Asymmetric Fads and Inefficient Plunges: Evaluating the Adaptive vs. Efficient Market Hypotheses

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Abstract

I propose the concept of inefficient plunges to characterize asymmetric deviations of the market price from the efficient price with the aim of examining the efficient market hypothesis. To gauge market inefficiency, I present an asymmetric Fads model, which allows for both inefficient plunges in the transitory component and a switching variance in the permanent component by embedding a Markov-switching process in an unobserved components model. Applying the model to the S&P 500 and the FTSE 350 reveals that inefficient plunges are deep, steep, and transient. This finding suggests that market inefficiency is a regime-dependent and asymmetric phenomenon, meaning that although the U.S. and U.K. stock markets are efficient during normal times, they are considerably below efficient prices during crises. Overall, the asymmetric Fads model proposed in this study supports the adaptive market hypothesis and casts doubt on the efficient market hypothesis.

Keywords: Inefficient plunges, Adaptive Market Hypothesis, Efficient Market Hypothesis, Rational Bubbles, Negative Bubbles, Asymmetric Unobserved Components Model.

JEL Classification: C32, C58, G14, G41.

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1. Introduction

I present an asymmetric Fads model to capture the evolution of stock market inefficiency, which is defined as the deviation of the market price from the efficient price. Assuming that the efficient price follows a random walk process with a drift, this model reveals regime-dependence and asymmetry in market inefficiency: negative deviations during crises tend to be deep and steep, whereas positive deviations during non-crisis (normal) periods are negligible. In fact, although the efficient price drops during crashes due to expectations of lower future cash flows, the market price tends to overreact and drop even further, such that it negatively deviates from the efficient price. I call these negative deviations of the market price from the efficient price "inefficient plunges", a new concept that is similar to the idea of "negative bubbles", proposed in a few recent studies, since they both refer to negative deviations of the market price from a hypothetical price. However, there is a distinction between these two concepts. Inefficient plunges in this study are defined as negative deviations from the efficient price, aiming to better gauge stock market inefficiency in the U.S. and U.K., whereas negative bubbles have been defined as negative deviations from the fundamental price.[1]

This study, by characterizing inefficient plunges, evaluates the Adaptive Market Hypothesis (AMH) proposed by Lo (2004, 2019) against the Efficient Market Hypothesis (EMH) put forth by Samuelson (1965) and Fama (1970). The EMH argues that since the market price encompasses all information currently available, the future price is unpredictable, and so the market price follows a random walk process with a drift, which is referred to as the Random Walk Hypothesis (RWH). This hypothesis is entirely built on the Rational Expectations Hypothesis (REH), put forth by Muth (1961) and Lucas (1978), which postulates that all investors have rational expectations. However, contrary to the REH, behavioural finance suggests that because of biases such as panic and overreaction amid crises and greed and overconfidence amid speculative bubbles, a sizeable portion of investors are not always rational (Simon, 1955; Arrow, 1982), and hence the market cannot always be efficient (Russell and Thaler, 1985). For instance, the Over-Reaction Hypothesis (ORH) proposed by De Bondt and Thaler (1985) demonstrates that investors have a tendency to overreact to events, especially negative news.

Considering the above disagreement, the AMH aims to reconcile the EMH with behavioural finance by describing a framework in which rationality and irrationality forces coexist and investors are not unboundedly and unchangingly rational. The main hypothesis of this framework states that market rationality and, consequently, market inefficiency do not remain constant but instead evolve over time. This hypothesis is supported by a growing empirical literature that applies a rolling window

[1] The efficient price is not necessarily equal to the fundamental price because the efficient price can exceed the fundamental price by the size of positive bubbles.

analysis to demonstrate time-variation in market inefficiency, particularly during crisis periods (see, e.g., Lim et al., 2008; Anagnostidis et al., 2016; Ito et al., 2016; Noda, 2016; Hill and Motegi, 2019).

Moreover, according to the established literature on speculative bubbles, which imposes a positive sign on bubbles, the occurrence of speculative bubbles is consistent with the REH and EMH. Hence, the market price that might contain positive bubbles must still be efficient. In this sense, Blanchard and Watson (1982), Tirole (1985), and Diba and Grossman (1988) present a model named Rational Bubbles to rationalize the formation of speculative bubbles. Nevertheless, a few theoretical models suggest that withdrawal of rational but uninformed investors and binding financial constraints cause negative bubbles to form during crises (see, e.g., Barlevy and Veronesi, 2003; Yuan, 2005; Cao et al., 2016; Emery, 2021), and a handful of empirical studies affirm the possibility of negative bubbles during crises, when the market price is lower than its fundamental value (see, e.g., Yan et al., 2012; Goetzmann and Kim, 2018). As a result, given the rationality of positive bubbles, one of the main culprits of market inefficiency can be attributed to negative bubbles during crises.

Based on the empirical literature mentioned above, there is a consensus about the EMH: the market price does not follow a random walk process with a drift, and the market is not always efficient, in particular during crises. However, there are still some unresolved questions about the dynamics of market inefficiency. In this regard, this study investigates whether market price deviations from the efficient price, which measure market inefficiency, are regime-dependent and asymmetric. To this end, I augment the conventional Fads model by taking asymmetric price deviations into account. In the proposed Unobserved Components (UC) model, I decompose the market price into its permanent and transitory components. The permanent component is specified as a random walk process with a drift to reflect the dynamics of the efficient price. Thus, the transitory component, by construction, stands for market inefficiency. To accommodate the asymmetry, I incorporate inefficient plunges in the transitory component and a concomitant switching variance in the permanent component by using a Markov-switching process. Under this setup, therefore, inefficient plunges characterize asymmetric market inefficiency during high-variance states. I then evaluate how well the proposed model, which I refer to as the asymmetric Fads model, explains stock market stylized facts, including asymmetric inefficiency and asymmetric volatility.

By applying the model to inflation-adjusted S&P 500 and FTSE 350, this study documents regime-dependence and asymmetry in market inefficiency since the plunging coefficients are -7.07 for the U.S. and -5.81 for the U.K. stock markets, with standard errors of 1.04 and 1.27. The likelihood ratios for testing the asymmetry are 47.4 and 17.2, respectively, which are far greater than the critical value of 10.8 for a conservative 0.1% significance level. Inefficient plunges are deep, steep, and transient, which means they reach a notable depth of 10% to 15% during crises, survive the arbitrage process for a notable duration of 4.8 and 5.9 months for the U.S. and U.K. stock markets, and then shrink and finally disappear within a couple of additional months, such that the expected duration is short for crisis periods and long for non-crisis periods. As a result, although the S&P 500 and FTSE 350 are efficient during normal times, they are remarkably below efficient prices during crisis periods, which comprise at least 20% of their sample. This finding draws a conclusion in favour of the AMH of Lo (2004, 2019) over the EMH of Samuelson (1965) and Fama (1970) and supports time-variation in market inefficiency with spikes during crises that has been previously recognized indirectly by using a rolling window analysis in a few studies.

Moreover, the observed time-variation in the drift term (inflation-adjusted long-run return) for the S&P 500 hints at the presence of positive bubbles in the permanent component. Also, the observation of significant inefficient plunges in the transitory components of both stock markets is in accordance with the possibility of negative bubbles; however, inefficient plunges are likely larger in magnitude than negative bubbles because the former are negative deviations from the efficient price while the latter are defined as negative deviations from the fundamental.

Finally, the asymmetric Fads model, by allowing for inefficient plunges and a concomitant switching variance, captures the following stock market's stylized facts: (1) deep and steep inefficient plunges accord with the asymmetric return distribution, meaning that returns are negatively skewed because downturns are sharper than upturns (Campbell and Hentschel, 1992; Hong and Stein, 2003; Adrian and Rosenberg, 2008, among others). (2) Inefficient plunges that appear alongside concomitant high-variance states are consistent with asymmetric volatility, implying that the onset of episodes of high volatility coincides with episodes of large negative returns (Nelson, 1991; Glosten et al., 1993; Jones et al., 2004; Avramov et al., 2006; Bollerslev et al., 2006; Liu et al., 2012; among others). (3) The fact that transient inefficient plunges are followed by rebounds that take a few months to fill the gaps between market and efficient prices aligns with volatility clustering, indicating that large fluctuations in prices are typically followed by further large fluctuations (Engle, 1982, 2004; Bollerslev, 1986), which occur more frequently during downturns (Ning et al., 2015).

This study makes contributions to the literature in two ways. First, it repurposes the UC model with Markov-switching to introduce and characterize the new concept of inefficient plunges, with the aim of examining the EMH. To my knowledge, this study is the first to capture asymmetric deviations of the market price from the efficient price by choosing to model at log levels rather than in differences. Second, the existing empirical literature on testing the EMH is insufficient to identify the nature of the time-variation in market inefficiency. By contrast, this study explicitly establishes that market inefficiency is asymmetric and pinpoints the level of market inefficiency at any given moment during crises or normal periods, in contrast to previous works that only provided an approximation of market inefficiency by performing correlation or random walk tests based on a rolling window analysis.

The remainder of this paper is organized as follows: Section 2 provides a review on the literature. Section 3 describes the data and methodology and justifies the specification of the asymmetric Fads model. Section 4 presents the results and robustness tests, and Section 5 provides the concluding remarks.

2. Literature review

This study lies at the crossroads of four branches in the literature of finance, including the Efficient Market Hypothesis (EMH), the Adaptive Market Hypothesis (AMH), Rational Bubbles, and the Fads model; each is discussed in this section.

The EMH, proposed by Samuelson (1965) and Fama (1970), states that the market price reflects all available public information. Hence, the price is unpredictable, and no investor can outperform the market since the future movement of the price depends only on the newly released information. If this is the case, the market price follows a random walk process with a drift, what is referred to as the RWH, or weak form of the EMH.

Empirically, although many studies have explored the RWH for different stock markets by using unit root, correlation, or variance ratio tests, Durusu-Ciftci et al. (2019) cite a lack of agreement about dynamics of stock market efficiency. The reason for such a debate is that the bulk of the literature has tested the RWH under the dubious assumption that market efficiency is an absolute all-or-nothing measure that remains constant over time, while it actually appears to be a fuzzy, rather than a binary, and time-varying phenomenon (Campbell et al., 1998; Lim and Brook, 2011, Ito et al., 2016).

Theoretically, EMH relies heavily on the REH, which was originally introduced by Muth (1961) and later popularized by Lucas (1978). The REH states that since all investors have rational expectations, the market price implied by the investors' behaviour is essentially the same as the price predicted by the theoretical model. In contrast, behavioural finance and economics argue that a sizable number of investors in the market are not rational (Simon, 1955; Kahneman and Tversky, 1979; Arrow, 1982; among others). In this context, the irrationality reflects itself in various forms of behavioural biases, including panic during crises and greed during speculative bubbles, each of which results in market inefficiency. As an illustration, De Bondt and Thaler (1985) presented the ORH, which propounds the idea that investors overreact to unexpected news. These overreactions contradict the EMH and advocate predictability because extreme movements in stock prices will be followed by subsequent price movements in the opposite direction.

The AMH, first introduced by Farmer and Lo (1999) and later formalized by Lo (2004, 2019), tries to reconcile the EMH with behavioural finance and economics by establishing a framework in which rationality and irrationality forces coexist. In this framework, investors are not unboundedly rational and they have a tendency to find satisfactory heuristic solutions, which are not necessarily optimal, to economic challenges. Since the solution of the old regime is not suitable for the new one, investors need to adapt their heuristics when the regime changes. This adaptation involves trial and error and inevitably behavioural biases in a sizable number of investors, who constitute an irrationality force that moves the price away from the efficient level. Inversely, the rest of the investors, who are rational and take advantage of arbitrage opportunities, form a rationality force that brings the price back to its efficient level. As a result, the AMH describes the confrontation between forces of rationality and irrationality. For example, during normal times, rationality predominates over irrationality, such that the market price is efficient. Conversely, irrationality prevails over rationality during crisis times and leads to a panic-driven deviation from efficiency. Afterwards, once the panic recedes, rationality will dominate again, and the market price will revert to its efficient level. This narrative attributes market inefficiency to market irrationality, whose magnitude is determined by the proportion of irrational investors in the market.

The AMH raises questions asking to what extent the irrationality force is stronger than the rationality force and how long the inefficient price survives the arbitrage process. The EMH and behavioural finance respond to these questions in two opposite directions. The former insists that the irrationality force is negligible and the inefficient price disappears immediately, but the latter maintains that the irrationality force is substantial and the inefficient price lasts for a notable period.[2] In response, the EMH proponents counter-argue that even if individual irrationality does exist, its effect on the market price is negligible as irrational investors account for a small portion of the market and arbitrageurs immediately bring the price back to the efficient level. This counter-argument is rejected by Russell and Thaler (1985), who state that the presence of some rational agents is not sufficient to guarantee the existence of a rational expectations equilibrium. Furthermore, Kindleberger (1989) and Lo (2004) provided some anecdotal examples of speculative bubbles, manias, and panic during market crashes, which suggest persistent deviations of the market price from the efficient price and thus cast doubt on market rationality at the aggregate level.

