

DivClust: Controlling Diversity in Deep Clustering

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Abstract

Clustering has been a major research topic in the field of machine learning, one to which Deep Learning has recently been applied with significant success. However, an aspect of clustering that is not addressed by existing deep clustering methods, is that of efficiently producing multiple, diverse partitionings for a given dataset. This is particularly important, as a diverse set of base clusterings are necessary for consensus clustering, which has been found to produce better and more robust results than relying on a single clustering. To address this gap, we propose DivClust, a diversity controlling loss that can be incorporated into existing deep clustering frameworks to produce multiple clusterings with the desired degree of diversity. We conduct experiments with multiple datasets and deep clustering frameworks and show that: a) our method effectively controls diversity across frameworks and datasets with very small additional computational cost, b) the sets of clusterings learned by DivClust include solutions that significantly outperform single-clustering baselines, and c) using an off-the-shelf consensus clustering algorithm, DivClust produces consensus clustering solutions that consistently outperform single-clustering baselines, effectively improving the performance of the base deep clustering framework. Code is available at <https://github.com/ManiadisG/DivClust>.

1. Introduction

The exponentially increasing volume of visual data, along with advances in computing power and the development of powerful Deep Neural Network architectures, have revived the interest in unsupervised learning with visual data. Deep clustering in particular has been an area where significant progress has been made in the recent years. Existing works focus on producing a single clustering, which is evaluated in terms of how well that clustering matches

the ground truth labels of the dataset in question. However, consensus, or ensemble, clustering remains under-studied in the context of deep clustering, despite the fact that it has been found to consistently improve performance over single clustering outcomes [3, 17, 45, 73].

Consensus clustering consists of two stages, specifically generating a set of base clusterings, and then applying a consensus algorithm to aggregate them. Identifying what properties ensembles should have in order to produce better outcomes in each setting has been an open problem [18]. However, research has found that inter-clustering diversity within the ensemble is an important, desirable factor [14, 20, 24, 34, 51], along with individual clustering quality, and that diversity should be moderated [15, 22, 51]. Furthermore, several works suggest that controlling diversity in ensembles is important toward studying its impact and determining its optimal level in each setting [22, 51].

The typical way to produce diverse clusterings is to promote diversity by clustering the data multiple times with different initializations/hyperparameters or subsets of the data [3, 17]. This approach, however, does not guarantee or control the degree of diversity, and is computationally costly, particularly in the context of deep clustering, where it would require the training of multiple models. Some methods have been proposed that find diverse clusterings by including diversity-related objectives to the clustering process, but those methods have only been applied to clustering precomputed features and cannot be trivially incorporated into Deep Learning frameworks. Other methods tackle diverse clustering by creating and clustering diverse feature subspaces, including some that apply this approach in the context of deep clustering [48, 61]. Those methods, however, do not control inter-clustering diversity. Rather, they influence it indirectly through the properties of the subspaces they create. Furthermore, typically, existing methods have been focusing on producing orthogonal clusterings or identifying clusterings based on independent attributes of relatively simple visual data (e.g. color/shape). Consequently, they are oriented toward *maximizing* inter-clustering diversity, which is not appropriate for consensus

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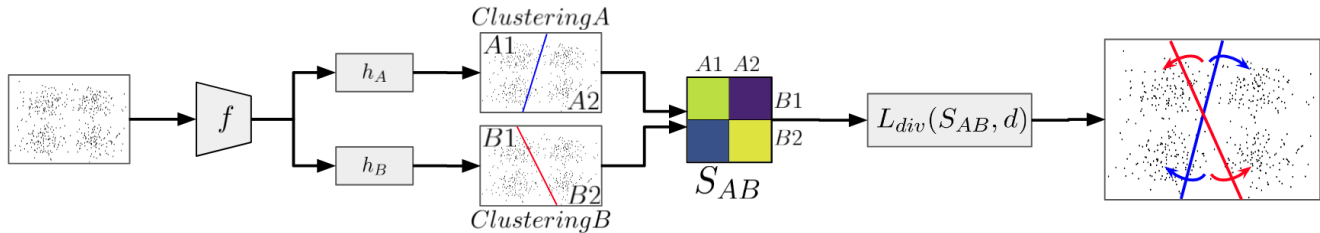


Figure 1. Overview of DivClust. Assuming clusterings A and B , the proposed diversity loss L_{div} calculates their similarity matrix S_{AB} and restricts the similarity between cluster pairs to be lower than a similarity upper bound d . In the figure, this is represented by the model adjusting the cluster boundaries to produce more diverse clusterings. Best seen in color.

clustering [15, 22, 51].

To tackle this gap, namely generating multiple clusterings with deep clustering frameworks efficiently and with the desired degree of diversity, we propose DivClust. Our method can be straightforwardly incorporated into existing deep clustering frameworks to learn multiple clusterings whose diversity is *explicitly controlled*. Specifically, the proposed method uses a single backbone for feature extraction, followed by multiple projection heads, each producing cluster assignments for a corresponding clustering. Given a user defined diversity target, in this work expressed in terms of the average NMI between clusterings, DivClust restricts inter-clustering similarity to be below an appropriate, dynamically estimated threshold. This is achieved with a novel loss component, which estimates inter-clustering similarity based on soft cluster assignments produced by the model, and penalizes values exceeding the threshold. Importantly, DivClust introduces minimal computational cost and requires no hyperparameter tuning with respect to the base deep clustering framework, which makes its use simple and computationally efficient.

Experiments on four datasets (CIFAR10, CIFAR100, Imagenet-10, Imagenet-Dogs) with three recent deep clustering methods (IIC [37], PICA [32], CC [44]) show that DivClust can effectively control inter-clustering diversity without reducing the quality of the clusterings. Furthermore, we demonstrate that, with the use of an off-the-shelf consensus clustering algorithm, the diverse base clusterings learned by DivClust produce consensus clustering solutions that outperform the base frameworks, effectively improving them with minimal computational cost. Notably, despite the sensitivity of consensus clustering to the properties of the ensemble, our method is robust across various diversity levels, outperforming baselines in most settings, often by large margins. Our work then provides a straightforward way for improving the performance of deep clustering frameworks, as well as a new tool for studying the impact of diversity in deep clustering ensembles [51].

In summary, DivClust: a) can be incorporated in ex-

isting deep clustering frameworks in a plug-and-play way with very small computational cost, b) can explicitly and effectively control inter-clustering diversity to satisfy user-defined targets, and c) learns clusterings that can improve the performance of deep clustering frameworks via consensus clustering.

2. Related Works

2.1. Deep Clustering

The term deep clustering refers to methods that cluster data while learning their features. They are generally divided into two categories, namely those that alternate training between clustering and feature learning and those that train both simultaneously.

