Asymmetric Co-fluctuations of U.S. Output and Unemployment: Friedman's Plucking Model and Okun's Law

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Abstract

We integrate Friedman's plucking model and the gap version of Okun's law by embedding output and the unemployment rate in a bivariate unobserved components model with Markov-switching to examine their asymmetric co-fluctuations in the U.S. economy. The results establish that output and unemployment are synchronously and proportionally characterized by the plucking property. Given that the labour market has been identified as the source of the plucking property, we suggest that the plucking property, through stable Okun's law, transmits from unemployment to output. Our proposed asymmetric bivariate model provides two additional results regarding two specification aspects of trend-cycle decomposition: First, we identify an unprecedented deceleration in U.S. potential output in the aftermath of the 2007−09 global financial crisis. Second, our model yields a robust estimation of parameters and components with insignificant correlations between shocks.

Keywords: Business Cycle Asymmetries, Friedman's Plucking Model, Okun's Law, Structural Break, Trend-Cycle Decomposition.

JEL Classification: C32, E32, F44, O47.

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1. Introduction

In contrast to the mainstream view in the business cycle literature, the plucking model proposed by Milton Friedman (1964, 1993) suggests an asymmetric cyclical component, meaning that output does not fluctuate around a long-run trend but instead is steeply plucked down below a ceiling, known as potential output, during recessions and gradually returns toward this ceiling during recoveries. This business cycle asymmetry is referred to as the plucking property or ceiling effect. In this regard, few studies have empirically established the business cycle asymmetry in U.S. output (see, e.g., Kim and Nelson, 1999a; Sinclair, 2010; Eo and Morley, 2022), which suggests that stabilization policies may raise economic welfare by affecting the total average of output.[1]

Likewise, the U.S. unemployment rate does not fluctuate around the trend, but it is characterized by steep jumps above the natural rate of unemployment during recessions and gradual decrements to its natural level during recoveries. Indeed, the unemployment rate distribution displays a remarkable positive skewness, which is referred to as deepness asymmetry by Neftci (1984) and Sichel (1993). Recently, Ferraro (2018), Dupraz et al. (2019), and Ferraro and Fiori (2023), by developing a number of equilibrium models for the unemployment rate, have documented that the source of the plucking property is search frictions and nominal wage rigidities in the U.S. labour market, yet their models do not consider the potential transmission of such a plucking property from unemployment to output.

Okun's law, first proposed by Arthur Okun (1962), is an empirical relation between U.S. output and unemployment rate gaps. Although some studies state that Okun's law has weakened over time or cast doubt on its stability during recessions (Gordon, 2010; Owyang and Sekhposyan, 2012; Grant, 2018), many other researchers conclude in favour of the stability of Okun's law (Galí et al., 2012; Daly et al., 2014; Ball et al., 2017; Michail, 2019; among many others). Therefore, given the stability of Okun's law, fluctuations in output and unemployment must be synchronous and proportional; that is what we call "co-fluctuations."

Considering Friedman's plucking model and Okun's law together, asymmetric fluctuations appear to be a common feature of both U.S. output and the unemployment rate. We refer to this phenomenon as "asymmetric co-fluctuations" of U.S. output and the unemployment rate. Since business cycle asymmetries are more pronounced in unemployment (see, e.g., Falk, 1986; Sichel, 1993; McKay and Reis, 2008), and the U.S. labour market is identified as the source of the plucking property, it seems that the plucking property transmits from unemployment to output.

[1] The asymmetric business cycle, instead of contraction and expansion, identifies recession and recovery as two phases of business cycles. The term recession refers to the period in which output recedes from its potential, and recovery conveys the period of returning to potential output. Therefore, stabilization policies, in this framework, have the potential to increase the total average of output by reducing fluctuations.

In this study, we incorporate Friedman's plucking model and Okun's law into a bivariate Unobserved Components (UC) model with Markov-switching to investigate the asymmetric co-fluctuations. This model comprises four specification aspects, each of which entails choosing between two alternative assumptions. The choices made on each of these aspects remarkably alter the features of the trend and cyclical components estimated by different studies. We, therefore, discuss our choices on each aspect by briefly reviewing the literature on trend-cycle decomposition.[2]

The first aspect is whether the fluctuations in U.S. output and the unemployment rate are asymmetric or symmetric. We select the asymmetric choice for this aspect since Kim and Nelson (1999a), Mills and Wang (2002), De Simone and Clarke (2007), Sinclair (2010), Morley and Piger (2012), and Eo and Morley (2022), by applying a univariate UC model with Markov-switching, confirm that output fluctuations in the U.S. and other advanced economies are asymmetric. The second aspect concerns whether unemployment must be included as an auxiliary within a bivariate model or not. Following Clark (1989) and Morley and Wong (2020), we select to include the unemployment rate in our model. As demonstrated by Gonzalez-Astudillo and Roberts (2022), this arrangement makes the features of the estimated output components robust to changes in the specification of other aspects, in particular the correlation between shocks.

The third aspect asks whether the trend growth of U.S. output is stochastic or deterministic. There is a plethora of evidence supporting the presence of a time-varying drift (trend growth) in the output of the U.S. and other advanced economies that reflects decline in productivity growth (Antolin-Diaz et al., 2017; Fernald et al., 2017). Therefore, we utilize stochastic trend growth, namely a random walk process, which is advocated by Clark (1987), Grant and Chan (2017a), and Kim and Chon (2020). Alternatively, following Perron and Wada (2009), Luo and Startz (2014), Grant and Chan (2017b), and Eo and Morley (2022), we adopt a specification for trend growth that accounts for a structural break. Finally, the fourth aspect asks whether the correlation between shocks to the trend and cyclical components matters or not. We exclude correlations in the main setup, yet we allow for correlations to verify the robustness of our results.[3]

We cast the proposed bivariate UC model with Markov-switching, which we name the asymmetric bivariate model for ease of writing, and a number of alternative models into their state-space forms to estimate parameters and log likelihood values.[4] For hypothesis testing, we use several pairwise

[2] Table A1 in Appendix A lists the choice of each study in the literature for each of the four specification aspects.
[3] There is a puzzle about the correlation between shocks to the trend and cyclical components, particularly in the univariate UC model. In this regard, Basistha (2007), Wada (2012), and Iwata and Li (2015) show that the univariate correlated UC model is subject to spurious correlation, meaning that the estimated correlation might pile-up towards unity even though the true correlation parameter is zero.
[4] Namely, we perform the symmetric bivariate UC model used by Clark (1989) and Gonzalez-Astudillo and Roberts (2022), the univariate UC model proposed by Clark (1987), the univariate correlated UC model of Morley et al. (2003), and the asymmetric univariate UC models of Kim and Nelson (1999a) and Sinclair (2010).

comparisons of log likelihoods estimated for the asymmetric bivariate model and those estimated for nested models. Estimation results establish the asymmetric co-fluctuations; indeed, grounded on the stable gap version of Okun's law, we observe that output and unemployment are synchronously and proportionally characterized by the plucking property. In particular, the estimated coefficients of the plucking property and Okun's law are 0.7 and -1.45, respectively, with small standard errors of 0.06 and 0.12. The likelihood ratio for testing the asymmetry is 91.2, which is substantially greater than the 0.1% critical value of 10.8. Also, estimated gaps that are large in magnitude and often negative for output and positive for unemployment verify the ceiling effect: output seldom ascends above the ceiling (potential output), and correspondingly, the unemployment rate seldom descends below the floor (natural rate). Further, the expected duration is about 3 quarters for recessions and 28 quarters for recoveries, suggesting that recessions are short and recoveries are long. Overall, co-fluctuations are asymmetric in amplitude, speed, and duration, which highlights that deep, steep, and transitory recessions tend to be followed by commensurate, gradual, and permanent recoveries. This conclusion is consistent with the empirical results of Neftci (1984), Sichel (1993), Friedman (1993), Kim and Nelson (1999a), Sinclair (2010), Morley and Piger (2012), and Eo and Morley (2022), who test cycle asymmetry for output and the unemployment rate separately.

Additionally, given that Ferraro (2018), Dupraz et al. (2019), and Ferraro and Fiori (2023) identify the U.S. labour market as the source of the plucking property, we propose that the plucking property transmits from the unemployment rate to output. To support this expression, we demonstrate that the gap version of Okun's law is indeed stable as long as asymmetric fluctuations of U.S. unemployment and stochastic trend growth in U.S. output are accommodated.

Moreover, we document a gradual decline in U.S. output trend growth that began in the 1960s, which is in accordance with Antolin-Diaz et al. (2017), Grant and Chan (2017a), Fernald et al. (2017), and Kim and Chon (2020). We, however, observe that this decline has been exacerbated as a result of an unprecedented deceleration in U.S. potential output in the aftermath of the 2007−09 financial crisis, which is consistent with Luo and Startz (2014), Grant and Chan (2017b), and Eo and Morley (2022), who find a structural break in trend growth around 2008. Concerning correlations between shocks, an asymmetric bivariate model that accounts for both asymmetry and co-fluctuations yields robust results with insignificant correlations. Finally, we apply a large number of bivariate and univariate models to U.S. real GDP and U.S. unemployment to verify the robustness of our proposed model. We also apply the proposed model to other time series, including U.S. real GDP per capita and U.K. real GDP. In addition, we modify the proposed model to explore the COVID-19 recession.

Establishing asymmetric co-fluctuations of U.S. output and the unemployment rate suggests that stabilization policies can raise the total average of output. Bearing this policy implication in mind, this study contributes to the literature in several ways. To our knowledge, we are the first to explain the asymmetric co-fluctuations by simultaneously characterizing the plucking property in U.S. output and unemployment. We, thus, augment the univariate UC models with Markov switching presented by Kim and Nelson (1999a), Sinclair (2010), and Eo and Morley (2022) by including unemployment within the bivariate model, and augment the bivariate UC models presented by Clark (1989), Grant (2018), and Gonzalez-Astudillo and Roberts (2022) by including a Markov-switching process in the model. The model of this study provides two additional results: an unprecedented deceleration in U.S. potential output and an insignificant correlation between shocks.

Last of all, jointly estimating the trends of output and unemployment while the plucking property is accounted for offers a measure for the natural rate defined as a floor (lower limit) of unemployment, which is linked to potential output as a ceiling (upper limit) of output. In light of the gap version of Okun's law, the natural rate in our model is the unemployment rate at which the output gap is zero, and accordingly, it is named the Zero Output Gap Rate of Unemployment (ZOGRU). This measure is a counterpart for the Non-Accelerating Inflation Rate of Unemployment (NAIRU), which applies the Phillips curve to estimate the unemployment rate at which the inflationary pressure is zero.

The remainder of this paper reviews the literature on asymmetric business cycles and Okun's law in Section 2. Section 3 describes the data and methodology and also specifies the bivariate model as well as univariate models applied to output and unemployment separately. In this section, we also present alternatives to our choices regarding four specification aspects of trend-cycle decomposition. Section 4 presents the results and discussion for the asymmetric bivariate model as well as several alternative bivariate and univariate models. Finally, Section 5 provides the conclusion.

2. Literature review

This study relates to three branches of existing literature: the business cycle, Okun's law, and trend-cycle decomposition. In this section, we describe the controversy between two schools of thought in the business cycle literature. We then review various attributes of business cycle asymmetries, and then turn to the disagreement over the stability of Okun's law. Regarding the literature on trend-cycle decomposition, the four specification aspects are discussed in detail in the methodology section.

2.1.1 Business cycles: Friedman's plucking model versus symmetric models

Typically, the business cycle literature supposes that output fluctuates symmetrically around a trend known as the natural level. In this view, the peak of an expansion is above the trend meaning that the economy can produce more than its natural level, and at the trough of a recession, output is below its natural level. For example, the Real Business Cycle (RBC) model, proposed by Kydland and Prescott (1982) and Long and Plosser (1983), regards technological shocks as the main drivers of symmetric fluctuations. Although technological shocks play a role, the standard RBC tends to overlook the role of adverse events such as wars, oil crises, financial crises, and the COVID-19 pandemic in shaping recessions and the role of responsive policy in fostering recoveries. Most trend-cycle decompositions also assume shocks to the cyclical component are symmetric (see, e.g., Beveridge and Nelson, 1981; Nelson and Plosser, 1982; Harvey, 1985; Clark, 1987, 1989; Morley et al., 2003; Perron and Wada, 2009; Grant and Chan, 2017a, 2017b; Grant, 2018; Kim and Chon, 2020; Kim and Kim, 2022). On the contrary, Friedman's plucking model (1993) suggests business cycle asymmetry by considering a ceiling of maximum feasible output referred to as the potential output determined by available resources. Here, output cannot go above the ceiling and most of the time it is close to the potential output except that occasionally it is plucked down by negative shocks during recessions. Then, during the subsequent recoveries, output returns toward its potential through a series of self-equilibrating forces known as the "bounce-back" effect.

The policy implications of these two perspectives are starkly conflicting. Under the symmetric cycle assumption, stabilization policy does not raise the average level of output; hence, the welfare gain of the stabilization policy is negligible (Lucas, 1987; 2003). For instance, RBC models see fluctuations as Pareto optimal responses of households and firms to productivity shocks rather than as welfare-reducing deviations from some ideal path. On the contrary, under the asymmetric cycle assumption, stabilization policy aims to not only dampen the fluctuations but also raise the average level of output. For instance, in the plucking model, fluctuations are negative deviations from the potential output since recessions and recoveries refer to periods of time when output recedes from and returns to its potential capacity. As a result, reducing fluctuations can increase the total average of output. This conclusion is drawn from a few other studies. DeLong and Summers (1988), by viewing fluctuations as gaps rather than cycles around the trend, support that stabilization policies in the U.S. can improve the average level of output. Benigno and Ricci (2011) also report that a reduction in macroeconomic volatility, as a result of better stabilization policies, can improve the long-run output gap, especially when wage inflation is low.

2.1.2. Business cycle asymmetries

Business cycle asymmetry was initially observed by Mitchell (1927) and Keynes (1936), who note that recessions take place briefly and violently, whereas there are no such sharp turning points during expansions. Thereafter, Friedman (1964) proposes a plucking model in which output bumps along the ceiling of maximum feasible production except that every now and then it is plucked down by cyclical contractions, and it then returns to its ceiling potential. Friedman (1993) finally reaffirms the idea of the plucking model by observing an asymmetrical correlation pattern in the U.S. and some other advanced economies.

By reviewing the limited literature on business cycle asymmetry, we distinguish various attributes of asymmetries, including correlation, deepness, steepness, and duration asymmetries. While these asymmetries are explained separately in different studies to focus on one attributes of the general concept, they are tightly related to each other such that they together describe the same phenomenon, Friedman's plucking property.[5]

The correlation asymmetry, noted by Friedman (1964, 1993), states that the amplitude of recessions is correlated with the amplitude of succeeding expansions, whereas the amplitude of expansions is uncorrelated with the amplitude of succeeding recessions. This asymmetry accords with the ceiling effect. When output is plucked down by negative shocks during a recession, the depth of the recession varies depending on the severity of those negative shocks. Therefore, the amplitude of the previous expansion is unrelated to the amplitude of the recession. Afterward, when the subsequent recovery starts, output cannot go above a ceiling named "potential output," so the amplitude of the subsequent expansion tends to be correlated with the amplitude of the recession.

A similar expression of correlation asymmetry states that the deeper the recessions, the stronger the subsequent recoveries, which is empirically supported by a few studies for U.S. output (Wynne and Balke, 1992; Beaudry and Koop, 1993; Fatás and Mihov, 2013). Goodwin and Sweeney (1993) and Fatás and Mihov (2013) also provide substantial support for the ceiling effect in the U.S. and other advanced economies. In addition, by analysing 26 episodes of business cycles beginning in 1882 and ending with the Great Recession, Bordo and Haubrich (2017) confirm that the recovery of output is stronger following those recessions that are deep and coincide with financial crises. Recently, Dupraz et al. (2019) present empirical evidence that the U.S. unemployment rate displays a striking plucking property, which means that the amplitude of recessions forecasts the amplitude of the subsequent recoveries but not vice versa.

Deepness asymmetry indicates that recession troughs are deep while expansion peaks are small in amplitude, and steepness asymmetry signifies that recessions are steep (violent) whereas expansions are gradual (mild). To test these two asymmetries in a time series, Sichel (1993) suggests measuring the distributional asymmetry of the series and its difference. For instance, output exhibits deepness if it displays negative skewness relative to the trend, and it exhibits steepness if its first difference displays negative skewness. In this regard, Sichel (1993) and Goodwin and Sweeney (1993) report a significant negative skewness in the distribution of the de-trended output. Jensen et al. (2020), by comparing the skewness of real output growth before and after 1984, suggest a deepening asymmetry since they detect a more negative skewness for the U.S. and other advanced economies after 1984. Moreover, unemployment as a counter-cyclical variable exhibits deepness asymmetry if it displays

[5] See Table A2 in Appendix A for a detailed explanation.

positive skewness, and it exhibits steepness if its first difference displays positive skewness. In this regard, Neftci (1984), Sichel (1993), and Dupraz et al. (2019) report that unemployment distribution displays a remarkable positive skewness. Ramsey and Rothman (1996) also document deepness and steepness asymmetries in output and unemployment of the U.S. and other advanced economies by relating the concept of time reversibility to deepness and steepness. Overall, the findings of the above studies imply that, during recessions, output falls deeply and unemployment jumps sharply, whereas during recoveries, they both gradually return to their trends.

Duration asymmetry states that recessions are short and recoveries are long. Given that recessions are deep, it is clear that the duration asymmetry is analogous to the concept of steepness asymmetry. In this regard, Neftci (1984) applies a Markov process to compare the transition probabilities between contractionary and expansionary states. He concludes that unemployment is characterized by sudden jumps during contraction and gradual decrements during expansion.

Empirically, excluding some basic statistical evidence provided by the abovementioned studies, there are very few studies that examine the plucking model by developing a rigorous econometric model. Kim and Nelson (1999a) develop a state-space model with Markov-switching to examine asymmetric fluctuations in U.S. output and conclude in favour of Friedman's plucking model against symmetric alternatives. Subsequently, Mills and Wang (2002) as well as De Simone and Clarke (2007) provide international evidence for the Friedman's plucking model. In addition, Sinclair (2010), by developing a correlated asymmetric model, demonstrates that ignoring business cycle asymmetry underestimates the amplitude of the cyclical component.

Theoretically, Ferraro (2018), Dupraz et al. (2019), and Ferraro and Fiori (2023), using equilibrium business cycle models, explain why the unemployment rate does not fluctuate around a trend but is characterized by steep jumps above the natural rate of unemployment during recessions. They reach the conclusion that search frictions and downward nominal wage rigidities in the U.S. labour market are the main sources of the asymmetry in the unemployment rate. This result accords with DeLong and Summers (1984), Falk (1986), Sichel (1993), and McKay and Reis (2008), who empirically show that business cycle asymmetries are more pronounced in unemployment than in output, implying that the primary source of the plucking property stems from the U.S. labour market. However, all the above studies do not accommodate two plausible possibilities: (1) the potential transmission of the plucking property from the unemployment rate to output, which is addressed by the model proposed in this study; and (2) other potential sources of asymmetry, such as binding financial constraints, an idea that is proposed by Jensen et al. (2020).

2.2. Okun's law

Okun's law is an empirical relationship between fluctuations in output and the unemployment rate that was first proposed by Arthur Okun (1962) and is well-established for numerous countries (Ball et al., 2017). The gap version of Okun's law is the relationship between the output gap (the deviation of output from potential output) and the unemployment gap (the deviation of unemployment from its natural rate), whereas the difference version of Okun's law explains the relationship between output growth and change in the unemployment rate.

Although some studies cast doubt on the stability of Okun's law during recessions, particularly the Great Recession (see, e.g., Gordon, 2010; Owyang and Sekhposyan, 2012; Basu and Foley, 2013; Valadkhani and Smyth, 2015; Berger et al., 2016; Grant, 2018), the bulk of the literature concludes in favour of its stability in the U.S., the U.K. and other advanced economies (see, e.g., Sögner and Stiassny, 2002; Daly et al., 2011; Galí et al., 2012; Daly et al., 2014; Economou and Psarianos, 2016; Ball et al., 2017; Michail, 2019). Overall, the results suggesting the obsolescence of Okun's law are indeed greatly exaggerated, and the departures from Okun's law are small and short-lived, such that Okun's law is alive (Daly et al., 2014; Ball et al., 2017).

In essence, the reason for the results suggesting instability of Okun's law reported by a few studies is perhaps caused by excluding two key features of U.S. output from their models: asymmetry in the cyclical component and the decline in trend growth. If these two features are left unaccounted for, their traces can be reflected in the form of an instability in Okun's law. For instance, Berger et al. (2016) report that the Okun's coefficient drops and bounces back during recessions. However, given the evidence provided for asymmetric fluctuations, this result can be caused by imposing a symmetric cycle assumption. In addition, the gap or difference versions of Okun's law applied by Owyang and Sekhposyan (2012), Basu and Foley (2013), and Grant (2018) impose a constant trend growth (drift). This assumption is not innocuous since there is evidence supporting a gradual decline in trend growth in the U.S. and other advanced economies. Consequently, if the stochastic drift is the true model and the deterministic drift is the false model, it is probable that the declining trend growth, which is not accounted for, reflects itself in the form of an instability in the Okun's coefficient. Additionally, the instability of Okun's law reported by Valadkhani and Smyth (2015) appears to be caused by imposing a number of assumptions. For example, they impose a single Markov-switching process to explain the regime-switching in both the Okun's law coefficient and the output shock volatility, whereas no theoretical or empirical evidence is presented to support this idea that Okun's law and volatility have the same regime-switching timing and dynamics.

3. Data and methodology

We use data on the seasonally adjusted real gross domestic product (GDPC1) and the unemployment rate for people aged 16 and over (UNRATE) from FRED (Federal Reserve Economic Data). The quarterly sample period runs from 1948Q1 to 2019Q4, though we extend the data until 2022Q4 to explore the COVID-19 recession. We use the natural log of quarterly real GDP multiplied by 100 and the quarterly unemployment rate as two observed series in the model. We calculate the quarterly unemployment rate as the average of rates of the three months within the corresponding quarter. For example, the rate of unemployment for the first quarter is the average of the unemployment rates for January, February, and March. Alternatively, to control for the lead-lag effect between output and the unemployment rate, we calculate the leading quarterly unemployment rate by finding the average of three months, two of which are within the same quarter and the other is in the subsequent quarter. As an illustration, the leading unemployment rate for the first quarter is calculated as the average of the unemployment rates for February, March, and April. We additionally apply the proposed model to U.S. real GDP per capita and U.K. real GDP.

As specified in Section 3.1, we run the asymmetric bivariate model to examine the asymmetric co-fluctuations. To model the asymmetry, we include a Markov-switching process in the unemployment cyclical component with the aim of capturing the plucking property. To model co-fluctuations, we apply a gap version of Okun's law, where the unemployment rate is placed on the right-hand side, with the intention of capturing the transmission of the plucking property from unemployment rate to output in the U.S. economy. For the purpose of pairwise comparison, we estimate several models, including the main asymmetric bivariate model and the symmetric bivariate model utilized by Clark (1989) and Gonzalez-Astudillo and Roberts (2022), in which the plucking coefficient is imposed to be zero.

Furthermore, as explained in Section 3.2, we run the asymmetric univariate model in the spirit of the plucking model proposed by Kim and Nelson (1999a), Sinclair (2010), and Morley and Piger (2012), where the asymmetry is modelled by using a Markov-switching process in the cyclical component. We separately apply this model to U.S. output and unemployment to gain insight into the asymmetric fluctuations of these two indicators and also verify that correlation is insignificant exclusively for the asymmetric bivariate model. For pairwise comparison, we impose the plucking coefficient to be zero in order to estimate several nested models, including the univariate uncorrelated UC model of Clark (1987), the univariate correlated UC model proposed by Morley et al. (2003), the univariate UC model with a break in trend growth proposed by Perron and Wada (2009), and also the univariate correlated UC model with a break in trend growth designed by Grant and Chan (2017b).

In total, to account for the wide variety of specifications, we estimate twenty-two bivariate models, fourteen univariate models for output, and five univariate models for unemployment. The detailed specifications of each of the models are presented in Tables B1, B2, and B3 in Appendix B.

We cast each model in a state-space form to estimate models by using the Kalman's (1960) filter. In symmetric models, we simply use the maximum likelihood method. For asymmetric models in the presence of the Markov-switching process of Hamilton (1989), we use Kim's (1994) approximate maximum likelihood method to make the Kalman filter operable.[6] We evaluate the proposed model against other alternatives by estimating the plucking coefficient, the Okun's law coefficient, and their standard errors, as well as deriving likelihood ratios based on pairwise comparisons of log likelihood values.

3.1. The bivariate model: Friedman's Plucking Model and Okun's Law

In the bivariate model, to distinguish between the components of output and the unemployment rate, we denote the observed series, unobserved trend component, and unobserved cyclical component for output by x_t, x_t^*, and x_t^c, and we respectively denote those variables for the unemployment rate by u_t, u_t^*, and u_t^c. In this setting, we decompose each of the output and the unemployment rate into a trend and a cyclical component, as specified in Eq. (1) and Eq. (2):

$$x_t = x_t^* + x_t^c \tag{1}$$

$$u_t = u_t^* + u_t^c \tag{2}$$

where x_t is the log of output and u_t is the unemployment rate. x_t^* and x_t^c are unobserved trend and cyclical components of output that play the roles of potential output and the output gap, respectively. Similarly, u_t^* and u_t^c are unobserved trend and cyclical components of the unemployment rate, which respectively play the roles of the natural rate of unemployment and the unemployment gap.

3.1.1. The trend components of output and the unemployment rate

We model the output trend as a random walk process with a drift term as follows:

$$x_t^* = \mu_{t-1} + x_{t-1}^* + \varepsilon_{x^*,t} \tag{3}$$

where $\varepsilon_{x^*,t} \sim N(0, \sigma_{x^*}^2)$ denotes the output trend shock and is assumed to be white noise and normally distributed, just like all of the other shocks (also called innovations) in this study. Further, μ_t stands for time-varying drift, which plays the role of trend growth and is specified in two alternative ways. First, since a substantial time-variation in U.S. output trend growth is documented (see, e.g., Antolin-

[6] For more explanation, especially the state-space representation of bivariate and univariate models, see Appendix B. For estimation methods and initial values for parameters and state variables, see Appendix C, chapters 3-5 of Kim and Nelson (1999b), and chapters 13 and 22 of Hamilton (1994).

Diaz et al., 2017; Fernald et al., 2017), we make use of a stochastic drift that evolves according to a random walk process:

$$\mu_t = \mu_{t-1} + \varepsilon_{\mu,t} \tag{4.a}$$

where $\varepsilon_{\mu,t} \sim N(0, \sigma_\mu^2)$ represents the shock to output trend growth, and it is assumed to be white noise and uncorrelated with all other shocks. The above setup accords with those of Clark (1987, 1989), Grant and Chan (2017a), and Kim and Chon (2020). Second, similar to Perron and Wada (2009) and Grant and Chan (2017b), we alternatively consider a non-stochastic drift with a structural break:

$$\mu_t = \gamma + \delta \mathbb{1}_t (t \geq T_\mu) \tag{4.b}$$

where $\mathbb{1}_t$ is an indicator function that takes the value of one after the break (T_μ) and zero otherwise. In this setup, γ is the trend growth of output before the break date, and $\gamma + \delta$ is the trend growth after the break date. Comparing two competing specifications for output trend growth in Eq. (4.a) and Eq. (4.b), we advocate the former since the stochastic drift lets the data speak for itself and enables the model to capture both the gradual decline and the potential break in U.S. output trend growth. This choice is consistent with Antolin-Diaz et al. (2017), who suggest that the random walk process is robust to misspecifications even if the actual process is characterized by discrete structural breaks.