The AMH proposes that the composition of the market is changing over time because investors with different attitudes and levels of irrationality are entering and leaving the market. Therefore, the level of market irrationality and, accordingly, market inefficiency are not constant but instead evolve over time. A rapidly growing literature within this context, by relaxing the assumption of constant market efficiency, supports the AMH and casts doubt on the EMH (Lim and Brook, 2011). For example, by applying a rolling window analysis, Ito and Sugiyama (2009) and Hill and Motegi (2019) document a time-variation in market inefficiency, particularly during financial crises, in the U.S. and U.K. stock

[2] In this sense, Grossman and Stiglitz (1980) argue that a perfectly efficient market is impossible because traders do not have any incentive to acquire costly information unless there is a profit-making arbitrage opportunity.

markets. Similarly, Lim et al. (2008), Anagnostidis et al. (2016), and Ito et al. (2016) demonstrated that financial crises adversely affect the level of efficiency in the European, Asian, and American stock markets. Lastly, Kim et al. (2011), Noda (2016), Le Tran and Leirvik (2019), and Mattera and Di Sciorio (2022) constructed time-varying indices for market inefficiency or return predictability by using methods that rely on overlapping or non-overlapping rolling windows. Overall, although the methods used in all the above studies are based on rolling window analysis, which is not able to exactly determine the extent to which the market is inefficient at any given moment in time, they all suggest that the market inefficiency is time-varying.

According to the third branch of the literature, speculative bubbles are defined as positive deviations of the market price from the fundamental price that are followed by a burst. For instance, Tulip Mania in 1637, the dot-com bubble in the late 1990s, and the real estate bubble in 2005 are three of the most notorious bubbles in which asset prices skyrocketed to unreasonably high levels and then collapsed. In this context, Blanchard and Watson (1982), Tirole (1985), and Diba and Grossman (1988) present the model of Rational Bubbles, as implied by the name, to rationalize the formation of speculative bubbles. According to Rational Bubbles, speculative bubbles occur even if all investors have rational expectations and are aware that the bubble will eventually burst.

For clarification, consider a simple example given by Blanchard and Watson (1982), in which the speculative bubble either survives by growing at a rate higher than the risk-free rate with probability $\mathbb{p}$ or bursts with probability $(1-\mathbb{p})$. Clearly, the conventional risk-return trade-off holds true in this example because an investor who tolerates the risk of bursting the bubble can be compensated with a return higher than the risk-free rate. Speculative bubbles, therefore, are not necessarily inconsistent with rationality (Arrow, 1982), because the market price may rationally deviate from its fundamental value when investors expect that the speculative bubble will grow further. On this basis, most of the literature on speculative bubbles, by imposing a positive sign on bubbles, implicitly rules out the possibility of negative bubbles.[3] Given this restriction, Blanchard and Watson (1982), Tirole (1985), Diba and Grossman (1988), Adam and Szafarz (1992), Lux and Sornette (2002), and Basse (2021), among others, substantiate the rationality of speculative bubbles. Therefore, even if the market price contains positive bubbles, it must still be efficient, and hence, negative bubbles during crises can be one of the potential determinants of market inefficiency and predictability.

Negative bubbles are defined as negative deviations of the market price from the fundamental. Few studies have explored negative bubbles. In particular, Yan et al. (2012) applied the Johansen et al. (2000) rational expectation bubbles model to the S&P 500, and Goetzmann and Kim (2018) analysed

[3] Few studies allow for both positive and negative bubbles, where asset prices might deviate from their fundamentals in either a positive or negative direction (see, e.g., Lux, 1995; Shiller, 2000).

101 global stocks to characterize a pattern of crash-and-rebound, which describes how large drops can lead to the formation of negative bubbles and how they are typically followed by strong rebounds. According to their results, the return following a crash is, on average, 10% higher than normal times. This 10% extra rise during the rebound is indeed the mirror image of the excessive drop (negative bubbles) during the crash. In a recent study, Emery (2021) has extended the adverse selection model of Akerlof (1970) to show that asymmetric information can lead to negative bubbles. The results of this study and a few other theoretical works (see, e.g., Barlevy and Veronesi, 2003; Yuan, 2005; Cao et al., 2016) demonstrate that a large withdrawal of rational but uninformed investors from the market and/or binding financial constraints can lead to a negative bubble to form during crashes.

The Fads model is a trend-cycle decomposition to examine the possibility of deviations of the market price from its fundamental price, which are caused by noisy traders who, based on fashions, fads, and sentiments, bid the price away from the fundamental (Shiller et al., 1984; Summers, 1986; Fama and French, 1988; Poterba and Summers, 1988; among others). In the conventional Fads model, the permanent component usually stands for the fundamental value and is specified as a random walk process with a drift. The transitory component (also known as the Fads component) is specified as an autoregressive process of order one or two to allow for potential mean-reversion. The estimation of a close-to-unity autoregressive coefficient in this setup suggests that market price deviations from the fundamental, measured as the Fads component, are transitory but persistent. Indeed, a significant and persistent Fads component rejects the EMH since it implies that the market price deviates from the fundamental price and slowly returns to it, which induces autocorrelation in returns, enables investors to make a predictable profit, and accordingly supports predictability.

The conventional Fads model, however, does not distinguish between positive and negative bubbles since it is subject to two caveats. The first caveat is that the Fads model imposes the transversality condition to dismiss the potential presence of positive bubbles inside the permanent component.[4] Consequently, the Fads model tends to confuse the permanent component with the fundamental price, whereas there are many studies stating that positive bubbles are not only possible but also rational. Hence, the Fads model ignores that the permanent component might exceed the fundamental price by the size of a positive bubble, which is fuelled by speculative activities during the boom phase of the bubble. As a result, given that Rational Bubbles and the Fads component are two distinct features (Camerer, 1989), and U.S. and U.K. stock markets exhibit traces of both (Schaller and Van Norden, 2002), unaccounted for positive bubbles in the conventional Fads model confound the measurement

[4] The efficient price is the sum of the fundamental price (the sum of discounted expected dividends) and speculative bubble (the present value of the expected resale price). The transversality condition requires that the present value of the expected resale price converge to zero as time goes to infinity.

of the permanent and transitory components. To tackle this shortcoming, one can consider requiring the permanent component to stand for the efficient price rather than the fundamental price.

The second shortcoming of the Fads model is that it disregards the asymmetry in deviations of the market price from the permanent component, while there is sufficient evidence suggesting that large and steep price movements tend to be downward rather than upward (Yuan, 2005). As a result, the Fads model is unable to characterize negative deviations as one of the main determinants of market inefficiency. In addition, although a few variants of the Fads model have introduced asymmetry into the UC model, since these models are specified in differences, they are unable to characterize the asymmetric deviations from efficiency, which calls for modelling at levels. For example, Turner et al. (1989) and Liu et al. (2012) allow for the asymmetry by including a Markov-switching process for the mean and variance of returns, and Kim and Kim (1996) allow for the asymmetry by including two independent Markov-switching processes, each accounting for one of the switching variances of shocks to the permanent and transitory components of returns.

Moreover, Turner et al. (1989) and Liu et al. (2012) exclude the transitory component that is essential for characterizing price deviations from the efficient price, and Kim and Kim (1996), while including a transitory component, do not account for the asymmetry in price deviation. Additionally, the former studies assume that the expected return remains constant within each regime, while the latter imposes a more restrictive assumption that the expected return is constant across all regimes. Their findings, anyhow, indirectly support the concept of asymmetric deviations because the former gives a hint that returns during high-variance states are much lower than returns during low-variance states, and the latter estimates a transitory component that is often negative.

3. Data and Methodology

This study uses monthly data of the S&P 500 index from 1948M1 to 2022M12 and the FTSE 350 index from 1986M1 to 2022M12, obtained from Bloomberg. I apply models to these indices without dividends reinvested, yet the results are not sensitive to the choice of dividend reinvestment. I adjust each monthly index for inflation by dividing it by the consumer price index of the corresponding country. For robustness tests, I also use nominal indices at daily, weekly, and monthly frequencies.

Regarding the application of the trend-cycle decomposition in finance and economics, the bulk of studies have used different versions of the UC models of Harvey (1985) and Clark (1987). The conventional Fads model, particularly, is a UC model that discards the possibility of asymmetric price deviations from efficiency, even though they seem plausible. Meanwhile, several studies in economics have applied the UC model with Markov-switching to explain business cycle asymmetries (see, e.g., Kim and Nelson, 1999a), and few studies in finance have used the UC model with Markov-switching to examine regime-dependent returns (Turner et al., 1989; Liu et al., 2012) and transient

Fads (Kim and Kim, 1996). With this in mind, I augment the Fads model to address its shortcomings and, more importantly, to repurpose the UC model with Markov-switching to examine the EMH.

I decompose the market price into the permanent and transitory components, which correspond to the efficient price and inefficient plunges, respectively. I preserve positive bubbles, together with the fundamental price, in the permanent component and specify it as a random walk process with a drift.[5] The transitory component (also called the Fads component) is specified as an autoregressive process of order two to account for the potential persistency during the price rebound. To accommodate the asymmetry, I incorporate inefficient plunges in the transitory component and a switching variance in the permanent component by employing a Markov-switching process of Hamilton (1989). I cast the model in a state-space form with Markov-switching to estimate it by using Kim's (1994) approximate maximum likelihood method.[6] I test the main model against symmetric alternatives based on a set of pairwise comparisons of the log likelihood values of competing models.

3.1. The asymmetric Fads model

Within a univariate model of trend-cycle decomposition, consider Eq. (1), in which the actual market price is decomposed into a permanent and a transitory component:

$$p_t = p_t^e + p_t^i \tag{1}$$

where the observable series, denoted by p_t, is the natural log of the market index. p_t^e and p_t^i are unobserved permanent and transitory components, which play the roles of the efficient price and inefficient plunges. Considering the possibility of speculative bubbles, the efficient price can contain positive bubbles, and hence it is not necessarily equal to the fundamental price. For this reason, the permanent component in this study characterizes the efficient price rather than the fundamental price.

3.1.1. The permanent component (efficient prices)

Given that speculative bubbles are rationalized by Blanchard and Watson (1982), Diba and Grossman (1988), and other scholars, the efficient price in this setup still follows a random walk process with

[5] According to the present value model and the law of iterated expectations, the efficient price is the sum of two elements: the fundamental price that is the sum of discounted expected dividends and the speculative bubble that is the discounted expected resale price at infinity. There are four rationales for keeping these two elements as a single unit inside the permanent component. First, the specification of the efficient price (the sum of the fundamental and the positive bubble) is given as a random walk with a drift, whereas there is no clear specification for each separately. Second, based on Blanchard and Watson (1982), the fundamental price and the positive bubble are not independent as they both increase during bubble formation and drop together during the burst. Thus, I circumvent decomposing them since it is exceedingly challenging, if not impossible. Third, Camerer (1989) warns against the confusion of Rational Bubbles with the Fads component. Hence, my model places positive deviations from fundamental, along with the fundamental price, inside the permanent component to characterize the efficient price and places negative deviations from efficiency in the transitory component to characterize inefficient plunges. Fourth, this study aims to examine the effect of negative deviations, rather than positive bubbles, on stock market inefficiency.

[6] See Appendices A and B, Kim and Nelson (1999b) and Hamilton (1994), which explain how to make the Kalman's (1960) filter operable.

a drift. Hence, similar to Fama and French (1988), Poterba and Summers (1988), and Eraker (2008), I specify the permanent component as follows:

$$p_t^e = \mu_{t-1} + p_{t-1}^e + \varepsilon_{p^e,t} \tag{2}$$

where $\varepsilon_{p^e,t} \sim N(0, \sigma_{p^e}^2)$ is the shock to the permanent component and is explained further in Section 3.1.3. In this setting, μ_t is the drift term that represents the long-run return and might be time-varying since, due to changes in expectations about future cash flows, long-run episodes of bull and bear markets are observed in the U.S. and U.K. stock markets. Accordingly, I specify the long-run return as a random walk:

$$\mu_t = \mu_{t-1} + \varepsilon_{\mu,t} \tag{3.a}$$

where $\varepsilon_{\mu,t} \sim N(0, \sigma_\mu^2)$ is the shock to the long-run return and is assumed to be uncorrelated with $\varepsilon_{p^e,t}$. Alternatively, following Summers (1986), Poterba and Summers (1988), Turner et al. (1989), among others, I can model the long-run return as a constant rate:

$$\mu_t = \mu \tag{3.b}$$

I run a version of the proposed model for each of the two above specifications for long-run return presented in Eq. (3.a) and Eq. (3.b). Nevertheless, I advocate the model with stochastic drift for two reasons. First, a stochastic drift in the form of a random walk, by capturing the time-variation in the long-run return, enables the model to characterize speculative bubbles. Second, the random walk is more robust to misspecifications and provides more flexibility (Antolin-Diaz et al., 2017).