Alternate learning: Methods following this approach generally utilize a two-step training regime repeated in regular intervals (e.g. per-epoch or per-step). First, sample pseudo-labels are produced based on representations extracted by the model (e.g. by feature clustering). Second, those pseudo-labels are utilized to improve the learned representations, typically by training the feature extraction model as a classifier. Those methods include DEC [64], DAC [8], DCCM [63], DDC [7], JULE [66], SCAN [57], ProPos [33] and SPICE [49], as well as DSC-N [36], IDFD [65] and MIX’EM [58], which propose ways to train models whose representations produce better outcomes when clustered. Other works in this area are DeepCluster [5], SeLa [1], PCL [42] and HCSC [21], though their primary focus is on representation learning.

Simultaneous learning: These methods jointly learn features and cluster assignments. They include ADC [23], IIC [37] and PICA [32], which train clustering models end-to-end with loss functions that enforce desired properties on the clusters assignments, ConCURL [11, 52], which builds on BYOL [19] with a loss maximizing the agreement of clusterings from transformed embeddings, DCCS [71], which leverages an adversarial component in the clustering process, and GatCluster [50], which proposes an attention

mechanism combined with four self-learning tasks. Finally, methods such as SCL [31], CC [44], GCC [72], TCC [53] and MiCE [56] leverage contrastive learning.

Although some deep clustering methods [1, 37, 52] use multiple clusterings, most do not explore the prevalence and impact of inter-clustering diversity, and none proposes ways to control it. Our work is, to the best of our knowledge, the first that addresses both issues.

2.2. Diverse Clustering

The most straightforward way of producing multiple, diverse clusterings is clustering the data multiple times. Typical methods to increase diversity include varying the clustering algorithm or its hyperparameters, using different initializations, and clustering a subset of the samples or features [3]. This approach, however, is a) computationally costly, in that it requires clustering the data multiple times, b) unreliable, as some ways to increase diversity might decrease the quality of clusters (e.g. using a subset of the data), and c) ineffective, as there is no guarantee that the desired degree of diversity will be achieved.

To tackle this, several methods have been proposed to create multiple, diverse clusterings [25]. We identify two main approaches of promoting inter-clustering diversity: a) explicitly, by optimizing for appropriate objectives, and b) implicitly, by optimizing for decorrelated/orthogonal feature subspaces, which, when clustered, lead to diverse clusterings. Methods in the first category include COALA [2], Meta Clustering [6], Dec-kmeans [35], MNMF [67], MSC [26], ADFT [10] and MultiCC [60]. Subspace clustering methods include MISC [59], ISAAC [69], NR-kmeans [47], RAOSC [70] and ENRC [48]. Distinctly, diverse clustering has also been explored in the context of multi-view data by OSC [9], MVMC [68], DMSMF [46], DMClusts [62], DiMSC [4], and DiMVMC [61].

To the best of our knowledge, except for DiMVMC and ENRC, *none* of the existing methods are compatible with Deep Learning, they require a learned feature space on which to be applied, and most have quadratic complexity relative to the number of samples. This restricts their use on real-life high dimensional data, where deep clustering produces better outcomes [56, 57]. Regarding DiMVMC and ENRC, they depend on autoencoder-based architectures and adapting them to more recent deep clustering frameworks, which perform significantly better, is not trivial. More importantly, they utilize subspace clustering, inheriting its limitations regarding controlling diversity. Specifically: a) no method has been proposed to infer *how* different the subspaces must be in order to lead to a *specific* degree of inter-clustering diversity, and b) subspace clustering methods inherit the randomness of the clustering algorithm applied to the subspaces (K-means for DiMVMC and ENRC), which further limits their control over the outcomes.

2.3. Consensus Clustering

The performance of clustering algorithms varies depending on the data and their properties, the algorithm itself, and its hyperparameters. This makes finding reliable clustering solutions particularly difficult. Consensus, or ensemble, clustering has emerged as a solution to this problem, specifically by combining the results of multiple, different clusterings, rather than relying on a single solution. This has been found to produce better and more robust outcomes than single-clustering approaches [3, 17, 18, 45]. The process of consensus clustering happens in two stages: a) multiple, diverse base clusterings are generated and b) those clusterings are aggregated using a consensus algorithm.

Generating diverse clusterings: The properties of the set of clusterings used by the consensus algorithm is a key factor for obtaining good performance. Multiple works [24, 40, 51] have found that both the quality of individual base clusterings and their diversity is critical, and that, indeed, clustering ensembles with a moderate degree of diversity lead to better outcomes [15, 20, 22]. Typical methods for ensemble generation include using different clustering algorithms [13], using different initializations of the same clustering algorithm or different hyperparameters (e.g. the number of clusters) [16, 22, 41], clustering with different subsets of the features [54], using random projections to diversify the feature space [14], and clustering with different subsets of the dataset [12, 13]. However, concrete methods for identifying optimal hyperparameters, such as the degree of diversity, the number of clusterings in the ensemble, and the method by which the ensemble is generated, remain elusive.

Consensus algorithms: Consensus algorithms aim to aggregate multiple, diverse clusterings to produce a single, robust solution. Various approaches to this problem have been proposed, such as using matrix factorization [43], distance minimization between clusterings [75], utilizing multiple views [55], graph learning [28, 73, 74] and matrix co-association [29, 38]. We note that, while improving consensus algorithms increase the robustness of consensus clustering overall, the stages of ensemble generation and its aggregation with consensus algorithms are largely independent.

Consensus Clustering & Deep Learning: Despite the established advantages of consensus clustering over single-clustering approaches, consensus clustering has not been explored in the context of deep clustering. A possible reason is the computational cost of generating multiple, diverse base clusterings, which would require training multiple models. The only work that has, to the best of our knowledge, applied consensus clustering in the deep clustering setting is DeepCluE [27]. Notably, however, the base clusterings used by DeepCluE are not all learned by the model. Rather, a single-clustering model is trained, and an ensemble is generated by clustering features from multiple layers of the model with U-SPEC [30]. Our work addresses

this gap, by proposing a way to train a single deep clustering model to generate multiple clusterings with controlled diversity and with minimal computational overhead.

3. Method

Overview: Our method consists of two components: a) A novel loss function that can be incorporated in deep clustering frameworks to control inter-clustering diversity by applying a threshold to cluster-wise similarities, and b) a method for dynamically estimating that threshold so that the clusterings learned by the model are sufficiently diverse, according to a user-defined metric.

More concretely, we assume a deep clustering model that learns K clusterings (typically a backbone encoder followed by K projection heads), a deep clustering framework and its loss function L_{main} , and a diversity target D^T set by the user, expressed as an upper bound to inter-clustering similarity¹ (i.e. the maximum acceptable similarity). In order to control the inter-clustering similarity D^R of the learned clusterings so that $D^T \leq D^R$, we propose a complementary loss L_{div} . Specifically, given soft cluster assignments for a pair of clusterings $A, B \in K$, we define the inter-clustering similarity matrix $S_{AB} \in \mathbb{R}^{C_A \times C_B}$, where C_A and C_B is the number of clusters in each clustering, and $S_{AB}(i, j) \in [0, 1]$ measures the similarity between clusters $i \in C_A$ and $j \in C_B$. It follows that decreasing the values of S_{AB} reduces the similarity between the clusters of A and B , and therefore increases their diversity. Accordingly, L_{div} utilizes S_{AB} in order to restrict inter-clustering similarity to be under an upper similarity bound d . The value of d is dynamically adjusted during training, decreasing when $D^R > D^T$ and increasing when $D^R \leq D^T$, thereby tightening and relaxing the loss function so that, overall and throughout training, inter-clustering similarity D^R remains at or under the desired level D^T .