The unemployment trend in our model is named ZOGRU since it measures the unemployment rate at which the output gap is zero and is specified as a random walk with a drift that allows for a break:

$$u_t^* = \eta + \theta \mathbb{1}_t (t \geq T_{u^*}) + u_{t-1}^* + \varepsilon_{u^*,t} \tag{5}$$

Under this equation, $\mathbb{1}_t$ is an indicator that takes the value of one after the break date (T_{u^*}) and zero otherwise. Thus, η is the drift before the break date, and $\eta + \theta$ is the drift after the break date. Our motivation for allowing for a break is the observed rise and decline in NAIRU, which measures the U.S. natural rate of unemployment, before and after the 1980s by Ball and Mankiw (2002), Semmler and Zhang (2006), and Basistha and Startz (2008). We include the shock to the unemployment trend, $\varepsilon_{u^*,t} \sim N(0, \sigma_{u^*}^2)$, to account for stochastic behaviour in the natural rate in the form of a random walk by following Gordon (1997), Staiger et al. (1997), Laubach (2001), Watson (2014), among others.[7]

3.1.2. The cyclical component of unemployment

To allow asymmetric fluctuations, we consider that shocks to the cyclical component are a mixture of asymmetric and symmetric shocks. Since asymmetric fluctuations are more pronounced in the unemployment rate (DeLong and Summers, 1984; Falk, 1986; Sichel, 1993; McKay and Reis, 2008), and the U.S. labour market is identified as the source of the plucking property (Ferraro, 2018; Dupraz

[7] If the variance of the unemployment trend shock ($\varepsilon_{u^*,t}$) in Eq. (5) is estimated to be zero ($\sigma_{u^*}^2 = 0$), one can argue that the natural rate of unemployment does not exhibit stochastic behaviour, and its variation can be explained by a drift term with a structural break.

et al., 2019; Ferraro and Fiori, 2023), we embed an unobservable, first-order, and two-state Markov-switching variable into the cyclical component of unemployment rather than output, as follows:

$$u_t^c = \pi_u S_t + \varphi_1 u_{t-1}^c + \varphi_2 u_{t-2}^c + \varepsilon_{u^c,t} \tag{6}$$

where π_u (the plucking coefficient) measures the amplitude of the asymmetric shock and is expected to be positive since the unemployment rate is counter-cyclical. In Eq. (6), φ_1 and φ_2 are coefficients of the AR(2) process. They allow for high persistence in unemployment, and their sum $(\varphi_1 + \varphi_2)$ is expected to be less than one. Furthermore, $\varepsilon_{u^c,t} \sim N(0, \sigma_{u^c}^2)$ is a typical symmetric cyclical shock to the unemployment rate.

In this model, S_t identifies the state of the economy: $S_t = 0$ during normal times and $S_t = 1$ during recessions. The state of the economy will be determined endogenously as it evolves according to the Markov-switching process proposed by Hamilton (1989):

$$\Pr[S_t = 1 | S_{t-1} = 1] = p \tag{7}$$

$$\Pr[S_t = 0 | S_{t-1} = 0] = q \tag{8}$$

In this approach, p and q determine the transition probabilities. p is the probability of staying in the recession, and thus, $1 - p$ is the probability of transitioning from the recession to the normal state. Similarly, q is the probability of staying in the normal state, and thus, $1 - q$ is the probability of transitioning from the normal to the recession state.

3.1.3. Okun's law and the cyclical component of output

After characterizing the asymmetric fluctuations of unemployment, similar to Clark (1989), Berger et al. (2016), Ball et al. (2017), Grant (2018), and Gonzalez-Astudillo and Roberts (2022), we utilize the gap version of Okun's law to capture the co-fluctuations of output and unemployment:

$$x_t^c = \beta u_t^c + \varepsilon_{x^c,t} \tag{9}$$

where β is the Okun's coefficient, and $\varepsilon_{x^c,t}$ is the Okun's residual (also called the remaining cyclical component in output), which is modelled as follows:

$$\varepsilon_{x^c,t} = \pi_x S_t + \psi \varepsilon_{x^c,t-1} + \xi_{x^c,t} \tag{10}$$

In Eq. (10), π_x is the output-specific plucking coefficient that gauges the part of the plucking property in output that is not explained by the plucking property in unemployment, and ψ is an autoregressive coefficient to control any persistency in the Okun's residuals. If the leftover plucking property in Okun's residuals is negligible, we conclude that the plucking property in output is sourced from the plucking property in unemployment. Overall, a significant and positive π_u, a significant and negative β, and a trivial leftover plucking property confirm the asymmetric co-fluctuations of U.S. output and unemployment, which indicates that output and unemployment are synchronously and proportionally characterized by the plucking property. Furthermore, $\xi_{x^c,t} \sim N(0, \sigma^2_{x^c})$ is the shock to the remaining cyclical component with a constant variance. For robustness tests, however, we allow this shock to have different variances before and after the great moderation, as follows:

$$\sigma^2_{x^c} = \sigma^2_{x^c,0}\mathbb{1}_t(t \leq T_\sigma) + \sigma^2_{x^c,1}\mathbb{1}_t(t \geq T_\sigma) \tag{11}$$

where $\mathbb{1}_t$ is an indicator function so that the variance before the break date is equal to $\sigma^2_{x^c,0}$ and after the break date is $\sigma^2_{x^c,1}$. In addition to exploring the above mentioned structural breaks, we test for a potential break in Okun's coefficient to address the concern about the stability of Okun's law.

3.1.4. The variance-covariance matrix of shocks

Considering five shocks to the components presented in Eq. (1) to Eq. (11), the variance-covariance matrix of shocks is:

$$\begin{bmatrix} \varepsilon_{x^*,t} \\ \varepsilon_{u^c,t} \\ \varepsilon_{\mu,t} \\ \varepsilon_{u^*,t} \\ \xi_{x^c,t} \end{bmatrix} \sim N\left(\mathbf{0}_{5\times 1}, \begin{bmatrix} \sigma^2_{x^*} & \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & 0 & 0 & \rho_{x^*,x^c}\sigma_{x^*}\sigma_{x^c} \\ \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & \sigma^2_{u^c} & 0 & 0 & \rho_{u^c,x^c}\sigma_{u^c}\sigma_{x^c} \\ 0 & 0 & \sigma^2_\mu & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{u^*} & 0 \\ \rho_{x^*,x^c}\sigma_{x^*}\sigma_{x^c} & \rho_{u^c,x^c}\sigma_{u^c}\sigma_{x^c} & 0 & 0 & \sigma^2_{x^c} \end{bmatrix}\right) \tag{12}$$

where ρ_{x^*,u^c} stands for the correlation between shocks to the output trend and the symmetric cyclical component, ρ_{x^*,x^c} is the correlation between shocks to the output trend and the remaining cyclical component, and ρ_{u^c,x^c} is the correlation between shocks to the symmetric cyclical component and the remaining cyclical component. In the main setup, we assume these three correlations are all zero; yet, for robustness, we relax the zero-correlation assumptions.[8]

Regarding other correlations, it is presumed that the shock to output trend growth ($\varepsilon_{\mu,t}$) and the shock to the unemployment trend ($\varepsilon_{u^*,t}$) are uncorrelated with all other shocks. The former assumption is common in the literature (see, e.g., Clark, 1987, 1989; Grant and Chan, 2017a; Antolin-Diaz et al., 2017; Kim and Chon, 2020). The latter is also reasonable for three reasons. First, the natural rate of unemployment is independent from temporary fluctuations since it is the structural unemployment rate that would prevail in the absence of any cyclical variations (Phelps, 1967; Friedman, 1968). Second, Gonzalez-Astudillo and Roberts (2022), using likelihood ratio tests, demonstrate that this assumption is innocuous. Third, it is common in the empirical literature to specify the unemployment trend as a random walk, with its shocks assumed to be uncorrelated with other shocks (see, e.g., Clark, 1989; Watson, 2014; Grant, 2018).[9]

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[8] Favourably, our results show that these three correlations are all insignificant, confirming that our model handles the pile-up issue related to spurious correlation.

[9] Additionally, our results of both asymmetric and symmetric univariate UC models applied to unemployment show that the correlation between shocks to the trend and the cyclical components of unemployment is insignificant.

3.2. The univariate model

In addition to the bivariate model, we run the univariate trend-cycle decomposition, in which a single variable of interest, either output or the unemployment rate, is decomposed into a trend and a cyclical component, as specified in Eq. (13):

$$z_t = z_t^* + z_t^c \tag{13}$$

where the observed series is denoted by z_t. Accordingly, z_t^* and z_t^c are unobserved trend and cyclical components. If we intend to decompose the log level of output, z_t^* and z_t^c play the roles of potential output and the output gap; and likewise, if we decompose the unemployment rate, z_t^* and z_t^c play the roles of the natural rate of unemployment and the unemployment gap, respectively.

3.2.1. The trend component

For U.S. output, we consider that the trend component is a random walk process with a drift:

$$z_t^* = \mu_{t-1} + z_{t-1}^* + \varepsilon_{z^*,t} \tag{14}$$

where $\varepsilon_{z^*,t} \sim N(0, \sigma_{z^*}^2)$ denotes the output trend shock and is assumed to be white noise and normally distributed like all other shocks. Similar to Section 3.1.1, μ_t is the drift term and is specified in two alternative ways to capture the time-variation in U.S. output trend growth. Firstly, we can model the output trend growth as a stochastic drift in the form of a random walk process, specified in Eq. (4.a), and secondly, we model the trend growth as a non-stochastic drift with a structural break, specified in Eq. (4.b).

For U.S. unemployment, it is observed that a measure of the natural rate of unemployment named NAIRU, has gradually increased from 5% in the 1950s to 6% in the 1980s, and thereafter it has been decreasing to the level of 4.5% until now. Therefore, we model the unemployment trend as a random walk with a drift term, which accounts for the potential structural break in the following manner:

$$z_t^* = \eta + \theta \mathbb{1}_t(t \geq T_{u^*}) + z_{t-1}^* + \varepsilon_{z^*,t} \tag{15}$$

where $\varepsilon_{z^*,t} \sim N(0, \sigma_{z^*}^2)$ stands for the shock to the unemployment trend and accounts for any potential stochastic variation in the unemployment trend.[10] Under this equation, $\mathbb{1}_t$ is an indicator function that takes the value of one after the break date (T_μ) and zero otherwise, and thus η is the drift before and $\eta + \theta$ is the drift after the break date. For references to studies that support the above specification for the unemployment trend, see Section 3.1.1 in the bivariate model.

[10] Our estimation results indicate that if the structural break in the unemployment trend is allowed for, the variance of the shock to the unemployment trend is near zero. On this basis, one could remove the unemployment trend shock by assuming that its variance is zero ($\sigma_{z^*}^2 = 0$).

3.2.2. The cyclical component

To accommodate asymmetric shocks, we incorporate a Markov switching process into the cycle and to allow for the possible high persistence of the cyclical component, we model it as an AR(2) process as follows:

$$z_t^c = \pi_z S_t + \varphi_1 z_{t-1}^c + \varphi_2 z_{t-2}^c + \varepsilon_{z^c,t} \tag{16}$$

where φ_1 and φ_2 are coefficients of the AR(2) process, whose sum ($\varphi_1 + \varphi_2$) is expected to be less than one, and π_z is the plucking coefficient measuring the amplitude of the asymmetric shocks. A significant π_z, which is expected to be negative for output and positive for the unemployment rate, confirms Friedman's plucking property for each of the above indicators separately. In this setup, the state of the economy (S_t) is zero during normal times and one during recessions and follows a first-order, and two-state Markov-switching process of Hamilton (1989) specified in Eq. (7) and Eq. (8). For the usual symmetric shock to the cyclical component in Eq. (16), we assume that $\varepsilon_{z^c,t} \sim N(0, \sigma_{z^c}^2)$ has a constant variance. However, for robustness tests of the model applied to output, its variance ($\sigma_{z^c}^2$) is allowed to be different before and after the great moderation as follows:

$$\sigma_{z^c}^2 = \sigma_{z^c,0}^2 \mathbb{1}_t(t \leq T_\sigma) + \sigma_{z^c,1}^2 \mathbb{1}_t(t \geq T_\sigma) \tag{17}$$

where $\mathbb{1}_t$ is an indicator function to capture the potential break in the variance of symmetric shocks to the output cyclical component, which is equal to $\sigma_{z^c,0}^2$ before and $\sigma_{z^c,1}^2$ after the break date (T_σ).

3.2.3. The variance-covariance matrix of shocks

Finally, the variance-covariance matrix of shocks is represented in Eq. (18):

$$\begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{\mu,t} \\ \varepsilon_{z^c,t} \end{bmatrix} \sim N\left(\mathbf{0}_{3\times 1}, \begin{bmatrix} \sigma_{z^*}^2 & 0 & \rho_{z^*,z^c}\sigma_{z^*}\sigma_{z^c} \\ 0 & \sigma_\mu^2 & 0 \\ \rho_{z^*,z^c}\sigma_{z^*}\sigma_{z^c} & 0 & \sigma_{z^c}^2 \end{bmatrix}\right) \tag{18}$$

Concerning the correlation in the proposed model, we maintain the assumption that all shocks are uncorrelated. For the model applied to output, we allow for correlation between shocks to the output trend and cyclical components (ρ_{z^*,z^c}) in robustness tests to investigate the possibility of non-zero correlation, as suggested by Morley et al. (2003) and Sinclair (2010). For those models applied to unemployment, since the natural rate represents structural unemployment that exists independently of all temporary and seasonal fluctuations (Phelps, 1967; Friedman, 1968), the correlation between shocks to the unemployment trend and cyclical components must be zero by definition. Nevertheless, we allow for this correlation (ρ_{z^*,z^c}) in the robustness tests to verify that it is indeed insignificant.

4. Results and discussion

We estimate twenty-two bivariate models, fourteen univariate models for output, and five univariate models for unemployment. We denote each model with an identifier and a descriptor. The descriptor consists of five parts, four of which express specification aspects for output and one of which denotes the specification for unemployment. For example, the identifier and descriptor of our proposed model are 1a and A-Bi-RW-SB-UC, which means that this model is **A**symmetric and **Bi**variate. The output trend growth is a **R**andom **W**alk, the natural rate of unemployment has a **S**tructural **B**reak, and finally, the model is **U**n**C**orrelated as the correlation between shocks is presumed to be zero. The full list of identifiers and descriptors of other models is presented in Tables B1, B2, and B3 in Appendix B.

In Section 4.1, we discuss the results of the bivariate models. To test for asymmetry, we compare the results of the proposed model 1a with those of its symmetric counterpart reported in column 4a of Table 1. Respecting time-variation in U.S. output trend growth, in addition to model 1a, which allows for stochastic trend growth, we implement model 2a, which accounts for a structural break in output trend growth, and model 3a, which imposes constant trend growth. Concerning the correlation, we execute models 1b, 2b, and 3b that are correlated versions of models 1a, 2a, and 3a, respectively. We also present the result of structural break tests and robustness tests for bivariate models in this section. In Sections 4.2 and 4.3, we present the findings of univariate models for output and unemployment. In Section 4.4, by extending the estimation period up to 2022Q4 and modifying the proposed model, we explore the COVID-19 recession as an epitome of the plucking property. We finally report the results for two additional series, U.S. output per capita and U.K. output in Section 4.5.

4.1. Results of the bivariate models

The results of the asymmetric bivariate model substantiate the asymmetric co-fluctuations of U.S. output and unemployment. As presented in Table 1, the Okun's law coefficient is $\beta = -1.45$ with a standard error of 0.12, implying that a 1% gap in unemployment is accompanied by a 1.45% gap in output. This supports their co-fluctuation, proving that fluctuations in U.S. output and unemployment are indeed synchronous and proportional. Furthermore, these co-fluctuations are asymmetric because the estimated plucking coefficient is $\pi_u = 0.70$ with a standard error of 0.06, and the product of two coefficients ($\beta \times \pi_u = -1.01$) gauges the plucking property in output. Given that the labour market is identified as the source of the plucking property (Ferraro, 2018; Dupraz et al., 2019; Ferraro and Fiori, 2023), our findings advocate the transmission of the plucking property from the unemployment rate to output. The top-left and bottom-left panels of Figure 1 depict potential output as a ceiling for output and the natural rate implied by our model (ZOGRU) as a floor for unemployment, which are estimated jointly. The top-right panel displays that estimated output gaps are deep, often negative, and rarely positive; and likewise, the bottom-right panel shows that unemployment gaps are large in amplitude, often positive, and rarely negative, which support the plucking property and the ceiling effect, proposed by Friedman (1993), in both economic indicators.

Moreover, the estimated transition probability reported in column 1a of Table 1 is low for recessions ($p = 0.660$) and is high for recoveries ($q = 0.965$). Thus, the expected duration is around 3 quarters for recessions and 28 quarters for recoveries. The sum of autoregressive coefficients estimated for the cyclical component ($\varphi_1 + \varphi_2$) is 0.93, suggesting a relatively persistent cyclical component and gradual recoveries. Hence, altogether, we highlight that the co-fluctuations of U.S. output and the unemployment rate are asymmetric in amplitude, speed, and duration, which implies that deep, steep, and transitory recessions will be followed by commensurate, gradual, and permanent recoveries. We now assess the four specification aspects in detail in the following subsections:

4.1.1. The first specification aspect (asymmetry vs. symmetry)

To substantiate asymmetric fluctuations, we compare the log likelihood of -11.9 for the asymmetric bivariate model in column 1a of Table 1 with that of -57.5 for its symmetric counterpart in column 4a. This comparison yields a likelihood ratio of 91.2, which is greater than the critical value of 10.8 for a conservative 0.1% significance level.[11] We, therefore, document that the plucking coefficient is significant and shocks to the cyclical component are asymmetric. This finding remains valid for other asymmetric bivariate models, such as 2a and 3a,[12] and accords with the results of asymmetric univariate models presented by Kim and Nelson (1999a), Sinclair (2010), and Eo and Morley (2022), who note the presence of asymmetric fluctuations in U.S. output.

4.1.2. The second specification aspect (bivariate vs. univariate models)

The resemblance of the results obtained from the proposed model presented in Figure 1 to those of its correlated counterpart shown in Figure D1 in Appendix D confirms that the asymmetric bivariate model generates cyclical components with substantial amplitude no matter whether the correlation is involved in the model or not. This casts doubt on the results of several symmetric univariate models applied by Beveridge and Nelson (1981), Nelson and Plosser (1982), Morley et al. (2003), Grant and

[11] In the presence of a Markov-switching process, testing hypotheses based on the likelihood ratio statistics is non-standard as the nuisance parameter is not identified under the null hypothesis, and consequently the asymptotic distribution of the likelihood ratio test is unknown and does not follow the standard χ^2 distribution. Few papers propose computationally burdensome simulation-based or bootstrap-based methods to test for Markov-switching that are operable for simple models (see, e.g., Hansen, 1992; Garcia, 1998; Di Sanzo, 2009). Because of the large dimension of our models and the forty-one different models estimated in this study, we maintain the use of the non-standard likelihood ratio test. Also, exceptionally large likelihood ratios derived for testing asymmetry in this study leave very little, if not no, doubt that co-fluctuations are asymmetric.

[12] By comparing the log likelihoods of -7.4 and -19.9 for asymmetric models 2a and 3a shown in Table 1 with values of -52.6 and -62.5 for their symmetric counterparts 5a and 6a, we favour the asymmetric models over symmetric models because the corresponding likelihood ratios are 90.4 and 85.2.

Chan (2017b), Kim and Chon (2020), and Kim and Kim (2022), implying that the variation in output is almost entirely driven by the supply-related trend and that the demand-related cyclical component is tiny and noisy.

We also investigate the possibility of Okun's law instability by comparing the log likelihood of model 8, which accounts for a potential break in Okun's coefficient, with the log likelihood of our proposed model 1a, where the Okun's coefficient is presumed to be stable. The top-left panel of Figure 2 plots the corresponding likelihood ratios for a sequence of breaks in the Okun's coefficient rolling from 1960 to 2010. Likelihood ratios are less than any reasonable threshold, such as QLR critical values of 8.9 and 7.2 for 5% and 10% significance levels. Therefore, we conclude that Okun's law is stable as long as the model heeds key specification aspects, such as the asymmetric cyclical component and stochastic trend growth. This result is in line with the findings of Galí et al. (2012), Daly et al. (2014), Ball et al. (2017) and Michail (2019), among others, while it is in opposition to Berger et al. (2016) and Grant (2018), among a few others.

4.1.3. The third specification aspect (stochastic vs. deterministic trend growth)

The results of models 1a and 2a provide persuasive evidence for time-variation in output trend growth in the U.S. in the form of both a gradual decline that began in the 1960s and a sharp structural break following the 2007–09 financial crisis. The middle-left panel of Figure 1 plots the dynamics of the trend growth over time: Annual trend growth gradually declined from about 4% in the 1960s to 2.6% in the mid-2000s, and then it fell from 2.6% to an unprecedented rate of 1.2% in the aftermath of the 2007–09 financial crisis. This flags up an unusual annual shortfall of 1.3 percentage points following the financial crisis by comparing the actual trend growth with long-run extrapolations from 1990 or 2009 as two counterfactuals.

Moreover, we explore an unknown break in U.S. output trend growth by rolling the break date in the central 70% of the sample. The middle-left panel of Figure 2 plots the corresponding likelihood ratio values for a sequence of structural breaks from 1960 to 2010 and exhibits two distinct episodes. First, from the mid-1960s to the late-1990s, a lot of moderately significant breaks occurred repeatedly in every period, which hints at the gradual decline in trend growth. Second, after the late-1990s, the likelihood ratios are highly significant and peak markedly twice in a row in 2006 and 2010, hinting at an unprecedented deceleration in U.S. potential output, which is displayed in the top-right panel of Figure D2 in Appendix D.

Comparing two competing specifications for trend growth, the bottom-left panel of Figure 2 shows that the random walk performs better than almost all models with a structural break date before 2000 and is also close to the best models with a selected break date near the 2007–09 financial crisis.

4.1.4. The fourth specification aspect (uncorrelated vs. correlated models)

Including both the plucking property (asymmetry) and Okun's law (co-fluctuation) in the asymmetric bivariate model makes the correlation irrelevant, which means that all correlations are insignificant, and the features of the trend and cyclical components are robust to the assumption about correlations. For instance, by comparing the log likelihood of -11.9 for the proposed model in column 1a with that of -11.4 reported for its correlated counterpart in column 1b of Table 1, we accept the null hypothesis that correlation between the trend and symmetric cyclical shocks (ρ_{x^*,u^c}) is zero, as the likelihood ratio of 1.0 is less than the critical values of 3.84 and 2.71 for 5% and 10% significance levels. This finding remains unchanged for the other asymmetric bivariate models 2a and 3a and for testing the other two correlations (ρ_{x^*,x^c} and ρ_{u^c,x^c}).[13] By contrast, the correlation between shocks is significant and affects the estimation of parameters and components in the symmetric bivariate model and the asymmetric univariate model.[14] Combining these results, we state that insignificant correlation can be achieved by accounting for both asymmetry and co-fluctuations, although including only one of them helps to alleviate the sensitivity of the results to the correlation between shocks, which has previously been reported by Gonzalez-Astudillo and Roberts (2022).

4.1.5. Structural breaks and robustness tests

We explore the robustness of our findings by estimating alternative models, each of which accounts for a rolling structural break in one of the following parameters: the Okun's coefficient, the volatility of shocks to the output cyclical component, output trend growth, and the drift in the unemployment trend. To find the unknown break date, we truncate the first and last ten years (15%) of the sample. We then sequentially estimate the log likelihood values for the model with a potential break, whose date rolls from 1960Q1 to 2010Q1 within the whole sample from 1950Q1 to 2019Q4. We compute the likelihood ratios by comparing the estimated log likelihood values for the unrestricted model with the value of the restricted model, where the break is unaccounted for. By comparing the supremum of likelihood ratios with a reasonable threshold, such as the Quandt Likelihood Ratio (QLR) critical values presented in Andrews (1993), we finally detect structural breaks.[15]

Each panel in Figure 2 illustrates likelihood ratios for a sequence of breaks in one of the parameters mentioned above. The top-left panel, as discussed before, dismisses the instability in the gap version

[13] The likelihood ratios for testing any other correlations are all close to 0.0, which are reported in note 5 of Table 1 and notes 4 and 5 of Table D1 in Appendix D.

[14] For the symmetric bivariate model, as stated in note 6 of Table D1 in Appendix D, we reject the null hypothesis of zero-correlation. Similarly, for the asymmetric univariate model, as explained in Section 4.2 and note 5 of Table 2, we reject the null hypothesis of zero-correlation between shocks.

[15] Since we take the supremum of log likelihood ratios the critical values to test for the unknown structural break are considerably larger than those of the usual likelihood ratio test. In this sense, we use a conservative 1% QLR critical value given that we apply the approximate, rather than exact, maximum likelihood method.

of Okun's law since likelihood ratios are less than the 5% QLR critical value. The top-right panel detects a break in the volatility of shocks to the remaining cyclical component in 1983, which is close to the break date of 1982 reported by Eo and Morley (2022) and hints at the great moderation—the decrease in the volatility of shocks to output that begun in the mid-1980s. In particular, the results of model 7 presented in Table D1 in Appendix D show that the volatility is equal to $\sigma^2_{x^c,0} = 0.55$ before the break and $\sigma^2_{x^c,1} = 0.04$ after the structural break. The middle-left panel identifies two sources of instability in U.S. trend growth: a gradual decline started in the 1960s, and a structural break occurred in 2009.

The middle-right panel reports a significant break in the drift of the natural rate of unemployment (ZOGRU) in 1981, which is accounted for in our proposed model. Accordingly, the bottom-left panel of Figure 1 depicts an increase in ZOGRU from 3% in the 1950s to 6% in the 1980s, followed by a gradual decrease to levels around 4.5% until now. Our ZOGRU estimation is similar to the estimated NAIRU, which measures the natural rate of unemployment, provided by Ball and Mankiw (2002), Semmler and Zhang (2006), and Basistha and Startz (2008), who document its rise and decline before and after the 1980s. Furthermore, since the estimated standard deviation of shocks to ZOGRU is not significant ($\sigma_{u^*} = 0.0004$ with a standard error of 0.02), the unemployment trend does not involve a stochastic element, and its variation could be explained by estimating a drift term with a structural break.