3.1.2. The transitory component (inefficient plunges)

Like Kim and Kim (1996), the transitory component comprises an autoregressive process of order two with coefficients φ_1 and φ_2. However, to accommodate asymmetric price deviations, I consider that shocks to the transitory component are a mixture of asymmetric and symmetric shocks. Thus, I incorporate an unobservable, first-order, and two-state Markov-switching process into the transitory component, which I call the asymmetric Fads component:

$$p_t^i = \pi_i S_t + \varphi_1\, p_{t-1}^i + \varphi_2\, p_{t-2}^i + \varepsilon_{p^i,t} \tag{4}$$

where $\varepsilon_{p^i,t} \sim N(0, \sigma_{p^i}^2)$ is the typical shock to the transitory component, and π_i is the inefficient plunge coefficient that measures the magnitude of inefficient plunges and is expected to be negative. The state of the stock market is denoted by S_t, an indicator that distinguishes between crisis periods when $S_t = 1$ and normal times when $S_t = 0$. This indicator will be determined endogenously as it evolves according to the Markov-switching process as in Hamilton (1989):

$$\Pr[S_t = 1 | S_{t-1} = 1] = p \tag{5}$$

$$\Pr[S_t = 0 | S_{t-1} = 0] = q \tag{6}$$

In this approach, p and q determine the transition probabilities. p is the probability of staying in the crisis, and thus, $(1-p)$ is the probability of transitioning from the crisis to a normal state. Similarly, q is the probability of staying in the normal state, and thus, $(1-q)$ is the probability of transitioning from the normal state to a crisis state. In this setting, the term $\pi_i S_t$ gauges the excessive drop in price (inefficient plunge) during crises, which has a role similar to the negative bubble in the model of Yan et al. (2012), adopted from the model of bubbles and crashes presented by Johansen et al. (2000).

3.1.3. The variance-covariance matrix of shocks

In this model, I maintain the assumption that all shocks (also known as innovations) are white noise, normally distributed, and also uncorrelated with each other. Regarding the variance of shocks to the permanent component, I follow the approach of Turner et al. (1989), Kim and Kim (1996), and Liu et al. (2012) who include a Markov-switching variance in the model. In this sense, $\varepsilon_{p^e,t} \sim N(0, \sigma^2_{p^e})$ is allowed to have a switching variance that is higher during crises than during normal periods:

$$\sigma^2_{p^e} = \sigma^2_{p^e,0}(1 - S_t) + \sigma^2_{p^e,1}(S_t) \tag{7}$$

where $\sigma^2_{p^e,1}$ and $\sigma^2_{p^e,0}$ are variances of shocks to the permanent component during crises and normal periods, respectively. I employ a single Markov-switching process for both inefficient plunges in the transitory component and the variance of shocks in the permanent component because asymmetric volatility implies a concomitant occurrence of price fall and volatility jump. Regarding the variance of shocks to the transitory component, $\varepsilon_{p^i,t} \sim N(0, \sigma^2_{p^i})$ is assumed to have a constant variance since the plunging coefficient in Eq. (4) accounts for the asymmetry in the price deviations. For robustness tests, however, I consider that this variance is also regime-dependent as follows:

$$\sigma^2_{p^i} = \sigma^2_{p^i,0}(1 - S_t) + \sigma^2_{p^i,1}(S_t) \tag{8}$$

where $\sigma^2_{p^i,1}$ and $\sigma^2_{p^i,0}$ are variances during crises and normal times.

Lastly, all shocks are assumed to be uncorrelated in this study. Given all the assumptions I made for the three shocks in this model, the variance-covariance matrix of shocks is:

$$\begin{bmatrix} \varepsilon_{p^e,t} \\ \varepsilon_{p^i,t} \\ \varepsilon_{\mu,t} \end{bmatrix} \sim N\left(\mathbf{0}_{3\times1}, \begin{bmatrix} \sigma^2_{p^e} & 0 & 0 \\ 0 & \sigma^2_{p^i} & 0 \\ 0 & 0 & \sigma^2_{\mu} \end{bmatrix}\right) \tag{9}$$

Concerning correlations, merely for the sake of robustness tests, I allow for correlation between each pair of shocks as specified in Appendix A. Favourably, the results presented in Appendix C indicate that these three correlations are all insignificant.

4. Results and discussion

I estimate several alternative models to test for asymmetry in the form of inefficient plunges in the transitory component and/or a switching variance in the permanent component using a set of pairwise comparisons of log likelihood values. As shown in the first two rows of Tables 1 and 2 and explained in Table A in Appendix A, each of the sixteen models is denoted by an identifier and a descriptor. The descriptor consists of two major parts. The first part expresses the model specification regarding the asymmetry in the transitory component as well as the asymmetry of the variance in the permanent component. And the second part determines whether the drift term (long-run return) is specified as a random walk process or a constant. For example, the identifier of the proposed model is 1a, and its descriptor is denoted by "A (IP-SV)-RW", which means that this model accounts for the **A**symmetry by allowing for both **I**nefficient **P**lunges in the transitory component and a **S**witching **V**ariance in the permanent component. In this model, the term "RW" hints that the long-run return is specified in the form of a **R**andom **W**alk process. To give more examples, the descriptor "A (IP)-Con" states that model 2b accounts for the **A**symmetry by allowing for **I**nefficient **P**lunges but not the asymmetry of the variance, and the descriptor "A (SV)-Con" expresses that model 3b accounts for the **A**symmetry by allowing for a **S**witching **V**ariance but not the asymmetry in the transitory component. The term "Con" also says these models impose a **Con**stant long-run return.

Tables 1 and 2 report parameters and log likelihood values estimated by several models for the S&P 500 and FTSE 350. The results of the asymmetric Fads model substantiate inefficient plunges since the plunging coefficients are $\pi_i = -7.07$ for the U.S. and $\pi_i = -5.81$ for the U.K. stock markets, with small standard errors of 1.04 and 1.27 and likelihood ratios of 47.4 and 17.2, respectively, which are much greater than the critical value of 10.8 for a 0.1% significance level. The switching variance of shocks to the permanent component during crises is $\sigma^2_{p^e,1} = 5.68^2$ for the U.S. and $\sigma^2_{p^e,1} = 5.73^2$ for the U.K. markets, which are notably greater than their counterpart values of $\sigma^2_{p^e,0} = 2.40^2$ for the U.S. and $\sigma^2_{p^e,0} = 1.78^2$ for the U.K. during normal periods.[7]

For the S&P 500, the transition probability reported in column 1a of Table 1 is low ($p = 0.793$) for crisis periods and high ($q = 0.956$) for normal periods; as a result, the expected duration is short (4.8 months) for crises and long (22.7 months) for normal periods. For the case of the FTSE 350, column

[7] In the presence of a Markov-switching process, testing hypotheses based on the likelihood ratio statistics is non-standard as the nuisance parameter is not identified under the null hypothesis and the asymptotic distribution of the likelihood ratio test does not follow a standard χ^2 distribution. Few studies have proposed several computationally burdensome simulation-based or bootstrap-based methods to test for Markov-switching that are operable for simple models (see, e.g., Hansen, 1992; Garcia, 1998; Di Sanzo, 2009). Because of the sixteen alternative models estimated for each stock market, to avoid computational burden, I maintain the use of the non-standard likelihood ratio test. In addition, likelihood ratios derived for testing asymmetry are extremely large and leave very little, if not no, doubt that market price deviations are asymmetric.

1a of Table 2 reports that transition probabilities are $p = 0.832$ and $q = 0.923$; hence, the expected duration is short (5.9 months) for crises and long (13.0 months) for normal periods. Additionally, the sum of autoregressive coefficients in the cyclical component $(\varphi_1 + \varphi_2)$ is 0.72 for the U.S. and 0.34 for the U.K. stock markets, implying a relatively persistent Fads component in both markets.

Figures 1 and 2 plot the permanent and transitory components of these two stock markets. The top-left panels demonstrate that the gaps between market prices and efficient prices are negligible during upturns but are noticeable during downturns. The top-right panels, by plotting these gaps, show that inefficient plunges are deep, almost always negative, and transient since the gaps between the market and efficient prices tend to emerge and vanish quickly. The depth of inefficient plunges reaches 10% and even 15% during crisis periods, which often coincide with NBER and ECRI recession dates for the U.S. and U.K. economies. A plunge continues for 4.8 months in the U.S. and 5.9 months in the U.K. stock markets, on average, and thereafter the corresponding gap shrinks and disappears within a couple of additional months. In this sense, the magnitude of inefficient plunges in the S&P 500 and FTSE 350 during 25% and 20% of their sample sizes, respectively, exceeds a threshold of 6% that is chosen only for illustrative purposes. By taking a neutral position about potential market exuberance during periods of speculative bubbles, I conclude that the U.S. and U.K. stock markets are inefficient at least 20% of the time that corresponds to downturns. Lastly, the bottom-right panels of Figures 1 and 2 display the probabilities of inefficient plunges accompanied by high-variance states.

Overall, results are in line with the AMH of Lo (2004) and in opposition to the EMH of Fama (1970). I highlight that deviations of U.S. and U.K. market prices from efficient prices, which measure the level of market inefficiency, are substantial, regime-dependent and asymmetric. This result supports the ORH of De Bondt and Thaler (1985), who report asymmetry in overreaction, as well as the results of rolling window analysis applied by Ito and Sugiyama (2009), Ito et al. (2016), Le Tran and Leirvik (2019), and Hill and Motegi (2019), who suggest that U.S. and U.K. stock markets are not efficient during crises. Furthermore, Synchrony of inefficient plunges in the transitory component and jumps in variance in the permanent component during crisis periods suggest that excessive price drops and volatility jumps occur concomitantly, hinting at the asymmetric volatility suggested by Turner et al. (1989), Nelson (1991), Jones et al. (2004), Avramov et al. (2006), and Liu et al. (2012.).

4.1. Inefficient plunges in the transitory component

To test if the plunging coefficient is significant ($\pi_i \ngeq 0$), I compare the log likelihoods for proposed models 1a and 1b, which account for the asymmetry by allowing for both inefficient plunges in the transitory component and a switching variance in the permanent component, with the log likelihood values for nested models 3a and 3b, which account for the asymmetry by allowing only a switching variance in the permanent component. For the S&P 500, by comparing log likelihoods of -2493.6

and -2492.5 for non-nested models 1a and 1b, presented in Table 1, with those values of -2517.3 and -2514.7 for nested models 3a and 3b, I report notable likelihood ratios of 47.4 and 44.4. For the FTSE 350, I compare log likelihoods of -1221.5 and -1218.9 for non-nested models 1a and 1b in Table 2 with values of -1230.1 and -1228.0 for nested models 3a and 3b and report likelihood ratios of 17.2 and 18.2. Since all likelihood ratios are greater than the critical value of 10.8 for a 0.1% significance level, I state that deviations of the actual market price from the efficient price, as a measure of market inefficiency, are regime-dependent and asymmetric in both stock markets.

Moreover, comparing log likelihood values of -2508.7 and -2506.0 for models 2a and 2b with those of -2560.4 and -2557.9 for symmetric models 4a and 4b presented in Table 1 for the S&P 500, and likewise comparing log likelihood values of -1240.5 and -1236.6 for models 2a and 2b with those of -1269.6 and -1265.2 for symmetric models 4a and 4b presented in Table 2 for the FTSE 350 reaffirm the presence of asymmetry in the form of inefficient plunges. The corresponding likelihood ratios are 103.4 and 103.8 for the U.S. and 58.2 and 57.2 for the U.K. stock markets, all exceedingly greater than the 0.1% critical value of 10.8.

