

## Highlights

### **Modeling Value of Information in Remote Sensing from Correlated Sources**

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- In scenarios with multiple sources, their correlation benefits the Age of Information
- When correlation is considered, we better speak of Value of Information (VoI)
- To track it, we exploit the Markov property of the resulting multidimensional process
- We present models with different complexity to analyze VoI under correlated sources
- If properly optimized, our VoI models are accurate and scalable to large networks

# Modeling Value of Information in Remote Sensing from Correlated Sources

Alberto Zancanaro<sup>a</sup>, Giulia Cisotto<sup>b</sup>, Leonardo Badia<sup>a,\*</sup>

<sup>a</sup>University of Padova, Dept. of Information Engineering, via Gradenigo 6B, 35131, Padova, Italy

<sup>b</sup>University of Milan Bicocca, Dept. of Informatics, Systems and Communication, viala Sarca 236, Milan, 20126, Italy

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## Abstract

This paper investigates data correlation in remote sensing networks and how it can be characterized through diverse models quantifying the Value of Information (VoI), a metric that describes how informative the data transmitted by the sensors are. For each sensor, the VoI evaluations comprise the average node-specific Age of Information (AoI), the average cost spent for sending updates, and the AoI of neighbor nodes, assumed to be correlated sources of information and therefore benefiting the VoI of other sensors nearby. We discuss how this metric can be tracked through a two-dimensional Markov chain, but we also show how this representation can be simplified by including the impact of neighbor nodes within the transition probabilities, so as to obtain a simpler model that gives the same insight in terms of VoI evaluations.

*Keywords:* Age of Information, Value of Information, Internet of Things, Data acquisition, Markov chains.

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## 1. Introduction

The upcoming next-generation networks are expected to include communications for a plethora of applications such as augmented/virtual reality, digital twins, remote sensing in extreme conditions, eHealth, smart industry and agriculture [1]. All of these rely on up-to-date information exchange throughout the network, possibly obtained

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\*Corresponding author

Email addresses: [alberto.zancanaro.1@phd.unipd.it](mailto:alberto.zancanaro.1@phd.unipd.it) (Alberto Zancanaro), [giulia.cisotto@unimib.it](mailto:giulia.cisotto@unimib.it) (Giulia Cisotto), [leonardo.badia@unipd.it](mailto:leonardo.badia@unipd.it) (Leonardo Badia)

leveraging Internet of Things (IoT) devices with limited individual capabilities, but pervasive diffusion. In this context, Age of Information (AoI) becomes a key performance metric [2, 3, 4, 5, 6]. Indeed, for many scenarios of industrial, agricultural, medical, or environmental monitoring and surveillance, updates might be sporadic and of limited size [7]. Thus, more than guaranteeing high throughput or low delivery delay, it is critical to ensure that the information about the underlying physical processes monitored is up-to-date [8].

In a scenario where a source transmits data to a destination, AoI is defined as the time elapsed since the most recent successful update received, and hence it captures the freshness of information from the destination's standpoint [9]. This concept can be expanded to consider that if updates come for free, it is straightforward to keep the AoI to low values, i.e., basically updating the information very frequently. However, if there are some costs associated with exchanging data, updates are not so frequent and the AoI increases. As a result, it can be argued whether the benefit of achieving low AoI (i.e., up-to-date information) is worth paying those costs [10, 11].

Throughout this paper, we will use a weighted combination of the AoI (possibly from multiple sources, as will be discussed later) and a transmission cost term, which we will consider to be our Value of Information (VoI). In the literature, the adoption of similar approaches is common to describe the benefits coming from an update compared to its cost, also sometimes including other components such as the stochastic decrease of uncertainty [12], a multi-parameter combination including timeliness and relevance [13], or non-linear transformations of the elapsed time as opposed to a linear AoI [14].

The generalization of this rationale to the case of multiple sensors coexisting in the same area opens some new challenges [15]. A basic extension of the underlying model would imply defining and tracking the AoI/VoI for the specific sensors separately, which is however appropriate only if they are associated with different physical processes, independent of one another [16]. In this case, when considering a specific AoI/VoI, only an update from the corresponding sensor can bring "fresher" information to the destination.

However, in many IoT applications, multiple sensors actually track correlated un-

derlying processes, sometimes even the same one [6, 17]. Since the general purpose of introducing AoI/VoI evaluations is to determine how often one should update [18], considering the AoI/VoI from multiple sources as totally unrelated would cause a storm of (often redundant) updates.

Our goal is instead to consider situations where correlation among sensors is explicitly kept into account to reduce unnecessary updates. For mission-critical and emergency monitoring, this would be particularly relevant to avoid network congestion in the precise moment an alert is to be raised, due to some recent updates suggesting a problem or malfunctioning [19, 20]. At the same time, for energy-constrained devices, limiting unnecessary exchanges of data can prolong the lifetime [21].

While we recognize the importance of efficient AoI/VoI management under correlated sensed data (e.g., from sensors in spatial proximity or tracking interconnected quantities), our investigations in the present paper are not directed towards mathematical optimization approaches but rather to the involved modeling aspects. In more detail, we consider a VoI model for data coming from a specific sensor, comprising three ingredients: (i) the AoI of the data received from that sensor; (ii) the transmission cost of the sensor; and (iii) a further AoI-related term to account for some recent information coming from other correlated sources, i.e., the most up-to-date “neighbor sensor” that can benefit the AoI of the sensor under consideration, to some extent.