Moreover, the middle-left panel of Figure D2 in Appendix D illustrates the small depth of the leftover plucking property in the Okun's residuals, which is attributable to the short lead-lag effect between output and unemployment.[16] Finally, our results are robust to choices made on the third and fourth specification aspects. For instance, the finding of asymmetry in business cycles stands up in models 1a, 2a, and 3a, no matter what the specification for U.S. trend growth is, and holds true for correlated models 1b, 1c, 1d, 1e, 2b, 2c, 3b, and 3c independent from the assumption about correlation. Also, regardless of the specifications for trend growth and correlation, the estimated Okun's coefficient is around -1.4 for most asymmetric models and is around -1.7 for most symmetric models.

4.2. Results of the univariate models for output

To confirm the asymmetric fluctuations of U.S. output, we compare the log likelihood of -336.9 for the asymmetric univariate model 1a with the value of -354.4 for its symmetric counterpart reported in column 4a of Table 2 and obtain a likelihood ratio of 35.0, which is greater than the 0.1% critical value of 10.8. The estimated plucking coefficient is $\pi_z = -1.67$ with a standard error of 0.19. This

[16] See the explanation in Appendix D for further discussion.

finding stands up in other asymmetric univariate models, restates the results of asymmetric bivariate models in this study, and accords with the results of univariate models in earlier studies by Kim and Nelson (1999a) and Sinclair (2010). The transition probabilities are $p = 0.644$ and $q = 0.956$, and the expected duration is about 3 quarters for recessions and about 23 quarters for recoveries, which are close to those estimated in the asymmetric bivariate model. As shown in the top-right panel of Figure 3 output gaps are deep, often negative, and rarely positive. In addition, the estimated sum of autoregressive coefficients for the cyclical component ($\varphi_1 + \varphi_2$) is 0.76 for model 1a, implying a moderate cyclical persistency in output.

4.3. Results of the univariate models for unemployment

The results in Figure 3 and Table 3 are derived from the asymmetric univariate model applied to the unemployment rate. The estimated plucking coefficient ($\pi_z = 0.75$) is close to $\pi_u = 0.70$ estimated in the asymmetric bivariate model. To confirm the asymmetry in fluctuations of U.S. unemployment, we compare the log likelihood of -4.9 for the asymmetric univariate model 1a with the value of -36.7 for its symmetric counterpart 2a and report a likelihood ratio of 63.6. The transition probabilities are $p = 0.63$ for recessions and $q = 0.97$ for recoveries, which are almost equal to those estimated by the asymmetric univariate and asymmetric bivariate models. The sum of autoregressive coefficients for the unemployment cyclical component ($\varphi_1 + \varphi_2$) is 0.94, which is greater than the 0.76 estimated for the output cyclical component and implies a higher persistency in unemployment. Comparing the middle-left and middle-right panels of Figure 3 suggests a remarkable resemblance between the plucking probabilities in U.S. output and the unemployment rate estimated by univariate models applied separately to these two indicators.

Both asymmetric and symmetric univariate models applied to the unemployment rate are robust to the assumption about the correlation between shocks to the trend and cyclical components simply because this correlation is estimated to be near zero. Comparing the log likelihoods of -4.9 and -36.7 for uncorrelated models 1a and 2a with those of -4.9 and -36.4 for their correlated counterparts 1b and 2b accepts the null hypothesis of zero-correlation because the corresponding likelihood ratios of 0.0 and 0.8 are negligible. This is consistent with the definition of the natural rate of unemployment, as it represents the structural unemployment rate that exists independently of all cyclical fluctuations (Phelps, 1967; Friedman, 1968). In fact, U.S. unemployment inherently encompasses cyclicality in the form of plucking property. This desirable feature introduces this indicator as a reliable proxy for measuring business cycles and a straightforward auxiliary to be included within a bivariate model to facilitate the trend-cycle decomposition of U.S. output.

4.4. Exploring the COVID-19 recession

In this section, we modify the main proposed model to account for the COVID-19 recession, which exemplifies the plucking property because it severely constrained output, led to spare capacity, and created a deep output gap in the U.S. and other economies. A sharp jump in the unemployment rate from 3.8% in 2020Q1 to 13% in 2020Q2 and a proportional steep pluck-down in output identify the COVID-19 recession as the deepest and shortest recession among post-World War II recessions in the U.S. As a result, Eq. (6) that applies a single plucking coefficient (π_u) for all recessions is unable to account for the unprecedented depth and duration of the COVID-19 recession. To deal with this issue, we follow a simple approach by adding a dummy variable for the COVID-19 pandemic and estimating the COVID-specific plucking coefficient ($\pi_{u,COVID}$) as follows:

$$u_t^c = \pi_u S_t + \pi_{u,COVID} \times \mathbb{1}_t(T_{COVID-start} \le t \le T_{COVID-end}) + \varphi_1 u_{t-1}^c + \varphi_2 u_{t-2}^c + \varepsilon_{u^c,t} \tag{19}$$

where $\mathbb{1}_t$ is an indicator that takes one during the COVID-19 pandemic and zero otherwise. We set the start of the COVID-19 pandemic ($T_{COVID-start}$) at 2020Q1 and the end ($T_{COVID-end}$) at 2021Q1, with unemployment remaining above 6%. By extending the estimation period up to 2022Q4, the model gauges 8.5% and 11% gaps in unemployment and output during the COVID-19 pandemic. The estimated components and parameters, presented in Figure 4 and Table D4 in Appendix D, are similar to those of the main model. However, the common plucking coefficient ($\pi_u = 1.33$) is larger than that of the proposed model to explain a portion of the greater depth of the COVID-19 recession compared to the depth of previous recessions. The COVID-specific plucking coefficient ($\pi_{u,COVID} = 1.38$ with a standard error of 0.32) is also remarkable because it helps the model explain the rest of the extra depth of the COVID-19 recession. In total, the additional plucking property during the COVID-19 recession is captured by both a larger common π_u and a sizeable $\pi_{u,COVID}$.[17]

4.5. Results for U.S. output per capita and U.K. output

In addition to U.S. real output, we apply the asymmetric bivariate model to two other macroeconomic time series: U.S. output per capita to demonstrate that its annual trend growth in the aftermath of the financial crisis is lower than 1%, and U.K. output to provide international evidence for asymmetric co-fluctuations, with the results shown in Table D5 in Appendix D. For U.S. output per capita, we report the same plucking coefficient, Okun's coefficient, and transition probabilities as reported for output. Based on Figure D3 in Appendix D, the components have very similar features as derived for output, and the annual trend growth of 0.7% for U.S. output per capita is incredibly low. For U.K.

[17] In our approach presented in Eq. (19), $\pi_{u,COVID}$ captures the additional plucking property during the COVID-19 pandemic. Another approach to modelling the plucking property during the COVID-19 pandemic is to incorporate two independent Markov-switching processes, one for previous recessions and another for the COVID-19 recession, whose depth is around double the average of previous recessions.

output, the estimated plucking coefficient and Okun's law coefficient are $\beta = -1.53$ and $\pi_u = 0.23$ with standard errors of 0.35 and 0.05, respectively. The estimated transition probabilities for U.K. output are $p = 0.88$ and $q = 0.97$, which indicate that although the amplitude of the U.K. plucking property is milder than that of the U.S., its recession duration is longer.

5. Concluding remarks

By embedding output and unemployment in a bivariate state-space model with a Markov-switching process, we integrate Friedman's plucking model and Okun's law. Estimating substantial plucking and Okun's law coefficients ($\pi_u = 0.70$ and $\beta = -1.45$) establishes the asymmetric co-fluctuations, stating that fluctuations in output and the unemployment rate are synchronously and proportionally characterized by the plucking property. Given that recent studies identified the labour market as the source of the plucking property, this study highlights the transmission of the plucking property from unemployment to output.

Our model also sheds light on four specification aspects of trend-cycle decomposition. The plucking property and ceiling effect are remarkable in both indicators because gaps are large in magnitude and often negative for output and positive for unemployment. By capturing business cycle asymmetries, our empirical findings indicate that recessions are deep, steep, and transitory and will be followed by commensurate, gradual, and permanent recoveries. Additionally, we demonstrate that the gap version of Okun's law is stable provided that asymmetric fluctuations and stochastic trend growth are both accommodated. We also document a gradual decline in trend growth that started in the 1960s as well as an unprecedented deceleration in U.S. potential output in the aftermath of the 2007−09 financial crisis. Furthermore, the asymmetric bivariate model that includes both Friedman's plucking property (asymmetry) and Okun's law (co-fluctuation) yields robust results with an insignificant correlation.

Moreover, by jointly estimating the trends of output and the unemployment rate and accounting for the plucking property in both indicators, our model provides a counterpart for the Non-Accelerating Inflation Rate of Unemployment (NAIRU) to measure the natural rate of unemployment. We call this new measure the Zero Output Gap Rate of Unemployment (ZOGRU), the unemployment rate at which the output gap is zero.

Concerning limitations, we impose a constant plucking coefficient for all recessions, while the depth of each recession differs from the others. The effect of this assumption, however, is moderate since the model has enough flexibility to adjust the duration of the state of the economy for an individual recession to capture its special depth. Further, other potential sources of asymmetry that are not taken into account, such as binding borrowing constraints, liquidity shortages, and credit crunches, open avenues for future research.

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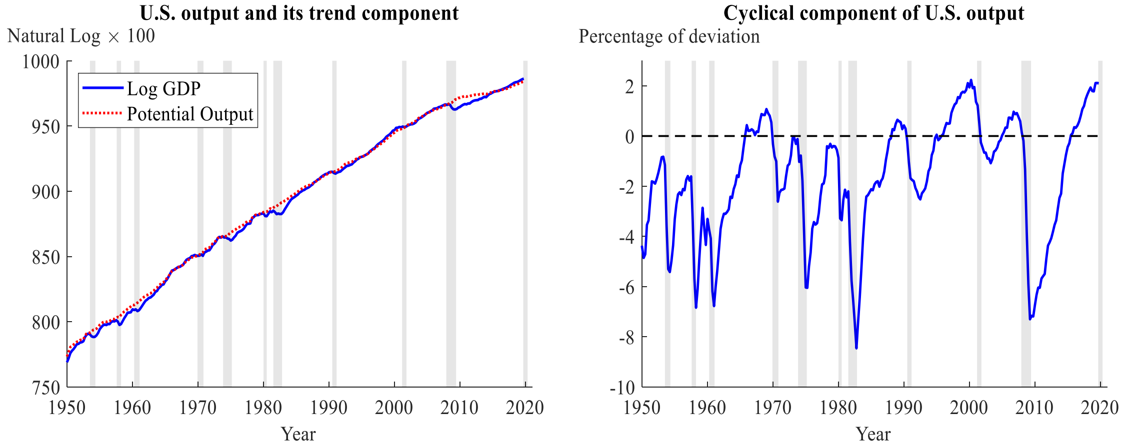
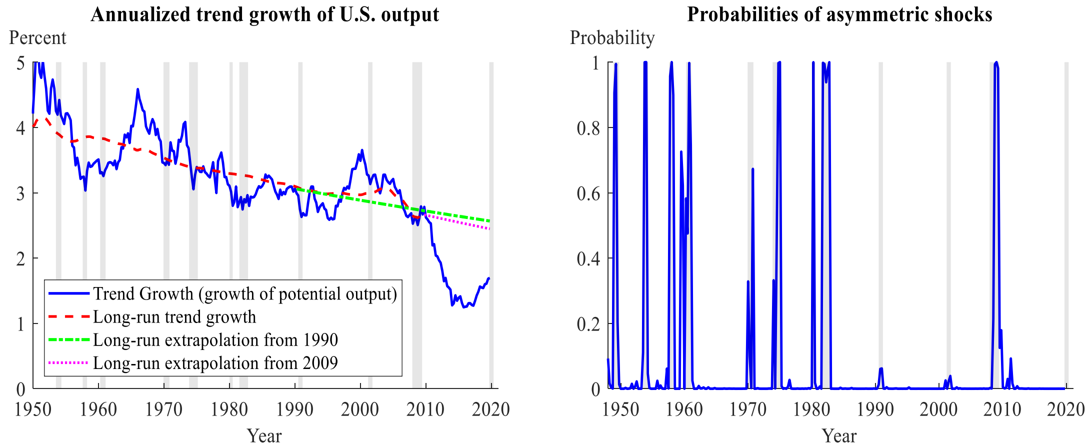
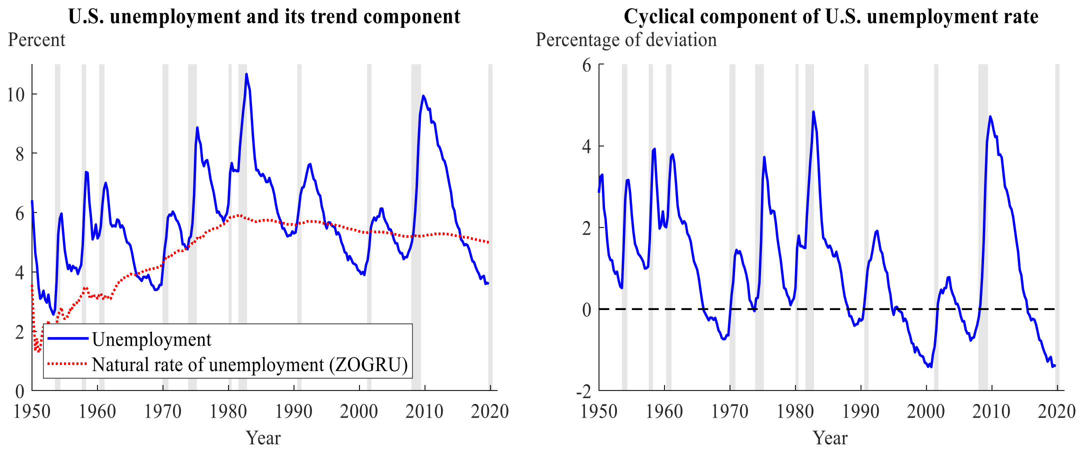
Figures



(a) Potential output (trend) and output gap (cyclical component)



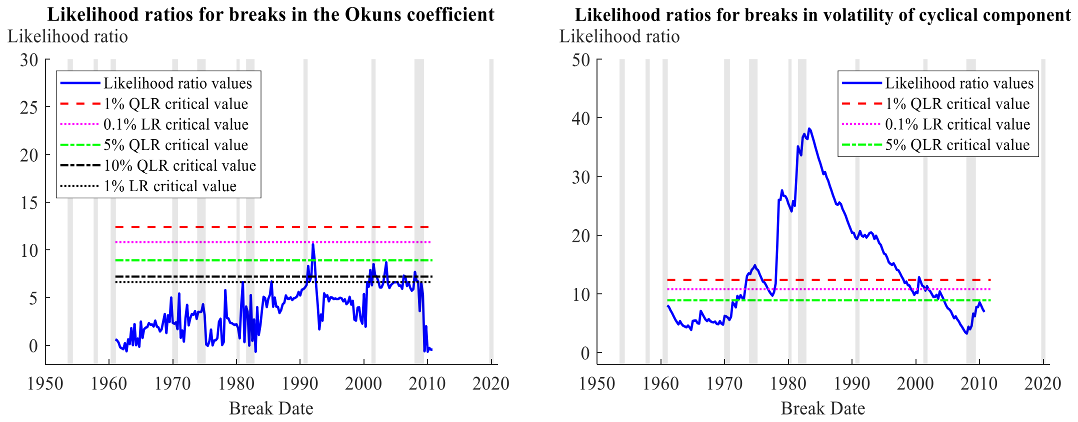
(b) Trend growth of output and the plucking probabilities for bivariate model



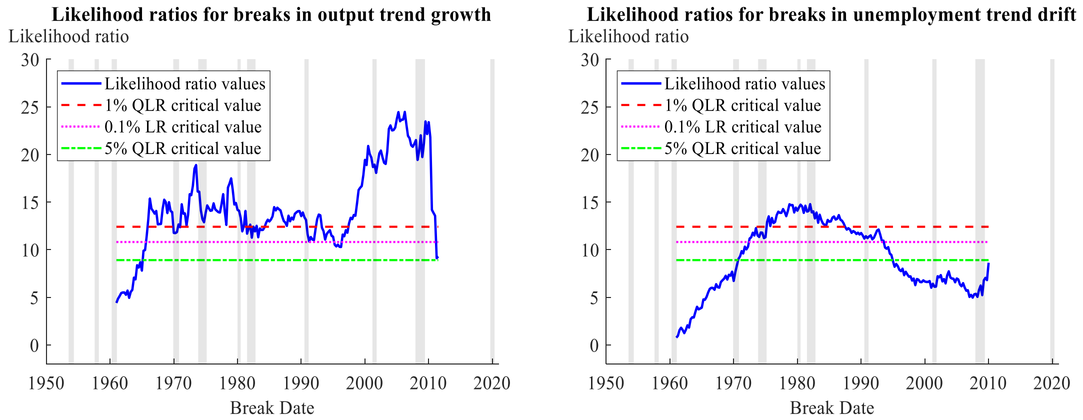
(c) Natural rate of unemployment (trend) and unemployment gap (cyclical component)

Figure 1: Results of the asymmetric bivariate model

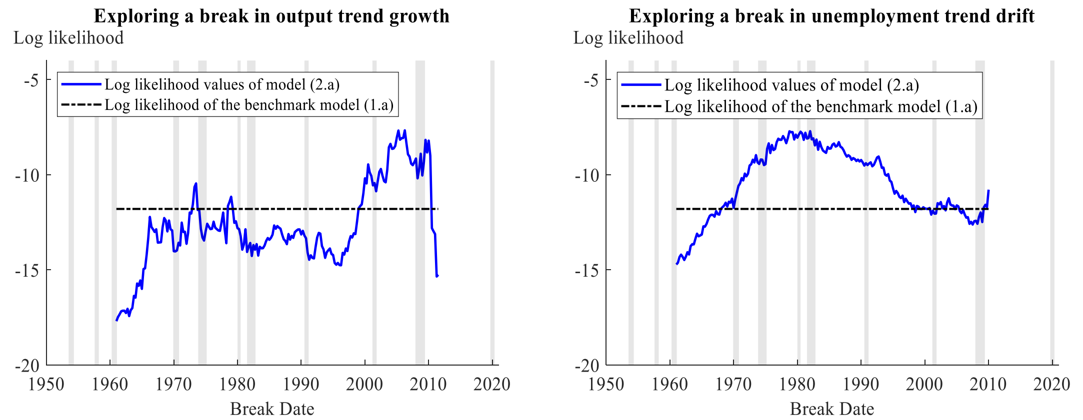
Notes:
(1) All panels plot the results of our proposed model (Asymmetric-Bivariate-RW-SB-UC).
(2) The top panels plot potential output and the output gap, and the middle-left panel plots the trend growth of output.
(3) The middle-right panel plots the plucking probabilities, which are estimated for both output and unemployment jointly.
(4) The bottom panels plot the trend and gap for unemployment.
(5) The shaded areas are the NBER recession dates. See Table E1 in Appendix E for details.



(a) Likelihood ratios at different break dates for Okun's law coefficient (left) and volatility (right)



(b) Likelihood ratios at different break dates for output trend growth (left) and unemployment trend drift (right)



(c) Log likelihood values of models 1a and 2a

Figure 2: Exploring structural breaks in the parameters for asymmetric bivariate model

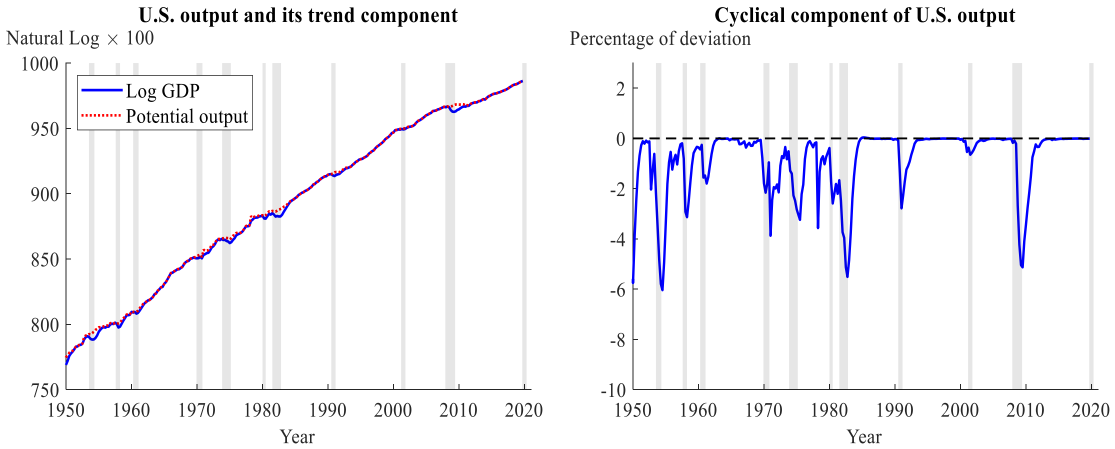
Notes:

(1) All panels plot the log likelihood values for the model with a break whose date rolls from 1960 to 2010.

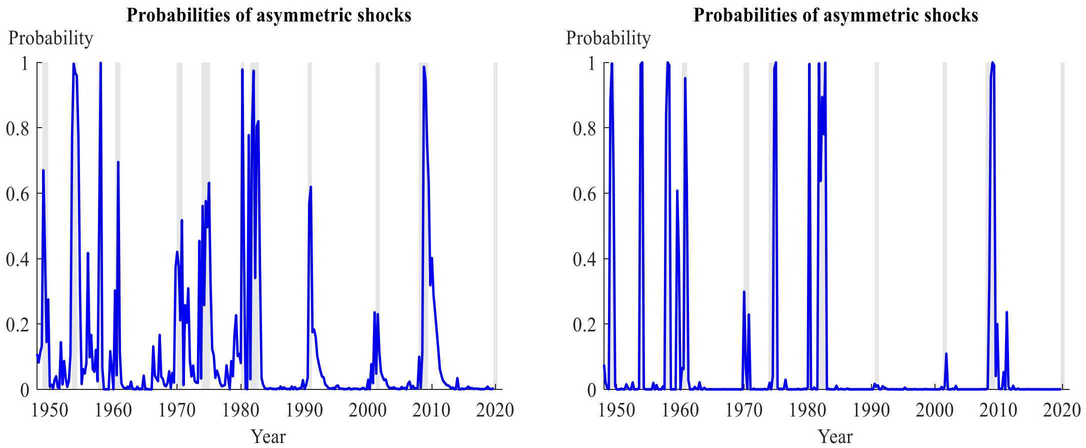
(2) The top-left panel plots the likelihood ratios for breaks in Okun's law, given the setup of the proposed model 1a. Since likelihood ratios are less than 5% QLR critical values, we rule out instability of Okun's law. In addition, likelihood ratios are even less than a 1% LR critical value of 6.63, which itself is less than the suitable critical value for the supremum of likelihood ratio among a sequence of breaks. The top-right panel plots the likelihood ratios for breaks in volatility of shocks to the remaining cyclical component on different dates.

(3) The middle panels plot likelihood ratio test statistics. In the middle-left panel, likelihood ratio compares the log likelihood value of model 2a with a break in trend growth with that of its counterpart model 3a with constant trend growth. The middle-right panel plots the likelihood ratios testing for structural breaks in the drift of the unemployment trend (ZOGRU) on different dates against a constant trend. The bottom-left panel shows the log likelihood values to detect a break in trend growth, conditioned on the setup of the model 2a. The bottom-right panel shows the log likelihoods to detect the break in the drift of the unemployment trend (ZOGRU), conditioned on a break in output trend growth in 2009. In both panels, the black dashed line represents the log likelihood of our proposed model, which specifies the trend growth as a random walk.

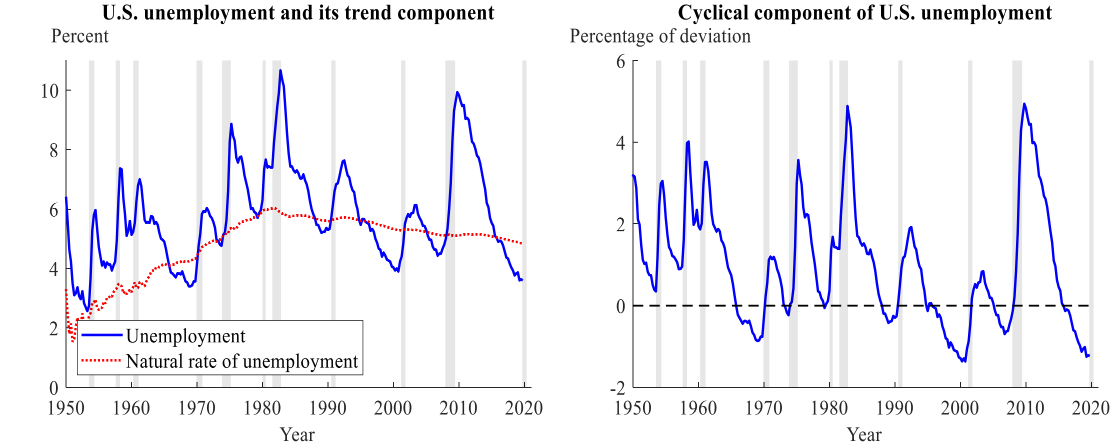
(4) The shaded areas are the NBER recession dates.