Defining the inter-clustering similarity matrix: Our method assumes a standard deep clustering architecture, consisting of an encoder f , followed by K projection heads h_1, \dots, h_K , each of which produces assignments for a clustering k . Specifically, let X be a set of N unlabeled samples. The encoder maps each sample $x \in X$ to a representation $f(x)$, and each projection head h_k maps $f(x)$ to C_k clusters, so that $p_k(x) = h_k(f(x)) \in \mathbb{R}^{C_k \times 1}$ represents a probability assignment vector mapping sample $x \in X$ to C_k clusters in clustering k . Without loss of generality, we assume that $C = C_k \forall k \in K$. Each clustering can then be represented by a cluster assignment matrix $P_k(X) = [p_k(x_1), p_k(x_2), \dots, p_k(x_N)] \in \mathbb{R}^{C \times N}$. The column $p_k(n)$, that is the probability assignment vector for the

¹It is trivial to modify our formulation to enforce a lower bound. However, experiments (see Sec. 4.2) showed that, when learning multiple clusterings, deep clustering frameworks inherently tend to converge to near-identical solutions, which made the lower bound scenario redundant.

n -th sample, encodes the degrees to which sample x_n is assigned to different clusters. The row vector $q_k(i) \in \mathbb{R}^N$ shows which samples are softly assigned to cluster $i \in C$. We refer to $q_k(i)$ as the cluster membership vector.

To quantify the similarity between clusterings A and B we define the inter-clustering similarity matrix $S_{AB} \in \mathbb{R}^{C \times C}$. We define each element $S_{AB}(i, j)$ as the cosine similarity between the cluster membership vector $q_A(i)$ of cluster $i \in A$ and the cluster membership vector $q_B(j)$ of cluster $j \in B$:

$$S_{AB}(i, j) = \frac{q_A(i) \cdot q_B(j)}{\|q_A(i)\|_2 \|q_B(j)\|_2} \quad (1)$$

This measure expresses the degree to which samples in the dataset are assigned similarly to clusters i and j . Specifically, $S_{AB}(i, j) = 0$ if $q_A(i) \perp q_B(j)$ and $S_{AB}(i, j) = 1$ if $q_A(i) = q_B(j)$. It is, therefore, a differentiable measure of the similarity of clusters i and j .

Defining the loss function: Based on the inter-clustering similarity matrix S_{AB} , we define DivClust’s loss to softly enforce that a clustering A does not have an *aggregate* cluster similarity with a clustering B greater than a similarity upper bound d . The aggregate similarity S_{AB}^{aggr} is defined as the average similarity of clustering A ’s clusters with their most similar cluster of clustering B (Eq. (2)). Using this metric, we propose L_{div} (Eq. (3)), a loss that regulates diversity between clusterings A and B by forcing that $S_{AB}^{aggr} < d$, for $d \in [0, 1]$. It is clear from Eq. (3) that $S_{AB}^{aggr} < d \Rightarrow L_{div}(A, B) = 0$, in which case the diversity requirement is satisfied and the loss has no impact. Conversely, $S_{AB}^{aggr} \geq d \Rightarrow L_{div}(A, B) > 0$, in which case the loss requires that inter-clustering similarity decreases.

$$S_{AB}^{aggr} = \frac{1}{C} \sum_{i=1}^C \max_j S_{AB}(i, j) \quad (2)$$

$$L_{div}(A, B) = [S_{AB}^{aggr} - d]_+ \quad (3)$$

Having defined the diversity loss L_{div} between two clusterings, we extend it to multiple clusterings K and combine it with the base deep clustering framework’s objective. For a clustering $k \in K$, we denote with $L_{main}(k)$ the loss of the base deep clustering framework for that clustering, and with $L_{div}(k, k')$ the diversity controlling loss between clusterings k and k' . We present the joint loss $L_{joint}(k)$ for each clustering k in Eq. (4), where $L_{main}(k)$ depends on cluster assignment matrix P_k , while $L_{div}(k, k')$ depends on P_k and $P_{k'}$. Accordingly, the model’s training loss L_{total} , seen in Eq. (5), is the average of L_{joint} over all clusterings.

$$L_{joint}(k) = L_{main}(k) + \frac{1}{K-1} \sum_{k'=1, k' \neq k}^K L_{div}(k, k') \quad (4)$$

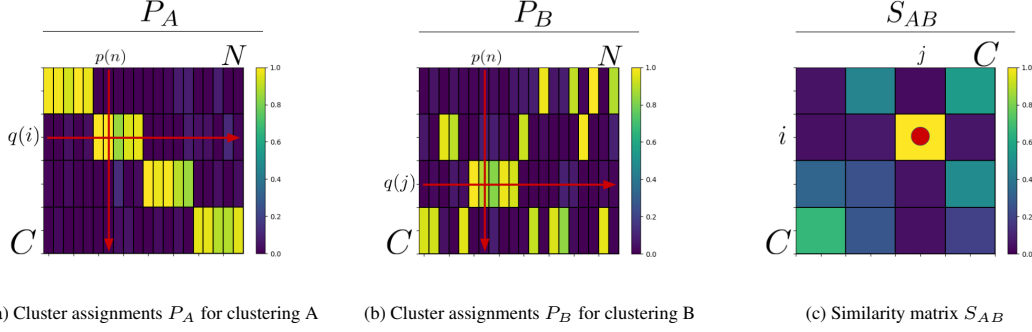


Figure 2. Examples of synthetic cluster assignments P_A , P_B and similarity matrix S_{AB} . Note that clusters $i \in A$ and $j \in B$ are softly assigned the same samples. Correspondingly, their similarity score $S_{AB}(i, j)$ is high (highlighted with red in Fig. 2c). Best seen in color.

$$L_{total} = \frac{1}{K} \sum_{k=1}^K L_{joint}(k) \quad (5)$$

The loss L_{total} is therefore a combination of the base deep clustering framework’s loss L_{main} for each clustering $k \in K$ and the loss L_{div} , which is used to control inter-clustering diversity. The proposed loss formulation is applicable to any deep clustering framework that produces cluster assignments through the model (as opposed to frameworks using offline methods such as MIX’EM [58]), which covers the majority of deep clustering frameworks outlined in Sec. 2.

Dynamic upper bound d : The proposed loss L_{div} controls inter-clustering diversity by restricting the values of S_{AB} according to the similarity upper bound d . However, the values of S_{AB} are calculated based on the cosine similarity of *soft* cluster assignments. This means that pairs of cluster assignment vectors i, j will have different similarity values $S_{AB}(i, j)$ depending on their sharpness, even if they point to the same cluster in terms of their corresponding hard assignment. It follows that S_{AB} and, accordingly, the impact of d , are dependent on the confidence of cluster assignments and vary throughout training and between experiments (as factors like the number of clusters and model capacity influence the confidence of cluster assignments). Therefore, d is an ambiguous and unintuitive metric for users to define diversity targets with.