For this VoI concept, we propose and compare two different models, both being discrete-time Markov chains (MCs) [22]. A preliminary investigation of these models was presented in [15], where we considered a detailed evolution of a two-dimensional state tracking the sensor of interest and adding one dimension to describe whether the most recent update from one of the neighbor sensors can also be useful to some extent. It can be shown that this model admits a coherent evaluation, especially in terms of when to update. However, a further simplification is possible, which is described by the second MC model, where the updates coming from neighbor sensors are merged with the updates from the sensor of interest in the transition probabilities. While this is an approximation, it is very good in terms of resulting evaluations, especially for what concerns the original purpose of assessing the updating frequency. Thus, it emerges as a practical instrument to be implemented in low-cost IoT devices to enable decentralized

network control [11].

With respect to the analysis presented in [15], where the connection between the Complete MC and the Scalar MC was only shown by means of trial-and-error procedures, here we obtain a closed-form characterization of the equivalence. As a result, we present an in-depth analysis showing how to set the hyper-parameter of the Scalar MC to obtain a behavior analogous to that of the Complete MC, which enables the representation of a scenario with correlated sensors with low complexity yet to a reasonable level of accuracy.

Moreover, another original contribution is that, beyond the reduction in computational complexity of the model, we also quantify the more concrete reduction in the *activity* of the sensors when leveraging the source correlation, which is another practical assessment of interest for real network scenarios.

The rest of this paper is organized as follows. In Section 2 we discuss the state of the art. Section 3 presents our methodology and introduces the definition of the VoI, and also the two MC models. Numerical results are presented in Section 4 to provide quantitative insight. Finally, Section 5 concludes the paper.

## 2. State of the art

Remote sensing systems with multiple quantities being monitored at the same time are an immediate generalization of standard AoI analyses [4, 6, 16]. Also, considering correlation in the monitored metrics has recently received significant attention, since it allows for a more meaningful representation of many IoT systems.

As examples, we report two scenarios in which a sensor network acquires data with medium to high correlation. Tables 1 and 2 show the Pearson's correlation coefficients between each pair of sensors in the two scenarios, respectively. Specifically, Table 1 shows Pearson's correlation coefficients between pairs of EEG channels located over the scalp of a healthy subject. Each multi-channel EEG recording was about 16 s long and obtained during a motor imagery task (further details are available in [23]). This scenario can be considered as an example of a homogeneous sensor network. On the other hand, Table 2 shows Pearson's correlation coefficients obtained from smart monitoring of a pothos plant for 10 minutes. Here, multi-modal (i.e., heterogeneous) sensing

Channel name	Fz	FC1	C3	C4	Pz	POz
Fz	–	0.95	0.71	0.72	0.48	0.3
FC1	0.95	–	0.84	0.71	0.56	0.37
C3	0.71	0.84	–	0.66	0.72	0.54
C4	0.72	0.71	0.66	–	0.73	0.56
Pz	0.48	0.56	0.72	0.73	–	0.92
POz	0.3	0.37	0.54	0.56	0.92	–

Table 1: Pearson’s correlation between time-series of EEG channels in healthy subjects. Each recording was 16 s long and obtained during a motor imagery task (further details in [23]). Values averaged across 2 subjects. Correlation values above 0.7 are highlighted in red.

Sensor type	$H$	$T$	$m_1$	$m_2$
$H$	–	0.91	0.48	0.03
$T$	0.91	–	-0.24	-0.12
$m_1$	0.48	-0.24	–	0.39
$m_2$	0.03	-0.12	0.39	–

Table 2: Pearson’s correlation of sensed metrics from a 10 minute measurement of a pothos plant, including ambient humidity  $H$ , room temperature  $T$ , and diffuse reflectances from one leaf in the wavelength range 1350–1650 nm ( $m_1$ ) and 1750–2150 nm ( $m_2$ ), respectively (further details in [24]). Correlation values above 0.7 are highlighted in red.

included an ambient humidity sensor, a room temperature sensor, and one short-wave infra-red portable spectrometer measuring the diffuse reflectance from one pothos’ leaf in the range 1350 – 2150 nm (further details are available in [24]). In both scenarios, high correlation values were obtained, especially in the case of homogeneous sensors due to their spatial proximity and a very complex system to monitor. It is worth noting that correlation values might significantly change over time even in homogeneous sensor networks, e.g., EEG captures the fast dynamic behavior of the different areas of the brain. We also computed the correlation between sensors in shorter recording times, obtaining even larger correlations, but we decided to report a single case study for each scenario for the sake of compactness.

At any rate, data correlation may represent a feature to exploit for several IoT systems. Most of the proposed investigations relate to how it can be used for a more performing management of the updates, in terms of scheduling efficiency or low energy consumption [21].

In [25], spatial correlation of information in a field under monitoring is investigated from the perspective of determining the optimal spatial density of sensing points to achieve adequate and timely coverage of the process, so the distribution of multiple

sources is itself the parameter to derive. The existence of many sensors is instead explicitly addressed as an aspect to manage in [26], and is investigated from a queueing theory perspective with continuous time. At the same time, [27] goes further and investigates proper scheduling of the sources within a similar scenario. In both these papers, there are multiple AoI values at the destination depending on one specific sensor only, without correlation. In [28], multiple sources are considered but they all monitor the same process (hence, their correlation makes them alternative to one another), their differences being instead in their energy consumption and reliability, which can be traded off for one another. Another paper considering multiple correlated sources is [29], where a joint allocation and scheduling problem is considered to minimize the AoI. However, the scenario is that of wireless cameras capturing different but possibly overlapping pictures, and the objective is a multi-view optimization that is decomposed into smaller problems.