(a) Potential output (trend) and output gap (cyclical component)



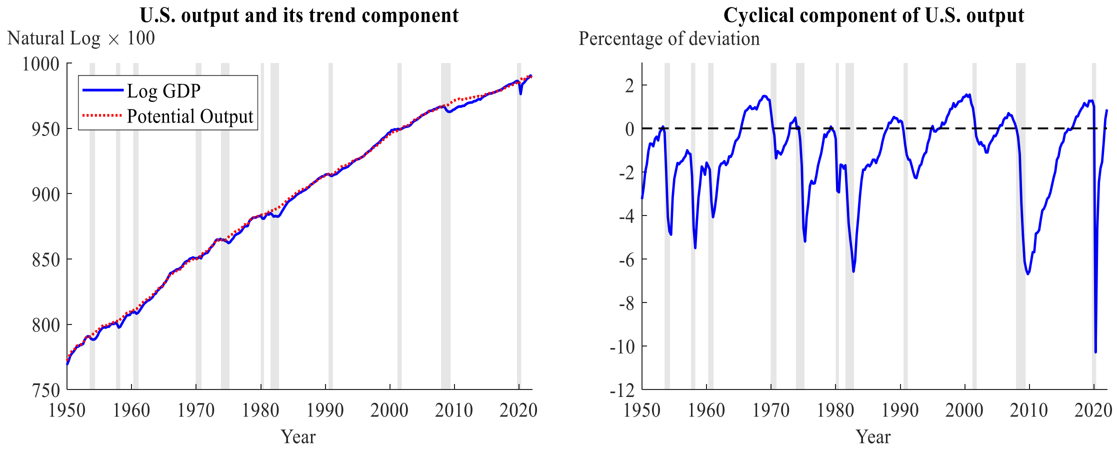
(b) Plucking probabilities for output (left) and unemployment (right)



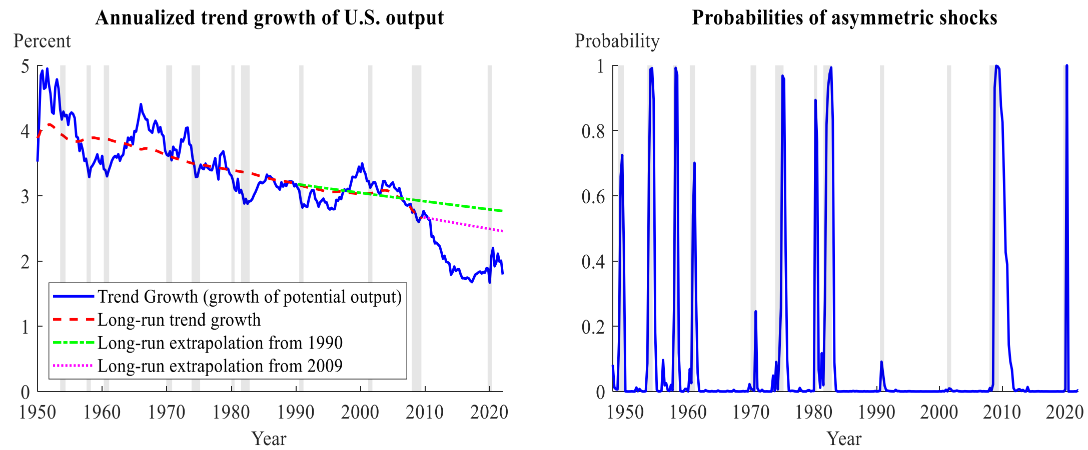
(c) Natural rate of unemployment (trend) and unemployment gap (cyclical components)

Figure 3: Comparing the results of the asymmetric univariate models for output and unemployment

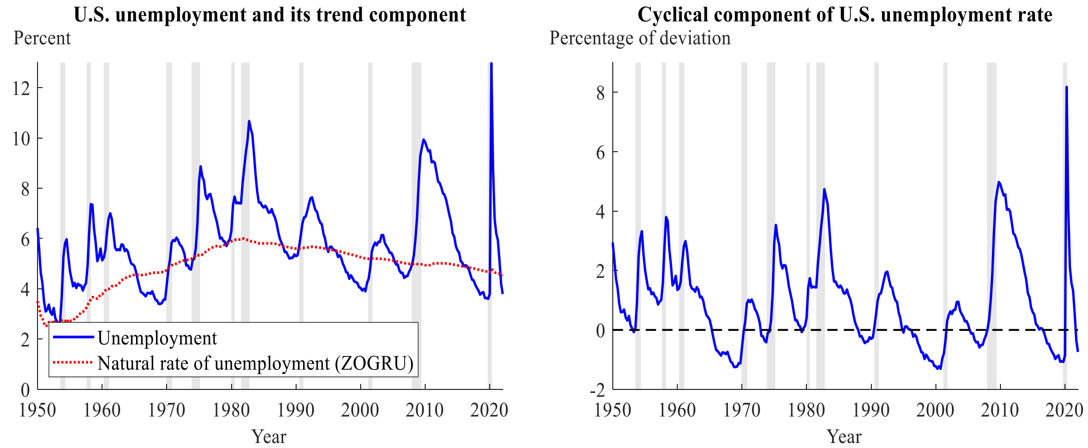
Notes:
(1) The top panels plot the results of the asymmetric univariate model for output with a stochastic (random walk) trend growth where shocks to the trend and cyclical components are uncorrelated (Asymmetric-Univariate-RW-UC). This model replicates the work of Kim and Nelson (1999a), and its correlated version is presented in Figure D4 in Appendix D.
(2) The middle panels plot the plucking probabilities for output and unemployment estimated separately.
(3) The bottom panels plot the results of the asymmetric univariate model for unemployment with a break in the drift of the unemployment trend where shocks to the trend and cyclical components are uncorrelated (Asymmetric-Univariate-SB-UC).
(4) The shaded areas are the NBER recession dates.



(a) Potential output (trend) and output gap (cyclical component)



(b) Trend growth of output and the plucking probabilities for bivariate model



(c) Natural rate of unemployment (trend) and unemployment gap (cyclical component)

Figure 4: Results of the asymmetric bivariate model, including the COVID-19 recession

Notes:

(1) All panels plot the results of the modified model (Asymmetric-Bivariate-RW-SB-UC-Mod), which is estimated based on Eq. (19) for the period of 1948Q1 to 2022Q4 to include the COVID-19 recession.

(2) The top panels plot potential output and the output gap, and the middle-left panel plots the trend growth of output.

(3) The middle-right panel plots the plucking probabilities, which are estimated for both output and unemployment jointly.

(4) The bottom panels plot the trend and gap for unemployment.

(5) The shaded areas are the NBER recession dates.

Tables

Table 1: Estimated parameters of the bivariate models

Models	1a	1b	2a	2b	3a	4a	5a	6a
Parameters	A-Bi-RW-SB-UC	A-Bi-RW-SB-C1	A-Bi-SB-SB-UC	A-Bi-SB-SB-C1	A-Bi-Con-SB-UC	S-Bi-RW-SB-UC	S-Bi-SB-SB-UC	S-Bi-Con-SB-UC
σ_{x^*}	0.44 (0.09)	0.39 (0.10)	0.51 (0.06)	0.51 (0.06)	0.62 (0.02)	0.56 (0.11)	0.54 (0.21)	0.65 (0.03)
σ_{u^c}	0.21 (0.01)	0.19 (0.02)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)	0.27 (0.01)	0.27 (0.01)	0.27 (0.01)
σ_{μ}	0.03 (0.01)	0.03 (0.01)	–	–	–	0.02 (0.01)	–	–
σ_{u^*}	0.00 (0.02)	0.07 (0.07)	0.00 (0.03)	0.00 (0.02)	0.00 (0.01)	0.02 (0.04)	0.02 (0.05)	0.02 (0.05)
σ_{x^c}	0.34 (0.08)	0.37 (0.08)	0.25 (0.08)	0.26 (0.09)	0.00 (0.11)	0.25 (0.18)	0.31 (0.35)	0.05 (0.05)
γ	T-V	T-V	0.83 (0.03)	0.82 (0.03)	0.75 (0.03)	T-V	0.82 (0.04)	0.75 (0.04)
δ	T-V	T-V	-0.49 (0.08)	-0.48 (0.08)	–	T-V	-0.49 (0.09)	–
η	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)
θ	-0.04 (0.01)	-0.03 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.03 (0.01)	-0.04 (0.01)	-0.04 (0.01)
φ_1	1.38 (0.04)	1.39 (0.06)	1.36 (0.04)	1.36 (0.04)	1.36 (0.04)	1.60 (0.04)	1.60 (0.04)	1.60 (0.04)
φ_2	-0.45 (0.04)	-0.45 (0.06)	-0.43 (0.04)	-0.42 (0.04)	-0.43 (0.04)	-0.65 (0.04)	-0.65 (0.04)	-0.65 (0.04)
π_u	0.70 (0.06)	0.70 (0.06)	0.69 (0.05)	0.70 (0.05)	0.69 (0.05)	–	–	–
p	0.66 (0.09)	0.66 (0.09)	0.67 (0.09)	0.67 (0.09)	0.66 (0.09)	–	–	–
q	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	–	–	–
β	-1.45 (0.12)	-1.33 (0.17)	-1.44 (0.13)	-1.36 (0.19)	-1.34 (0.14)	-1.73 (0.10)	-1.78 (0.11)	-1.72 (0.11)
π_x	-1.01 (0.15)	-1.04 (0.18)	-1.05 (0.15)	-1.09 (0.17)	-1.04 (0.17)	–	–	–
ψ_x	0.49 (0.10)	0.54 (0.12)	0.50 (0.11)	0.53 (0.12)	0.54 (0.10)	0.56 (0.24)	0.73 (0.33)	0.77 (0.16)
ρ_{x^*,u^c}	–	-0.26 (0.27)	–	-0.06 (0.11)	–	–	–	–
ρ_{x^*,x^c}	–	–	–	–	–	–	–	–
ρ_{u^c,x^c}	–	–	–	–	–	–	–	–
Log likelihood	-11.9	-11.4	-7.4	-7.2	-19.9	-57.5	-52.6	-62.5

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) Standard errors of the estimated parameters are reported in parenthesis.
(c) Numerical values for parameters denoted by 0.00 are respectively 0.0004 for model 1a, 0.00001 for model 2a, 0.0002 for model 2b, and 0.0001 and 0.001 for model 3a.

Notes:
(1) The estimation period runs from 1948Q1 to 2019Q4. We estimate twenty-two bivariate models on the basis of choices on the four specification aspects. We denote each model with a term consisting of five parts, four of which is related to one of the specification aspects of output and the other denotes the specification of the unemployment trend. For example, A-Bi-RW-SB-UC represents an Asymmetric-Bivariate-Random Walk-Structural Break-Uncorrelated model. For this model, the shocks are asymmetric, there are two variables (output and the unemployment rate), the trend growth is presumed to be a random walk (stochastic), the drift in unemployment trend has a structural break, and the correlation between shocks to the trend and cyclical components is presumed to be zero. For the list of models and their specifications, see Table B1 in Appendix B. The results of the other fourteen models are presented in Table D1 in Appendix D.

Table 1: The notes Continue

(2) For all models, the structural break in the drift of the unemployment trend (natural rate of unemployment) in 1981Q1 is accounted for. The break date is identified based on likelihood ratio statistics estimated for a sequence of breaks from 1960 to 2010, which spiked around 1981, as shown in the middle-right panel of Figure 2. For models 2a, 2b, and 5a, the structural break in trend growth in 2009Q3 is accounted for. The break date is determined based on likelihood ratio statistics estimated for a sequence of breaks from 1960 to 2010, which spiked around the 2007−09 financial crisis, as shown in the middle-left panel of Figure 2.

(3) By pairwise comparison of the log likelihood values of -11.9, -7.4, and -19.9 reported for asymmetric models 1a, 2a, and 3a with values of -57.5, -52.6, and -62.5 for their symmetric counterpart models 4a, 5a, and 6a, respectively, we favour the asymmetric models over symmetric models. The corresponding likelihood ratios of 91.2, 90.4, and 85.2 are all exceedingly greater than the critical values of 10.8 for a 0.1% significance level. The comparison between likelihood ratios of -11.4 for the asymmetric correlated model 1b and -49.1 for the symmetric correlated model 4b, which is presented in Table D1 in Appendix D, bears a likelihood ratio of 75.4, indicating that including the correlation in the model does not change the result.

(4) We compare the log likelihood values of -7.4 and -52.6 reported for models 2a and 5a with values of -19.9 and -62.5 for their counterpart models 3a and 6a, where the trend growth is assumed to be constant. We reject the null hypothesis of constant drift in the asymmetric and symmetric bivariate models because the likelihood ratios of 25.0 for the asymmetric and 19.8 for the symmetric models are greater than the critical value of 10.8 for 0.1% significance level. Regarding the models with stochastic trend growth, the log likelihood values of -11.9 and -57.5 for models 1a and 4a are respectively near to values of -7.4 and -52.6 for their counterpart models with a structural break in trend growth in 2009, which maximizes the approximate log likelihood with respect to the break date. In addition, by comparing the log likelihood values of -11.9 and -57.5 for models 1a and 3a with values of -19.9 and -62.5 for models 3a and 6a with constant trend growth, we observe a considerable improvement in the log likelihood value. We also report the log likelihood of -22.1 for a model named 3a′, which is presented in Table D1 in Appendix D and is fully nested in model 1a. We therefore favour the model with stochastic drift over the model with constant drift. Thus, the random walk is capable of accommodating unknown breaks in trend growth and competing with the best model selected among models with structural breaks.

(5) We relax the assumption of zero correlation between shocks to the output trend and the symmetric cyclical component (ρ_{x^*,u^c}) by estimating models 1b and 2b. By comparing the log likelihood values of -11.9 and -7.4 for uncorrelated models 1a and 2a with values of -11.4 and -7.2 for their correlated counterpart models 1b and 2b, respectively, we accept the null hypothesis of zero-correlation in the asymmetric bivariate model. Indeed, the likelihood ratio values of 1.0 for model 1 and 0.4 for model 2 are less than critical values of 3.84 and 2.71 for 5% and even 10% significance levels. Further, as shown in Table D1 in Appendix D, we find the other correlations between shocks to the output trend and the remaining cyclical component (ρ_{x^*,x^c}) in model 1c, and between shocks to the symmetric cyclical component and the remaining cyclical component (ρ_{u^c,x^c}) placed in the model 1d. are negligible, with log likelihood ratios of 0.0. Finally, by running model 1e, we show that all three correlations are jointly insignificant. Overall, relaxing the zero-correlation assumption does not change the estimated parameters, confirming that the correlation is irrelevant in the asymmetric bivariate model.

Table 2: Estimated parameters of the univariate models for output

Models	1a	1b	2a	2b	3a	4a	5a	6a
Parameters	A-Uni-RW-UC	A-Uni-RW-C	A-Uni-SB-UC	A-Uni-SB-C	A-Uni-Con-UC	S-Uni-RW-UC	S-Uni-SB-UC	S-Uni-Con-UC
σ_{z^*}	0.63 (0.03)	1.06 (0.13)	0.68 (0.03)	1.04 (0.11)	0.70 (0.03)	0.60 (0.07)	0.62 (0.07)	0.67 (0.06)
σ_{z^c}	0.00 (0.15)	0.64 (0.16)	0.002 (0.17)	0.62 (0.16)	0.001 (0.08)	0.50 (0.09)	0.50 (0.08)	0.45 (0.07)
σ_μ	0.06 (0.02)	0.01 (0.01)	–	–	–	0.02 (0.01)	–	–
γ	T-V	T-V	0.82 (0.04)	0.85 (0.06)	0.77 (0.04)	T-V	0.82 (0.04)	0.77 (0.04)
δ	T-V	T-V	-0.38 (0.11)	-0.40 (0.17)	–	T-V	-0.35 (0.13)	–
φ_1	1.11 (0.08)	1.14 (0.07)	1.10 (0.07)	1.09 (0.07)	1.05 (0.09)	1.58 (0.10)	1.57 (0.08)	1.61 (0.11)
φ_2	-0.35 (0.08)	-0.40 (0.07)	-0.28 (0.07)	-0.35 (0.07)	-0.39 (0.09)	-0.67 (0.09)	-0.65 (0.08)	-0.69 (0.07)
π_z	-1.67 (0.19)	-1.88 (0.27)	-1.71 (0.20)	-1.82 (0.20)	-1.70 (0.26)	–	–	–
p	0.64 (0.06)	0.60 (0.09)	0.64 (0.08)	0.62 (0.09)	0.91 (0.02)	–	–	–
q	0.95 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.97 (0.01)	–	–	–
ρ_{z^*,z^c}	–	-0.88 (0.05)	–	-0.85 (0.06)	–	–	–	–
Log likelihood	-336.9	-332.6	-334.7	-329.3	-341.0	-354.4	-352.6	-357.1

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) Standard errors of the estimated parameters are reported in parenthesis.

Notes:

(1) The estimation period runs from 1948Q1 to 2019Q4. We estimate fourteen univariate models on the basis of choices on the four specification aspects. We denote each model with a term consisting of four parts, each of which is related to one of the specification aspects. For example, A-Uni-RW-UC means Asymmetric-Univariate-Random Walk-Uncorrelated. Indeed, in this model, shocks are asymmetric, there is only one variable, the trend growth is presumed to be a random walk (stochastic), and the correlation between shocks to the trend and cyclical components is presumed to be zero. For the list of models and specifications, see Table B2 in Appendix B. The results of the other six models are presented in Table D2 in Appendix D.

(2) In models 2a, 2b, and 5a, a break in trend growth in 2009Q3 is accounted for. The break date is determined based on likelihood ratio statistics estimated for a sequence of break dates from 1960 to 2010, which spiked around the 2007−09 financial crisis, as shown in the middle-left panel of Figure 2 and in the left panel of Figure D10 in Appendix D.

(3) By pairwise comparison of the log likelihood values of -336.9, -334.7, and -341.0 reported for asymmetric models 1a, 2a, and 3a with values of -354.4, -352.6, and -357.1 for their symmetric counterparts 4a, 5a, and 6a, respectively, we favour the asymmetric models over symmetric models. The corresponding likelihood ratios of 35.0, 35.8, and 32.2 are all considerably greater than the critical value of 10.8 for a 0.1% significance level.

(4) We compare the log likelihood values of -334.7 and -352.6 reported for models 2a and 5a with values of -341.0 and -357.1 for their counterpart models 3a and 6a, where the trend growth is assumed to be constant. We reject the null hypothesis of constant drift in the symmetric and asymmetric univariate models because the likelihood ratios of 12.6 for the asymmetric and 9.0 for the symmetric models are greater than the critical value of 6.63 for a 1% significance level. Regarding the models with stochastic trend growth, the log likelihoods of -336.9 and -354.4 for models 1a and 4a are close to those of their counterpart models with a break in trend growth in 2009, which relatively maximizes the log likelihood with respect to the break date. In addition, by comparing the log likelihoods of -336.9 and -354.4 for models 1a and 4a with those of -341.0 and -357.1 for models 3a and 6a with constant trend growth, we observe a considerable improvement in log likelihood. We additionally report the log likelihood of -344.1 for a model named 3a′, which is presented in Table D2 in Appendix D and is fully nested in model 1a. We therefore favour the model with stochastic drift over the model with constant drift. As a result, the random walk is indeed capable of accommodating unknown breaks in trend growth and competing with a good model among models with structural breaks.

(5) By comparing the log likelihood values of the uncorrelated and correlated versions of each of the models 1 and 2, we reject the null hypothesis of zero-correlation in the asymmetric univariate model because the likelihood ratio values of 8.6 for model 1 and 10.8 for model 2 are greater than the 1% critical value of 6.63. Although the correlation is significant, the change in the estimation of other parameters and features of the trend and cyclical components is mild when the business cycle asymmetry is accounted for. For example, the trend and cyclical components shown in Figure 3 for the uncorrelated asymmetric model are similar to those in Figure D4 in Appendix D for the correlated asymmetric model. Likewise, the trend and cyclical components of the uncorrelated and correlated models in the left and right panels of Figure D5 in Appendix D are very similar.

Table 3: Estimated parameters of the univariate models for unemployment

Models	1a	1b	2a	2b	3
Parameters	A-Uni-SB-UC	A-Uni-SB-C	S-Uni-SB-UC	S-Uni-SB-C	A-Uni-SB0-UC
σ_{z^*}	0.0001 (0.02)	0.001 (0.03)	0.0001 (0.03)	0.03 (0.05)	–
σ_{z^c}	0.21 (0.009)	0.21 (0.02)	0.27 (0.01)	0.28 (0.04)	0.21 (0.009)
η	0.03 (0.006)	0.03 (0.006)	0.02 (0.008)	0.02 (0.009)	0.03 (0.006)
θ	-0.04 (0.009)	-0.04 (0.009)	-0.03 (0.01)	-0.03 (0.01)	-0.04 (0.009)
φ_1	1.38 (0.04)	1.38 (0.04)	1.60 (0.04)	1.58 (0.06)	1.38 (0.04)
φ_2	-0.44 (0.04)	-0.44 (0.04)	-0.65 (0.04)	-0.64 (0.07)	-0.44 (0.04)
π_z	0.75 (0.06)	0.75 (0.06)	–	–	0.75 (0.06)
p	0.63 (0.11)	0.63 (0.11)	–	–	0.63 (0.11)
q	0.97 (0.01)	0.97 (0.01)	–	–	0.97 (0.01)
ρ_{z^*,z^c}	–	0.58 (10.75)	–	-0.60 (0.80)	–
Log likelihood	-4.9	-4.9	-36.7	-36.4	-4.9

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) Standard errors of the estimated parameters are reported in parenthesis.

Notes:

(1) The estimation period is from 1948Q1 to 2019Q4. We estimate five univariate models for unemployment. We denote each model with a term consisting of four parts, each of which is related to each specification aspect. For example, A-Uni-SB-UC means Asymmetric-Univariate-Structural Break-Uncorrelated. Indeed, in this model, shocks are asymmetric, there is one variable, the drift in unemployment trend has a break, and the correlation between shocks to the trend and cyclical components is presumed to be zero. For the list of models and specifications, see Table B3 in Appendix B.

(2) For all models, the structural break in the drift of the unemployment trend (natural rate of unemployment) in 1981Q1 is accounted for. The break date is identified based on likelihood ratio statistics estimated for a sequence of breaks from 1960 to 2020, which spiked around 1981, as shown in the middle-right panel of Figure 2 and in the right panel of Figure D10 in Appendix D.

(3) By pairwise comparison of log likelihood values of -4.9 and -4.9 reported for two asymmetric models 1a and 1b with values of -36.7 and -36.4 reported for their symmetric counterpart models 2a and 2b, we favour the asymmetric models over symmetric models. The corresponding likelihood ratios of 63.6 and 63.0 are exceedingly greater than the critical value of 10.8 for a 0.1% significance level.

(4) By comparing the log likelihood of -4.9 reported for the asymmetric uncorrelated model 1a with the value of -4.9 for its correlated counterpart 1b, we accept the null hypothesis of zero-correlation because the likelihood ratio is 0.0.

(5) By comparing the log likelihood of -36.7 reported for the symmetric uncorrelated model 2a with the value of -36.4 for its correlated counterpart 2b, we accept the null hypothesis of zero-correlation because the negligible likelihood ratio of 0.8 is less than the critical values of 3.84 and even 2.71 for 5% and 10% significance levels.

(6) Model 3, which is nested in model 1a, assumes that the variance of shocks to the unemployment trend must be zero ($\sigma_{z^*}^2 = 0$), yet its estimated parameters are identical to those of model 1a.

Supplementary Appendices to

Asymmetric Co-fluctuations of U.S. Output and Unemployment: Friedman's Plucking Model and Okun's Law

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Appendix A: Summary of empirical literature

Table A1: Four specification aspects in existing trend-cycle decompositions

Choices on four specification aspects		Models and Authors
		Asymmetric Bivariate Model
1. Asymmetric 2. Bivariate	3. Stochastic trend growth 4. Insignificant Correlation	This study
		Symmetric Bivariate Model
1. Symmetric 2. Bivariate	3. Stochastic trend growth 4. Correlated shocks	Clark (1989), Gonzalez-Astudillo and Roberts (2022)
1. Symmetric 2. Bivariate	3. Stochastic trend growth 4. Uncorrelated shocks	Berger et al. (2016), Fernald et al. (2017),
1. Symmetric 2. Bivariate	3. Constant trend growth 4. Uncorrelated shocks	Owyang and Sekhposyan (2012), Grant (2018)
		Asymmetric Univariate Model
1. Asymmetric 2. Univariate	3. Stochastic trend growth 4. Uncorrelated shocks	Kim and Nelson (1999a), Mills and Wang (2002), De Simone and Clarke (2007), Morley and Piger (2012)
1. Asymmetric 2. Univariate	3. Structural break in trend growth 4. Uncorrelated shocks	Eo and Morley (2022)
1. Asymmetric 2. Univariate	3. Constant trend growth 4. Correlated shocks	Sinclair (2010)
		Symmetric Univariate Model
1. Symmetric 2. Univariate	3. Stochastic trend growth 4. Correlated shocks	Kim and Chon (2020), Kim and Kim (2022)
1. Symmetric 2. Univariate	3. Stochastic trend growth 4. Uncorrelated shocks	Harvey (1985), Clark (1987), Grant and Chan (2017a)
1. Symmetric 2. Univariate	3. Structural break in trend growth 4. Correlated shocks	Luo and Startz (2014)
1. Symmetric 2. Univariate	3. Structural break in trend growth 4. Uncorrelated shocks	Perron and Wada (2009), Grant and Chan (2017b)
1. Symmetric 2. Univariate	3. Constant trend growth 4. Correlated shocks	Beveridge and Nelson (1981), Nelson and Plosser (1982), Morley et al. (2003)

Notes:
(1) This table categorizes the literature on trend-cycle decomposition by determining the decision made by each study regarding each of the four specification aspects. These four aspects are: whether the cyclical component is asymmetric or symmetric; whether unemployment must be included within a bivariate model; whether the trend growth is stochastic or deterministic; and whether the correlation between shocks to the trend and cyclical components is relevant or not.
(2) If a study performs different setups for one or two of the specification aspects, the above table refers to the main model it uses.

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[*] Corresponding author. See the website https://sites.google.com/view/mohammaddehghani for data and code. For details about the method and parameter constraints, see the comments in the MATLAB code.

Table A2: Four attributes of business cycle asymmetries

Definition	Related literature
Correlation asymmetry (ceiling effect)	
The amplitude of recessions is strongly correlated with the amplitude of succeeding expansions, but the amplitude of expansions is uncorrelated with the amplitude of succeeding recessions.	Friedman (1964, 1993), Wynne and Balke (1992), Beaudry and Koop (1993), Goodwin and Sweeney (1993), Fatás and Mihov (2013), Bordo and Haubrich (2017), Dupraz et al. (2019).
Deepness asymmetry	
● Recession troughs are deep and expansion peaks are small in amplitude. ● Output displays a negative skewness relative to the trend. ● The unemployment rate displays a positive skewness.	Neftci (1984), DeLong and Summers (1984), Sichel (1993), Goodwin and Sweeney (1993).
Steepness asymmetry	
● Recessions are steep (violent) and expansions are gradual (mild). ● Output growth (first difference) displays a negative skewness. ● Unemployment growth (first difference) displays a positive skewness.	DeLong and Summers (1984), Falk (1986), Sichel (1993), McKay and Reis (2008), Jensen et al. (2020).
Duration asymmetry	
Recessions are short and recoveries are long.	Neftci (1984)

Notes:

(1) This table reviews four attributes of business cycle asymmetries considered in different studies.

(2) The output gap skewness of -0.93 and -0.4 as well as unemployment gap skewness of +0.75 and +0.94 reported in Figure D11 in Appendix D, provides preliminary evidence for asymmetries in output and unemployment.

(3) The Markov-switching process has the potential to capture all types of asymmetries: A significant plucking coefficient with the addition of estimating output gaps that are often negative and rarely positive confirms the ceiling effect (correlation asymmetry). Estimating deep output gaps with a short expected duration for recessions and a long expected duration for recoveries implies deepness, steepness, and duration asymmetries.

(4) Besides the asymmetries explained in the above table, other studies have defined alternative asymmetries. For example, McQueen and Thorley (1993) explore the sharpness symmetry, which means that peaks are sharp and troughs are round for the unemployment rate. Another classification suggests two asymmetries: asymmetry around the vertical line, and asymmetry around the horizontal line. In this sense, correlation and deepness are asymmetries around the horizontal line and steepness and duration are asymmetries around the vertical line.