To tackle this issue and to provide a reliable and intuitive method for defining diversity objectives, we propose dynamically determining the value of the threshold d during training. Concretely, let D be an inter-clustering similarity metric chosen by the user. In this work, we use avg. Normalized Mutual Information (NMI), a well established metric for estimating inter-clustering similarity.

$$D = \frac{1}{(K-1)(K/2)} \sum_{k=1}^{K-1} \sum_{k'=k+1}^K NMI(P_k^h, P_{k'}^h) \quad (6)$$

where $P_k^h \in \mathbb{Z}^N$ is the hard cluster assignment vector for N samples in clustering $k \in K$ and $NMI(P_k^h, P_{k'}^h)$ represents the NMI between k and k' . $D \in [0, 1]$, with higher values indicating more similar clusterings.

Assuming a user-defined similarity target D^T , expressed as a value of metric D , we denote with D^R the measured inter-clustering similarity of the clusterings learned by the model, expressed in the same metric. DivClust’s objective is to control inter-clustering diversity, which translates to learning clusterings such that $D^R \leq D^T$. Accordingly, appropriate thresholds d must be used during training. Under the assumption that D^R decreases monotonically w.r.t. d , we propose the following update rule for d :

$$d_{s+1} = \begin{cases} \max(d_s(1-m), 0), & \text{if } D^R > D^T \\ \min(d_s(1+m), 1), & \text{if } D^R \leq D^T \end{cases}, \quad (7)$$

where d_s and d_{s+1} are the values of the threshold d for the current and the next steps, and $m \in (0, 1)$ regulates the magnitude of the update steps. Following this update rule, we decrease d when the measured inter-clustering similarity D^R needs to decrease, and increase it otherwise. For computational efficiency, instead of calculating D^R over the entire dataset in every training step, we do so every 20 iterations on a memory bank of $M = 10,000$ cluster assignments – the latter is updated at every step in a FIFO manner. We set the hyperparameter m to $m = 0.01$ in all experiments.

4. Experiments

We conduct several experiments to evaluate DivClust’s adaptability, its effectiveness in controlling diversity, and the quality of the resulting clusterings. First, to show that DivClust effectively controls inter-clustering diversity and produces high quality clusterings with various frameworks, we combine it with IIC [37], PICA [32] and CC [44], and apply it to CIFAR10 with various diversity targets D^T . Subsequently, we focus on the best framework of the

Framework	Clusterings	D^T	D^R	CNF	Mean Acc.	Max. Acc.	DivClust Acc.
IIC	1	-	-	0.997	0.442	0.442	0.442
	20	1.	0.983	0.996	0.526	0.526	0.526
	20	0.95	0.939	0.998	0.531	0.537	0.533
	20	0.9	0.888	0.997	0.568	0.59	0.578
	20	0.8	0.8	0.997	0.611	0.678	0.653
	20	0.7	0.694	0.996	0.566	0.637	0.685
PICA	1	-	-	0.906	0.533	0.533	0.533
	20	1.	0.991	0.814	0.597	0.597	0.596
	20	0.95	0.931	0.826	0.624	0.631	0.625
	20	0.9	0.891	0.841	0.648	0.665	0.652
	20	0.8	0.817	0.828	0.598	0.635	0.595
	20	0.7	0.703	0.824	0.625	0.691	0.671
CC	1	-	-	0.936	0.764	0.764	0.764
	20	1.	0.976	0.934	0.763	0.763	0.763
	20	0.95	0.946	0.934	0.762	0.773	0.76
	20	0.9	0.9	0.931	0.794	0.818	0.789
	20	0.8	0.814	0.93	0.762	0.847	0.819
	20	0.7	0.699	0.927	0.703	0.818	0.815

Table 1. Results for IIC, PICA and CC applied on CIFAR10 with DivClust. CNF and Mean Acc. are calculated by averaging the corresponding metrics over all clusterings, while Max Acc. refers to the best performing base clustering. The DivClust Acc. metric measures the accuracy of a consensus clustering produced with the *DivClust C* method.

D^T	D^R			
	CIFAR10	CIFAR100	ImageNet-10	ImageNet-Dogs
1.	0.976	0.939	0.987	0.941
0.95	0.946	0.926	0.948	0.945
0.9	0.9	0.848	0.897	0.87
0.8	0.814	0.806	0.807	0.795
0.7	0.699	0.705	0.696	0.702

Table 2. Avg. inter-clustering similarity scores D^R for clustering sets produced by DivClust combined with CC for various diversity targets D^T . The objective of DivClust is that $D^R \leq D^T$.

three, namely CC, and conduct experiments on 4 datasets (CIFAR10, CIFAR100, Imagenet-10 and Imagenet-Dogs). Our findings demonstrate that, across frameworks and datasets, DivClust can: a) effectively control diversity and b) improve clustering outcomes over the base frameworks and alternative ensembling methods.

4.1. Experiments setup

Datasets: We conduct experiments with 4 standard datasets in deep clustering: CIFAR10, CIFAR100 [39] (evaluating on the 20 superclasses), ImageNet-10 and ImageNet-Dogs [8].

Metrics: Inter-clustering similarity is measured by averaging the NMI between clusterings to calculate the inter-clustering NMI metric D (Eq. (6)), with higher values indicating more similar clusterings. We denote with D^T the

diversity target set by the user and with D^R the measured inter-clustering similarity after training. When DivClust is applied we want that $D^R \leq D^T$. Clustering quality is evaluated based on overlap of the clusterings with the dataset’s ground truth labels, using the Accuracy (ACC), Normalized Mutual Information (NMI) and Adjusted Rand Index (ARI) metrics. We also report the avg. cluster assignment confidence (CNF), which measures cluster separability. For all four metrics greater values are better, 1 being optimal.

Implementation & Training: DivClust is incorporated into the base frameworks as described in Sec. 3, by adding DivClust’s loss to their objective and duplicating projection heads h to produce multiple clusterings. The models were trained following the configurations (model architecture, training duration, hyperparameters etc.) suggested in their respective papers [32, 37, 44], unless stated otherwise. PICA and IIC were trained without overclustering. We set the number of clusterings to $K = 20$, following convention in consensus clustering [73], and the number of clusters C to the number of classes for each dataset, following convention for deep clustering evaluation [32, 37, 44].