From the perspective of scheduling IoT devices to minimize the average AoI, keeping into account a multiplicity of correlated sources and exploiting their correlation, the main reference is [9], where this problem, also considering different types of devices, is formulated as an infinite horizon average cost Markov decision process. The difference between all these papers and our present contribution is that we do not seek an optimization exploiting the correlation of neighboring sources, but rather we discuss its quintessential characterization and we propose a low-complexity representation, which can in turn be exploited in simple IoT contexts to determine efficient updating patterns.

To this end, it is worth mentioning that we specifically focus on discrete-time MC models, so our investigation can be seen as an extension of [22]. The use of such models allows for matrix-geometric approaches and is convenient in many scenarios, where a discrete time axis can be considered [30]. Also, this would make it immediate to merge these investigations with the special cases where the source or the channel follows an embedded MC [31, 32].

Finally, the idea of VoI is often addressed as an expansion to the plain concept of AoI and, as discussed, is subject to different interpretations [10, 12, 13]. Our stance in the present paper is that VoI is introduced as an extension of the AoI and we will explicitly mean it to represent a linear combination of the information freshness of cor-

related data coming from different sensors and a cost term [30], even though different expressions can be used to this end, with similar meaning but more complex math.

### 3. Methodology

Consider a system with multiple sensors sending data to a single receiver/collection point. We can think of the different sensors as all monitoring correlated metrics of an underlying process of interest. This can be the result of spatial correlation, which would possibly expand to a specific geometric structure of the sensor placement, such as a lattice or a grid [25, 33], or a logical relationship among the underlying metrics [9], as would be the case for biometric sensors for the same individual - in this specific case, the relevance of health tracking metrics would be directly connected to the AoI of at least some of them, when not all [7].

We now detail the analysis of such a system. The symbols and notations adopted can be tracked in table 3.

Symbol	Explanations
$p$	Transmission probability (for a single source)
$c$	Transmission cost
$N$	Number of neighbor sources
$\delta$	Age of Information (AoI)
$V$	Value of information (VoI)
$\alpha$	Hyper-parameter to weight the benefit from neighbor fresh information (Complete MC)
$q$	Degree of correlation (Scalar MC)

Table 3: Abbreviation table

We focus on a specific device  $i$ , and summarize the correlation of its tracked metric with other measurements from  $N$  different “neighbor sensors” in set  $\mathcal{N}$  that can also be tracked. All the sensors in  $\mathcal{N} \cup \{i\}$  adopt similar policies for sending updates, acting without any coordination but just being aware of their mutual correlation. The receiver is interested in getting information about the process status of sensor  $i$  but also somehow benefits whenever the information in the neighbors is fresh. For this analysis, we consider a discrete-time axis divided into slots of the same duration called *epochs*; in



each epoch, sources can either update or stay idle, and we assume that their transmissions are without collisions. So the age of information for source  $i$  is

$$\delta_i(t) = t - \max_{\tau_i^{(k)} \leq t} \{\tau_i^{(k)}\} \quad (1)$$

where  $\{\tau_i^{(1)}, \tau_i^{(2)}, \dots, \tau_i^{(n)}, \dots\}$  are the epochs where the  $i$ th source sent an update. We remark that with this notation, the AoI assumes value 0 for all the epochs  $t = \tau_i^{(k)}$  where an update is performed.

We consider the simplest possible operating model for the sources, i.e., they decide in a random fashion, and independently of one another, whether to send an update or stay idle during the current time slot, and this decision is made with identical and independent distributed (i.i.d.) probability  $p$ . Thus, the average AoI at the receiver's side for the current source can be computed as a direct consequence of its geometric distribution (starting from 0), i.e., [30]

$$\mathbb{E}[\delta_i] = \frac{1}{p} - 1 \quad (2)$$

If we choose  $\mathbb{E}[\delta_i](p)$  as a metric to optimize, it is trivial to note that the function is minimized for  $p^* = 1$ . But this is unrealistic in a real scenario since the sensing/transmission operations ought to be kept limited to avoid unnecessary expenditures and strain on the sensor. To account for this, we may include a cost term proportional to the transmission rate according to a parameter  $c > 0$ . In this case, we can define a penalty function

$$K_1(p) = \mathbb{E}[\delta_i] + cp = \frac{1}{p} - 1 + cp \quad (3)$$

where both terms combined into  $K_1(p)$  (the expected AoI and the average transmission cost) are better when set to a low value. Thus, our goal may be seen as to find a penalty-minimizing update probability  $p^*$ , which is promptly found as  $p^* = \sqrt{1/c}$ . Even though the result is immediate, it properly accounts for the intuition that a higher cost decreases  $p^*$ . This implies that  $c$  can be seen either as an external parameter related to information update costs, e.g., in terms of energy consumed [34] or as a virtual tunable parameter that is used to regulate the frequency of updates in a distributed fashion [30].

This setting can be extended from a modeling standpoint by considering and exploiting the presence of multiple sources providing correlated information, which is

our main aim in the present paper. Note that we are not just interested in extending the model per se, but rather we want to investigate how correlation affects the system performance. In particular, we look to address how correlation can determine a possible further decrease in the transmission probability, other than what is expressed by the cost, since a situation where fresh information coming from a neighbor sensor is strongly correlated may make it pointless to update for the sensor of interest too.

Thus, we extend this analysis by involving the  $N$  neighbors of the sensor of interest, which are assumed to follow the same rule of updating with probability  $p$ . Now, we consider that the processes monitored by  $N$  neighbors are correlated with that of the sensor of interest and if one of them sends an update this can be in some way also useful to the sensor of interest itself.