Appendix B: State-space representations

B.1: Bivariate state-space model with Markov-switching

We cast the bivariate model explained in Eq. (1) to Eq. (12) in a state-space form. The observation equation, the transition equation, and variance covariance matrix of error terms are as follows:

$$\begin{bmatrix} x_t \\ u_t \end{bmatrix} = \begin{bmatrix} 1 & \beta & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_t^* \\ u_t^c \\ u_{t-1}^c \\ \mu_t \\ u_t^* \\ \varepsilon_{x^c,t} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{B.1.1}$$

$$\begin{bmatrix} x_t^* \\ u_t^c \\ u_{t-1}^c \\ \mu_t \\ u_t^* \\ \varepsilon_{x^c,t} \end{bmatrix} = \begin{bmatrix} 0 \\ \pi_u S_t \\ 0 \\ 0 \\ \eta + \theta \mathbb{1}_t(t \geq T_{u^*}) \\ \pi_x S_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \varphi_1 & \varphi_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi \end{bmatrix} \begin{bmatrix} x_{t-1}^* \\ u_{t-1}^c \\ u_{t-2}^c \\ \mu_{t-1} \\ u_{t-1}^* \\ \varepsilon_{x^c,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{x^*,t} \\ \varepsilon_{u^c,t} \\ 0 \\ \varepsilon_{\mu,t} \\ \varepsilon_{u^*,t} \\ \xi_{x^c,t} \end{bmatrix} \tag{B.1.2}$$

$$\begin{bmatrix} \varepsilon_{x^*,t} \\ \varepsilon_{u^c,t} \\ 0 \\ \varepsilon_{\mu,t} \\ \varepsilon_{u^*,t} \\ \xi_{x^c,t} \end{bmatrix} \sim N(\mathbf{0}_{6\times 1}, \begin{bmatrix} \sigma_{x^*}^2 & \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & 0 & 0 & 0 & \rho_{x^*,x^c}\sigma_{x^*}\sigma_{x^c} \\ \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & \sigma_{u^c}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\mu^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{u^*}^2 & 0 \\ \rho_{x^*,x^c}\sigma_{x^*}\sigma_{x^c} & 0 & 0 & 0 & 0 & \sigma_{x^c}^2 \end{bmatrix} \tag{B.1.3}$$

In the above model, we consider natural log GDP multiplied by 100 and the unemployment rate as the observed series (x_t and u_t). To test for asymmetry, we derive the restricted symmetric model by imposing $\pi_u = 0$ on the unrestricted asymmetric model.

There are multiple state-space representations for the bivariate model. In addition to the above one, for example, we could consider a measurement error for Okun's law and cast the bivariate model in the state-space form explained below, which estimates very similar parameters as the first form.

$$\begin{bmatrix} x_t \\ u_t \end{bmatrix} = \begin{bmatrix} 1 & \beta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_t^* \\ u_t^c \\ u_{t-1}^c \\ \mu_t \\ u_t^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{x^c,t} \\ 0 \end{bmatrix} \tag{B.1.4}$$

$$\begin{bmatrix} x_t^* \\ u_t^c \\ u_{t-1}^c \\ \mu_t \\ u_t^* \end{bmatrix} = \begin{bmatrix} 0 \\ \pi_u S_t \\ 0 \\ 0 \\ \eta + \theta \mathbb{1}_t(t \geq T_{u^*}) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & \varphi_1 & \varphi_2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{t-1}^* \\ u_{t-1}^c \\ u_{t-2}^c \\ \mu_{t-1} \\ u_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{x^*,t} \\ \varepsilon_{u^c,t} \\ 0 \\ \varepsilon_{\mu,t} \\ \varepsilon_{u^*,t} \end{bmatrix} \tag{B.1.5}$$

$$\begin{bmatrix} \varepsilon_{x^c,t} \\ 0 \end{bmatrix} \sim N(\mathbf{0}_{2\times 1}, \begin{bmatrix} \sigma_{x^c}^2 & 0 \\ 0 & 0 \end{bmatrix} \tag{B.1.6}$$

$$\begin{bmatrix} \varepsilon_{x^*,t} \\ \varepsilon_{u^c,t} \\ 0 \\ \varepsilon_{\mu,t} \\ \varepsilon_{u^*,t} \end{bmatrix} \sim N\left(\mathbf{0}_{5\times1}, \begin{bmatrix} \sigma_{x^*}^2 & \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & 0 & 0 & 0 \\ \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & \sigma_{u^c}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\mu}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{u^*}^2 \end{bmatrix}\right) \tag{B.1.7}$$

Table B1: Specification of 22 bivariate models

Model name	Tables and Figures	Related model in the literature
Model 1a: Asymmetric-Bivariate-RW-SB-UC	Table 1, Figures 1, 2	--
Model 1b: Asymmetric-Bivariate-RW-SB-C1 (ρ_{x^*,u^c})	Table 1, Figure D1	--
Model 1c: Asymmetric-Bivariate-RW-SB-C2 (ρ_{x^*,x^c})	Table D1	--
Model 1d: Asymmetric-Bivariate-RW-SB-C3 (ρ_{u^c,x^c})	Table D1	--
Model 1e: Asymmetric-Bivariate-RW-SB-C123 ($\rho_{x^*,u^c}, \rho_{x^*,x^c}, \rho_{u^c,x^c}$)	Table D1	--
Model 2a: Asymmetric-Bivariate-SB-SB-UC	Table 1	--
Model 2b: Asymmetric-Bivariate-SB-SB-C1 (ρ_{x^*,u^c})	Table 1	--
Model 2c: Asymmetric-Bivariate-SB-SB-C2 (ρ_{x^*,x^c})	Table D1	--
Model 3a: Asymmetric-Bivariate-Con-SB-UC (5 state variable)	Table 1	--
Model 3a′: Asymmetric-Bivariate-Con-SB-UC (6 state variables)	Table D1	--
Model 3b: Asymmetric-Bivariate-Con-SB-C1 (ρ_{x^*,u^c})	Table D1	--
Model 3c: Asymmetric-Bivariate-Con-SB-C2 (ρ_{x^*,x^c})	Table D1	--
Model 4a: Symmetric-Bivariate-RW-SB-UC	Table 1	(Clark, 1989)
Model 4b: Symmetric-Bivariate-RW-SB-C1 (ρ_{x^*,u^c})	Table D1	--
Model 4c: Symmetric-Bivariate-RW-SB-C2 (ρ_{x^*,x^c})	Table D1	--
Model 5a: Symmetric-Bivariate-SB-SB-UC	Table 1	--
Model 5b: Symmetric-Bivariate-SB-SB-C1 (ρ_{x^*,u^c})	Table D1	--
Model 6a: Symmetric-Bivariate-Con-SB-UC	Table 1	--
Model 6b: Symmetric-Bivariate-Con-SB-C1 (ρ_{x^*,u^c})	Table D1	(Gonzalez-Astudillo and Roberts, 2022)
Model 7: Asymmetric-Bivariate-RW-SB-UC-SB	Figure 2	--
Model 8: Asymmetric-Bivariate-SB-RW-SB-UC	Figure 2	--
Model 9: Asymmetric-Bivariate-RW-SB0-UC	Table D1	--

Notes:

(1) We estimate twenty-two different bivariate models on the basis of different choices on the four specification aspects. Accordingly, we denote each model with an identifier and a descriptor. The descriptor consists of five parts, four of which are related to each specification aspect. The first part determines whether the model is asymmetric or symmetric, and the second part shows whether the model is univariate or bivariate. The third part hints at the specification of output trend growth (stochastic, structural break, or constant). The fourth part shows that we take a structural break in drift of the natural rate of unemployment into account. The last part indicates whether the model is uncorrelated or correlated. For example, model 1a, which is the main model and is denoted by Asymmetric-Bivariate-RW-SB-UC, means the model is asymmetric and bivariate. The trend growth is presumed to be a random walk (stochastic) process, and the natural rate has a structural break. The correlation between shocks to the trend and cyclical components is also assumed to be zero.

(2) The second SB, as an additional part of model 7, expresses a structural break in the volatility of shocks to the remaining cyclical component. This model is used to find a break in the variance of the Okun's law residual to explain the change in variances before and after the great moderation.

(3) The first SB, as an additional part of model 8, expresses a structural break in Okun's coefficient. This model is used to investigate instability in Okun's law, particularly following the global financial crisis.

(4) The term SB0 shown in model 9 indicates that model 9 is nested in model 1a by assuming that the variance of shocks to the unemployment trend is zero ($\sigma_{u^*}^2 = 0$). See footnote 7 for more information.

(5) We present the results of the bolded models in Table 1, and the rest are presented in Table D1 in Appendix D.

B.2: Univariate state-space model with Markov-switching

To estimate the output trend and cyclical components, we cast the univariate model explained in Eq. (13) to Eq. (18) in a state-space form. The observation equation, the transition equation, and variance covariance matrix of error terms are as follows:

$$[z_t] = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \\ \mu_t \end{bmatrix} + [0] \tag{B.2.1}$$

$$\begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \\ \mu_t \end{bmatrix} = \begin{bmatrix} 0 \\ \pi_z S_t \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \varphi_1 & \varphi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_{t-1}^* \\ z_{t-1}^c \\ z_{t-2}^c \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \\ \varepsilon_{\mu,t} \end{bmatrix} \tag{B.2.2}$$

$$\begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \\ \varepsilon_{\mu,t} \end{bmatrix} \sim N\left(\mathbf{0}_{4\times 1}, \begin{bmatrix} \sigma_{z^*}^2 & \rho\sigma_{z^*}\sigma_{z^c} & 0 & 0 \\ \rho\sigma_{z^*}\sigma_{z^c} & \sigma_{z^c}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\mu^2 \end{bmatrix}\right) \tag{B.2.3}$$

In the above model, we consider natural log GDP multiplied by 100 as the observed series (z_t). To test for asymmetry, we derive the restricted symmetric model by imposing $\pi_z = 0$ on the unrestricted asymmetric model. We then estimate this nested model by using Kalman's (1960) filter. In the above setup, we consider a stochastic trend growth that moves according to a random walk. Alternatively, if we model the trend growth as a non-stochastic drift with a structural break, the corresponding state-space representation would be:

$$[z_t] = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \end{bmatrix} + [0] \tag{B.2.4}$$

$$\begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \end{bmatrix} = \begin{bmatrix} \gamma + \delta \mathbb{1}_t(t \geq T_\mu) \\ \pi_z S_t \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \varphi_1 & \varphi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_{t-1}^* \\ z_{t-1}^c \\ z_{t-2}^c \end{bmatrix} + \begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \end{bmatrix} \tag{B.2.5}$$

$$\begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \end{bmatrix} \sim N\left(\mathbf{0}_{3\times 1}, \begin{bmatrix} \sigma_{z^*}^2 & \rho\sigma_{z^*}\sigma_{z^c} & 0 \\ \rho\sigma_{z^*}\sigma_{z^c} & \sigma_{z^c}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right) \tag{B.2.6}$$

For the trend and cyclical components for unemployment, we use the following state-space form:

$$[z_t] = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \end{bmatrix} + [0] \tag{B.2.7}$$

$$\begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \end{bmatrix} = \begin{bmatrix} \eta + \theta \mathbb{1}_t(t \geq T_\mu)) \\ \pi_z S_t \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \varphi_1 & \varphi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_{t-1}^* \\ z_{t-1}^c \\ z_{t-2}^c \end{bmatrix} + \begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \end{bmatrix} \tag{B.2.8}$$

$$\begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \end{bmatrix} \sim N(\mathbf{0}_{3\times1}, \begin{bmatrix} \sigma^2_{z^*} & \rho\sigma_{z^*}\sigma_{z^c} & 0 \\ \rho\sigma_{z^*}\sigma_{z^c} & \sigma^2_{z^c} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{B.2.9}$$

Where the unemployment rate is considered as the observed series (z_t) and the structural break for the natural rate of unemployment is accounted for. In above models, we could find the break date by exploring the supremum of the log likelihood ratios calculated for a sequence of structural breaks rolling from 1960 to 2010.

Table B2: Specification of 14 univariate models for output

Model name	Tables and Figures	Related model in the literature
Model 1a: Asymmetric-Univariate-RW-UC	Table 2, Figure 3	(Kim and Nelson, 1999a)
Model 1b: Asymmetric-Univariate-RW-C (ρ_{z^*,z^c})	Table 2, Figure D4	--
Model 2a: Asymmetric-Univariate-SB-UC	Table 2, Figure D5	--
Model 2b: Asymmetric-Univariate-SB-C (ρ_{z^*,z^c})	Table 2, Figure D5	--
Model 3a: Asymmetric-Univariate-Con-UC (3 state variables)	Table 2, Figure D6	--
Model 3a′: Asymmetric-Univariate-Con-UC (4 state variables)	Table D2	--
Model 3b: Asymmetric-Univariate-Con-C (ρ_{z^*,z^c})	Table D2, Figure D6	(Sinclair, 2010)
Model 4a: Symmetric-Univariate-RW-UC	Table 2, Figure D7	(Clark, 1987)
Model 4b: Symmetric-Univariate-RW-C (ρ_{z^*,z^c})	Table D2, Figure D7	--
Model 5a: Symmetric-Univariate-SB-UC	Table 2, Figure D7	(Perron and Wada, 2009)
Model 5b: Symmetric-Univariate-SB-C (ρ_{z^*,z^c})	Table D2, Figure D7	--
Model 6a: Symmetric-Univariate-Con-UC	Table 2, Figure D8	--
Model 6b: Symmetric-Univariate-Con-C (ρ_{z^*,z^c})	Table D2, Figure D8	(Morley, 2003)
Model 7: Asymmetric-Univariate-RW-UC-SB	Figure D10	--

Table B3: Specification of 5 univariate models for unemployment

Model name	Tables and Figures	Related model in the literature
Model 1a: Asymmetric-Univariate-SB-UC	Table 3, Figure 3	--
Model 1b: Asymmetric-Univariate-SB-C (ρ_{z^*,z^c})	Table 3, Figure D4	--
Model 2a: Symmetric-Univariate-SB-UC	Table 3, Figure D9	--
Model 2b: Symmetric-Univariate-SB-C (ρ_{z^*,z^c})	Table 3, Figure D9	--
Model 3: Asymmetric-Univariate-SB0-UC	Table 3	--

Notes:

(1) We estimate fourteen univariate models for output and five univariate models for unemployment. We show each model with an identifier and a descriptor. The descriptor consists of four parts, each of which is related to each specification aspect. The first part determines whether the model is asymmetric or symmetric, and the second part re-emphasises that the model is univariate. The third part hints at the specification of output trend growth (stochastic, structural break, or constant). Regarding the unemployment rate, the third part shows that we take a structural break in drift term of the natural rate of unemployment into account. The last part indicates whether the model is uncorrelated or correlated. For example, model 1a, which is denoted by Asymmetric-Univariate-RW-UC, means the model is asymmetric, univariate, the trend growth is random walk, and the correlation between shocks to the trend and cyclical components is presumed to be zero.

(2) The term SB, as an additional part of model 7 that is applied to output, expresses a structural break in the variance of the cyclical component to explain the change in variances before and after the great moderation. This model is used to find the break in residual volatility.

(3) The term SB0 shown in model 3 that is applied to unemployment indicates that model 3 is nested in model 1a by assuming that the variance of shocks to the unemployment trend is zero ($\sigma^2_{z^*} = 0$). See footnote 7 for more information.

(4) We present the results of the bold models in Tables 2 and 3, and the rest are presented in Table D2 in Appendix D.

Appendix C: Approximate maximum likelihood method

For asymmetric models in the presence of the Markov-switching process of Hamilton (1989), we use Kim's (1994) approximate maximum likelihood method to make the Kalman's (1960) filter operable. For more explanation, see chapters 4 and 5 of Kim and Nelson's (1999b) and chapters 13 and 22 of Hamilton (1994). For symmetric models, we use the maximum likelihood method, performed by the Kalman's (1960) filter as explained in chapters 2 and 3 of Kim and Nelson (1999b) and chapter 5 of Hamilton (1994).

We need to impose a set of constraints on parameters, which are explained carefully in the first part of Appendix C. We also consider a set of initial values for parameters as well as state variables. For the former, all initial values for parameters are presented in Tables C1, C2, and C3 in the second part of Appendix C. For the latter, we use the first observation for trend components, zero for cyclical components, and 3.2% for annual trend growth to determine the prior values for state variables. The prior variances of state variables are set to be 10. The results are robust to changes in prior values of state variables and their variances. For example, we can use a wilder guess by setting the variances of state variables equal to 1000, which bears the same estimation for parameters.

C.1: Parameters constraints

We employ a numerical optimization procedure to maximize the approximate log likelihood function subject to a set of constraints. We need to impose these constraints on some of the parameters, namely coefficients, probabilities, and standard deviations of shocks. To this end, we account for constraints by using a transformation function, $T(\omega)$, which transforms a vector of unconstrained parameters $\omega = [\omega_1, \dots, \omega_{20}]'$ to a vector of constrained parameters $\Omega = [\Omega_1, \dots, \Omega_{20}]'$ presented below:

$$\Omega = [\sigma_{x^*}, \sigma_{u^c}, \sigma_\mu, \sigma_{u^*}, \sigma_{x^c}, \gamma, \delta, \eta, \theta, \varphi_1, \varphi_2, \pi_u, p, q, \beta, \pi_x, \psi_x, \rho_{x^*,u^c}, \rho_{x^*,x^c}, \rho_{u^c,x^c}]' \quad \text{(C.1.1)}$$

where $\Omega = T(\omega)$ is the vector of parameters of interest that we want to estimate and $T(\omega)$ is a vector function, whose elements are continuous transformation functions $T_i(\omega)$ for $i = 1, \dots, 20$. We know that performing unconstrained optimization with respect to ω is equivalent to performing constrained optimization with respect to Ω. We therefore adopt an unconstrained optimization with respect to the vector ω, where the objective (approximate log likelihood) function is considered as a function of the transformation function. We define each element of the transformation function as follows:

First, for coefficients and standard deviations of shocks that should be positive, we use an exponential transformation suggested by Kim and Nelson (1999b). For example,

$$\sigma_{x^*} = exp(\omega_1) \quad \text{(C.1.2)}$$

where σ_{x^*} is the standard deviation (square root of variance) of shocks to the trend component of output and is assumed to be positive. Similarly, for other standard deviations, including σ_{u^c}, σ_μ, σ_{u^*}, and σ_{x^c} and for coefficients γ, η, and π_u that are expected to be positive and for other coefficients δ, θ, β, and π_x that are expected to be negative, we use an exponential transformation. For example, $\pi_u = exp(\omega_{12})$ ensures a positive plucking coefficient for unemployment, and $\beta = -exp(\omega_{15})$ ensures a negative Okun's law coefficient.

Second, to have transition probabilities in the [0 1] interval, we exert the following transformations:

$$p = \frac{exp(\omega_{13})}{1 + exp(\omega_{13})} \quad \text{and} \quad q = \frac{exp(\omega_{14})}{1 + exp(\omega_{14})} \quad \text{(C.1.3)}$$

Third, for the coefficient of the autoregressive process of order one, we use Eq. (C1..4):

$$\psi_x = \frac{\omega_{17}}{1 + |\omega_{17}|} \quad \text{(C.1.4)}$$

Clearly, ψ_x lies in the stationary region since $-1 < \psi_x < 1$. For coefficients of the autoregressive process of order two, we need to set the values of φ_1 and φ_2 within the stationary region that means the roots of the lag polynomial $(1 - \varphi_1 L - \varphi_2 L^2 = 0)$ must lie outside the unit circle. In this sense, we use the following transformations proposed by Morley et al. (2003):

$$\varphi_1 = 2\kappa_1 \quad \text{and} \quad \varphi_2 = -(\kappa_1^2 + \kappa_2) \tag{C.1.5.a}$$

where κ_1 and κ_2 are determined below:

$$\kappa_1 = \frac{\omega_{10}}{1 + |\omega_{10}|} \quad \text{and} \quad \kappa_2 = \frac{(1 - |\kappa_1|) \times \omega_{11}}{1 + |\omega_{11}|} + |\kappa_1| - \kappa_1^2 \tag{C.1.6.a}$$

For these two coefficients of the autoregressive process, we can take an alternative transformation proposed by Kim and Nelson (1999b):

$$\varphi_1 = \kappa_1 + \kappa_2 \quad \text{and} \quad \varphi_2 = \kappa_1 \times \kappa_2 \tag{C.1.5.b}$$

where κ_1 and κ_2 are determined below:

$$\kappa_1 = \frac{\omega_{10}}{1 + |\omega_{10}|} \quad \text{and} \quad \kappa_2 = \frac{\omega_{11}}{1 + |\omega_{11}|} \tag{C.1.6.b}$$

However, transformations in Eq. (C.1.5) and Eq. (C.1.6) impose a further restriction that the roots of the autoregressive polynomial are real numbers.

Fourth, for correlation coefficients, we consider Eq. (C.1.7):

$$\rho_{x^*,u^c} = \frac{\omega_{18}}{1 + |\omega_{18}|} \tag{C.1.7}$$

where ρ_{x^*,u^c} is the correlation between shocks and clearly satisfies the condition $-1 < \rho_{x^*,u^c} < 1$.

Finally, for correlated models, we can use alternative constraints for standard deviations of shocks to components and their correlation. In this setting, we use a Cholesky factorization like Hamilton (1994) and Morley et al. (2003), which is presented as follows:

$$\sigma_{x^*} = \sqrt{[P'P]_{1,1}} \text{ and } \sigma_{u^c} = \sqrt{[P'P]_{2,2}} \text{ and } \rho_{x^*,u^c} = \frac{[P'P]_{1,2}}{\sqrt{[P'P]_{1,1}} \times \sqrt{[P'P]_{2,2}}} \tag{C.1.8}$$

In Eq. (C.1.8), $[P'P]_{i,j}$ is the element on the row i and column j of the symmetric positive definite matrix denoted by $[P'P]_{2\times2}$, which is known as the Cholesky factorization. To derive the elements of $[P'P]_{2\times2}$, we first need to construct the Cholesky factor ($P_{2\times2}$), a lower triangular matrix with positive diagonal elements $P_{1,1} = exp(\omega_1)$ and $P_{2,2} = exp(\omega_2)$ and an off-diagonal element of $P_{1,2} = \omega_{18}$.

It is worth noting that the results of this study are robust to the choice of the transformation functions when there are two alternatives. In addition, the plucking property (asymmetry) and Okun's law (co-fluctuations) are two pronounced features of U.S. macroeconomics so that excluding each or both of their corresponding constraints ($\pi_u > 0$ and $\beta < 0$) does not change the estimated parameters.

C.2: Tables of initial values for parameters

Table C1: Initial values (after-transformation) for the parameters of the bivariate model

	1a	1b	2a	2b	3a	4a	5a	6a
Parameters	A-Bi-RW-SB-UC	A-Bi-RW-SB-C1	A-Bi-SB-SB-UC	A-Bi-SB-SB-C1	A-Bi-Con-SB-UC	S-Bi-RW-SB-UC	S-Bi-SB-SB-UC	S-Bi-Con-SB-UC
σ_{x^*}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
σ_{u^c}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
σ_{μ}	0.50	0.50	–	–	–	0.50	–	–
σ_{u^*}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
σ_{x^c}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
γ	T-V	T-V	0.80	0.80	0.80	T-V	0.80	0.80
δ	T-V	T-V	-0.30	-0.30	–	T-V	-0.30	–
η	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
θ	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
φ_1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
φ_2	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4
π_u	1.8	1.8	1.8	1.8	1.8	–	–	–
p	0.70	0.70	0.70	0.70	0.70	–	–	–
q	0.90	0.90	0.90	0.90	0.90	–	–	–
β	-1.50	-1.50	-1.50	-1.50	-1.50	-1.50	-1.50	-1.50
π_x	-1.8	-1.8	-1.8	-1.8	-1.8	–	–	–
ψ_x	0.50	0.50	0.50	0.50	0.50	0.50	0.70*	0.75*
ρ_{x^*,u^c}	–	0.50	–	0.50	–	–	–	–
ρ_{x^*,x^c}	–	–	–	–	–	–	–	–
ρ_{u^c,x^c}	–	–	–	–	–	–	–	–

Notes:

(1) The results of the main proposed model as well as other models are unbelievably robust to the choice of the initial values for each parameter. Therefore, we use the same initial values for almost all models. Alternatively, one can follow a hierarchical method to find initial values, meaning that the researcher estimates the simplest model (symmetric univariate model) first and keeps the estimated parameters to use as a best guess for the initial values for less restricted models.

(2) We set a few initial values for a few models different from those in other models to avoid deriving imaginary standard errors in one or two parameters. These initial values are denoted by asterisks.

Table C2: Initial values (after-transformation) for the parameters of the univariate model for output

Models	1a	1b	2a	2b	3a	4a	5a	6a
Parameters	A-Uni-RW-UC	A-Uni-RW-C	A-Uni-SB-UC	A-Uni-SB-C	A-Uni-Con-UC	S-Uni-RW-UC	S-Uni-SB-UC	S-Uni-Con-UC
σ_{z^*}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
σ_{z^c}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
σ_{μ}	0.50	0.50	–	–	–	0.50	–	–
γ	T-V	T-V	0.75	0.75	0.75	T-V	0.75	0.75
δ	T-V	T-V	-0.30	-0.30	–	T-V	-0.30	–
φ_1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
φ_2	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4
π_z	-1.80	-1.80	-1.80	-1.80	-1.80	–	–	–
p	0.70	0.70	0.70	0.70	0.70	–	–	–
q	0.90	0.90	0.90	0.90	0.90	–	–	–
ρ_{z^*,z^c}	–	0.50	–	0.50	–	–	–	–

Table C3: Initial values (after-transformation) for the parameters of the univariate model for unemployment

Models	1a	1b	2a	2b	3
Parameters	A-Uni-SB-UC	A-Uni-SB-C	S-Uni-SB-UC	S-Uni-SB-C	A-Uni-SB-UC
σ_{z^*}	0.50	0.50	0.50	0.50	–
σ_{z^c}	0.50	0.50	0.50	0.50	0.50
η	0.03	0.03	0.03	0.03	0.03
θ	-0.03	-0.03	-0.03	-0.03	-0.03
φ_1	1.2	1.2	1.2	1.2	1.2
φ_2	-0.4	-0.4	-0.4	-0.4	-0.4
π_z	1.8	1.8	–	–	1.8
p	0.70	0.70	–	–	0.70
q	0.90	0.90	–	–	0.90
ρ_{z^*,z^c}	–	0.65*	–	0.20*	–

Notes:
(1) The results of all models for both indicators are robust to the choice of the initial values for each parameter.
(2) We use the same initial values for all models for both indicators that are almost the same as the initial values for the bivariate model. For correlations between shocks to the unemployment trend and cyclical components in models 1b and 2b, we use different initial values to avoid deriving imaginary standard errors in one or two parameters. These initial values are denoted by asterisks.

Appendix D: Additional empirical results

Some additional figures and tables are presented and explained here in Appendix D. Figure D1 shows the results of model 1b, the correlated version of the asymmetric bivariate model. The insignificant correlation between shocks says that the features of our proposed model are robust no matter whether the correlation is involved in the model or not. Panels of Figure D2 present supplementary results of the asymmetric bivariate model to establish its results. The top panels display trend growth and trend acceleration to call attention to an unprecedented deceleration in U.S. potential output. The middle-left panel shows that the depth of the leftover plucking property in the Okun's residuals is small. This implies that the plucking property in output is sourced from the plucking property in unemployment. To be substantiated, we provide additional evidence confirming that the leftover plucking property in the main proposed model is attributable to the lead-lag effect between output and unemployment. The middle-right panel, in this regard, indicates that controlling the 1-month lead-lag effect between output and unemployment entirely removes the leftover plucking property in the Okun's residuals. In addition, as reported in Table D3, while the output-specific plucking coefficient is notable in the main model ($\pi_x = -1.01$), it is negligible in the model with the 1-month lead-lag effect controlled ($\pi_x = -0.46$), and is finally zero in models with the 2-month or 3-month lead-lag effect controlled ($\pi_x = -0.000$). As a result, considering that the unemployment rate lags behind output for only one month, we infer that their co-fluctuations are sufficiently synchronous. The bottom panels of Figure D2 also present routine diagnostic tests on error terms by showing that shocks to the Okun's residuals are zero-mean noise and their autocorrelation functions are fast decaying.