4.2. Switching variance in the permanent component

To test if the switching variance differs across regimes ($\sigma^2_{p^e,1} \neq \sigma^2_{p^e,0}$), I compare the log likelihood values of proposed models 1a and 1b with those of their nested models 2a and 2b, which account for the asymmetry by allowing only inefficient plunges in the transitory component. I corroborate the hypothesis of switching variance by deriving likelihood ratios of 30.2 and 27.0 for the S&P 500 and 38.0 and 35.4 for the FTSE 350, which are all greater than the 0.1% critical value of 10.8. Similarly, the likelihood ratios of 86.2 and 86.4 for the S&P 500 and 79.0 and 74.4 for the FTSE 350, derived by comparing log likelihoods for models 3a and 3b with those reported for symmetric models 4a and 4b, reaffirm the presence of a switching variance in the permanent component. If one compares the likelihood ratios of the two pairwise comparisons stated above, such as comparing 30.2 with 86.2 for the U.S. market or comparing 38.0 with 79.0 for the U.K. market, it is clear that the likelihood ratios for testing the hypothesis of switching variance are lower for the pairwise comparison of models in which inefficient plunges are included. This result identifies inefficient plunges as an alternative for switching variance to explain the asymmetry.

To differentiate two sources of asymmetry (inefficient plunges and switching variance), I compare models 2a and 2b with their competing models 3a and 3b. Although these models are non-nested, comparing their log likelihoods highlights the importance of including both inefficient plunges and a switching variance to characterize stock market dynamics. Conducting this comparison for the S&P 500 hints that models with inefficient plunges perform better than models with a switching variance in maximizing the log likelihood function. Conversely, models with a switching variance outperform models with inefficient plunges for the FTSE 350.

4.3. Time-varying long-run return

To inspect the time-variation in the U.S. and U.K. inflation-adjusted long-run return, I compare the log likelihoods of -2493.6 and -1221.5 for model 1a in Tables 1 and 2 with those values of -2495.1 and -1222.4 for the nested model 1b′, whose result is presented in Tables C1 and C2 in Appendix C. Since the likelihood ratios of 3.0 for the U.S. and 1.8 for the U.K. are not larger than the critical value of 3.84 for a 5% significance level, it appears that a random walk process with a deterministic drift is sufficient to capture the dynamics of the efficient price in these two stock markets.

However, the estimation of $\sigma_\mu = 0.05$ for the standard deviation of shocks to the inflation-adjusted long-run return, with a standard error of 0.02, along with the bottom-left panel of Figure 1, implies a considerable time-variation in the U.S. inflation-adjusted long-run return because it wanders away from its annual average of 4.8% due to long-run episodes of bull and bear markets. Namely, the inflation-adjusted long-run return gradually declined from 10% in the 1950s to -5% in the 1970s and early 1980s, which corresponds to the episode of high Fed fund rates and the Volcker mandate. Then it returned to rates near its average of 4.8% in the late 1980s and early 1990s. Afterward, the return soared to an unreasonable rate of 15% in the late 1990s, resulting from the dot-com bubble. With the exception of the low rates in the aftermath of the 2007–09 financial crisis, the inflation-adjusted long-run return has been between 4% and 10% from the early 2000s until now. For the FTSE 350, the estimated standard deviation of shocks to the long-run return is $\sigma_\mu = 0.03$, with a standard error of 0.02. The bottom-left panel of Figure 2 shows that the inflation-adjusted long-run return of the FTSE 350 continually declined from 15% in the mid-1980s to 6% in the early 2010s and then later to a desperately low rate of 2.5% in the early 2020s. This declining long-term return is attributable to the fact that the U.K. stock market comprises traditional constituents, which are companies with fewer growth opportunities compared to those in the U.S. stock market.

4.4. Robustness tests

Applying alternative models and using the daily and weekly S&P 500 and FTSE 350 indices confirm that the results obtained in this study are robust to changes in model specifications and frequencies. Both the plunging coefficient and switching variance are significant in both markets, regardless of the assumption about the long-run return, the variance of shocks to the transitory component, and the correlations between shocks. For example, I relax the assumption of constant variance for shocks to the transitory component by applying Eq. (8) and running models 1c and 1d, where the asymmetry is accounted for by allowing inefficient plunges in the transitory component as well as two switching variances of shocks to the permanent and transitory components. Comparing the log likelihoods for models 1a and 1b, presented in Tables 1 and 2, with those of models 1c and 1d in Tables C1 and C2 in Appendix C verifies that this variance is not switching. Further, the results of models 1e, 1f, and 1g, each of which allows for one of the correlations between each pair of shocks, suggest that they are all insignificant.

It may be criticized as being too restrictive to impose a single Markov-switching process to account for both inefficient plunges in the transitory component and a switching variance in the permanent component. This assumption, however, seems innocuous considering the evidence given in Figure C1 in Appendix C, which juxtaposes the probability of inefficient plunges implied by model 2a with the probability of high-variance states implied by model 3a for the S&P 500 and FTSE 350. The dynamics of these two probabilities are quite similar, suggesting that inefficient plunges and switches in variance occur concomitantly, although jumps in variance seem more persistent than inefficient plunges.

The results remain unchanged for higher data frequencies. By applying the main models to the daily and weekly S&P 500 and FTSE 350, whose results are presented in Tables C3 and C4 in Appendix C, I confirm the frequency-independence of the main finding. Finally, regarding the order of the autoregressive process in the transitory component, I estimate models with an autoregressive process of order one, which bear almost the same results as those in models with order two. Comparing the log likelihood of model 1a with an autoregressive process of order two to that of its counterpart model with an autoregressive process of order one yields likelihood ratios of 6.6 for the S&P 500 and 0.0 for the FTSE 350. For the sake of coherence, therefore, I use the model with an autoregressive process of order two for both markets.[8]

[8] These results and other results of applying several models to monthly indices with dividends reinvested and monthly nominal indices (not adjusted for inflation) are available upon request.

5. Concluding remarks

I define the new concept of inefficient plunges as negative deviations of the actual market price from the efficient price to measure the level of market inefficiency. To establish regime-dependence and asymmetry in market inefficiency, I present an asymmetric Fads model in which inefficient plunges in the transitory component and a concomitant switching variance in the permanent component are both allowed for by employing a Markov-switching process.

By applying the proposed model to the monthly inflation-adjusted S&P 500 and FTSE 350, I report substantial inefficient plunges in the U.S. and U.K. stock markets. The switching variance of shocks to the permanent component during crisis periods is considerably greater than its value during normal periods in both stock markets. The expected duration is relatively short (4.8 months for the U.S. and 5.9 months for the U.K.) for crisis periods and long (22.7 months for the U.S. and 13.0 months for the U.K.) for normal periods.

Since the estimated inefficient plunges are deep, often negative, steep, and transient, I conclude that market inefficiency is regime-dependent and asymmetric, meaning that although the U.S. and U.K. stock markets are efficient during normal times, they are far below efficient prices during crises. These results support the AMH against the EMH and also suggest the possibility of negative bubbles.

Concerning limitations, this study does not identify a specific driver for market inefficiency, yet in diagnosing inefficient plunges as a symptom, it hints at three potential competing drivers: market irrationality, market unawareness, and financial frictions that are proposed by a few studies explained briefly in the literature review. In addition, since this study aims to examine the influence of negative deviations on market inefficiency, it remains silent about potential market exuberance during periods of speculative bubbles, when the market price positively deviates from its fundamental. However, given that speculative bubbles are rationalized in the Rational Bubbles model, positive deviations do not appear to be inconsistent with efficiency. Lastly, the proposed model can be applied to examine the asymmetry in a variety of financial markets, such as stocks, futures, options, and currencies. Also, developing a model to incorporate both positive and negative bubbles is a worthy proposal for future research.

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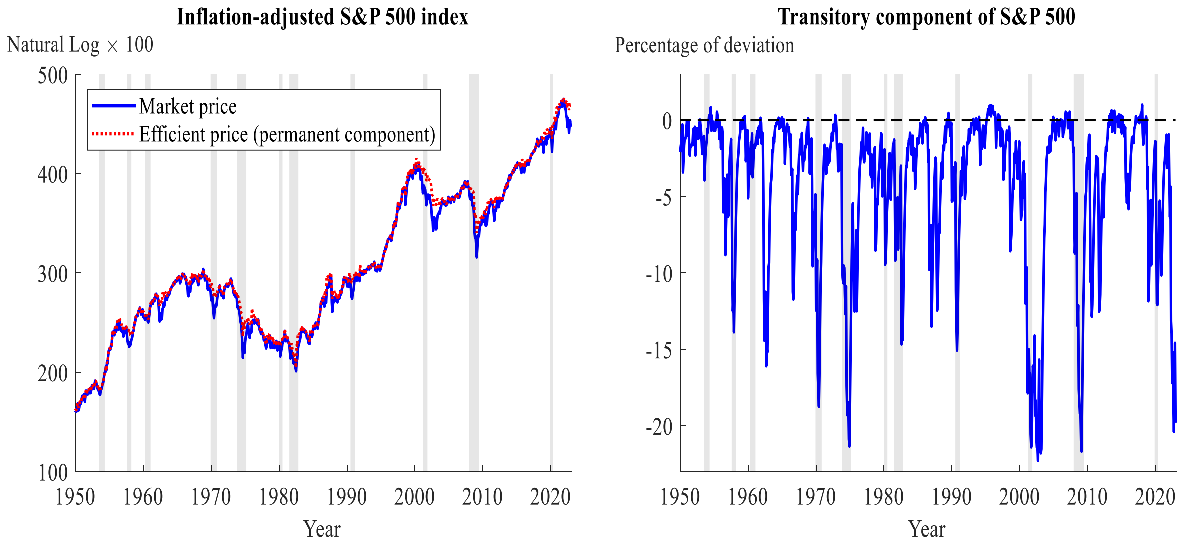
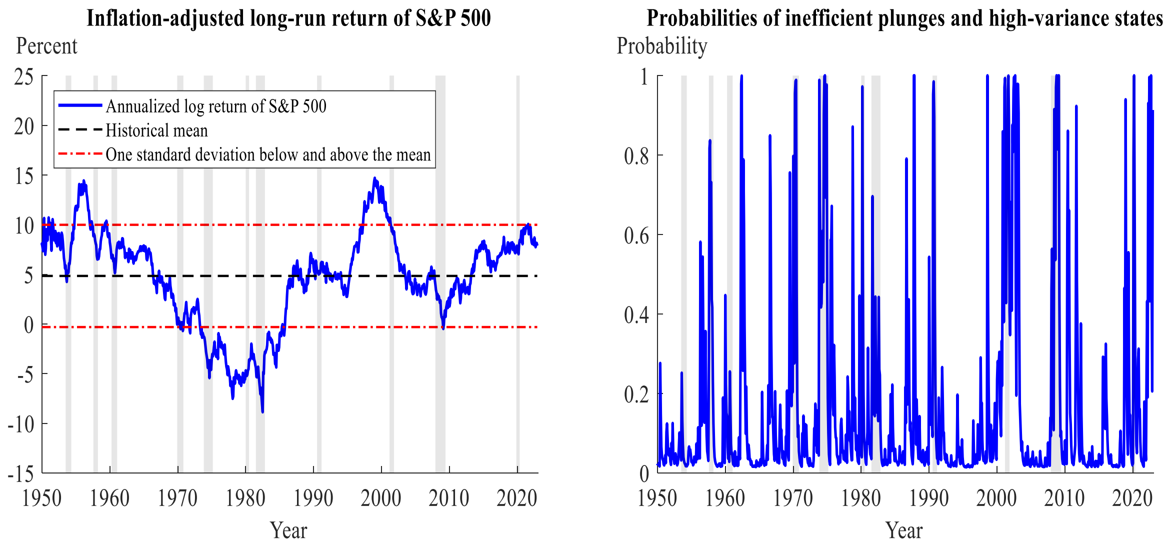
Figures



(a) Estimated efficient price (left) and inefficient plunges (right)



(b) Long-run return (left) and probabilities of asymmetric deviations (right)

Figure 1: The results of the asymmetric Fads model for the S&P 500

Notes:

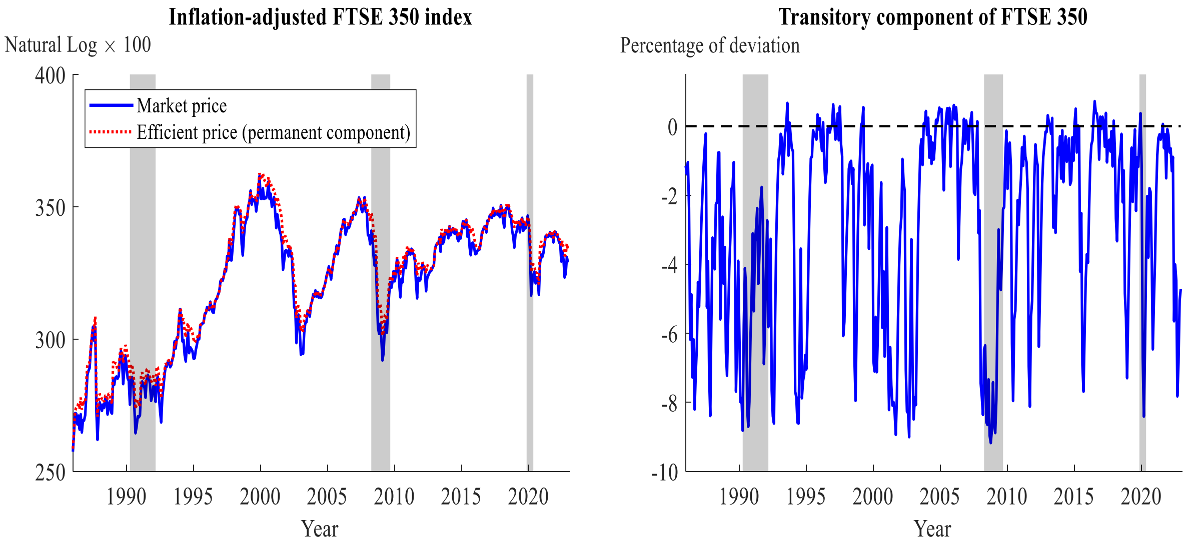
(1) All panels plot the results of model 1a with the descriptor A (IP-SV)-RW.