In order to quantify this usefulness, we decrease the AoI of the sensor of interest, which in (2) is introduced as a penalty (the lower, the better). Therefore, we insert a factor that measures the difference between the age of the sensor of interest and the age of the most up-to-date neighbor, whenever this is lower than the age of the sensor of interest. Otherwise, i.e., if the sensor of interest is more up-to-date than its entire neighborhood, there is no benefit achieved by exploiting the correlation. We obtain the following quantification that we regard as the VoI  $V_i$  of sensor  $i$  in the presence of correlation, i.e.,

$$V_i = \mathbb{E}[\delta_i - \alpha(\delta_i - \min_{j \in N} \delta_j) \cdot \mathbb{1}(\min_{j \in N} \delta_j < \delta_i)] \quad (4)$$

where  $\alpha$  is a hyper-parameter used to weigh the benefit that the most up-to-date neighbors has fresher information, and  $\mathbb{1}(\cdot)$  denotes a characteristic function (equal to 1 if the Boolean condition is true, 0 if false). In other words,  $\alpha$  quantifies (and embeds into our VoI formulation) the degree of correlation among the sources. Indeed,  $\alpha = 0$  would describe a scenario where the sources track disconnected values that have no relationship to one another, and therefore the presence of an up-to-date value in a neighbor sensor is not useful at all. Conversely,  $\alpha > 0$  implies that the sources are deterministically correlated so that when a neighbor source has low AoI, this benefits the source of interest as well, to the point that for  $\alpha = 1$  an update of another neighboring source works as well to decrease the VoI to 0.

For the sake of readability, in the following, we will consider the VoI as the expression of  $V_i$  as per (4), since it quantifies the freshness, but accounts for multiple sensors. In addition, we will always combine this term with an extra component describing the cost, which is explicitly kept separate to highlight the trade-off, as the network management is interested in having good value for a low cost. Finally, note that, despite the name that is chosen in accordance with the literature, the term  $V_i$  is based on an age evaluation, so the objective ought to be that of minimizing it, or keeping it low.

Since we assume that all the sensors behave identically for what concerns their updating policy, symmetry reasons dictate that the sensor of interest is actually the one with the freshest information in  $1/(N + 1)$  of the cases, in which case the benefit of exploiting the correlated information is 0. In the remaining cases, i.e., a fraction of  $N/(N + 1)$ , the neighbor with the lowest AoI brings instead a decrease in the VoI of the sensor of interest. The value of such AoI is the minimum of  $N$  geometrically distributed variables, thus we can extend (4) to

$$V_i = \frac{1}{p} - \frac{\alpha N}{N + 1} \left( \frac{1}{p} - \frac{1}{1 - (1 - p)^N} \right) - 1 \quad (5)$$

In the end, our VoI framework can be used to describe effective network management that minimizes a penalty function combining freshness and cost, i.e.,

$$\min K_2(p) = V_i + cp \quad (6)$$

which results in an optimal transmission probability  $p^*$ , promptly found as the solution of setting a first-order derivative to 0. This means that, for sufficiently high cost  $c$ ,  $p^*$  is the value for which  $dV_i/dp = -c$ .

To expand the model from just an average value computation to a full-fledged statistical characterization, we can actually use a discrete-time MC jointly tracking the AoI of the sensor of interest and its neighborhood, which would generalize to cases where the VoI does not follow from a linear combination of ages through weight  $\alpha$ . Remarkably, since the AoI values can be seen as rewards of renewal processes, whose cycles relate to an update from either the sensor of interest or one of the neighbors, such an MC would precisely track the VoI according to our proposed definition.

### 3.1. Complete MC model

To model the usefulness of the neighbors we propose a *triangular* MC that works in the following way. First, we remark that the AoI is in principle unlimited, but for a tractable numerical evaluation, we might prefer to set the maximum AoI to a large value  $M$ . This is the only approximation introduced in the model, and can easily be neglected if  $M$  is sufficiently large.

In the resulting MC,  $M+1$  states are then used to track the states where the sensor of interest  $i$  is the most up-to-date and then  $M(M+1)/2$  states where at least one neighbor has fresher information than node  $i$ , in which case we simply track the AoI of the *most up-to-date neighbor*.

A graphical representation of this is shown in Figure 1. Two sets of  $M+1$  nodes and  $M(M+1)/2$  nodes represent the respective cases where (i) the sensor of interest  $i$  is most up-to-date, in which case we track its AoI as the state; or, (ii) another neighbor sensor is, in which case we track the information of both AoI values of the node  $i$  and that neighbor. Thus, each state  $k \in \mathcal{S}$ , with  $|\mathcal{S}| = (M+1)(M+2)/2$ , of the MC is associated with either just value  $A_k$ , which represents the AoI for the node  $i$  at state  $k$ , or  $A_k$  and  $B_k$ , the latter being the AoI of the most up-to-date neighbor at state  $k$ , if lower than  $A_k$ .

Given the probability of updating  $p$  (assumed to be the same for all nodes) and the number of neighbors  $N$ , we can create the triangular MC depicted, with 3 possible transitions: (i) the sensor of interest sends an update, whose probability is  $p$ ; (ii) node  $i$  does not update, but at least one of its neighbors does, which happens with probability  $(1-p)(1-(1-p)^N)$ ; or (iii) no one sends an update, and the transition probability for this event is  $(1-p)^{N+1}$ .