Additionally, Figure D3 and Table D5 present the results of applying the asymmetric bivariate model to the U.S. GDP per capita and the U.K. GDP. Tables D1 and D2 are extensions of Tables 1 and 2, each of which presents the results of several alternative bivariate and univariate models, respectively. Table D3 lists the estimated parameters of the proposed model with the lead-lag effect controlled, and Table D4 lists the results for the modified model that accommodates the COVID-19 recession.

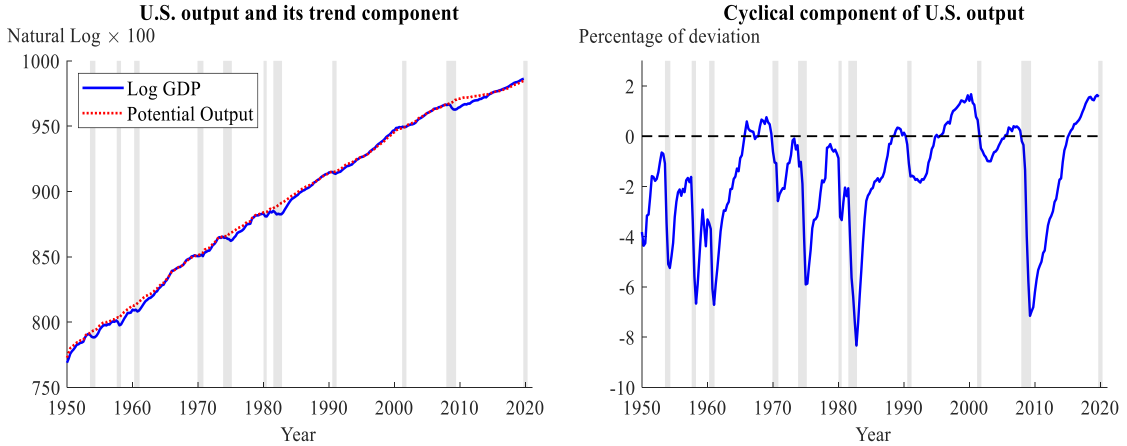
We also uncover the consequences of misspecifications in the asymmetric univariate model. Figure 3, left panels of Figure D4, and Figure D5 highlight that because a time-varying trend growth is accommodated in models 1a or 2a, the features of the estimated cyclical component are sensible and similar to those of asymmetric bivariate models. Conversely, the middle-left panel of Figure D6 reveals that an unaccounted for break in trend growth in model 3a compels the plucking probability to stay at one, which brings a paradoxical result: a permanent output gap in the transitory component. Concerning the correlation between shocks, we derive the likelihood ratio values of 8.6, 10.8, and 19.0 by comparing pairwise the log likelihoods of -336.9, -334.7, and -341.0 for the uncorrelated asymmetric models 1a, 2a, and 3a with those values of -332.6, -329.3, and -331.5 for their correlated counterpart models 1b, 2b, and 3b. The correlation is therefore relevant in the asymmetric univariate model, yet similarities of the features of trend and cyclical components of the uncorrelated and correlated models in the left and right panels of Figure D5 in Appendix D shows that embedding the asymmetry reduces the univariate model sensitivity to the assumption about the correlation.

We finally show that the counter-intuitive and sensitive features of the estimated components in the symmetric univariate models are consequences of two misspecifications related to trend growth and correlation. Figure D8 in Appendix D displays the trend and cyclical components estimated by model 6a on the left panel and its correlated counterpart 6b that replicates the results derived by Morley et al. (2003) on the right panel, where in both models the trend growth is assumed to be deterministic. Such an assumption leads to a cyclical component that clearly exhibits a downward leftover trend in the bottom-left panel and an unusually small amplitude in the bottom-right panel.[18] As a result, the features of components in a model with constant trend growth are dubious since they are sensitive to the assumption about the correlation. Figure D7 shows that relaxing the assumption of constant trend growth yields more intuitive and less sensitive results. The cyclical components of model 4a with stochastic trend growth and model 5a with a break in trend growth in the left panels do not contain a noticeable leftover trend. Further, although the cyclical components of correlated models 4b and 5b in the right panels have clearly smaller amplitudes compared to their uncorrelated counterparts, they are not diminutive and noisy like those in model 6a.

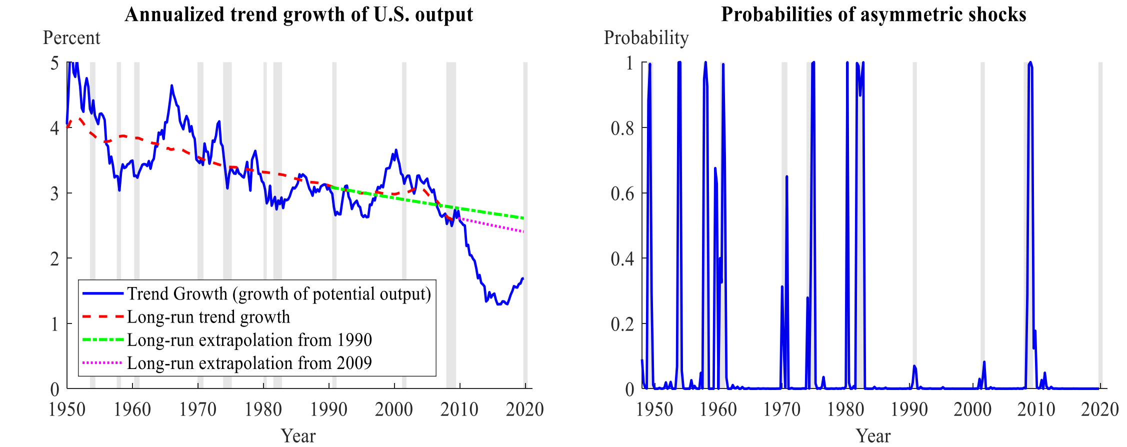
Figure D9 in Appendix D presents results of the univariate model applied to the unemployment rate, indicating that there is a very mild increase in the natural rate of unemployment from the 1950s to the 1980s, and then a mild reduction until now. Figure D10 provides evidence for the occurrence of breaks in output trend growth and unemployment trend drift. The left panel shows two clusters of repeatedly highly significant breaks, one in the 1970s and another in the 2000s, with multiple local peaks in 1973, 1978, 2000, and 2006, which supports the use of stochastic rather than deterministic drift to characterize the dynamics of trend growth in the U.S. The right panel reaffirms a significant break in the drift term of natural rate of unemployment occurred in 1981Q1. Finally, Figure D11 reports the output gap skewness of -0.93 and -0.4 and the unemployment gap skewness of +0.75 and +0.94 as initial evidence for asymmetries in output and unemployment.

[18] Constant trend growth is improper because comparing the log likelihoods of -354.4 and -352.6 for models 4a and 5a with the value of -357.1 for model 6a, reaffirms the presence of stochastic trend growth and a break in 2009, as the likelihood ratios of 5.4 and 9.0 are greater than the critical value of 3.84 for a 5% significance level.

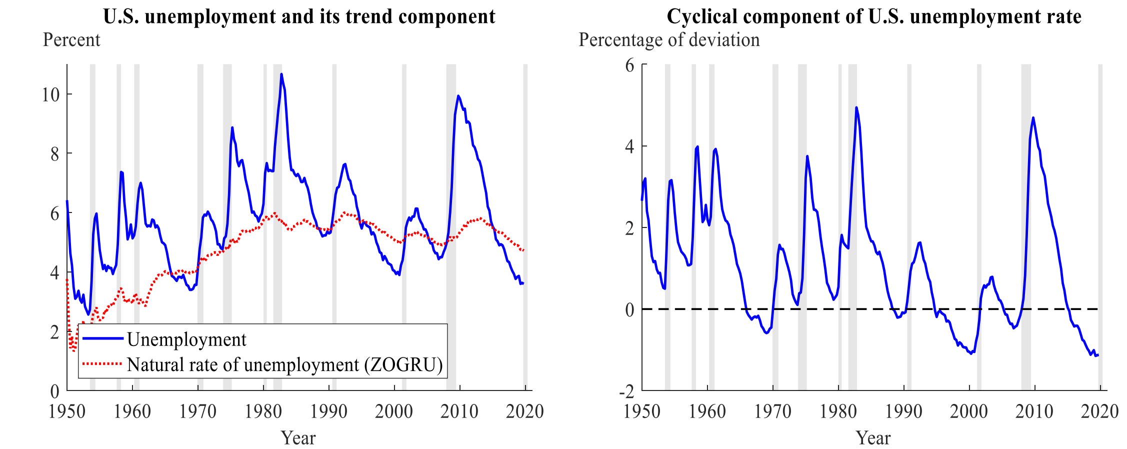
Appendix D: Additional figures



(a) Potential output (trend) and output gap (cyclical component)



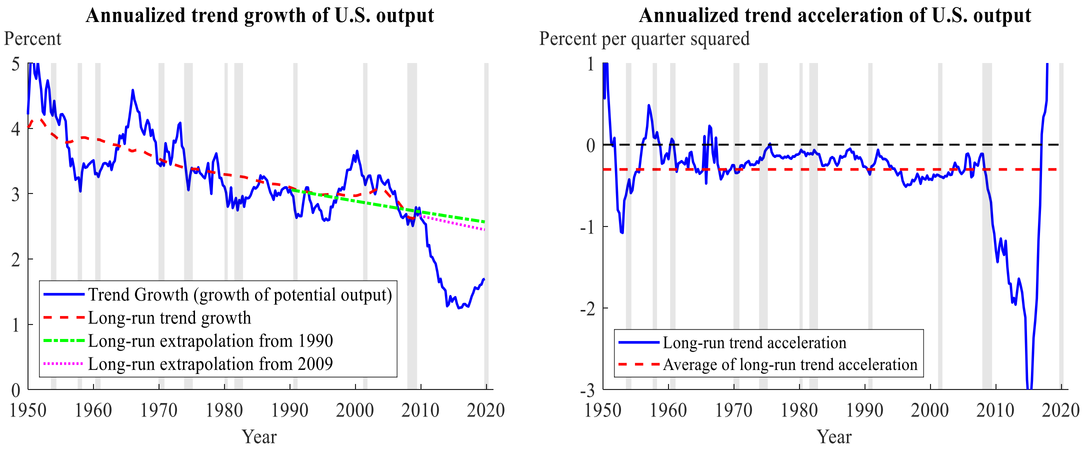
(b) Trend growth of output and the plucking probabilities for bivariate model



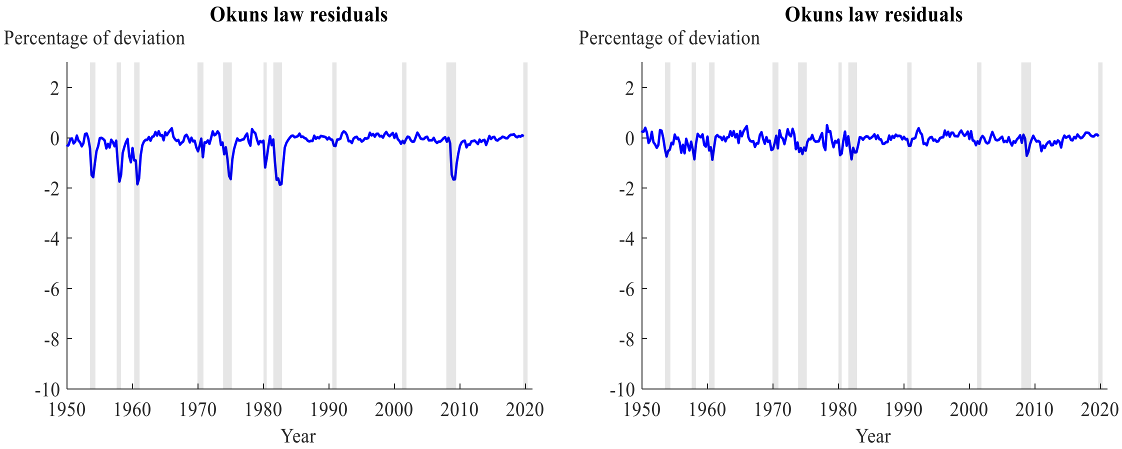
(c) Natural rate of unemployment (trend) and unemployment gap (cyclical component)

Figure D1: Results of the correlated version of the asymmetric bivariate model

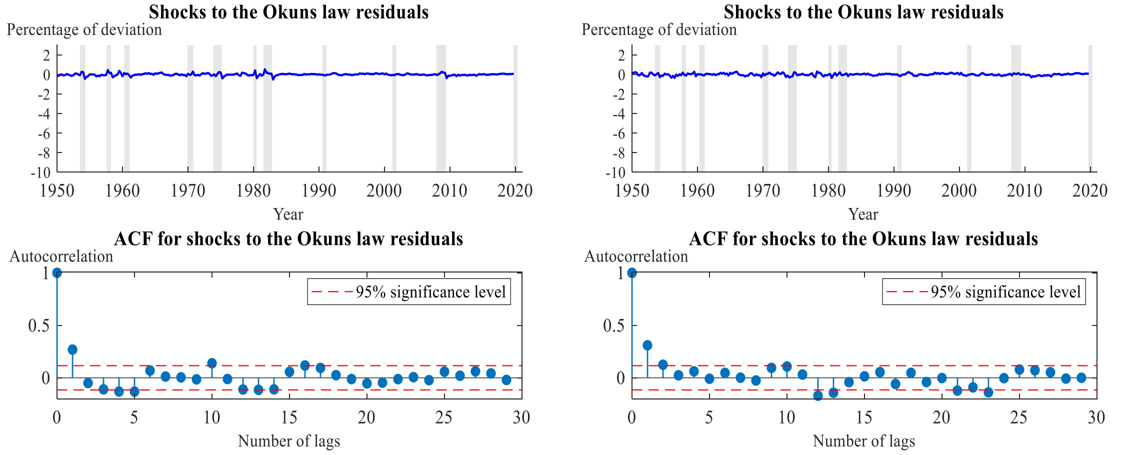
Notes:
(1) All panels plot the results of the correlated version of our proposed model (Asymmetric-Bivariate-RW-SB-C1).
(2) The top panels plot potential output and the output gap, and the middle-left panel plots the trend growth of output.
(3) The middle-right panel plots the plucking probabilities, which are estimated for both output and unemployment jointly.
(4) The bottom panels plot the trend and gap for unemployment.
(5) The shaded areas are the NBER recession dates.



(a) Trend growth and trend acceleration of U.S. output



(b) Okun's law residuals without (left) and with (right) the 1-month lead-lag effect controlled



(c) Diagnostic of shocks to the Okun's residuals for model without (left) and with (right) the 1-month lead-lag effect controlled

Figure D2: Supplementary results of the asymmetric bivariate model

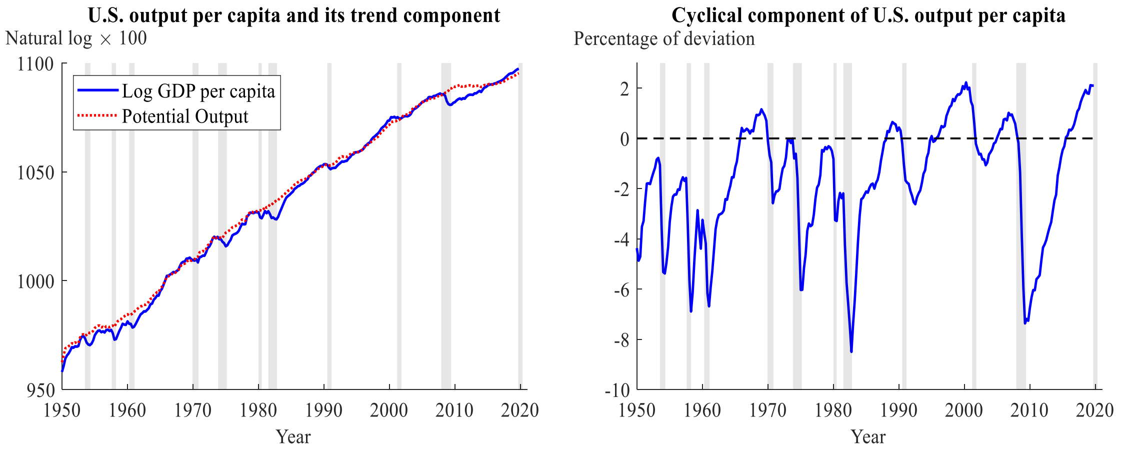
Notes:

(1) The top panels plot trend growth and trend acceleration (growth of trend growth) of U.S. output.

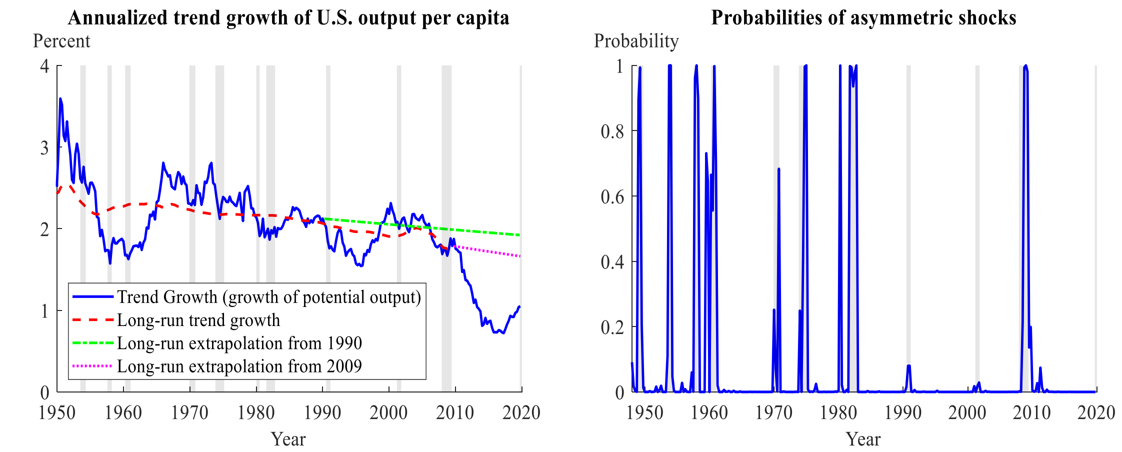
(2) The middle panel plots the Okun's law residuals. In the left panel, although some part of the plucking property remains in the residuals because it is not explained by Okun's law, the depth of the residuals is small in amplitude. The right panel shows controlling the 1-month lead-lag effect between output and unemployment entirely removes the leftover plucking property. This supports that the plucking property in output is mainly sourced from the plucking property in the unemployment rate.

(3) The shocks to the Okuns law residuals are zero-mean noise and its autocorrelation function decays very fast.

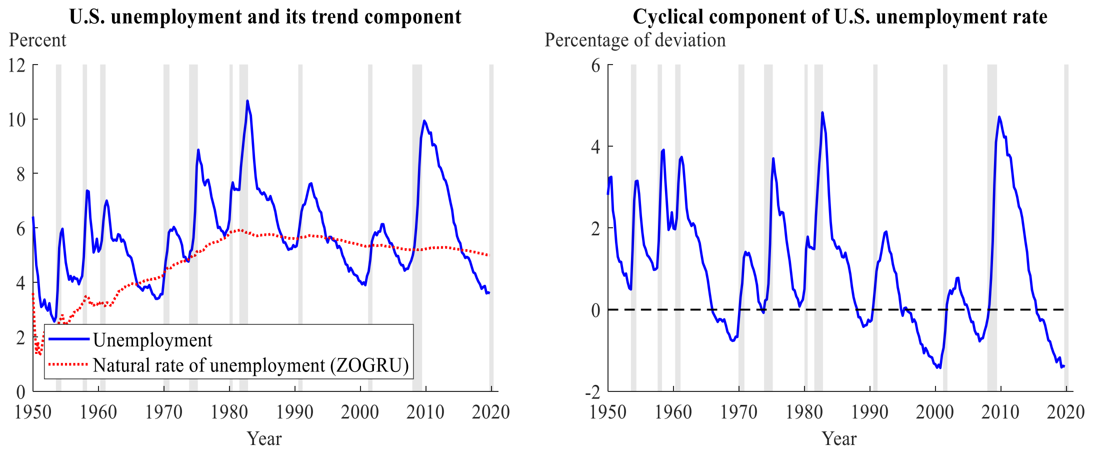
(4) The shaded areas are the NBER recession dates.



(a) Potential and gap for GDP per capita



(b) Trend growth of output per capita and the plucking probabilities for bivariate model



(c) Natural rate of unemployment (trend) and unemployment gap (cyclical component)

Figure D3: Results of the asymmetric bivariate model for GDP per capita

Notes:

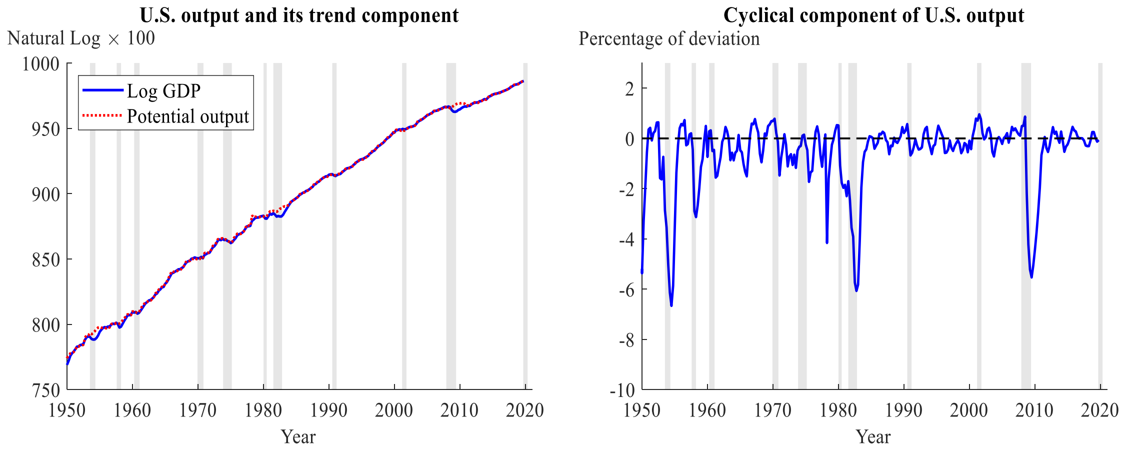
(1) All panels plot the results of our proposed model (Asymmetric-Bivariate-RW-SB-UC), applied to the U.S. GDP per capita.

(2) The top panels plot the potential and gap for GDP per capita, and the middle-left panel plots the trend growth of output per capita.

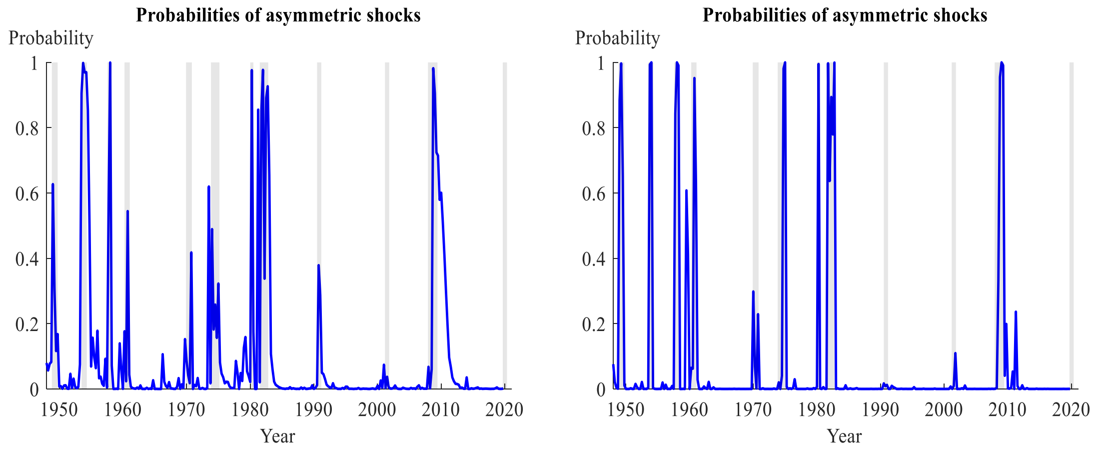
(3) The middle-right panel plots the plucking probabilities, which are estimated for both output and unemployment jointly.

(4) The bottom panels plot the trend and gap for unemployment.

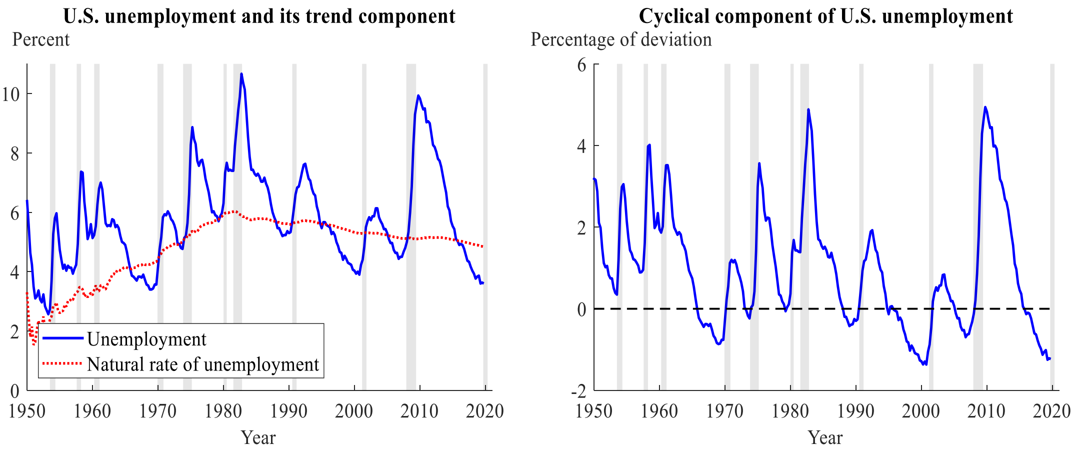
(5) The shaded areas are the NBER recession dates.