Consensus Clustering: To extract single clustering solutions we examine three methods: a) selecting the clustering k with the lowest corresponding loss $L_{main}(k)$ (**DivClust A**), b) using the consensus clustering algorithm SCCBG [73] to aggregate clusterings (**DivClust B**), and c) a combination of the two, where we select the 10 best clusterings with regard to their loss and then apply SCCBG (**DivClust C**). For clarity and space, we present in the paper

Dataset	D^T	CIFAR10			CIFAR100			ImageNet-10			ImageNet-Dogs		
Metric	NMI	NMI	ACC	ARI	NMI	ACC	ARI	NMI	ACC	ARI	NMI	ACC	ARI
K-means	-	0.087	0.229	0.049	0.084	0.130	0.028	0.119	0.241	0.057	0.55	0.105	0.020
AC	-	0.105	0.228	0.065	0.098	0.138	0.034	0.138	0.242	0.067	0.037	0.139	0.021
NMF	-	0.081	0.190	0.034	0.079	0.118	0.026	0.132	0.230	0.065	0.044	0.118	0.016
AE	-	0.237	0.314	0.169	0.100	0.165	0.048	0.210	0.317	0.152	0.104	0.185	0.073
DAE	-	0.251	0.297	0.163	0.111	0.151	0.046	0.206	0.304	0.138	0.104	0.190	0.078
DCGAN	-	0.265	0.315	0.176	0.120	0.151	0.045	0.225	0.346	0.157	0.121	0.174	0.078
DeCNN	-	0.240	0.282	0.174	0.092	0.133	0.038	0.186	0.313	0.142	0.098	0.175	0.073
VAE	-	0.245	0.291	0.167	0.108	0.152	0.040	0.193	0.334	0.168	0.107	0.179	0.079
JULE	-	0.192	0.272	0.138	0.103	0.137	0.033	0.175	0.300	0.138	0.054	0.138	0.028
DEC	-	0.257	0.301	0.161	0.136	0.185	0.050	0.282	0.381	0.203	0.122	0.195	0.079
DAC	-	0.396	0.522	0.306	0.185	0.238	0.088	0.394	0.527	0.302	0.219	0.275	0.111
ADC	-	-	0.325	-	-	0.160	-	-	0.530	-	-	-	-
DDC	-	0.424	0.524	0.329	-	-	-	0.433	0.577	0.345	-	-	-
DCCM	-	0.496	0.623	0.408	0.285	0.327	0.173	0.608	0.710	0.555	0.321	0.383	0.182
IIC	-	-	0.617	-	-	0.257	-	-	-	-	-	-	-
PICA	-	0.591	0.696	0.512	0.310	0.337	0.171	0.802	0.870	0.761	0.352	0.352	0.201
CC	-	0.705	0.790	0.637	0.431	0.429	0.266	0.859	0.893	0.822	0.445	0.429	0.274
CC-Kmeans	-	0.654	0.698	0.523	0.429	0.405	0.235	0.792	0.841	0.669	0.457	0.444	0.284
CC-Kmeans/S	-	0.674	0.69	0.554	0.428	0.402	0.228	0.792	0.842	0.673	0.456	0.444	0.283
CC-Kmeans/F	-	0.684	0.762	0.599	0.438	0.409	0.210	0.797	0.847	0.685	0.458	0.444	0.285
DeepCluE	-	0.727	0.764	0.646	0.472	0.457	0.288	0.882	0.924	0.856	0.448	0.416	0.273
DivClust C	1.	0.678	0.763	0.604	0.418	0.424	0.257	<u>0.86</u>	<u>0.895</u>	<u>0.825</u>	<u>0.459</u>	<u>0.451</u>	<u>0.298</u>
	0.95	0.677	0.76	0.602	0.431	<u>0.434</u>	<u>0.276</u>	0.891	0.936	0.878	<u>0.461</u>	<u>0.451</u>	<u>0.297</u>
	0.9	0.678	<u>0.789</u>	<u>0.641</u>	0.422	0.426	0.258	<u>0.879</u>	<u>0.92</u>	<u>0.859</u>	<u>0.48</u>	<u>0.487</u>	<u>0.332</u>
	0.8	<u>0.724</u>	0.819	0.681	0.422	0.414	0.26	<u>0.879</u>	<u>0.918</u>	<u>0.851</u>	<u>0.458</u>	<u>0.448</u>	<u>0.296</u>
	0.7	<u>0.71</u>	<u>0.815</u>	<u>0.675</u>	<u>0.44</u>	<u>0.437</u>	<u>0.283</u>	<u>0.85</u>	<u>0.90</u>	<u>0.819</u>	0.516	0.529	0.376

Table 3. Results combining DivClust with CC for various diversity targets D^T . We underline DivClust results that outperform the single-clustering baseline CC, and note with **bold** the best results for each metric across all methods and diversity levels. We emphasize that the NMI in this table measures the similarity between the single clustering produced by each method and the ground truth classes. The NMI values representing inter-clustering similarity D^R in ensembles produced by DivClust for the same experiments are presented in Tab. 2.

results only for the hybrid aggregation method **DivClust C**, which we found to be the most robust. Detailed results for all three approaches are provided in supplementary Tab. 5.

4.2. Results

Initially, we apply IIC, PICA and CC on CIFAR10, and present the outcomes in Tab. 1. We find that, for all three frameworks, DivClust effectively controls diversity, as D^R is consistently close to or lower than D^T . Furthermore, results indicate that DivClust is *necessary* to produce diverse clusterings in deep clustering frameworks, as, without it, they tend to converge to near identical solutions (when $D^T = 1$, $D^R \rightarrow 1$). Regarding cluster separability, assignment confidence CNF remains high for various diversity targets D^T , despite the increased complexity of optimizing both the main deep clustering loss and DivClust’s objective. Finally, we observe that, for most diversity targets D^T , the mean and max. accuracy, as well as the consensus clustering accuracy produced by the aggregation method **DivClust**

C, increase relative to the single clustering model. Notably, consensus clustering accuracy is higher than the mean clustering accuracy for most cases, which highlights the effectiveness of our approach. We stress that identifying clusterings in the ensemble whose performance matches the mean or max. accuracy is not trivial, which is why consensus clustering is necessary to reach a single clustering solution.

Having established that DivClust is effective across frameworks, we focus on CC and apply it on CIFAR10, CIFAR100, ImageNet-Dogs and Imagenet-10. We compare DivClust with the standard implementation of CC, which is trained to learn a single clustering (**CC**), as well as with alternative methods of ensemble clustering. Specifically, we apply the typical methods of ensemble generation by extracting the features learned by the single-clustering CC model and running K-means 20 times on the entire dataset (**CC-Kmeans**), on random subsets of the dataset (**CC-Kmeans/S**) and on random subsets of the feature space (**CC-Kmeans/F**), following [3]. In all three cases, SCCBG

is used to aggregate the resulting clusterings. Furthermore, we compare with **DeepCluE** [27], to the best of our knowledge the only other work that examines consensus clustering in the context of deep clustering, and which is also built on top of CC, allowing for a fair comparison. We note that DeepCluE is not mutually exclusive with DivClust, and could be used jointly with it.

Inter-clustering similarity scores D^R for this set of experiments are presented in Tab. 2, where it is seen that DivClust successfully controls diversity. Results for consensus clustering, the main task for which DivClust is intended, are presented in Tab. 3 for CC across 4 datasets, where we also include results from other deep clustering frameworks for reference. Detailed results, including aggregation methods DivClust A and DivClust B, as well as mean/max scores for DivClust’s clustering ensembles, are provided in supplementary Tab. 5. Results in Tab. 3 demonstrate that, for most diversity targets D^T , DivClust outperforms the single-clustering baseline CC and typical ensemble generation methods, and is competitive with the alternative consensus clustering method DeepCluE. Notably, DivClust is competitive with the baseline across diversity levels. This robustness is very significant, given that identifying what properties (including diversity) lead to optimal outcomes in clustering ensembles is an open problem [15, 22].