Here, we can exploit the renewal properties of AoI that whenever a node (either the sensor of interest or a neighbor) performs an update, it necessarily becomes the most up-to-date, whereas if no sensor updates, the most up-to-date node remains the same as the previous epoch. This proves that the resulting system still has the Markov property and as such can be tracked by an MC. At the same time, the choice of tracking just the most up-to-date neighbor allows for a precise description without any approximation, but with just an increased dimensionality (two AoI values instead of  $N+1$ ).

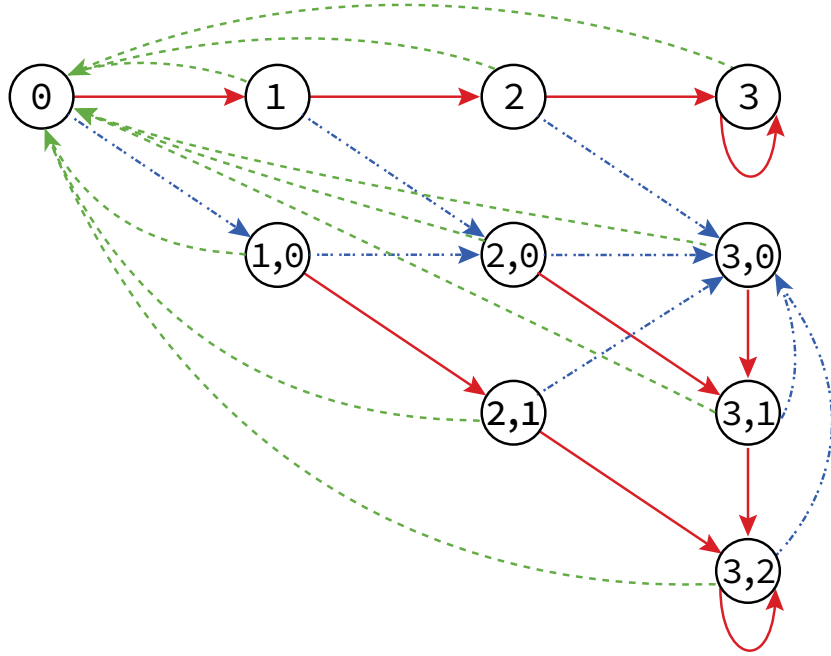


Figure 1: Example of a complete MC model. Solid red arrows, blue dotted arrows, and green dotted arrows mean updates from no node, a neighbor but not the sensor of interest, and the sensor of interest, respectively. States  $X$  in the top row imply that the sensor of interest is the most up-to-date with AoI  $X$ , while states  $X, Y$  denote the AoI values of the sensor of interest and the most up-to-date neighbor, respectively.

Thus, the VoI of sensor  $i$  can be computed after evaluating the steady-state probabilities  $\pi_k$  of the chain being in state  $k$  through

$$V_i = \sum_{k=0}^{\frac{(M+1)(M+2)}{2}} A_k \cdot \pi_k - \alpha \sum_{k=M+1}^{\frac{(M+1)(M+2)}{2}} (A_k - B_k) \cdot \pi_k \quad (7)$$

where the first summation considers the average AoI of node  $i$  for all the states, whereas the second one accounts for the cases where a neighbor is more up-to-date which decreases the VoI as per (4).

Aside from numerical limitation for solving the MC, this model is *exact* and allows full track of the VoI model presented above. We also remark that, differently from the solution of (5) that only applies to the case of updates with i.i.d. probability  $p$  for all epochs and all sensors, the MC model can actually be extended to more elaborate scenarios where the update rule is optimized [6, 12, 27, 29].

### 3.2. Scalar state MC

We now propose a simplification to the previous MC. Instead of having a triangular structure with two-dimensional states, we use a scalar state MC, where each node represents only an equivalent VoI of the node  $i$ . The assumption is that we merge transitions including an update, assumed to happen with probability  $t$ , regardless of whether it is coming from the sensor of interest or one of the neighbors, where the latter case is clearly weighted with a coefficient  $q < 1$ .

Within this model, we can consider only 2 transitions: the MC advances to the next state with probability  $1-t$  and returns to state 0 with probability  $t$ . The transition matrix  $\mathbf{P}$  of this MC is

$$\mathbf{P} = \begin{bmatrix} t & 1-t & 0 & 0 & \dots \\ t & 0 & 1-t & 0 & \dots \\ t & 0 & 0 & 1-t & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad (8)$$

To take into account the usefulness of the neighbors we consider:

$$t = p + (1-p)(1-(1-p)^N)q \quad (9)$$

where  $p$  is the probability of update,  $N$  is the number of neighbors and  $q$  is the probability that the update of a neighbor is useful, in which case the VoI is reset to 0 even though the update does not come from sensor  $i$ . In this way, when at least one neighbor updates its status, this update becomes also useful for the sensor  $i$  with probability  $q$  as it is fully informative and resets its AoI.

Analogously to the previous complete MC, we can set an upper limit  $M$  to the AoI values tracked, and once evaluated the stationary distribution vector  $\boldsymbol{\pi}$  it is possible to compute the VoI as

$$V_i = \sum_{k=0}^M k \cdot \pi_k \quad (10)$$

Note that  $q = 0$  and hence  $t = p$  leads back to the basic model of (2).