(a) Potential output (trend) and output gap (cyclical component)



(b) Plucking probabilities for output (left) and unemployment (right)



(c) Natural rate of unemployment (trend) and the unemployment gap (cyclical components)

Figure D4: Comparing results of the asymmetric univariate models for output and unemployment (correlated models)

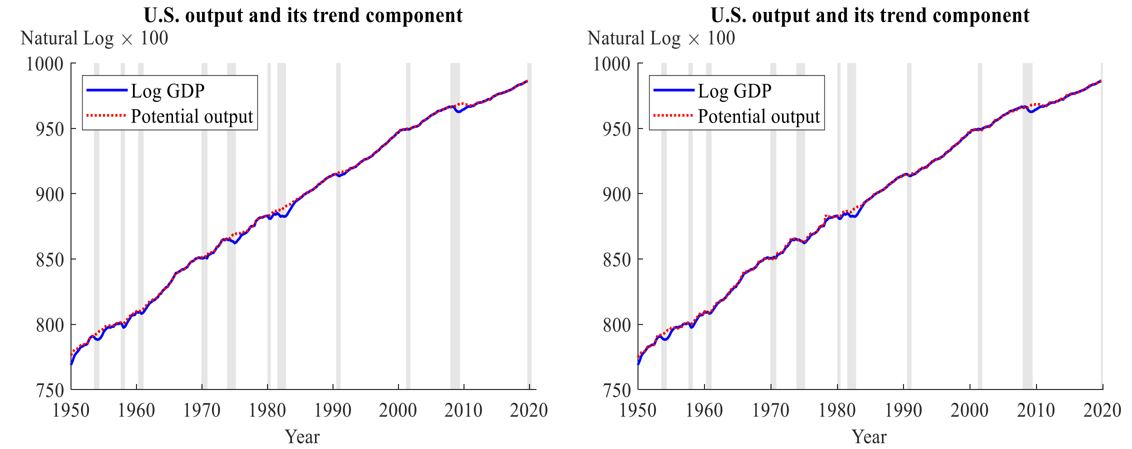
Notes:

(1) The top panels plot the results of the asymmetric univariate model for output with a stochastic (random walk) trend growth, where shocks to the trend and cyclical components are correlated (Asymmetric-Univariate-RW-C).

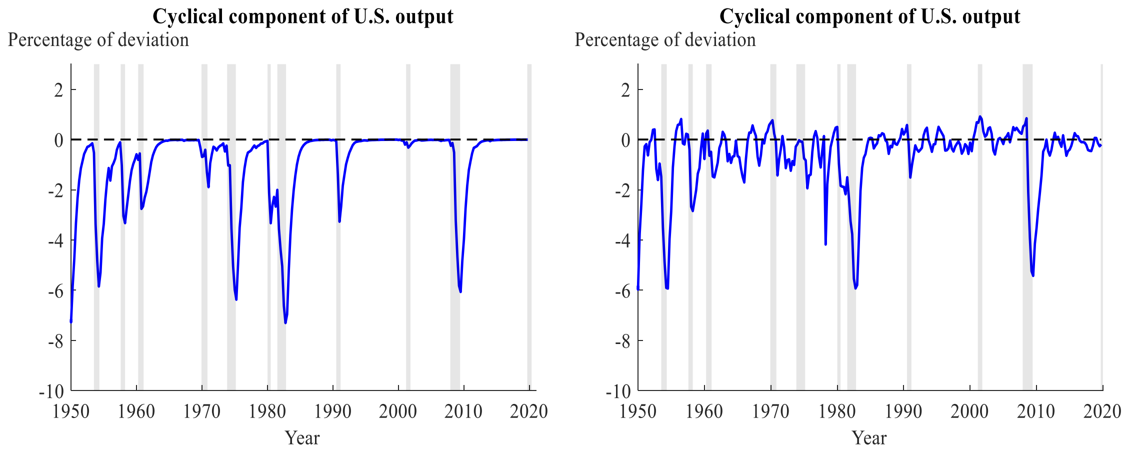
(2) The middle panels plot the plucking probabilities for output and unemployment estimated in two separate models.

(3) The bottom panels plot the results of the asymmetric univariate model for unemployment with a break in the drift of the unemployment trend, where shocks to the trend and cyclical components are correlated (Asymmetric-Univariate-SB-C).

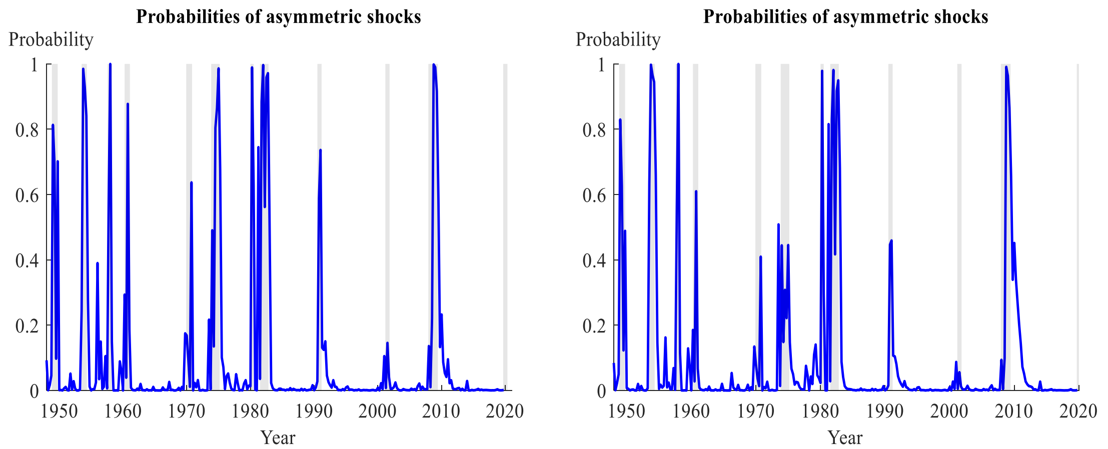
(4) The shaded areas are the NBER recession dates.



(a) The trend component



(b) Output gap (cyclical component)



(c) Plucking probabilities

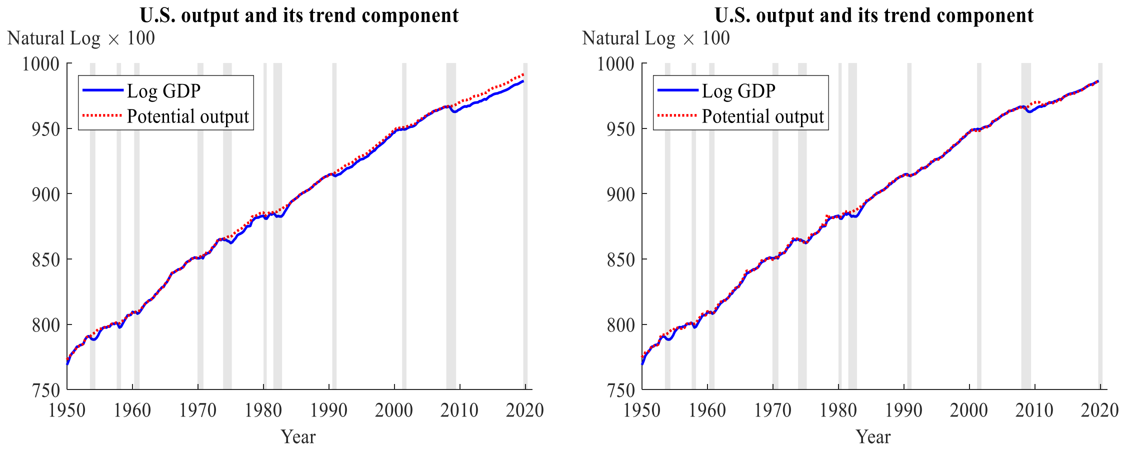
Figure D5: Results of the asymmetric univariate models with a break in trend growth

Notes:

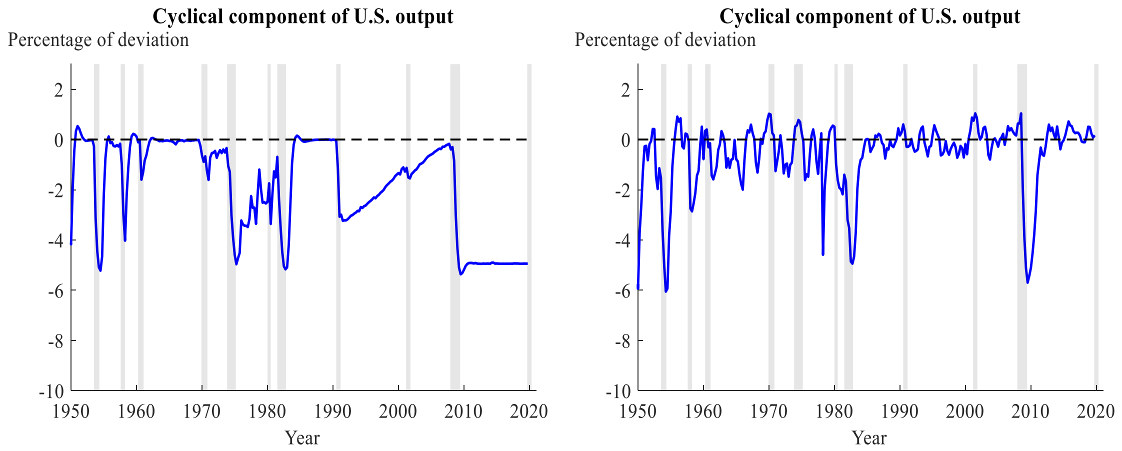
(1) The left panels plot the results of the asymmetric univariate model with uncorrelated shocks.

(2) The right panels plot the results of the asymmetric univariate model with correlated shocks.

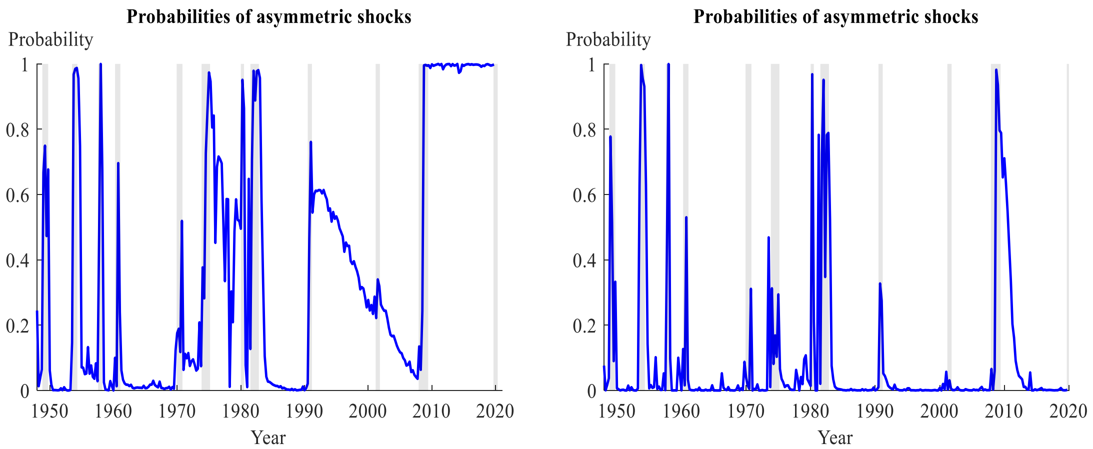
(3) The shaded areas are the NBER recession dates.



(a) The trend component



(b) Output gap (cyclical component)



(c) Plucking probabilities

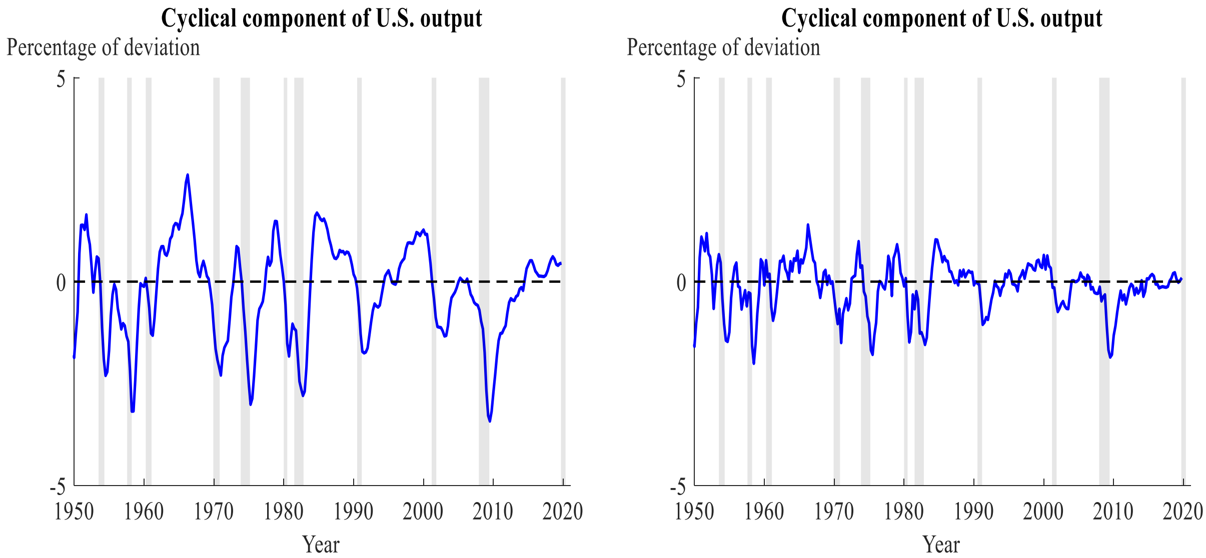
Figure D6: Results of the asymmetric univariate models with constant trend growth

Notes:

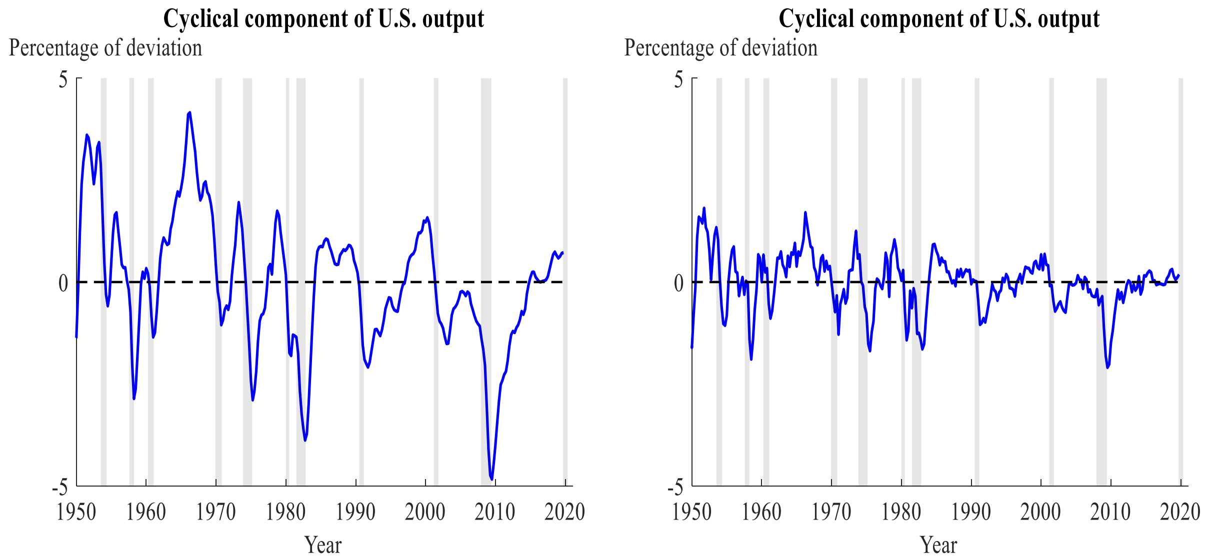
(1) The left panels plot the results of the asymmetric univariate model with uncorrelated shocks.

(2) The right panels plot the results of the asymmetric univariate model with correlated shocks, which replicates the work of Sinclair (2010).

(3) The shaded areas are the NBER recession dates.



(a) The cyclical component of uncorrelated (left) and correlated (right) models, with stochastic trend growth



(b) The cyclical component of uncorrelated (left) and correlated (right) models, with a break in trend growth

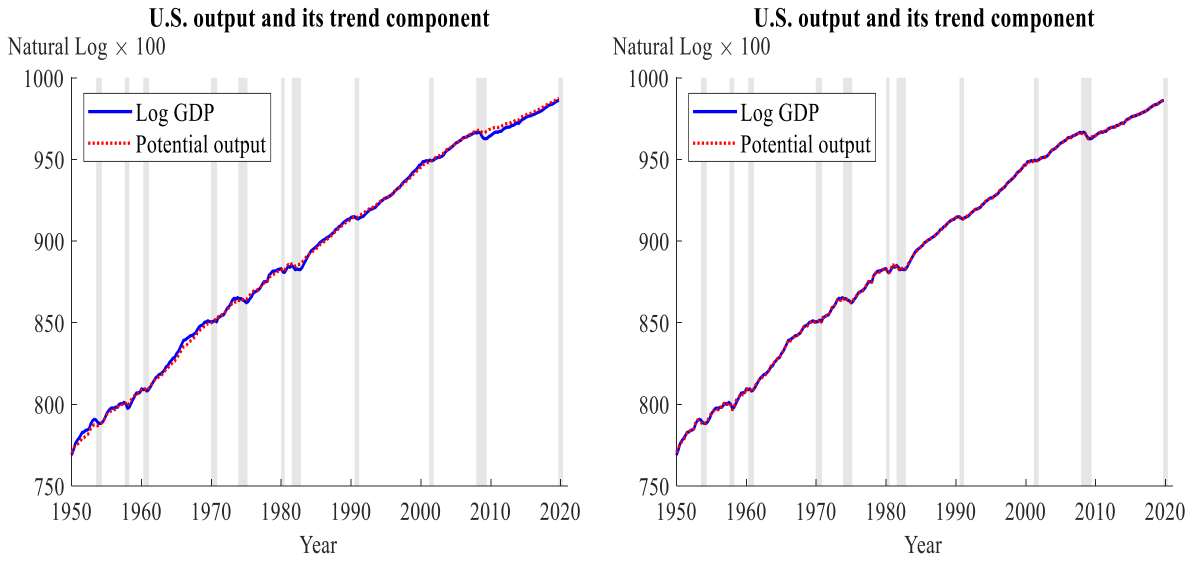
Figure D7. Results of the symmetric univariate models with inconstant trend growth for U.S. output

Notes:

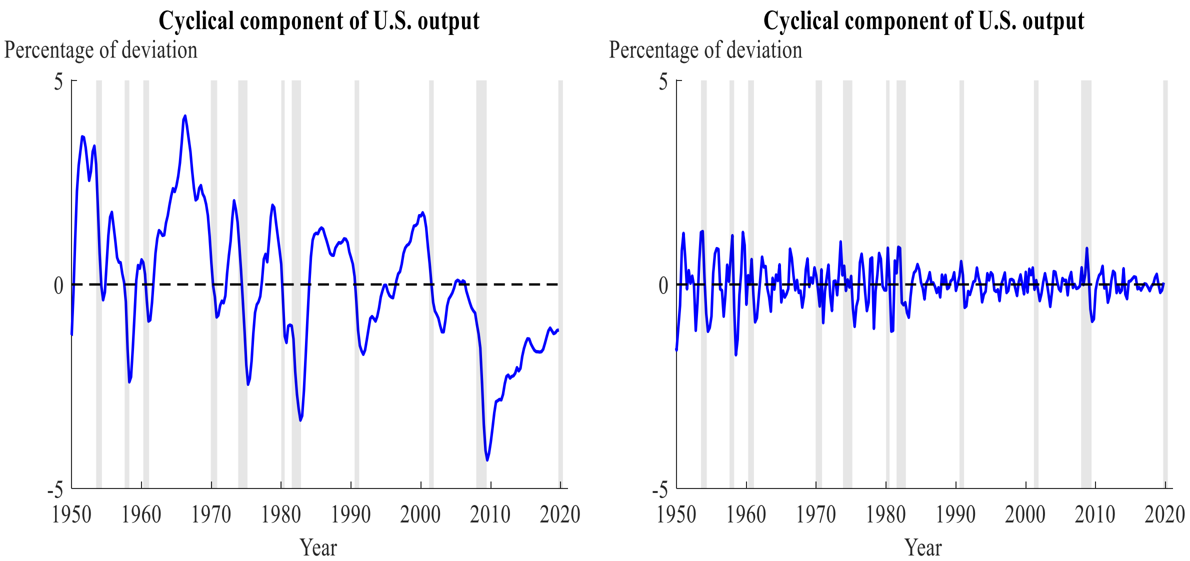
(1) The top panels are the results of the model with stochastic (random walk) trend growth where shocks to the trend and cyclical components are uncorrelated (Symmetric-Univariate-RW-UC) in the left panel and correlated (Symmetric-Univariate-RW-C) in the right panel. The former replicates the work of Clark (1987).

(2) The bottom panels are the results of the model with a structural break in trend growth in 2009, where shocks to the trend and cyclical components are uncorrelated (Symmetric-Univariate-SB-UC) in the left and correlated in the right panels (Symmetric-Univariate-SB-C). They are similar to those of Perron and Wada (2009) and Grant and Chan (2017b) with a break in 2009.

(3) The shaded areas are the NBER recession dates.



(a) The trend component of uncorrelated (left) and correlated (right) models



(b) The cyclical component of uncorrelated (left) and correlated (right) models

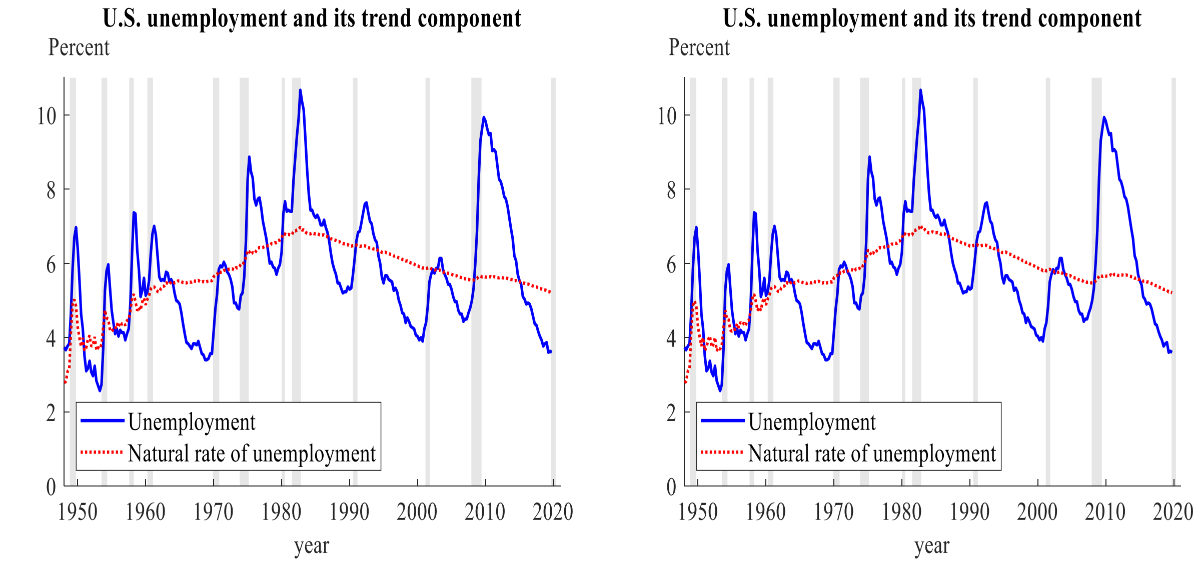
Figure D8: Results of the symmetric univariate models with constant trend growth for U.S. output

Notes:

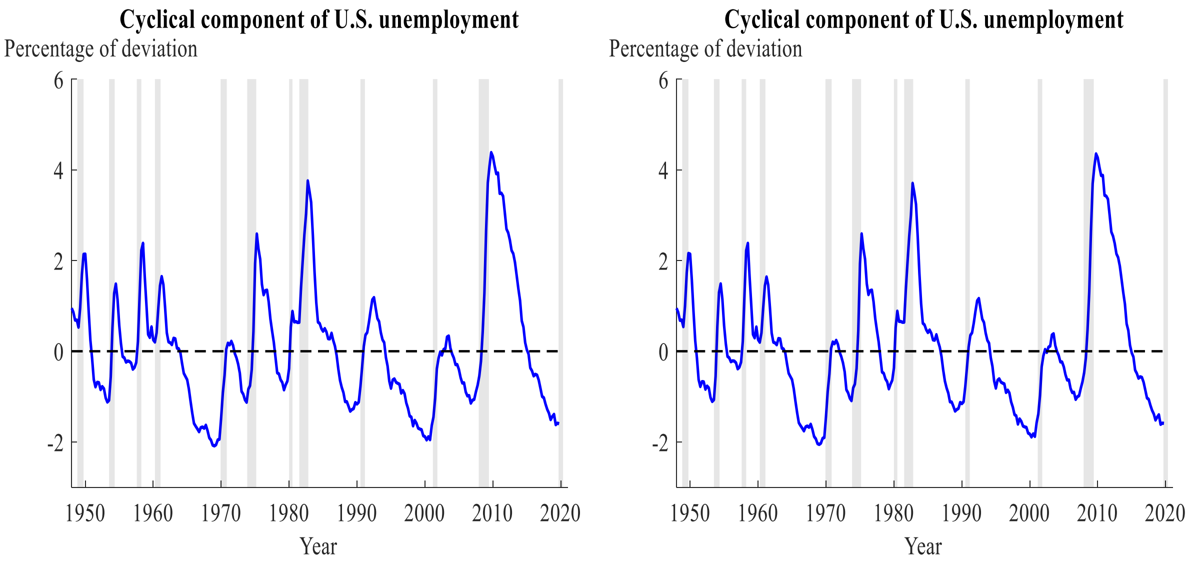
(1) The left panels plot the results of the symmetric univariate model with constant trend growth, where shocks to the trend and cyclical components are uncorrelated (Symmetric-Univariate-Con-UC).

(2) The right panels plot the results of the symmetric univariate model with constant trend growth, where shocks to the trend and cyclical components are correlated (Symmetric-Univariate-Con-C), and replicates those of Morley et al. (2003).

(3) The shaded areas are the NBER recession dates.



(a) The trend component of uncorrelated (left) and correlated (right) models



(b) The cyclical component of uncorrelated (left) and correlated (right) models

Figure D9: Results of the symmetric univariate model for U.S. unemployment

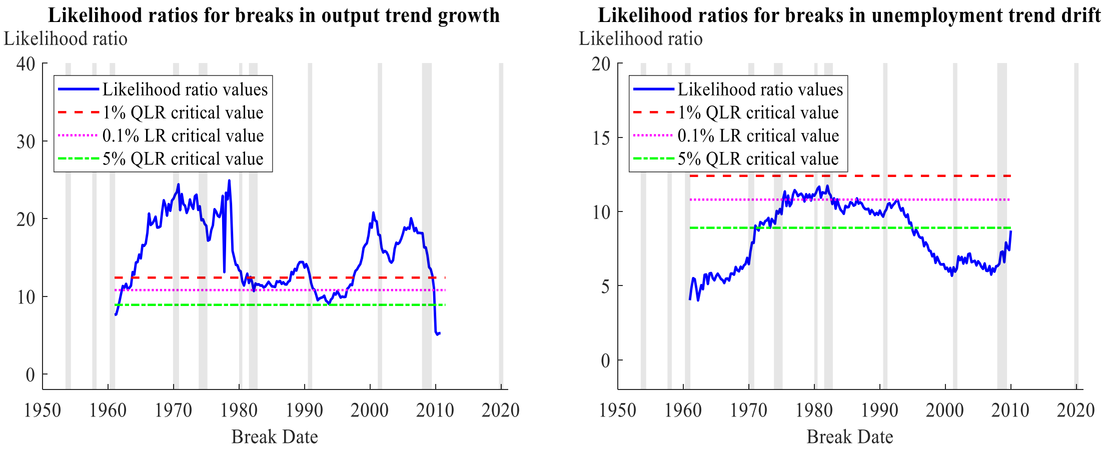
Notes:

(1) The left panels plot the results of the model with a structural break in the drift of the unemployment trend (natural rate) and uncorrelated shocks to the trend and cyclical components (Symmetric-Univariate-SB-UC). Similar setting is applied to the asymmetric bivariate model.