To summarize, the results of Tabs. 1 to 3 demonstrate that DivClust: a) effectively controls inter-clustering diversity in deep clustering frameworks in accordance with user-defined objectives, b) does not degrade the quality of the clusterings and in fact produces better solutions than single-clustering models, and c), it can be used with consensus clustering to identify single-clustering solutions superior to those of the corresponding single-clustering frameworks.

5. Discussion

5.1. Diversity Control & Consensus Clustering Performance

Results presented in Sec. 4 demonstrate the effectiveness of DivClust both in controlling inter-clustering diversity and in producing clustering ensembles that lead to consensus clustering outcomes superior to single clustering baselines.

Specifically, Tabs. 1 and 2 show that the inter-clustering similarity D^R of ensembles produced by DivClust is consistently lower than the targets D^T . In the few cases where $D^R \geq D^T$, it is by very small margins (the greatest deviation was +0.017), which may be attributed to our use of a memory bank to estimate D^R for the update rule of the threshold d . Regarding consensus clustering accuracy, despite the sensitivity of consensus clustering to the properties of the ensembles and, specifically, to different inter-clustering diversity levels, our method proves particularly robust to varying the diversity targets D^T , outperforming

baselines for most settings. This indicates that DivClust learns clusterings with a good quality-diversity trade-off and can be reliably used for consensus clustering.

Finally, we note that DivClust’s ability to explicitly control inter-clustering diversity can facilitate future research on the impact of diversity in clustering ensembles and toward methodologies that determine desirable diversity levels for specific settings [51].

5.2. Complexity & Computational Cost

The complexity of DivClust’s objective is $O(nK^2C^2)$, where n is the batch size, K is the number of clusterings, and C is the number of clusters in each clustering. Importantly, the cost of DivClust relates only to the computation of the loss and the additional projection heads, and is therefore *fixed* for fixed n , K , C values, regardless of model size and data dimensionality, which are generally the computational bottleneck in Deep Learning applications. Therefore, DivClust is scalable to large datasets. Finally, we note that, in practice, the computational overhead introduced by DivClust is minimal. Specifically, for experiments in this work, DivClust learned ensembles with $K = 20$ clusterings, with training time increasing between 10%-50% relative to the time it took to train the baseline single-clustering models. Contrasted with the alternative of training a single-clustering model 20 times (which would not allow for controlling diversity), DivClust provides an efficient approach for applying consensus clustering with deep clustering frameworks. A detailed analysis on complexity and runtimes is provided in supplementary Sec. 3.

6. Conclusion

We introduce DivClust, a method that can be incorporated into existing deep clustering frameworks to learn multiple clusterings while controlling inter-clustering diversity. To the best of our knowledge, this is the first method that can explicitly control inter-clustering diversity based on user-defined targets, and that is compatible with deep clustering frameworks that learn features and clusters end-to-end. Our experiments, conducted with multiple datasets and deep clustering frameworks, confirm the effectiveness of DivClust in controlling inter-clustering diversity and its adaptability, in terms of it being compatible with various frameworks without requiring modifications and/or hyperparameter tuning. Furthermore, results demonstrate that DivClust learns high quality clusterings, which, in the context of consensus clustering, lead to improved performance compared to single clustering baselines and alternative ensemble clustering methods.

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References

- [1] Yuki M. Asano, Christian Rupprecht, and Andrea Vedaldi. Self-labelling via simultaneous clustering and representation learning. In *International Conference on Learning Representations (ICLR)*, 2020. 2, 3
- [2] Eric Bae and James Bailey. Coala: A novel approach for the extraction of an alternate clustering of high quality and high dissimilarity. In *Sixth International Conference on Data Mining (ICDM'06)*, pages 53–62. IEEE, 2006. 3
- [3] Tossapon Boongoen and Natthakan Iam-On. Cluster ensembles: A survey of approaches with recent extensions and applications. *Computer Science Review*, 28:1–25, 2018. 1, 3, 7
- [4] Xiaochun Cao, Changqing Zhang, Huazhu Fu, Si Liu, and Hua Zhang. Diversity-induced multi-view subspace clustering. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 586–594, 2015. 3
- [5] Mathilde Caron, Piotr Bojanowski, Armand Joulin, and Matthijs Douze. Deep clustering for unsupervised learning of visual features. In *Proceedings of the European Conference on Computer Vision (ECCV)*, pages 132–149, 2018. 2
- [6] Rich Caruana, Mohamed Elhawary, Nam Nguyen, and Casey Smith. Meta clustering. In *Sixth International Conference on Data Mining (ICDM'06)*, pages 107–118. IEEE, 2006. 3
- [7] Jianlong Chang, Yiwen Guo, Lingfeng Wang, Gaofeng Meng, Shiming Xiang, and Chunhong Pan. Deep discriminative clustering analysis. *arXiv preprint arXiv:1905.01681*, 2019. 2
- [8] Jianlong Chang, Lingfeng Wang, Gaofeng Meng, Shiming Xiang, and Chunhong Pan. Deep adaptive image clustering. In *Proceedings of the IEEE international conference on computer vision*, pages 5879–5887, 2017. 2, 6
- [9] Ying Cui, Xiaoli Z Fern, and Jennifer G Dy. Non-redundant multi-view clustering via orthogonalization. In *Seventh IEEE International Conference on Data Mining (ICDM 2007)*, pages 133–142. IEEE, 2007. 3
- [10] Ian Davidson and Zijie Qi. Finding alternative clusterings using constraints. In *2008 Eighth IEEE International Conference on Data Mining*, pages 773–778. IEEE, 2008. 3
- [11] Aniket Anand Deshmukh, Jayanth Reddy Regatti, Eren Manavoglu, and Urun Dogan. Representation learning for clustering via building consensus. *Machine Learning*, pages 1–38, 2022. 2
- [12] Carlotta Domeniconi and Muna Al-Razgan. Weighted cluster ensembles: Methods and analysis. *ACM Transactions on Knowledge Discovery from Data (TKDD)*, 2(4):1–40, 2009. 3
- [13] Sandrine Dudoit and Jane Fridlyand. Bagging to improve the accuracy of a clustering procedure. *Bioinformatics*, 19(9):1090–1099, 2003. 3
- [14] Xiaoli Z Fern and Carla E Brodley. Random projection for high dimensional data clustering: A cluster ensemble approach. In *Proceedings of the 20th international conference on machine learning (ICML-03)*, pages 186–193, 2003. 1, 3
- [15] Xiaoli Z Fern and Wei Lin. Cluster ensemble selection. *Statistical Analysis and Data Mining: The ASA Data Science Journal*, 1(3):128–141, 2008. 1, 2, 3, 8
- [16] Ana LN Fred and Anil K Jain. Combining multiple clusterings using evidence accumulation. *IEEE transactions on pattern analysis and machine intelligence*, 27(6):835–850, 2005. 3
- [17] Reza Ghaemi, Md Nasir Sulaiman, Hamidah Ibrahim, and Norwati Mustapha. A survey: clustering ensembles techniques. *International Journal of Computer and Information Engineering*, 3(2):365–374, 2009. 1, 3
- [18] Keyvan Ghalipour, Ebrahim Akbari, Seyed Saeed Hamidi, Malrey Lee, and Rasul Enayatifar. From clustering to clustering ensemble selection: A review. *Engineering Applications of Artificial Intelligence*, 104:104388, 2021. 1, 3
- [19] Jean-Bastien Grill, Florian Strub, Florent Altché, Corentin Tallec, Pierre Richemond, Elena Buchatskaya, Carl Doersch, Bernardo Avila Pires, Zhaohan Guo, Mohammad Gheshlaghi Azar, et al. Bootstrap your own latent—a new approach to self-supervised learning. *Advances in neural information processing systems*, 33:21271–21284, 2020. 2
- [20] Francesco Gullo, Andrea Tagarelli, and Sergio Greco. Diversity-based weighting schemes for clustering ensembles. In *Proceedings of the 2009 SIAM international conference on data mining*. SIAM, 2009. 1, 3
- [21] Yuanfan Guo, Minghao Xu, Jiawen Li, Bingbing Ni, Xuanyu Zhu, Zhenbang Sun, and Yi Xu. Hcsc: Hierarchical contrastive selective coding. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 9706–9715, 2022. 2
- [22] Stefan T Hadjitodorov, Ludmila I Kuncheva, and Ludmila P Todorova. Moderate diversity for better cluster ensembles. *Information Fusion*, 7(3):264–275, 2006. 1, 2, 3, 8
- [23] Philip Haeusser, Johannes Plapp, Vladimir Golkov, Elie Aljalbout, and Daniel Cremers. Associative deep clustering: Training a classification network with no labels. In *German Conference on Pattern Recognition*, pages 18–32. Springer, 2018. 2
- [24] Seyed Saeed Hamidi, Ebrahim Akbari, and Homayun Motameni. The impact of diversity on clustering ensemble using χ^2 criterion. *International Journal of Nonlinear Analysis and Applications*, 2022. 1, 3
- [25] Juhua Hu and Jian Pei. Subspace multi-clustering: a review. *Knowledge and information systems*, 56(2):257–284, 2018. 3
- [26] Juhua Hu, Qi Qian, Jian Pei, Rong Jin, and Shenghuo Zhu. Finding multiple stable clusterings. *Knowledge and Information Systems*, 51(3):991–1021, 2017. 3
- [27] Dong Huang, Ding-Hua Chen, Xiangji Chen, Chang-Dong Wang, and Jian-Huang Lai. Deepclue: Enhanced image clustering via multi-layer ensembles in deep neural networks. *arXiv preprint arXiv:2206.00359*, 2022. 3, 8
- [28] Dong Huang, Chang-Dong Wang, and Jian-Huang Lai. Locally weighted ensemble clustering. *IEEE transactions on cybernetics*, 48(5):1460–1473, 2017. 3
- [29] Dong Huang, Chang-Dong Wang, Hongxing Peng, Jianhuang Lai, and Chee-Keong Kwoh. Enhanced ensemble clustering via fast propagation of cluster-wise similarities. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 51(1):508–520, 2018. 3