(2) The top panels plot permanent and transitory components.

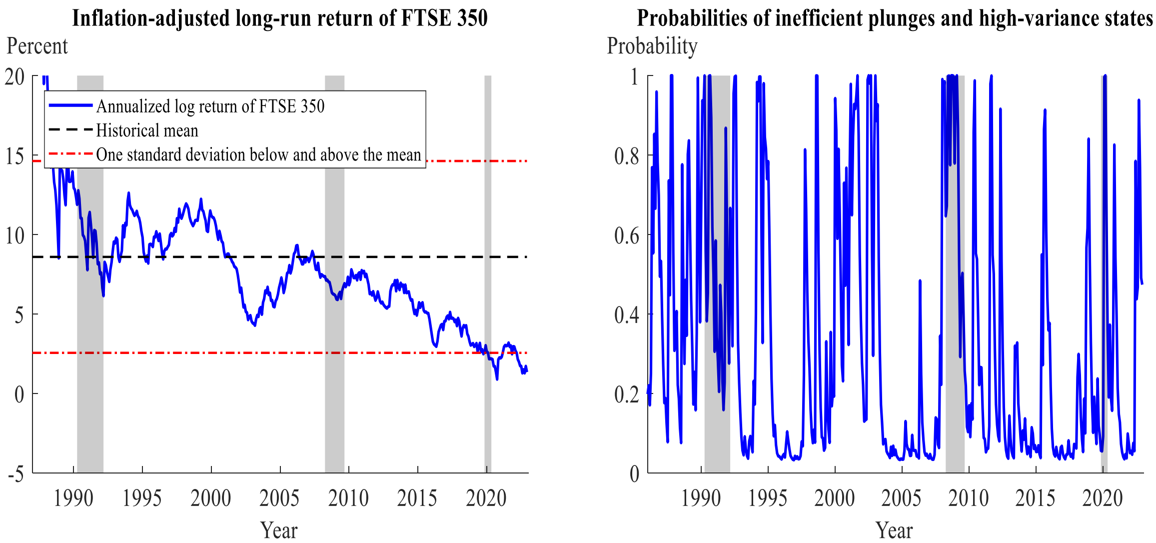
(3) The bottom-left panel plots inflation-adjusted long-run return (trend growth of price).

(4) The bottom-right panel plots the plunging probabilities.

(5) The shaded areas are the NBER recession dates. See Table D1 in Appendix D for details.



(a) Estimated efficient price (left) and inefficient plunges (right)



(b) Long-run return (left) and probabilities of asymmetric deviations (right)

Figure 2: The results of the asymmetric Fads model for the FTSE 350

Notes:

(1) All panels plot the results of model 1a with the descriptor A (IP-SV)-RW.

(2) The top panels plot permanent and transitory components.

(3) The bottom-left panel plots inflation-adjusted long-run return (trend growth of price).

(4) The bottom-right panel plots the plunging probabilities.

(5) The shaded areas are the ECRI recession dates. See Table D2 in Appendix D for details.

Tables

Table 1: Estimated parameters of different models for the S&P 500

Models	1a	1b	2a	2b	3a	3b	4a	4b
Parameters	A (IP-SV)-RW	A (IP-SV)-Con	A (IP)-RW	A (IP)-Con	A (SV)-RW	A (SV)-Con	S-RW	S-Con
$\sigma_{p^e,0}$	2.40 (0.41)	2.71 (0.30)	3.44 (0.10)	3.43 (0.09)	2.95 (0.40)	2.91 (0.29)	4.26 (0.10)	4.25 (0.11)
$\sigma_{p^e,1}$	5.68 (0.46)	5.86 (0.43)	–	–	5.53 (0.37)	5.52 (0.36)	–	–
$\sigma_{p^i,0}$	1.81 (0.44)	1.42 (0.43)	0.03 (0.85i)	0.17 (0.85i)	0.15 (5.94)	0.42 (1.33)	0.09 (0.86)	0.14 (0.91)
$\sigma_{p^i,1}$	–	–	–	–	–	–	–	–
σ_μ	0.05 (0.02)	–	0.00 (0.01)	–	0.00 (0.01)	–	0.00 (0.01)	–
μ	T-V	0.42 (0.11)	T-V	0.37 (0.12)	T-V	0.54 (0.13)	T-V	0.33 (0.14)
φ_1	0.56 (0.09)	0.52 (0.09)	0.68 (0.06)	0.68 (0.06)	0.60 (1.13)	0.61 (0.42)	0.62 (4.17)	0.62 (2.31)
φ_2	0.16 (0.07)	0.19 (0.08)	0.10 (0.06)	0.10 (0.06)	0.12 (3.21)	0.14 (0.51)	0.10 (4.12)	0.12 (2.01)
π_i	-7.07 (1.04)	-6.93 (1.03)	-10.42 (0.67)	-10.41 (0.67)	–	–	–	–
p	0.79 (0.07)	0.80 (0.07)	0.48 (0.08)	0.48 (0.08)	0.95 (0.02)	0.95 (0.02)	–	–
q	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	–	–
Log likelihood	-2493.6	-2492.5	-2508.7	-2506.0	-2517.3	-2514.7	-2560.4	-2557.9

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) The standard errors of the estimated parameters are in parenthesis. Those with the letter i for models 2a and 2b are imaginary numbers.
(c) Numerical values for parameters denoted by 0.00 are respectively 0.000005 for model 2a, 0.0007 for model 3a, and 0.0007 for model 4a.

Notes:

(1) The estimation period runs from 1948M1 to 2022M12. I estimate 16 models, each of which is denoted by an identifier and a descriptor. The descriptor consists of two parts. The first part expresses the specification of the model regarding the asymmetry, and the second part determines the specification of the long-run return. For example, the descriptor of model 1a is denoted by "A (IP-SV)-RW", which means this model allows for the **A**symmetry by including both **I**nefficient **P**lunges in the transitory component and a **S**witching **V**ariance in the permanent component. In this model, the long-run return is specified in the form of a **R**andom **W**alk process. See the first paragraph of Section 4 and Table A in Appendix A for further explanation.

(2) To test for asymmetry in the transitory component, I compare the log likelihood values for models 1a and 1b, in which the asymmetry is accounted for by including both inefficient plunges in the transitory component and a switching variance in the permanent component, with those values for models 3a and 3b, in which the asymmetry is accounted for by including only a switching variance in the permanent component. A pairwise comparison of the log likelihoods of -2493.6 and -2492.5 reported for models 1a and 1b with values of -2517.3 and -2515.7 for models 3a and 3b, respectively, favours the asymmetric Fads models over asymmetric variance models. The corresponding likelihood ratios of 47.4 and 44.4 are substantially greater than the critical value of 10.8 for a 0.1% significance level.

(3) Additionally, comparing the log likelihoods for asymmetric models 1a, 2a, and 3a with the log likelihood of the symmetric model 4a supports the presence of the asymmetry in the form of both inefficient plunges in the transitory component and a switching variance in the permanent component. The corresponding likelihood ratios of 133.6, 103.4, and 86.2 are substantially greater than the critical value of 10.8 for a 0.1% significance level.

(4) I also assessed the asymmetry by comparing two competing models. Models 2a and 2b allow for the asymmetry by including only inefficient plunges in the transitory component, while models 3a and 3b allow for the asymmetry by including only a switching variance in the permanent component. Although these models are non-nested, by comparing the log likelihood values of -2508.7 and -2506.0 reported for models 2a and 2b with values of -2517.3 and -2514.7 for models 3a and 3b, I highlight the importance of including a switching variance in addition to the inefficient plunges.

(5) $\sigma_{p^i,1}$ is estimated in model 1c, where I relax the assumption of constant variance for shocks to the transitory component. The result, presented in Table C1 in Appendix C, shows that this variance does not switch when the inefficient plunges are included.

Table 2: Estimated parameters of different models for the FTSE 350

Models	1a	1b	2a	2b	3a	3b	4a	4b
Parameters	A (IP-SV)-RW	A (IP-SV)-Con	A (IP)-RW	A (IP)-Con	A (SV)-RW	A (SV)-Con	S-RW	S-Con
$\sigma_{p^e,0}$	1.78 (0.37)	1.89 (0.37)	3.59 (0.16)	2.89 (0.45)	0.05 (1.32)	1.50 (0.99)	3.92 (0.41)	0.03 (1.01)
$\sigma_{p^e,1}$	5.73 (0.48)	5.65 (0.48)	–	–	5.48 (0.40)	5.63 (0.45)	–	–
$\sigma_{p^i,0}$	1.55 (0.28)	1.53 (0.27)	0.06 (0.76i)	2.05 (0.61)	2.17 (0.17)	1.50 (1.25)	1.93 (0.61)	4.40 (0.15)
$\sigma_{p^i,1}$	–	–	–	–	–	–	–	–
σ_μ	0.03 (0.02)	–	0.00 (0.01)	–	0.02 (0.01)	–	0.00 (0.04)	–
μ	T-V	0.39 (0.16)	T-V	0.14 (0.14)	T-V	0.61 (0.14)	T-V	0.15 (0.05)
φ_1	0.33 (0.11)	0.35 (0.11)	0.74 (0.10)	0.73 (0.09)	0.69 (0.10)	0.43 (0.99)	0.93 (0.04)	1.03 (0.05)
φ_2	0.01 (0.10)	0.02 (0.11)	0.07 (0.09)	0.11 (0.07)	0.17 (0.09)	0.21 (0.37)	-0.19 (0.12)	-0.06 (0.05)
π_i	-5.81 (1.27)	-6.30 (1.33)	-11.19 (1.13)	-11.35 (1.04)	–	–	–	–
p	0.83 (0.07)	0.83 (0.07)	0.44 (0.13)	0.49 (0.13)	0.92 (0.04)	0.92 (0.04)	–	–
q	0.92 (0.03)	0.93 (0.03)	0.96 (0.01)	0.97 (0.01)	0.92 (0.03)	0.92 (0.03)	–	–
Log likelihood	-1221.5	-1218.9	-1240.5	-1236.6	-1230.1	-1228.0	-1269.6	-1265.2

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) The standard errors of the estimated parameters are in parenthesis. That with the letter i for model 2a is an imaginary number.
(c) Numerical values for parameters denoted by 0.00 are respectively 0.00000002 for model 2a, and 0.004 for model 4a.

Notes:

(1) The estimation period runs from 1986M1 to 2022M12. I estimate 16 models, each of which is denoted by an identifier and a descriptor. The descriptor consists of two parts. The first part expresses the specification of the model regarding the asymmetry, and the second part determines the specification of the long-run return. For example, the descriptor of model 1a is denoted by "A (IP-SV)-RW", which means this model allows for the **A**symmetry by including both **I**nefficient **P**lunges in the transitory component and a **S**witching **V**ariance in the permanent component. In this model, the long-run return is specified in the form of a **R**andom **W**alk process. See the first paragraph of Section 4 and Table A in Appendix A for further explanation.

(2) To test for asymmetry in the transitory component, I compare the log likelihood values for models 1a and 1b, in which the asymmetry is accounted for by including both inefficient plunges in the transitory component and a switching variance in the permanent component, with those values for models 3a and 3b, where the asymmetry is accounted for by including only a switching variance in the permanent component. A pairwise comparison of the log likelihoods of -1221.5 and -1218.9 reported for models 1a and 1b with values of -1230.1 and -1228.0 for models 3a and 3b, respectively, strongly favours the asymmetric Fads models over asymmetric variance models. The corresponding likelihood ratios of 17.2 and 18.2 are substantially greater than the critical value of 10.8 for a 0.1% significance level.