The scalar state MC can be made very similar to the complete MC through careful fine-tuning of the parameter  $q$  as a function of  $\alpha$ , so as to match the performance at least

in the average sense. One can link the value of  $q$  and  $\alpha$  given the number of neighbors  $N$  and the transmission probability  $p$ . By matching the average VoI of the two chains we obtain

$$\frac{1}{p} - \frac{\alpha N}{N+1} \left( \frac{1}{p} - \frac{1}{1-(1-p)^N} \right) - 1 = \frac{1}{p + (1-p)(1-(1-p)^N)q} \quad (11)$$

where the left side of the equality comes from (5) as the average VoI, while the right side derives from (3) as the average VoI for the scalar state MC. Then it is possible to solve the equation for  $\alpha$  or  $q$ . With a few simple algebraic steps we arrive at

$$q(\alpha, p, N) = \frac{1}{Z_2(p, N)} \left( \frac{p}{1-p \cdot Z_1(p, N) \cdot \alpha} - p \right) \quad (12)$$

where

$$Z_1(p, N) = \frac{N}{N+1} \left( \frac{1}{p} - \frac{1}{1-(1-p)^N} \right) \quad (13)$$

and

$$Z_2(p, N) = (1-p)(1-(1-p)^N) \quad (14)$$

Both  $Z_1(p, N)$  and  $Z_2(p, N)$  are functions only of  $p$  and  $N$  and so once these two parameters are set, they act as constant terms of (12).

It may be worth investigating possible approximations derived in this way, that evaluate  $q$  through interpolations of different values in  $p$  and  $N$  so as to decrease its parametric dependence. As visible from Figure 2, where we plot  $q(\alpha, p, N)$  as given by (12), with  $\alpha$  on the x-axis and for some fixed values of  $p$  and  $N$ , the value of  $q(\alpha, p, N)$  is increasing in  $\alpha$ , and such an increase is approximately linear, at least for low values of  $\alpha$ , which, as will be argued next, are the most relevant for our analysis. This suggests a way to decrease the dependencies of (12) on 3 different parameters through approximations.

As a reference, we give two examples of such approximations. In the first one, we take  $q(\alpha, p, N)$  through a linear interpolation and we create a function that depends only on the parameter  $\alpha$ . We denote this linear approximation as  $q_{\text{lin}}$ . To do this, we apply an ordinary least square regression [35] to (12) for multiple values of  $p$ ,  $q$  and  $\alpha$ , in order to minimize the objective function:

$$\ell(b_0, b_1) = \sum_{i=1}^N (q_i - (b_1 \cdot \alpha + b_0))^2 \quad (15)$$

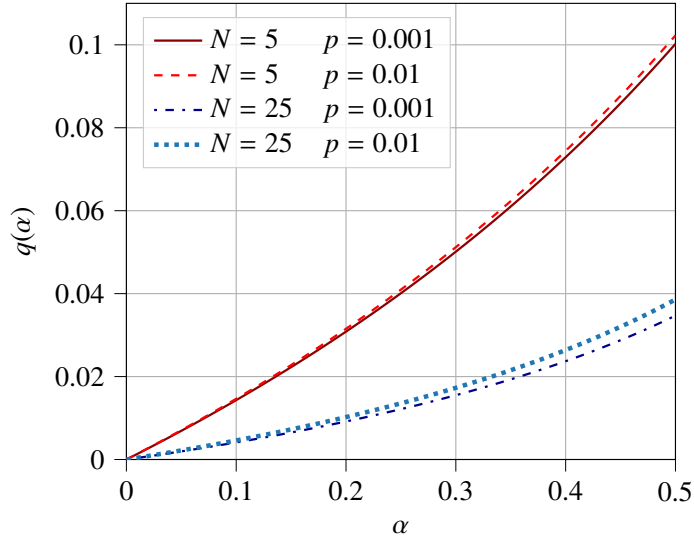


Figure 2: Comparison of correlation parameters,  $q$  vs.  $\alpha$ , for various combinations of  $p$  and  $N$

where  $q_i$  are samples from (12) and

$$q_{\text{lin}}(\alpha) = b_1 \cdot \alpha + b_0 \quad (16)$$

is the linear approximation of  $q(\alpha, p, N)$ . We search for the coefficients  $b_0, b_1$  minimizing the objective, i.e.,  $(b_0, b_1) = \arg \min_{b_0, b_1} (\ell(b_0, b_1))$

In the second approximation, we adopt a polynomial interpolation for (12). We define a multivariate quadratic function that depends on  $p$  and  $\alpha$  [36], which we call  $q_{\text{quad}}$ , and, as above, find its coefficients through least square minimization. In the end, we obtain the equation:

$$q_{\text{quad}}(\alpha, p) = b_1 \cdot \alpha + b_2 \cdot p + b_3 \cdot \alpha^2 + b_4 \cdot \alpha \cdot p + b_5 \cdot p^2 + b_0 \quad (17)$$

The values for the coefficients of both interpolations are reported in Table 4.

It is important to notice that in a general context, one can expect  $N$  and  $p$  to be within a certain range, at least as an order of magnitude. Thus, our interpolations that are shown for any value of  $N$  and  $p$  are certainly a worst-case scenario in practical contexts. As shown numerically, these approximations still work very well, thus one can expect that even a coarser approximation with a rough estimate of  $N$  and  $p$  would obtain good results in practical network deployments.



$q_{\text{lin}}(\alpha)$	
$b_1$	0.1996146
$b_0$	-0.0012262
$q_{\text{quad}}(\alpha, p)$	
$b_1$	0.0021396
$b_2$	0.0084879
$b_3$	0.1356926
$b_4$	0.9935651
$b_5$	-0.014334
$b_0$	$-2.51 \cdot 10^{-5}$

Table 4: Values of the coefficients of the interpolations.