(2) The right panels plot the results of the model with a structural break in drift in unemployment rate trend (natural rate) and correlated shocks to the trend and cyclical components (Symmetric-Univariate-SB-C).

(3) The resemblance of the components of uncorrelated and correlated models indicates an insignificant correlation (-0.60 with a standard error of 0.80) for the unemployment rate.

(4) The shaded areas are the NBER recession dates.



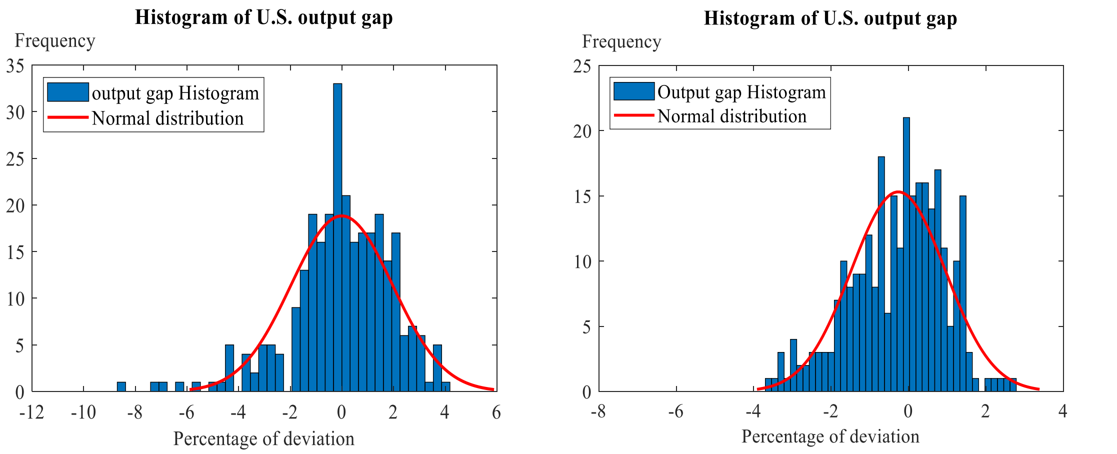
Likelihood ratios at different break dates for trend growth (left) and the unemployment trend (right)

Figure D10: Exploring structural breaks in the parameters for asymmetric univariate model

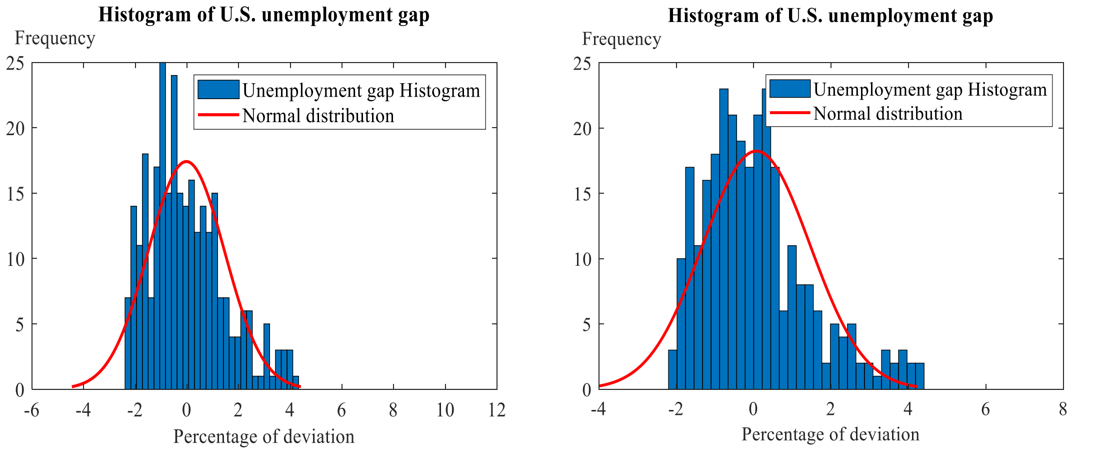
Notes:

(1) Both panels plot likelihood ratio values for a sequence of breaks rolling from 1960 to 2010. The left panel plots the likelihood ratios testing for breaks in trend growth on different dates against a constant trend growth. The right panel plots the likelihood ratios testing for breaks in the drift of the unemployment trend on different dates against a constant trend.

(2) The shaded areas are the NBER recession dates.



(a) The histogram of U.S. output gap estimated by Tukey's bi-weight filter (left) and UC model (right)



(b) The histogram of U.S. unemployment gap by Tukey's bi-weight filter (left) and UC model (right)

Figure D11: Histogram of output gap and unemployment gap.

Notes:

(1) The output gap skewness is -0.93 for the left and -0.41 for the right panel.

(2) The unemployment gap skewness is +0.75 for the left and +0.94 for the right panel.

Appendix D: Additional tables

Table D1 (Continue of Table 1): Estimated parameters of the bivariate models

Models	1c	1d	1e	2c	3a′	3b	3c
Parameters	A-Bi-RW-SB-C2	A-Bi-RW-SB-C3	A-Bi-RW-SB-C123	A-Bi-SB-SB-C2	A-Bi-Con-SB-UC	A-Bi-Con-SB-C1	A-Bi-Con-SB-C2
σ_{x^*}	0.44 (0.09)	0.43 (0.10)	0.44 (0.10)	0.52 (0.06)	0.62 (0.03)	0.63 (0.03)	0.70 (0.09)
σ_{u^c}	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)
σ_{μ}	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	–	–	–	–
σ_{u^*}	0.00 (0.02)	0.00 (0.03)	0.00 (0.03)	0.00 (0.02)	0.00 (0.01)	0.00 (0.03)	0.00 (0.02)
σ_{x^c}	0.18 (2.50)	0.35 (0.09)	0.20 (1.25)	0.28 (1.57)	0.00 (0.10)	0.00 (0.14)	0.13 (0.51)
γ	T-V	T-V	T-V	0.83 (0.03)	–	0.76 (0.03)	0.76 (0.04)
δ	T-V	T-V	T-V	-0.50 (0.08)	–	–	–
η	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.04 (0.01)	0.03 (0.01)	0.04 (0.01)
θ	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)
φ_1	1.38 (0.04)	1.38 (0.04)	1.37 (0.04)	1.36 (0.04)	1.38 (0.04)	1.35 (0.04)	1.37 (0.04)
φ_2	-0.44 (0.04)	-0.44 (0.04)	-0.44 (0.04)	-0.43 (0.04)	-0.44 (0.04)	-0.42 (0.04)	-0.44 (0.04)
π_u	0.70 (0.06)	0.70 (0.06)	0.70 (0.06)	0.70 (0.06)	0.70 (0.06)	0.71 (0.06)	0.70 (0.05)
p	0.66 (0.09)	0.66 (0.09)	0.66 (0.09)	0.67 (0.09)	0.66 (0.09)	0.66 (0.09)	0.66 (0.09)
q	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)
β	-1.44 (0.12)	-1.42 (0.14)	-1.30 (0.17)	-1.44 (0.13)	-1.39 (0.14)	-1.04 (0.29)	-1.29 (0.15)
π_x	-1.01 (0.15)	-1.01 (0.16)	-1.07 (0.17)	-1.05 (0.16)	-1.00 (0.17)	-1.20 (0.25)	-1.05 (0.17)
ψ_x	0.49 (0.10)	0.51 (0.12)	0.52 (0.12)	0.51 (0.11)	0.53 (0.11)	0.66 (0.12)	0.61 (0.11)
ρ_{x^*,u^c}	–	–	-0.24 (0.18)	–	–	-0.16 (0.12)	–
ρ_{x^*,x^c}	0.66 (16.6)	–	-0.65 (7.97)	-0.08 (3.59)	–	–	-0.69 (1.69)
ρ_{u^c,x^c}	–	-0.04 (0.12)	-0.25 (1.67)	–	–	–	–
Log likelihood	-11.9	-11.9	-10.9	-7.4	-22.1	-18.8	-19.2

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) Standard errors of the estimated parameters are reported in parenthesis.
(c) Numerical values for parameters denoted by 0.00 are 0.0001 for model 1c, 0.0002 for model 1d 0.0005 for model 1e, 0.001 for model 2c, .0000003 and 0.0006 for model 3a′, 0.0002 and 0.001 for model 3b, and 0.0006 for model 3c.

Notes:

(1) The estimation period runs from 1948Q1 to 2019Q4. See Table 1 for the main results and explanations.

(2) For all models, the structural break in the drift of the unemployment trend in 1981Q1 is accounted for. For models 2b and 5c, the structural break in trend growth in 2009Q3 is accounted for.

(3) Model 3a′ is another version of model 3a with almost identical estimation of parameters. Since in the former, we treat the drift term (constant trend growth) as a state variable and in the latter, the drift term is estimated as a parameter; model 3a′ is fully nested in model 1a, but model 3a is not.

(4) Models 1c, 2c, and 3c are correlated counterparts of asymmetric bivariate models 1a, 2a, and 3a, respectively, which are presented in Table 1. By comparing the log likelihoods of -11.9, -7.4, and -19.9 for the uncorrelated models with values of -11.9, -7.4, and -19.2 for correlated models, we accept the null hypothesis suggesting that the correlation between shocks to the output trend and remaining cyclical component (ρ_{x^*,x^c}) is negligible, as likelihood ratios of 0.0, 0.0, and 1.4 are zero or near zero.

Table D1 continued: Estimated parameters of the bivariate models

Models	4b	4c	5b	6b	7	8	9
Parameters	S-Bi-RW-SB-C1	S-Bi-RW-SB-C2	S-Bi-SB-SB-C1	S-Bi-Con-SB-C1	A-Bi-RW-SB-UC-SB	A-Bi-SB-RW-SB-UC	A-Bi-RW-SB0-UC
σ_{x^*}	0.55 (0.07)	0.57 (0.13)	0.56 (0.06)	0.64 (0.04)	0.42 (0.20)	0.44 (0.09)	0.44 (0.09)
σ_{u^c}	0.23 (0.02)	0.27 (0.01)	0.23 (0.02)	0.22 (0.01)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)
σ_{μ}	0.02 (0.01)	0.02 (0.01)	–	–	0.03 (0.02)	0.03 (0.01)	0.03 (0.01)
σ_{u^*}	0.14 (0.02)	0.03 (0.04)	0.14 (0.02)	0.14 (0.02)	0.00 (0.03)	0.00 (0.02)	–
σ_{x^c}	0.21 (0.14)	0.27 (0.01)	0.18 (0.13)	0.06 (0.18)	0.550 (0.13) 0.039 (1.50)	0.34 (0.08)	0.34 (0.08)
γ	T-V	T-V	0.83 (0.04)	0.78 (0.04)	T-V	T-V	T-V
δ	T-V	T-V	-0.43 (0.10)	–	T-V	T-V	T-V
η	0.02 (0.02)	0.02 (0.01)	0.02 (0.02)	0.02 (0.02)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)
θ	-0.03 (0.02)	-0.03 (0.01)	-0.03 (0.02)	-0.05 (0.02)	-0.03 (0.01)	-0.04 (0.01)	-0.04 (0.01)
φ_1	1.71 (0.04)	1.61 (0.04)	1.71 (0.04)	1.70 (0.04)	1.40 (0.06)	1.38 (0.04)	1.38 (0.04)
φ_2	-0.74 (0.04)	-0.67 (0.05)	-0.74 (0.04)	-0.73 (0.04)	-0.47 (0.05)	-0.44 (0.04)	-0.45 (0.04)
π_u	–	–	–	–	0.70 (0.06)	0.70 (0.06)	0.70 (0.06)
p	–	–	–	–	0.64 (0.11)	0.66 (0.09)	0.66 (0.09)
q	–	–	–	–	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)
β	-1.38 (0.15)	-1.74 (0.10)	-1.42 (0.16)	-1.27 (0.17)	-1.33 (0.13)	-1.45 (0.12) 0.03 (0.15)	-1.45 (0.12)
π_x	–	–	–	–	-1.07 (0.20)	-1.00 (0.16)	-1.01 (0.15)
ψ_x	0.12 (0.67)	0.55 (0.48)	0.06 (0.80)	0.51 (0.74)	0.58 (0.13)	0.49 (0.11)	0.49 (0.10)
ρ_{x^*,u^c}	-0.65 (0.14)	–	-0.63 (0.15)	-0.65 (0.13)	–	–	–
ρ_{x^*,x^c}	–	-0.07 (0.52)	–	–	–	–	–
ρ_{u^c,x^c}	–	–	–	–	–	–	–
Log likelihood	-49.1	-57.5	-46.2	-54.4	8.7	-11.9	-11.9

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) Standard errors of the estimated parameters are reported in parenthesis.
(c) Numerical values for parameters denoted by 0.00 are 0.0006 for model 7, and 0.0002 for model 8.

Notes continued:

(5) Model 1d is the correlated counterpart of asymmetric bivariate model 1a, which is presented in Table 1. By comparing the log likelihood of -11.9 for the uncorrelated model with the value of -11.9 for the correlated model, we ensure that the correlation between shocks to the symmetric cyclical component and the remaining cyclical component (ρ_{u^c,x^c}) is negligible as its likelihood ratio is 0.0. Further, model 1e allows for all correlations to test if three correlations are jointly significant or not. Comparing the log likelihood of -11.9 for model 1a with the value of -10.9 for model 1e indicates a negligible correlation as the likelihood ratio of 2.0 is smaller than the critical values of 7.81 and 6.25 for 5% and even 10% significance levels with three restrictions.

(6) Models 4b, 5b, and 6b are the correlated counterparts of symmetric bivariate models 4a, 5a, and 6a, respectively. By comparing the log likelihood values of -57.5, -52.6, and -62.5 for the uncorrelated models 4a, 5a, and 6a with values of -49.1, -46.2, and -54.4 for the correlated models 4b, 5b, and 6b, we reject the null hypothesis of zero-correlation between shocks to the trend and symmetric cyclical components (ρ_{x^*,u^c}), because likelihood ratios of 17.0, 12.8, and 16.2 are greater than the 0.1% critical value of 10.8.

(7) In model 7, the estimated volatility of shocks to the cyclical component before and after 1983 is allowed to be different. In model 8, a break in Okun's coefficient in 2009 is allowed, although it is insignificant. Model 9 assumes that the variance of shocks to the unemployment trend is zero ($\sigma_{u^*}^2 = 0$), yet its estimated parameters are identical to those of model 1a.

Table D2 (Continue of Table 2): Estimated parameters of the univariate models for output

Models	3a′	3b	4b	5b	6b	7
Parameters	A-Uni-Con-UC	A-Uni-Con-C	S-Uni-RW-C	S-Uni-SB-C	S-Uni-Con-C	A-Uni-Con-C-SB
σ_{z^*}	0.69 (0.03)	1.13 (0.12)	0.93 (0.19)	0.99 (0.19)	1.18 (0.15)	0.40 (0.03)
σ_{z^c}	0.00 (0.12)	0.70 (0.16)	0.80 (0.20)	0.86 (0.22)	0.79 (0.25)	0.950 (0.08) 0.003 (0.29)
σ_{μ}	–	–	0.02 (0.01)	–	–	0.05 (0.02)
γ	–	0.78 (0.06)	T-V	0.83 (0.06)	0.78 (0.07)	–
δ	–	–	T-V	-0.31 (0.17)	–	–
φ_1	1.12 (0.09)	1.09 (0.07)	1.44 (0.10)	1.44 (0.10)	1.24 (0.20)	1.36 (0.10)
φ_2	-0.29 (0.08)	-0.36 (0.07)	-0.57 (0.07)	-0.56 (0.09)	-0.63 (0.17)	-0.45 (0.08)
π_z	-1.62 (0.22)	-1.88 (0.23)	–	–	–	-1.21 (0.24)
p	0.60 (0.07)	0.61 (0.09)	–	–	–	0.40 (0.26)
q	0.96 (0.01)	0.96 (0.01)	–	–	–	0.97 (0.02)
ρ_{z^*,z^c}	–	-0.88 (0.04)	-0.71 (0.18)	-0.76 (0.14)	-0.92 (0.08)	–
Log likelihood	-344.1	-331.5	-353.2	-350.5	-349.4	-305.4

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) Standard errors of the estimated parameters are reported in parenthesis.
(c) Numerical values for the parameter denoted by 0.00 is 0.001 for model 3a′.

Notes:
(1) The estimation period runs from 1948Q1 to 2019Q4. See Table 2 for the main results and explanations.

(2) For model 5b, the structural break in trend growth in 2009Q3 is accounted for.

(3) Model 3a′ is another version of model 3a. It is clear that their estimations for parameters are very similar. The former treats the drift term (constant trend growth) as a state variable, and the latter estimates the drift as a parameter. Hence, model 3a′ is fully nested in model 1a, but model 3a is not.

(4) Models 4b and 5b are correlated counterparts of univariate models 4a and 5a, respectively, which are presented in Table 2. By comparing the log likelihoods of the uncorrelated models (-354.4 and -352.6) with those of the correlated models (-353.2 and -350.5), we accept the null hypothesis of zero-correlation because the likelihood ratios of 2.4 and 4.2 are less than the 1% critical value of 6.63.

(5) Model 6b is the correlated counterpart of the univariate model 6a presented in Table 2. By comparing the log likelihood of the uncorrelated model (-357.1) with that of the correlated model (-349.4), we reject the null hypothesis of zero-correlation because the likelihood ratio of 15.4 is greater than the 1% critical value of 6.63.

(6) In model 7, the estimated volatility of shocks to the cyclical component before and after 1983 is allowed to be different.

Table D3: Estimated parameters of the asymmetric bivariate model, with controlled lead-lag effect

Models	1a	1-month	2-month	3-month
Parameters	A-Bi-RW-SB-UC	A-Bi-RW-SB-UC	A-Bi-RW-SB-UC	A-Bi-RW-SB-UC
σ_{x^*}	0.44 (0.09)	0.34 (0.11)	0.36 (0.09)	0.33 (0.14)
σ_{u^c}	0.21 (0.01)	0.17 (0.01)	0.16 (0.01)	0.15 (0.01)
σ_{μ}	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)
σ_{u^*}	0.00 (0.02)	0.11 (0.01)	0.12 (0.01)	0.13 (0.01)
σ_{x^c}	0.34 (0.08)	0.38 (0.08)	0.35 (0.07)	0.44 (0.09)
γ	T-V	T-V	T-V	T-V
δ	T-V	T-V	T-V	T-V
η	0.03 (0.01)	0.02 (0.01)	0.03 (0.01)	0.03 (0.01)
θ	-0.04 (0.01)	-0.03 (0.01)	-0.03 (0.01)	-0.03 (0.01)
φ_1	1.38 (0.04)	1.47 (0.04)	1.48 (0.04)	1.47 (0.04)
φ_2	-0.45 (0.04)	-0.53 (0.04)	-0.55 (0.04)	-0.54 (0.04)
π_u	0.70 (0.06)	0.66 (0.06)	0.62 (0.05)	0.65 (0.06)
p	0.66 (0.09)	0.57 (0.13)	0.61 (0.11)	0.61 (0.11)
q	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)
β	-1.45 (0.12)	-1.86 (0.13)	-1.96 (0.10)	-1.91 (0.10)
π_x	-1.01 (0.15)	-0.46 (0.18)	-0.000 (0.001)	-0.000 (0.001)
ψ_x	0.49 (0.10)	0.39 (0.20)	0.12 (0.21)	0.28 (0.20)
Log likelihood	-11.9	-22.3	-23.3	-38.0

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) Standard errors of the estimated parameters are reported in parenthesis.

Notes:

(1) The estimation period runs from 1948Q1 to 2019Q4.

(2) All columns present the results of the proposed model 1a with different observed series for unemployment. The first column is the model without controlling the lead-lag effect between output and the unemployment rate. In this model, we simply calculate the quarterly unemployment rate as the average of the rates of the three months within the corresponding quarter. On the other hand, columns two, three, and four present the estimation results when the 1-month, 2-month, and 3-month lead-lag effect is accounted for. For example, we use 1-month leading unemployment as the data for the observed series to estimate the 1-month model. We calculate the 1-month leading unemployment rate for each quarter by taking an average of three months, two of which are within the same quarter, and the other one is in the subsequent quarter.

(3) The output-specific plucking coefficient (π_x) is significant. This suggests that some minor part of the plucking property is not explained by Okun's law. However, it is clear that this leftover plucking property is related to the lead-lag effect between output and the unemployment rate because by controlling the 1-month lead-lag effect, the coefficient will be less significant, and finally, by controlling the 2-month lead-lag effect, the output-specific plucking coefficient will be zero.

(4) Log likelihood values in this table are not comparable since the data inputs for unemployment are different.

Table D4: Estimated parameters of the bivariate model, including the COVID-19 recession

Models	1a1	1a2
Parameters	A-Bi-RW-SB-UC	A-Bi-RW-SB-UC
σ_{x^*}	0.60 (0.03)	0.60 (0.03)
σ_{u^c}	0.58 (0.02)	0.56 (0.02)
σ_{μ}	0.02 (0.01)	0.02 (0.01)
σ_{u^*}	0.00 (0.01)	0.00 (0.02)
σ_{x^c}	0.00 (1.52)	0.00 (0.12)
γ	T-V	T-V
δ	T-V	T-V
η	0.03 (0.01)	0.03 (0.01)
θ	-0.04 (0.01)	-0.04 (0.01)
φ_1	0.75 (0.05)	0.69 (0.05)
φ_2	0.09 (0.05)	0.06 (0.05)
π_u	1.33 (0.17)	1.26 (0.14)
p	0.65 (0.09)	0.76 (0.07)
q	0.96 (0.01)	0.96 (0.01)
β	-1.18 (0.06)	-1.18 (0.06)
π_x	-1.38 (0.16)	-1.35 (0.16)
ψ_x	-0.12 (0.17)	-0.17 (0.15)
$\pi_{u,Covid-19}$	0.46 (0.28)	1.38 (0.32)
ρ_{x^*,u^c}	–	–
ρ_{x^*,x^c}	–	–
ρ_{u^c,x^c}	–	–
Log likelihood	-310.3	-305.8

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) Standard errors of the estimated parameters are reported in parenthesis.
(c) Numerical values for parameters denoted by 0.00 are 0.0004, 0.003, 0.00017 and 0.00024, respectively.

Notes:

(1) The estimation period is from 1948Q1 to 2022Q4. The table reports the results of the modified model (Asymmetric-Bivariate-RW-SB-UC-Mod), which accounts for the COVID-19 recession by using Eq. (19) that includes a dummy for the COVID-19 recession. The start of the COVID-19 pandemic ($T_{COVID-start}$) is assumed to be 2020Q1 for model 1a1 and 2020Q2 for model 1a2. The end of the COVID-19 pandemic ($T_{COVID-end}$) is 2021Q1 for both models.

(2) The COVID-specific plucking coefficient ($\pi_{u,COVID}$) is sizeable as the depth of the Covid-19 recession is greater than those of previous recessions. The estimated common plucking coefficient (π_u) is also larger than that of the main proposed model to explain a portion of the greater depth of the COVID-19 recession compared to that of previous recessions.

(3) The log likelihood value of these modified models are not comparable with that of the main model because the data inputs for output and the unemployment rate are different from those of the main model.

(4) We set initial values for plucking parameters and one of the standard deviations (σ_{x^c}) different from those presented in Table C1 for the proposed model 1a to avoid deriving imaginary standard errors for σ_{x^c}. The selected initial values are 1.1, -1.1, 1.1, and 0.45 for π_u, π_x, $\pi_{u,Covid-19}$, and σ_{x^c}, respectively.

Table D5: Estimated parameters of the bivariate model, for U.S. output per capita and U.K. output

Models	1a for U.S. GDP per capita	1a for U.K. GDP
Parameters	A-Bi-RW-SB-UC	A-Bi-RW-SB-UC
σ_{x^*}	0.50 (0.09)	0.75 (0.13)
σ_{u^c}	0.21 (0.01)	0.10 (0.01)
σ_{μ}	0.02 (0.01)	0.02 (0.01)
σ_{u^*}	0.00 (0.02)	0.08 (0.01)
σ_{x^c}	0.29 (0.10)	0.35 (0.22)
γ	T-V	T-V
δ	T-V	T-V
η	0.03 (0.01)	0.04 (0.01)
θ	-0.04 (0.01)	-0.05 (0.02)
φ_1	1.38 (0.05)	1.62 (0.07)
φ_2	-0.44 (0.04)	-0.65 (0.06)
π_u	0.70 (0.05)	0.23 (0.05)
p	0.66 (0.09)	0.88 (0.07)
q	0.96 (0.01)	0.97 (0.02)
β	-1.45 (0.12)	-1.53 (0.35)
π_x	-1.02 (0.16)	-0.77 (0.52)
ψ_x	0.48 (0.11)	0.60 (0.28)
ρ_{x^*,u^c}	–	–
ρ_{x^*,x^c}	–	–
ρ_{u^c,x^c}	–	–
Log likelihood	-13.4	28.2

(a) T-V means that the model considers a time-varying state variable for the corresponding parameter.
(b) Standard errors of the estimated parameters are reported in parenthesis.

Note:

The estimation period runs from 1948Q1 to 2019Q4 for U.S. GDP per capita and spans from 1955Q1 to 2019Q4 for the U.K. GDP.

Appendix E: Business cycle dates

Table E1: Dates of the U.S. Business Cycles (Peak and Trough)

N	ECRI[a]	NBER[b]	Description
1	1957M8-1958M4	1957M8-1958M4	--
2	1960M4-1961M2	1960M4-1961M2	--
3	1969M12-1970M11	1969M12-1970M11	--
4	1973M11-1975M3	1973M11-1975M3	First Oil Crisis
5	1980M1-1980M7	1980M1-1980M7	Second Oil Crisis
6	1981M7-1982M11	1981M7-1982M11	Early 1980s recession
7	1990M7-1991M3	1990M7-1991M3	Early 1990s recession
8	2001M3-2001M11	2001M3-2001M11	Early 2000s recession
9	2007M12-2009M6	2007M12-2009M6	Global crisis and recession
10	2020M2-2020M4	2020M2-2020M4	COVID-19 recession

(a) Economic Cycle Research Institute
(b) National Bureau of Economic Research

Table E2: Dates of the U.K. Business Cycles (Peak and Trough)

N	ECRI[a]	NIESR[b]	Description
1	-	1951M3-1952M8	--
2	-	1955M12-1958M11	--
3	-	1961M3-1963M1	--
4	1974M9-1975M8	1973M1-1975M3	First Oil Crisis
5	1979M6-1981M5	1979M2-1982M4	Second Oil Crisis
6	-	1984M1-1984M3	--
7	-	1988M4-1992M2	Early 1990s recession
8	1990M5-1992M3	-	Early 1990s recession
9	2008M5-2010M1	-	Global crisis and recession
10	2019M10-2020M4	-	COVID-19 recession

(a) Economic Cycle Research Institute
(b) National Institute of Economic and Social Research

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