(3) Additionally, comparing the log likelihoods for asymmetric models 1a, 2a, and 3a with the log likelihood of the symmetric model 4a supports the presence of the asymmetry in the form of both inefficient plunges in the transitory component and a switching variance in the permanent component. The corresponding likelihood ratios of 96.2, 58.2, and 79.0 are all substantially greater than the critical value of 10.8 for a 0.1% significance level.

(4) I also assessed the asymmetry by comparing two competing models. Models 2a and 2b allow for the asymmetry by including only inefficient plunges in the transitory component, while models 3a and 3b allow for the asymmetry by including only a switching variance in the permanent component. Although these models are non-nested, by comparing the log likelihood values of -1240.5 and -1236.6 reported for models 2a and 2b with values of -1230.1 and -1228.0 for models 3a and 3b, I highlight the importance of including a switching variance in addition to the inefficient plunges.

(5) $\sigma_{p^i,1}$ is estimated in model 1c, where I relax the assumption of constant variance for shocks to the transitory component. The result, presented in Table C2 in Appendix C, shows that this variance does not switch when the inefficient plunges are included.

Supplementary Appendix to

Asymmetric Fads and Inefficient Plunges: Evaluating Adaptive vs. Efficient Market Hypotheses

Mohammad Dehghani[†,*]

Appendix A: Univariate state-space model with Markov-switching

For estimation, I cast the asymmetric model specified in Eq. (1) to Eq. (9) into a state-space form. The observation equation, the transition equation, and variance covariance matrix of error terms are:

$$[p_t] = [1 \quad 1 \quad 0 \quad 0]\begin{bmatrix} p_t^e \\ p_t^i \\ p_{t-1}^i \\ \mu_t \end{bmatrix} + [0] \tag{A.1}$$

$$\begin{bmatrix} p_t^e \\ p_t^i \\ p_{t-1}^i \\ \mu_t \end{bmatrix} = \begin{bmatrix} 0 \\ \pi_i S_t \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \varphi_1 & \varphi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} p_{t-1}^e \\ p_{t-1}^i \\ p_{t-2}^i \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{p^e,t} \\ \varepsilon_{p^i,t} \\ 0 \\ \varepsilon_{\mu,t} \end{bmatrix} \tag{A.2}$$

$$\begin{bmatrix} \varepsilon_{p^e,t} \\ \varepsilon_{p^i,t} \\ 0 \\ \varepsilon_{\mu,t} \end{bmatrix} \sim N(\mathbf{0}_{4\times1}, \begin{bmatrix} \sigma_{p^e}^2 & 0 & 0 & 0 \\ 0 & \sigma_{p^i}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\mu^2 \end{bmatrix} \tag{A.3.a}$$

In the above model, I consider the natural log price multiplied by 100 as the observed series (p_t). In this setup, I take a stochastic long-run return (μ_t) that evolves according to a random walk process. To test for asymmetry, I derive the symmetric model by imposing $\pi_i = 0$ on the unrestricted model to compare the log likelihoods of the nested and non-nested models. In Eq. (A.3.a), all shocks are assumed to be uncorrelated. However, as described in Eq. (A.3.b) for robustness tests, I allow for correlation between each pair of shocks:

$$\begin{bmatrix} \varepsilon_{p^e,t} \\ \varepsilon_{p^i,t} \\ 0 \\ \varepsilon_{\mu,t} \end{bmatrix} \sim N(\mathbf{0}_{4\times1}, \begin{bmatrix} \sigma_{p^e}^2 & \rho_{p^e,p^i}\sigma_{p^e}\sigma_{p^i} & 0 & \rho_{p^e,\mu}\sigma_{p^e}\sigma_\mu \\ \rho_{p^e,p^i}\sigma_{p^e}\sigma_{p^i} & \sigma_{p^i}^2 & 0 & \rho_{p^i,\mu}\sigma_{p^i}\sigma_\mu \\ 0 & 0 & 0 & 0 \\ \rho_{p^e,\mu}\sigma_{p^e}\sigma_\mu & \rho_{p^i,\mu}\sigma_{p^i}\sigma_\mu & 0 & \sigma_\mu^2 \end{bmatrix} \tag{A.3.b}$$

where ρ_{p^e,p^i} stands for the correlation between shocks to the permanent and transitory components; $\rho_{p^e,\mu}$ stands for the correlation between shocks to the permanent component and the long-run return; and $\rho_{p^i,\mu}$ denotes the correlation between shocks to the transitory component and the long-run return.

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* Corresponding author. See the website: https://sites.google.com/view/mohammaddehghani for data and code. For details about the method and parameter constraints, see the comments in the MATLAB code.

Table A: Specification of 16 models for stock market index

Model name	Tables and Figures
Model 1a: Asymmetric (IP-SV)-RW	Tables 1, 2, Figures 1, 2
Model 1b: Asymmetric (IP-SV)-Con (3 states variables)	Tables 1, 2
Model 1b′: Asymmetric (IP-SV)-Con (4 state variables)	Tables C1, C2
Model 1c: Asymmetric (IP-SVPT)-RW	Tables C1, C2
Model 1d: Asymmetric (IP-SVPT)-Con	Tables C1, C2
Model 1e: Asymmetric (IP-SVPT)-RW-Corr1	Tables C1, C2
Model 1f: Asymmetric (IP-SVPT)-RW-Corr2	Tables C1, C2
Model 1g: Asymmetric (IP-SVPT)-RW-Corr3	Tables C1, C2
Model 2a: Asymmetric (IP)-RW	Tables 1, 2, Figure C1
Model 2b: Asymmetric (IP)-Con	Tables 1, 2
Model 3a: Asymmetric (SV)-RW	Tables 1, 2, Figure C1
Model 3b: Asymmetric (SV)-Con	Tables 1, 2
Model 3c: Asymmetric (SVPT)-RW	Tables C1, C2
Model 3d: Asymmetric (SVPT)-Con	Tables C1, C2
Model 4a: Symmetric-RW	Tables 1, 2
Model 4b: Symmetric-Con	Tables 1, 2

Notes:

(1) I estimate sixteen models. I denote each model with an identifier and a descriptor. The descriptor consists of two major parts. The first part expresses the specification of the model regarding the asymmetry in the transitory component as well as the asymmetry of the variance of shocks in the permanent component. The second part states whether the drift term (long-run return) is specified as a random walk process or a constant. For example, the identifier of the main model is 1a, and its descriptor is "A (IP-SV)-RW", which means that this model accounts for the **A**symmetry by including both **I**nefficient **P**lunges in the transitory component and a **S**witching **V**ariance in the permanent component. The term "RW" also hints that the long-run return is specified as a **R**andom **W**alk.

(2) The last part of correlated models 1e, 1f, and 1g shows which non-zero correlation is allowed for. For example, model 1e, which is denoted by "A (IP-SV)-RW-Corr1", means that the model allows for the correlation between shocks to the permanent and transitory components (ρ_{p^e,p^i}). Similarly, model 1f allows for the correlation between shocks to the permanent component and the long-run return ($\rho_{p^e,\mu}$), and model 1g permits the correlation between shocks to the transitory component and the long-run return ($\rho_{p^i,\mu}$).

(3) We present the results of the bold models applied to the S&P 500 and FTSE 350 in Tables 1 and 2, and the rest are presented in Tables C1 and C2 in Appendix C.

(4) The proposed model in this study is similar to the model presented by Kim and Nelson (1999a), which was applied to the U.S. GDP in the economics literature.

(5) The proposed model in this study is closest to the models presented by Turner et al. (1989), Kim and Kim (1996), and Liu et al. (2012), but with two important distinctions. First, while my model is suitable to capture the asymmetric deviations from efficiency since it is specified at log levels, their models are not designed to explore the EMH since they are specified in differences. Second, Turner et al. (1989) and Liu et al. (2012) exclude the Fads (transitory) component, and Kim and Kim (1996) do not account for the asymmetry in the Fads component. These shortcomings are addressed in my model by incorporating both inefficient plunges in the transitory component and a switching variance in the permanent component.

(6) The conventional Fads model is similar to symmetric models 4a and 4b.

Appendix B: Approximate maximum likelihood and constraints

For asymmetric models in the presence of the Markov-switching process of Hamilton (1989), I use Kim's (1994) approximate maximum likelihood method to make the Kalman's (1960) filter operable. For more explanation, see chapters 4 and 5 of Kim and Nelson (1999b) and chapters 13 and 22 of Hamilton (1994). For symmetric models, I use the maximum likelihood method, performed by using the Kalman filter as explained in chapters 2 and 3 of Kim and Nelson (1999b) and chapter 5 of Hamilton (1994).

I need to impose a set of constraints on parameters, which are explained thoroughly in the second part of Appendix B. I consider a set of initial values for parameters as well as state variables. For the former, the initial values for parameters are presented in Tables B1 and B2 in Appendix B. For the latter, I select the first observation for the permanent component, zero for the transitory component, and 4.8% for the annual long-run return to determine the prior values for the corresponding state variables. The prior variances of state variables are set to be 10. The results are robust to changes in the prior values of state variables and their variances. For example, I used a wilder guess by setting the variances of state variables equal to 1000 and could find the same estimation for parameters.

Parameters constraints

I employ a numerical optimization procedure to maximize the approximate log likelihood function subject to a set of constraints. Hence, I impose constraints on some of the coefficients, probabilities, and standard deviations of shocks. To this end, I account for constraints by using a transformation function, $T(\omega)$, which transforms a vector of unconstrained parameters $\omega = [\omega_1, \ldots, \omega_{10}]'$ to a vector of constrained parameters $\Omega = [\Omega_1, \ldots, \Omega_{10}]'$ presented below:

$$\Omega = [\sigma_{p^e,0}, \sigma_{p^e,1}, \sigma_{p^i}, \sigma_\mu, \mu, \varphi_1, \varphi_2, \pi_i, p, q]' \tag{B.1}$$

where $\Omega = T(\omega)$ is the vector containing parameters and $T(\omega)$ is a vector function, whose elements are transformation functions $T_i(\omega)$ for $i = 1, \ldots, 10$. Since performing unconstrained optimization with respect to ω is equivalent to performing constrained optimization with respect to Ω, I adopt an unconstrained optimization with respect to the vector ω, where the objective (approximate log likelihood) function is considered as a function of the transformation function. I define each element of the transformation function as follows.

First, for coefficients and standard deviations of shocks that must be positive, I use an exponential transformation suggested by Kim and Nelson (1999b). For example,

$$\sigma_{p^e,0} = exp(\omega_1) \tag{B.2}$$

In Eq. (B.2), $\sigma_{p^e,0}$ is the standard deviation (square root of variance) of shocks to the permanent component during normal times, which must be positive. Similarly, for other standard deviations, including $\sigma_{p^e,1}$, σ_{p^i}, and σ_μ that are positive and for the coefficient π_i that is expected to be negative, I apply an exponential transformation. For example, $\pi_i = -exp(\omega_{12})$ ensures a negative plunging coefficient.

Second, to have transition probabilities in the [0 1] interval, we exert the following transformations:

$$p = \frac{exp(\omega_{13})}{1 + exp(\omega_{13})} \quad \text{and} \quad q = \frac{exp(\omega_{14})}{1 + exp(\omega_{14})} \tag{B.3}$$

Third, for coefficients of the autoregressive process of order two, I need to set the values of φ_1 and φ_2 within the stationary region that means the roots of the lag polynomial $(1 - \varphi_1 L - \varphi_2 L^2 = 0)$ must lie outside the unit circle. In this sense, I apply the transformations proposed by Morley et al. (2003):

$$\varphi_1 = 2\kappa_1 \quad \text{and} \quad \varphi_2 = -(\kappa_1^2 + \kappa_2) \tag{B.4.a}$$

where κ_1 and κ_2 are determined as follows:

$$\kappa_1 = \frac{\omega_{10}}{1 + |\omega_{10}|} \quad \text{and} \quad \kappa_2 = \frac{(1 - |\kappa_1|) \times \omega_{11}}{1 + |\omega_{11}|} + |\kappa_1| - \kappa_1^2 \tag{B.5.a}$$

For these two coefficients of the autoregressive process, one can take two alternative transformations proposed by Kim and Nelson (1999b):

$$\varphi_1 = \kappa_1 + \kappa_2 \quad \text{and} \quad \varphi_2 = \kappa_1 \times \kappa_2 \tag{B.4.b}$$

where κ_1 and κ_2 are determined below:

$$\kappa_1 = \frac{\omega_{10}}{1 + |\omega_{10}|} \quad \text{and} \quad \kappa_2 = \frac{\omega_{11}}{1 + |\omega_{11}|} \tag{B.5.b}$$

However, these two transformations impose a further restriction that the roots of the autoregressive polynomial are real numbers. Hence, I apply Eq. (B.4.a) and Eq. (B.5.a) as restrictions on coefficients of the autoregressive process of order two.