From the update formula of the scalar MC is it also possible to evaluate the load reduction, i.e. the average reduction in the number of transmission due to the correlation of the nodes. We take the probability  $t$  of resetting the information as per (9), which includes the summation of two terms. The first one, equal to  $p$ , corresponds to an update sent by the sensor itself, whereas the second one  $((1 - p)(1 - (1 - p)^N)q)$  indicate that the AoI of the node is reset by an update sent by at least 1 of the  $N$  neighbors. Thus, the resulting transmission rate of the node can be quantified as:

$$\frac{p}{t} = \frac{p}{p + (1 - p)(1 - (1 - p)^N)q} \quad (18)$$

#### 4. Results

We now present some numerical results to give a quantitative insight into how the different models capture the underlying VoI and how suitable they are to perform some simple management of the involved parameters. In particular, we are interested in seeing whether setting a proper cost parameter  $c$  can tune the update probabilities of the sensors, also when correlation among multiple sensors is kept into account with  $\alpha = 0.1$ , which describes a limited but noticeable influence. In all actuality, this choice of  $\alpha$  can be seen as the real interesting scenario, as a higher value would make our proposal of merging contributions from different sources even more accurate, whereas a lower value would possibly describe a correlation so weak that is not worth considering.

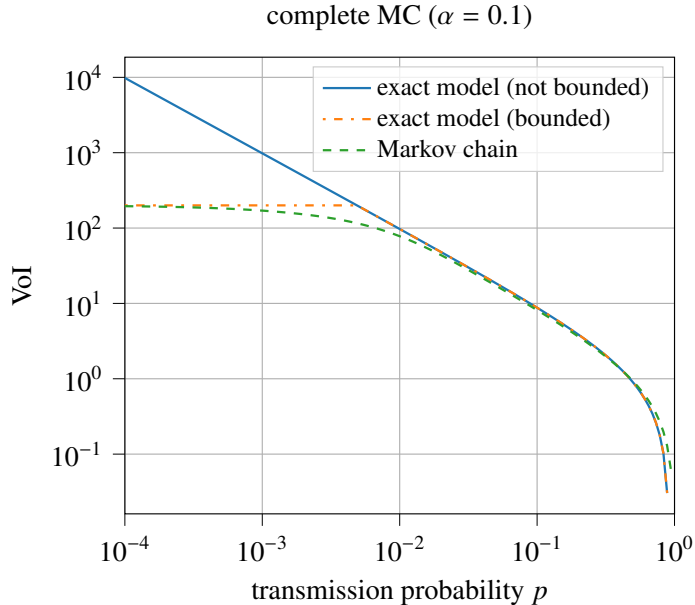


Figure 3: Comparison between VoI computed with complete MC and the corresponding theoretical model with  $\alpha = 0.1$

We first test if the two MCs correspond to the underlying theoretical model. To do this, we choose a set of parameters ( $N = 10$ ,  $q = 0.01$ , and  $\alpha = 0.1$ ) and we compute the VoI both theoretically through (5) and via the MCs as per (7) and (10). The value of  $q = 0.01$  is chosen according to (12) for  $N = 10$  and  $\alpha = 0.1$ , with  $p$  chosen as numerically fitting most of the values of  $p^*$  in the region of interest.

The results are plotted in Figures 3 and 4 for the complete and simplified MC, respectively. For both models, we can observe that the MC corresponds to the theoretical model, up to a computational bound imposed by the maximum number of states of the MCs. To have a more robust comparison we plot together with the unbounded theoretical VoI a bounded version defined as

$$V_{i,bound}(p) = \max(V_i(p), M) \quad (19)$$

with  $M$  the maximum AoI used in the MCs.

#### 4.1. Numerical Evaluations

In Figures 5 and 6, we report the numerical results obtained for the complete (triangular) MC, showing the VoI versus the transmission probability  $p$  and how the VoI

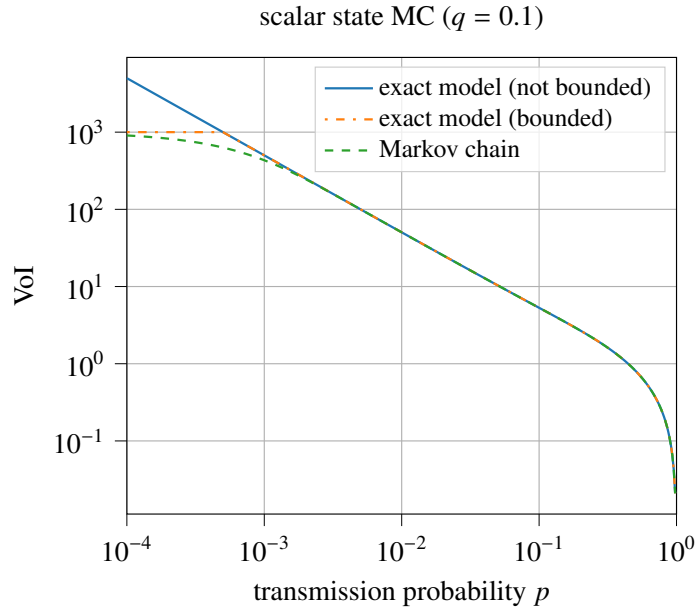


Figure 4: Comparison between VoI computed with scalar state MC and the corresponding theoretical model with  $q = 0.1$