- [30] Dong Huang, Chang-Dong Wang, Jian-Sheng Wu, Jian-Huang Lai, and Chee-Keong Kwoh. Ultra-scalable spectral clustering and ensemble clustering. *IEEE Transactions on Knowledge and Data Engineering*, 32(6):1212–1226, 2019. 3
- [31] Jiabo Huang and Shaogang Gong. Deep clustering by semantic contrastive learning. In *33rd British Machine Vision Conference 2022, BMVC 2022, London, UK, November 21–24, 2022*. BMVA Press, 2022. 3
- [32] Jiabo Huang, Shaogang Gong, and Xiatian Zhu. Deep semantic clustering by partition confidence maximisation. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 8849–8858, 2020. 2, 5, 6
- [33] Zhizhong Huang, Jie Chen, Junping Zhang, and Hongming Shan. Learning representation for clustering via prototype scattering and positive sampling. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2022. 2
- [34] Natthakan Iam-On, Tossapon Boongoen, Simon Garrett, and Chris Price. A link-based approach to the cluster ensemble problem. *IEEE transactions on pattern analysis and machine intelligence*, 33(12):2396–2409, 2011. 1
- [35] Prateek Jain, Raghu Meka, and Inderjit S Dhillon. Simultaneous unsupervised learning of disparate clusterings. *Statistical Analysis and Data Mining: The ASA Data Science Journal*, 1(3):195–210, 2008. 3
- [36] Pan Ji, Tong Zhang, Hongdong Li, Mathieu Salzmann, and Ian Reid. Deep subspace clustering networks. *Advances in neural information processing systems*, 30, 2017. 2
- [37] Xu Ji, João F Henriques, and Andrea Vedaldi. Invariant information clustering for unsupervised image classification and segmentation. In *Proceedings of the IEEE International Conference on Computer Vision*, pages 9865–9874, 2019. 2, 3, 5, 6
- [38] Yuheng Jia, Sirui Tao, Ran Wang, and Yongheng Wang. Ensemble clustering via co-association matrix self-enhancement. *IEEE Transactions on Neural Networks and Learning Systems*, 2023. 3
- [39] Alex Krizhevsky, Geoffrey Hinton, et al. Learning multiple layers of features from tiny images. 2009. 6
- [40] Ludmila I Kuncheva and Stefan Todorov Hadjitodorov. Using diversity in cluster ensembles. In *2004 IEEE international conference on systems, man and cybernetics (IEEE Cat. No. 04CH37583)*, volume 2, pages 1214–1219. IEEE, 2004. 3
- [41] Ludmila I Kuncheva and Dmitry P Vetrov. Evaluation of stability of k-means cluster ensembles with respect to random initialization. *IEEE transactions on pattern analysis and machine intelligence*, 28(11):1798–1808, 2006. 3
- [42] Junnan Li, Pan Zhou, Caiming Xiong, and Steven C.H. Hoi. Prototypical contrastive learning of unsupervised representations. In *ICLR*, 2021. 2
- [43] Tao Li, Chris Ding, and Michael I Jordan. Solving consensus and semi-supervised clustering problems using nonnegative matrix factorization. In *Seventh IEEE International Conference on Data Mining (ICDM 2007)*, pages 577–582. IEEE, 2007. 3
- [44] Yunfan Li, Peng Hu, Zitao Liu, Dezhong Peng, Joey Tianyi Zhou, and Xi Peng. Contrastive clustering. In *2021 AAAI Conference on Artificial Intelligence (AAAI)*, 2021. 2, 3, 5, 6
- [45] Hongfu Liu, Zhiqiang Tao, and Zhengming Ding. Consensus clustering: an embedding perspective, extension and beyond. *arXiv preprint arXiv:1906.00120*, 2019. 1, 3
- [46] Jiaqi Ma, Yipeng Zhang, and Lefei Zhang. Discriminative subspace matrix factorization for multiview data clustering. *Pattern Recognition*, 111:107676. 3
- [47] Dominik Mautz, Wei Ye, Claudia Plant, and Christian Böhm. Discovering non-redundant k-means clusterings in optimal subspaces. In *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, pages 1973–1982, 2018. 3
- [48] Lukas Miklautz, Dominik Mautz, Muzaffer Can Altinigneli, Christian Böhm, and Claudia Plant. Deep embedded non-redundant clustering. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 5174–5181, 2020. 1, 3
- [49] Chuang Niu, Hongming Shan, and Ge Wang. Spice: Semantic pseudo-labeling for image clustering. *IEEE Transactions on Image Processing*, 31:7264–7278, 2022. 2
- [50] Chuang Niu, Jun Zhang, Ge Wang, and Jimin Liang. Gat-cluster: Self-supervised gaussian-attention network for image clustering. In *European Conference on Computer Vision*, pages 735–751. Springer, 2020. 2
- [51] Milton Pividori, Georgina Stegmayer, and Diego H Milone. Diversity control for improving the analysis of consensus clustering. *Information Sciences*, 361:120–134, 2016. 1, 2, 3, 8
- [52] Jayanth Reddy Regatti, Aniket Anand Deshmukh, Eren Manavoglu, and Urun Dogan. Consensus clustering with unsupervised representation learning. In *2021 International Joint Conference on Neural Networks (IJCNN)*. IEEE, 2021. 2, 3
- [53] Yuming Shen, Ziyi Shen, Menghan Wang, Jie Qin, Philip Torr, and Ling Shao. You never cluster alone. *Advances in Neural Information Processing Systems*, 34, 2021. 3
- [54] Alexander Strehl and Joydeep Ghosh. Cluster ensembles—a knowledge reuse framework for combining multiple partitions. *Journal of machine learning research*, 3(Dec):583–617, 2002. 3
- [55] Zhiqiang Tao, Hongfu Liu, Sheng Li, Zhengming Ding, and Yun Fu. From ensemble clustering to multi-view clustering. In *IJCAI*, 2017. 3
- [56] Tsung Wei Tsai, Chongxuan Li, and Jun Zhu. Mice: Mixture of contrastive experts for unsupervised image clustering. In *International Conference on Learning Representations*, 2020. 3
- [57] Wouter Van Gansbeke, Simon Vandenhende, Stamatios Georgoulis, Marc Proesmans, and Luc Van Gool. Scan: Learning to classify images without labels. In *European Conference on Computer Vision*, pages 268–285. Springer, 2020. 2, 3
- [58] Ali Varamesh and Tinne Tuytelaars. Mix'em: Unsupervised image classification using a mixture of embeddings. In *Proceedings of the Asian Conference on Computer Vision*, 2020. 2, 5