Fourth, for correlation coefficients, I consider Eq. (B.6):

$$\rho_{p^e,p^i} = \frac{\omega_{18}}{1 + |\omega_{18}|} \tag{B.6}$$

where ρ_{p^e,p^i} is the correlation between shocks and clearly satisfies the condition $-1 < \rho_{p^e,p^i} < 1$.

It is worth noting that inefficient plunges (negative bubbles) do exist no matter whether the constraint on the plunging coefficient is imposed or not. Indeed, the phenomenon of inefficient plunges is such a pronounced feature of the U.S. and U.K. stock markets that excluding its corresponding constraints (π_i<0) does not change the estimated parameters.

Appendix B: Tables of initial values for parameters

Table B1: Initial values (after-transformation) for the model parameters used for the S&P 500

Models	1a	1b	2a	2b	3a	3b	4a	4b
Parameters	A (IP-SV)-RW	A (IP-SV)-Con	A (IP)-RW	A (IP)-Con	A (SV)-RW	A (SV)-Con	S-RW	S-Con
$\sigma_{p^e,0}$	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
$\sigma_{p^e,1}$	1.50	1.50	–	–	1.50	1.50	–	–
$\sigma_{p^i,0}$	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
$\sigma_{p^i,1}$	–	–	–	–	–	–	–	–
σ_{μ}	0.75	–	0.75	–	0.75	–	0.75	–
μ	T-V	0.75	T-V	0.75	T-V	0.75	T-V	0.75
φ_1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
φ_2	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4
π_i	-1.8	-1.8	-1.8	-1.8	–	–	–	–
p	0.70	0.70	0.70	0.70	0.70	0.70	–	–
q	0.90	0.90	0.90	0.90	0.90	0.90	–	–

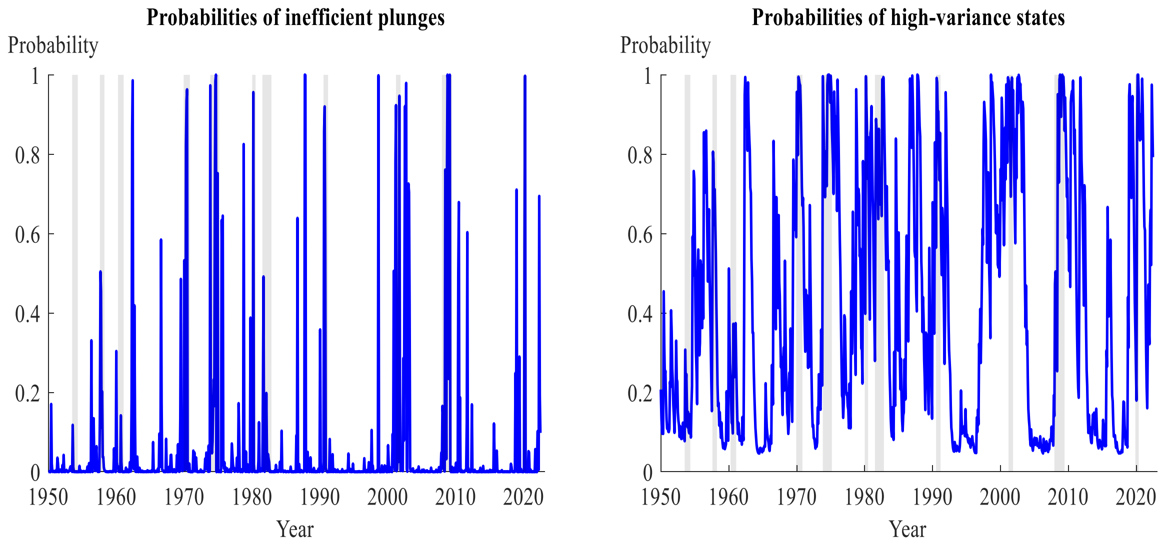
Notes:
(1) The results of all models are robust to the choice of the initial values for each parameter.
(2) I use the same initial values for almost all models. For models 1a, 1b, 3a, and 3b, I select higher initial values for variances during crises (1.50^2).

Table B2: Initial values (after-transformation) for the model parameters used for the FTSE 350

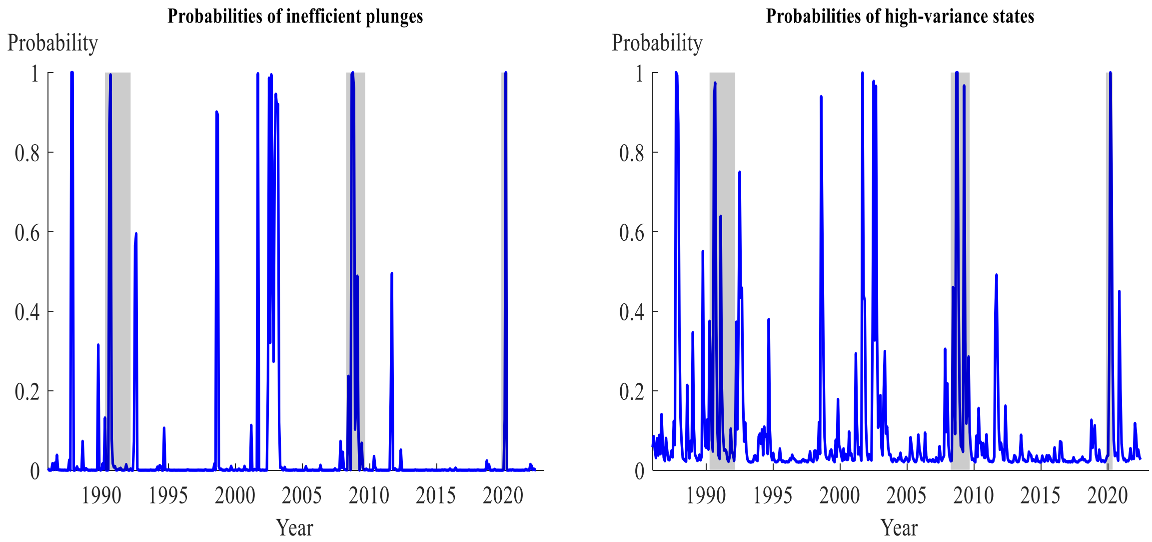
Models	1a	1b	2a	2b	3a	3b	4a	4b
Parameters	A (IP-SV)-RW	A (IP-SV)-Con	A (IP)-RW	A (IP)-Con	A (SV)-RW	A (SV)-Con	S-RW	S-Con
$\sigma_{p^e,0}$	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
$\sigma_{p^e,1}$	1.50	1.50	–	–	1.50	1.50	–	–
$\sigma_{p^i,0}$	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
$\sigma_{p^i,1}$	–	–	–	–	–	–	–	–
σ_{μ}	0.75	–	0.75	–	0.75	–	0.75	–
μ	T-V	0.75	T-V	0.75	T-V	0.75	T-V	0.75
φ_1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
φ_2	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4
π_i	-1.8	-1.8	-1.8	-1.8	–	–	–	–
p	0.70	0.70	0.70	0.70	0.70	0.70	–	–
q	0.90	0.90	0.90	0.90	0.90	0.90	–	–

Notes:
(1) The results of all models are robust to the choice of the initial values for each parameter.
(2) I use the same initial values for all models that are the same as the initial values for the S&P 500. For models 1a, 1b, 3a, and 3b, I select higher initial values for variances during crises (1.50^2).

Appendix C: Additional figures



(a) Probabilities estimated separately for model 2a in the left and for model 3a in the right (S&P 500)



(b) Probabilities estimated separately for model 2a in the left and for model 3a in the right (FTSE 350)

Figure C1: Synchrony of probabilities of inefficient plunges and high variance states

Notes:

(1) The top-left and bottom-left panels plot probabilities of asymmetric deviations in model 2a, which allows for only inefficient plunges in the transitory component.

(2) The top-right and bottom-right panels plot probabilities of asymmetric variance in model 3a, which allows for only switching variance in the permanent component.

(3) The shaded areas are the NBER and ECRI recession dates in the top and bottom panels, respectively. See Tables D1 and D2 in Appendix D for details.

Appendix C: Additional tables

Table C1 (Continue of Table 1): Estimated parameters of different models for the S&P 500

Models	1b′	1c	1d	1e	1f	1g	3c	3d
Parameters	A (IP-SV)-Con	A (IP-SVPT)-RW	A (IP-SVPT)-Con	A (IP-SV)-RW-C1	A (IP-SV)-RW-C2	A (IP-SV)-RW-C3	A (SVPT)-RW	A (SVPT)-Con
$\sigma_{p^e,0}$	2.72 (0.30)	2.44 (0.40)	2.72 (0.30)	2.98 (0.61)	2.41 (0.42)	2.46 (0.30)	2.85 (0.32)	2.89 (0.70)
$\sigma_{p^e,1}$	5.88 (0.43)	5.18 (1.02)	5.71 (0.87)	2.83 (2.06)	5.69 (0.45)	5.70 (0.44)	5.53 (0.35)	5.53 (0.34)
$\sigma_{p^i,0}$	1.42 (0.43)	1.78 (0.43)	1.41 (0.43)	3.54 (1.86)	1.82 (0.44)	1.77 (0.37)	0.68 (0.88)	0.50 (3.18)
$\sigma_{p^i,1}$	–	2.84 (1.58)	1.88 (2.09)	–	–	–	0.02 (0.95)	0.24 (1.02)
σ_μ	–	0.05 (0.03)	–	0.06 (0.03)	0.05 (0.02)	0.05 (0.02)	0.00 (0.01)	–
μ	–	T-V	0.41 (0.11)	T-V	T-V	T-V	T-V	0.54 (0.14)
φ_1	0.53 (0.09)	0.58 (0.09)	0.53 (0.10)	0.65 (0.10)	0.57 (0.09)	0.56 (0.08)	0.53 (0.36)	0.56 (0.48)
φ_2	0.19 (0.08)	0.16 (0.06)	0.19 (0.08)	0.12 (0.06)	0.16 (0.07)	0.17 (0.07)	0.16 (0.28)	0.18 (0.46)
π_i	-6.96 (1.04)	-6.91 (1.07)	-6.85 (1.10)	-7.00 (1.07)	-7.10 (1.05)	-7.08 (1.04)	–	–
p	0.80 (0.06)	0.78 (0.08)	0.79 (0.07)	0.73 (0.08)	0.80 (0.06)	0.79 (0.06)	0.95 (0.02)	0.95 (0.02)
q	0.96 (0.01)	0.95 (0.01)	0.95 (0.01)	0.95 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)
ρ_{p^e,p^i}	–	–	–	-0.63 (0.39) 0.71 (0.45i)	–	–	–	–
$\rho_{p^e,\mu}$	–	–	–	–	0.49 (1.50) 0.42 (1.56i)	–	–	–
$\rho_{p^i,\mu}$	–	–	–	–	–	0.50 (2.18i)	–	–
Log likelihood	-2495.1	-2493.5	-2492.5	-2493.1	-2493.6	-2493.6	-2517.2	-2514.7

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) The standard errors of the parameters are reported in parenthesis. Those with the letter i for models 1e, 1f, and 1g are imaginary numbers.
(c) The numerical value for the parameter denoted by 0.00 is 0.00006 for model 3c.

Notes:

(1) The estimation period runs from 1948M1 to 2022M12. See Table 1 for the main results and explanations.

(2) Model 1b′ is another version of model 1b with similar estimations of parameters. Since in the former, we treat the drift term (constant long-run return) as a state variable and in the latter, the drift term is estimated as a parameter, model 1b′ is fully nested in model 1a, but model 1b is not. Comparing log likelihood values of -2493.6 and -2495.1 suggests that a random walk with a deterministic drift is sufficient to capture the dynamics of the efficient price.

(3) The term "SVPT" for models 1c and 1d says that the **S**witching **V**ariance is allowed for two variances, one for shocks to the **P**ermanent component and another for shocks to the **T**ransitory component. Hence, models 1c and 1d account for the asymmetry by including inefficient plunges in the transitory component, one switching variance in the permanent component, and another switching variance in the transitory component. For these two models, I relax the assumption of constant variance for shocks to the transitory component by including a switching variance for these shocks based on Eq. (8). The result shows that the variance of shocks to the transitory component is not switching. In particular, comparing the log likelihoods of -2493.6 and -2492.5 for models 1a and 1b in Table 1 with the values of -2493.5 and -2492.5 for models 1c and 1d bears likelihood ratio values of 0.2 and 0.0, confirming that the variance of shocks to the transitory component is not switching.

