring when the theoretical variance of the wealth gap is lower for a more open calibration of trade integration *vis-à-vis* a more closed one. Theoretical variances are calculated accounting for independent Home and Foreign productivity shocks, with persistence $\rho = 0.8$. Average, Most Integrated and Least Integrated calibrations for trade integration are defined as in Table 1.

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