- [59] Xing Wang, Jun Wang, Carlotta Domeniconi, Guoxian Yu, Guoqiang Xiao, and Maozu Guo. Multiple independent subspace clusterings. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 33, pages 5353–5360, 2019. 3
- [60] Xing Wang, Guoxian Yu, Carlotta Domeniconi, Jun Wang, Zhiwen Yu, and Zili Zhang. Multiple co-clusterings. In *2018 IEEE International Conference on Data Mining (ICDM)*, pages 1308–1313. IEEE, 2018. 3
- [61] Shaowei Wei, Jun Wang, Guoxian Yu, Carlotta Domeniconi, and Xiangliang Zhang. Deep incomplete multi-view multiple clusterings. In *2020 IEEE International Conference on Data Mining (ICDM)*, pages 651–660. IEEE, 2020. 1, 3
- [62] Shaowei Wei, Jun Wang, Guoxian Yu, Carlotta Domeniconi, and Xiangliang Zhang. Multi-view multiple clusterings using deep matrix factorization. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 6348–6355, 2020. 3
- [63] Jianlong Wu, Keyu Long, Fei Wang, Chen Qian, Cheng Li, Zhouchen Lin, and Hongbin Zha. Deep comprehensive correlation mining for image clustering. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pages 8150–8159, 2019. 2
- [64] Junyuan Xie, Ross Girshick, and Ali Farhadi. Unsupervised deep embedding for clustering analysis. In *International conference on machine learning*, pages 478–487. PMLR, 2016. 2
- [65] Kouta Nakata Yaling Tao, Kentaro Takagi. Clustering-friendly representation learning via instance discrimination and feature decorrelation. *Proceedings of ICLR 2021*, 2021. 2
- [66] Jianwei Yang, Devi Parikh, and Dhruv Batra. Joint unsupervised learning of deep representations and image clusters. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 5147–5156, 2016. 2
- [67] Sen Yang and Lijun Zhang. Non-redundant multiple clustering by nonnegative matrix factorization. *Machine Learning*, 106(5):695–712, 2017. 3
- [68] Shixin Yao, Guoxian Yu, Jun Wang, Carlotta Domeniconi, and Xiangliang Zhang. Multi-view multiple clustering. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI-19*, pages 4121–4127. International Joint Conferences on Artificial Intelligence Organization, 7 2019. 3
- [69] Wei Ye, Samuel Maurus, Nina Hubig, and Claudia Plant. Generalized independent subspace clustering. In *2016 IEEE 16th International Conference on Data Mining (ICDM)*, pages 569–578. IEEE, 2016. 3
- [70] Zhong Zhang, Chongming Gao, Chongzhi Liu, Qinli Yang, and Junming Shao. Towards robust arbitrarily oriented subspace clustering. In *International Conference on Database Systems for Advanced Applications*, pages 276–291. Springer, 2019. 3
- [71] Junjie Zhao, Donghuan Lu, Kai Ma, Yu Zhang, and Yefeng Zheng. Deep image clustering with category-style representation. In *European Conference on Computer Vision*, pages 54–70. Springer, 2020. 2
- [72] Huasong Zhong, Jianlong Wu, Chong Chen, Jianqiang Huang, Minghua Deng, Liqiang Nie, Zhouchen Lin, and Xian-Sheng Hua. Graph contrastive clustering. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pages 9224–9233, 2021. 3
- [73] Peng Zhou, Liang Du, and Xuejun Li. Self-paced consensus clustering with bipartite graph. In *Proceedings of the Twenty-Ninth International Conference on International Joint Conferences on Artificial Intelligence*, pages 2133–2139, 2021. 1, 3, 6
- [74] Peng Zhou, Liang Du, Yi-Dong Shen, and Xuejun Li. Tri-level robust clustering ensemble with multiple graph learning. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 35, pages 11125–11133, 2021. 3
- [75] Peng Zhou, Liang Du, Hanmo Wang, Lei Shi, and Yi-Dong Shen. Learning a robust consensus matrix for clustering ensemble via kullback-leibler divergence minimization. In *Twenty-Fourth International Joint Conference on Artificial Intelligence*, 2015. 3