(4) To test for asymmetry in the form of inefficient plunges, I compare the log likelihood values for models 1c and 1d, in which the asymmetry is accounted for by including both inefficient plunges in the transitory component and two switching variances for shocks to the permanent and transitory components, with those values for models 3c and 3d, in which the asymmetry is accommodated only by including two switching variances for shocks to the permanent and transitory components. A pairwise comparison of the log likelihoods of -2493.5 and -2492.5 reported for models 1c and 1d with values of -2517.2 and -2514.7 for models 3c and 3d, respectively, favours the asymmetric Fads models over asymmetric variance models. The corresponding likelihood ratios of 47.0 and 43.8 are substantially greater than the critical value of 10.8 for a 0.1% significance level.

(5) Comparing the log likelihood of -2493.6 for the proposed model in column 1a of Table 1 with values of -2493.1, -2493.6, and -2493.6 for correlated models 1e, 1f, and 1g ensures that the correlations between shocks (ρ_{p^e,p^i}, $\rho_{p^e,\mu}$, and $\rho_{p^i,\mu}$) are all negligible as the corresponding likelihood ratios are 1.0, 0.0, and 0.0.

Table C2 (Continue of Table 2): Estimated parameters of different models for the FTSE 350

Models	1b′	1c	1d	1e	1f	1g	3c	3d
Parameters	A (IP-SV)-Con	A (IP-SVPT)-RW	A (IP-SVPT)-Con	A (IP-SV)-RW-C1	A (IP-SV)-RW-C2	A (IP-SV)-RW-C3	A (SVPT)-RW	A (SVPT)-Con
$\sigma_{p^e,0}$	1.96 (0.36)	1.84 (0.35)	2.00 (0.33)	1.79 (0.36)	1.77 (0.36)	1.80 (0.06i)	0.05 (1.17)	1.36 (0.53)
$\sigma_{p^e,1}$	5.73 (0.48)	5.96 (0.47)	5.89 (0.49)	6.47 (0.93)	6.70 (0.47)	5.74 (0.46)	5.85 (0.37)	5.79 (0.36)
$\sigma_{p^i,0}$	1.46 (0.28)	1.58 (0.25)	1.54 (0.26)	0.99 (1.54)	1.53 (0.28)	1.54 (0.25)	2.19 (0.16)	1.66 (0.49)
$\sigma_{p^i,1}$	–	0.03 (2.09)	0.16 (2.66)	–	–	–	0.00 (0.82)	0.00 (0.30)
σ_μ	–	0.03 (0.02)	–	0.03 (0.02)	0.02 (0.01)	0.03 (0.02)	0.03 (0.01)	–
μ	–	T-V	0.38 (0.17)	T-V	T-V	T-V	T-V	0.64 (0.13)
φ_1	0.33 (0.11)	0.32 (0.09)	0.35 (0.11)	0.32 (0.10)	0.32 (0.11)	0.33 (0.11)	0.65 (0.09)	0.48 (0.27)
φ_2	0.01 (0.11)	-0.00 (0.09)	0.01 (0.11)	-0.01 (0.13)	0.01 (0.10)	0.01 (0.10)	0.21 (0.09)	0.31 (0.16)
π_i	-6.15 (1.33)	-6.32 (1.16)	-6.95 (1.16)	-6.64 (1.37)	-5.75 (1.32)	-5.81 (1.27)	–	–
p	0.83 (0.07)	0.83 (0.07)	0.83 (0.07)	0.82 (0.07)	0.84 (0.06)	0.83 (0.07)	0.92 (0.04)	0.93 (0.04)
q	0.93 (0.03)	0.93 (0.02)	0.94 (0.02)	0.93 (0.02)	0.92 (0.03)	0.92 (0.03)	0.92 (0.03)	0.92 (0.03)
ρ_{p^e,p^i}	–	–	–	0.67 (2.22) -0.64 (0.95)	–	–	–	–
$\rho_{p^e,\mu}$	–	–	–	–	-0.96 (0.35) 0.98 (0.12)	–	–	–
$\rho_{p^i,\mu}$	–	–	–	–	–	0.52 (10.04i)	–	–
Log likelihood	-1222.4	-1221.1	-1218.7	-1220.8	-1220.5	-1221.4	-1228.7	-1226.8

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) The standard errors of the parameters are reported in parenthesis. Those with the letter i for the model 1g are imaginary numbers.
(c) Numerical values for parameters denoted by 0.00 are respectively -0.004 for model 1c, 0.001 for model 3c, and 0.0005 for model 3d.

Notes:

(1) The estimation period runs from 1986M1 to 2022M12. See Table 2 for the main results and explanations.

(2) Model 1b′ is another version of model 1b with similar estimations of parameters. Since in the former, we treat the drift term (constant long-run return) as a state variable and in the latter, the drift is estimated as a parameter, model 1b′ is fully nested in model 1a, but model 1b is not. Comparing log likelihood values of -1221.5 and -1222.4 suggests that a random walk with a deterministic drift is sufficient to capture the dynamics of the efficient price.

(3) The term "SVPT" for models 1c and 1d says that the **S**witching **V**ariance is allowed for two variances, one for shocks to the **P**ermanent component and another for shocks to the **T**ransitory component. Hence, models 1c and 1d account for the asymmetry by including inefficient plunges in the transitory component, one switching variance in the permanent component, and another switching variance in the transitory component. In these two models, I relax the assumption of constant variance for shocks to the transitory component by including a switching variance for these shocks based on Eq. (8). The result shows that the variance of shocks to the transitory component is not switching. In particular, comparing the log likelihood values of -1221.5 and -1218.9 for models 1a and 1b in Table 2 with the values of -1221.1 and -1218.7 for models 1c and 1d bears likelihood ratios of 0.8 and 0.2, confirming that the variance of shocks to the transitory component is not switching.

(4) To test for asymmetry in the form of inefficient plunges, I compare the log likelihood values for models 1c and 1d, in which the asymmetry is accounted for by including both inefficient plunges in the transitory component and two switching variances for shocks to the permanent and transitory components, with those values for models 3c and 3d, in which the asymmetry is accommodated only by including two switching variances for shocks to the permanent and transitory components. A pairwise comparison of the log likelihoods of -1221.1 and -1218.7 reported for models 1c and 1d with values of -1228.7 and -1226.8 for models 3c and 3d, respectively, favours the asymmetric Fads models over asymmetric variance models. The corresponding likelihood ratios of 17.6 and 14.6 are greater than the critical value of 10.8 for a 0.1% significance level.

(5) Comparing the log likelihood of -1221.5 for the proposed model in column 1a of Table 2 with values of -1220.8, -1220.5, and -1221.4 for correlated models 1e, 1f, and 1g ensures that the correlations between shocks (ρ_{p^e,p^i}, $\rho_{p^e,\mu}$, and $\rho_{p^i,\mu}$) are all negligible as the corresponding likelihood ratios are 1.4, 2.0, and 0.2.

Table C3: Estimated parameters of different models for the weekly and daily S&P 500

Models	1a (weekly)	1b (weekly)	1a (daily)	1b (daily)
Parameters	A (IP-SV)-RW	A (IP-SV)-Con	A (IP-SV)-RW	A (IP-SV)-Con
$\sigma_{p^e,0}$	1.38 (0.03)	1.38 (0.03)	0.50 (0.02)	0.51 (0.02)
$\sigma_{p^e,1}$	3.38 (0.10)	3.39 (0.10)	1.64 (0.03)	1.64 (0.03)
$\sigma_{p^i,0}$	0.19 (0.10)	0.19 (0.10)	0.34 (0.03)	0.33 (0.03)
$\sigma_{p^i,1}$	–	–	–	–
σ_{μ}	0.00 (0.001)	–	0.00 (0.001)	–
μ	T-V	0.18 (0.03)	T-V	0.04 (0.005)
φ_1	0.19 (0.08)	0.19 (0.08)	1.13 (0.03)	1.14 (0.03)
φ_2	0.49 (0.09)	0.49 (0.09)	-0.24 (0.03)	-0.25 (0.03)
π_i	-2.54 (0.26)	-2.56 (0.27)	-0.37 (0.04)	-0.38 (0.05)
p	0.90 (0.01)	0.91 (0.01)	0.96 (0.004)	0.96 (0.004)
q	0.97 (0.01)	0.97 (0.01)	0.99 (0.001)	0.99 (0.001)
Log likelihood	-7981.8	-7977.0	-23173.1	-23171.7

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) The standard errors of the estimated parameters are in parenthesis.
(c) Numerical values for parameters denoted by 0.00 are respectively 0.00006 for model 1a (weekly) and 0.0006 for model 1a (daily).

Notes:

(1) The estimation period runs from 1948W1 to 2022W52 and from 1948D1 to 2022D251 for daily data.

(2) For weekly data, the log likelihood values of models 3a and 3b are -8026.0 and -8021.2, respectively. Hence, the likelihood ratio for testing inefficient plunges is 88.4 for model 1a and 88.4 for model 1b.

(3) For daily data, the log likelihood values of models 3a and 3b are -23230.3 and -23235.8, respectively. Hence, the likelihood ratio for testing inefficient plunges is 114.4 for model 1a and 128.2 for model 1b.

Table C4: Estimated parameters of different models for the weekly and daily FTSE 350

Models	1a (weekly)	1b (weekly)	1a (daily)	1b (daily)
Parameters	A (IP-SV)-RW	A (IP-SV)-Con	A (IP-SV)-RW	A (IP-SV)-Con
$\sigma_{p^e,0}$	1.59 (0.06)	1.58 (0.06)	0.66 (0.02)	0.66 (0.02)
$\sigma_{p^e,1}$	4.25 (0.32)	4.23 (0.32)	1.80 (0.04)	1.80 (0.04)
$\sigma_{p^i,0}$	0.20 (0.12)	0.20 (0.12)	0.20 (0.04)	0.20 (0.04)
$\sigma_{p^i,1}$	–	–	–	–
σ_{μ}	0.00 (0.001)	–	0.00 (0.001)	–
μ	T-V	0.12 (0.04)	T-V	0.04 (0.01)
φ_1	0.08 (0.08)	0.07 (0.08)	0.95 (0.10)	0.92 (0.10)
φ_2	0.51 (0.08)	0.51 (0.08)	-0.27 (0.09)	-0.27 (0.09)
π_i	-3.11 (0.47)	-3.10 (0.45)	-0.86 (0.14)	-0.88 (0.14)
p	0.84 (0.04)	0.83 (0.03)	0.96 (0.01)	0.96 (0.01)
q	0.97 (0.01)	0.97 (0.01)	0.99 (0.01)	0.99 (0.01)
Log likelihood	-4093.6	-4088.5	-12259.4	-12255.2

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) The standard errors of the estimated parameters are in parenthesis.
(c) Numerical values for parameters denoted by 0.00 are respectively 0.002 for model 1a (weekly) and 0.0002 for model 1a (daily).

Notes:

(1) The estimation period runs from 1986W1 to 2022W52 for weekly data and from 1986D1 to 2022D251 for daily data.

(2) For weekly data, the log likelihood values of models 3a and 3b are -4114.6 and -4110.2, respectively. Hence, the likelihood ratio for testing inefficient plunges is 42.0 for model 1a and 43.4 for model 1b.

(3) For daily data, the log likelihood values of models 3a and 3b are -12295.0 and -12291.2, respectively. Hence, the likelihood ratio for testing inefficient plunges is 71.2 for model 1a and 72.0 for model 1b.

Appendix D: Business cycle dates

Table D1: Dates of the U.S. Business Cycles (Peak and Trough)

N	ECRI*	NBER**	Description
1	1957M8-1958M4	1957M8-1958M4	--
2	1960M4-1961M2	1960M4-1961M2	--
3	1969M12-1970M11	1969M12-1970M11	--
4	1973M11-1975M3	1973M11-1975M3	First Oil Crisis
5	1980M1-1980M7	1980M1-1980M7	Second Oil Crisis
6	1981M7-1982M11	1981M7-1982M11	Early 1980s recession
7	1990M7-1991M3	1990M7-1991M3	Early 1990s recession
8	2001M3-2001M11	2001M3-2001M11	Early 2000s recession
9	2007M12-2009M6	2007M12-2009M6	Global crisis and recession
10	2020M2-2020M4	2020M2-2020M4	COVID-19 recession

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Table D2: Dates of the U.K. Business Cycles (Peak and Trough)

N	ECRI*	NIESR**	Description
1	-	1951M3-1952M8	--
2	-	1955M12-1958M11	--
3	-	1961M3-1963M1	--
4	1974M9-1975M8	1973M1-1975M3	First Oil Crisis
5	1979M6-1981M5	1979M2-1982M4	Second Oil Crisis
6	-	1984M1-1984M3	--
7	-	1988M4-1992M2	Early 1990s recession
8	1990M5-1992M3	-	Early 1990s recession
9	2008M5-2010M1	-	Global crisis and recession
10	2019M10-2020M4	-	COVID-19 recession

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