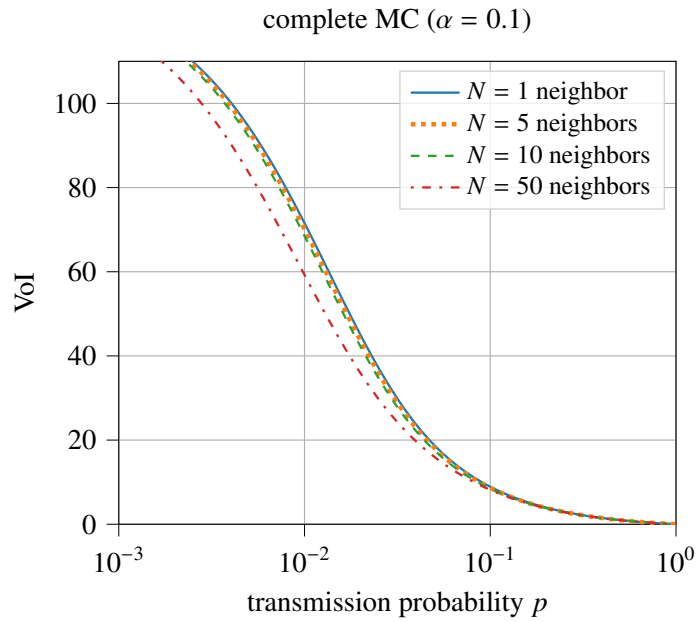


Figure 5: VoI of the complete MC with  $\alpha = 0.1$  vs. the transmission probability  $p$ .

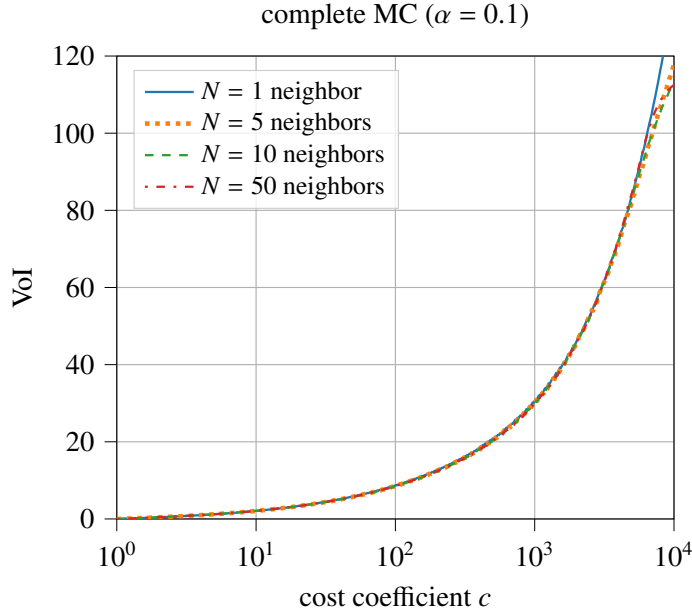


Figure 6: VoI of the complete MC with  $\alpha = 0.1$  vs. the cost coefficient  $c$ .

can be set according to the cost coefficient  $c$ . These results were obtained for  $\alpha = 0.1$ .

In Figures 7 and 8 we plot the numerical results obtained for the simplified scalar state MC. More precisely, the figures show the VoI versus the transmission probability  $p$  and how VoI is set as a function of the cost coefficient  $c$ , respectively.

In both Figures 6 and 8, we associate every value of  $c$  with the optimal transmission probability  $p^*$ , to further compute the VoI. To obtain this probability, we compute the derivative of the VoI in  $p$  (i.e., the curve in Figures 5 and 7) and set it equal to  $-c$ , since the overall penalty is set from (6) as  $K_2(p) = V_i + cp$ .

From the inspection of Figure 5, we can also infer how the number of neighbors influences the VoI. More precisely, a higher number of neighbors tends to decrease the VoI. This is coherent with the intuitive idea that, the more nodes in the network, the more likely is that they help each other with the updates. This effect is more relevant for intermediate values of  $p$ , since for  $p \simeq 1$  or  $p \simeq 0$ , the impact of correlated sensors on the updates is relatively limited.

Figure 6 shows how the VoI changes, based on the cost factor  $c$ . Also in this case,

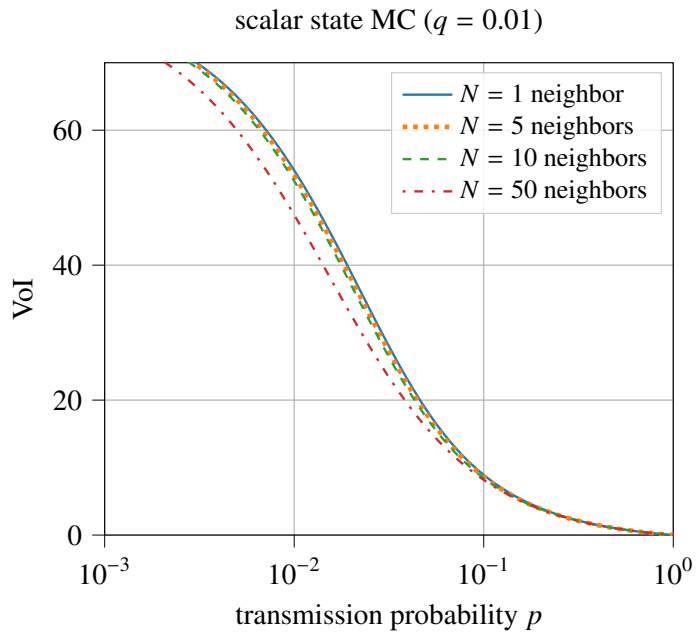


Figure 7: VoI of the scalar state MC with  $q = 0.01$  vs. the transmission probability  $p$ .

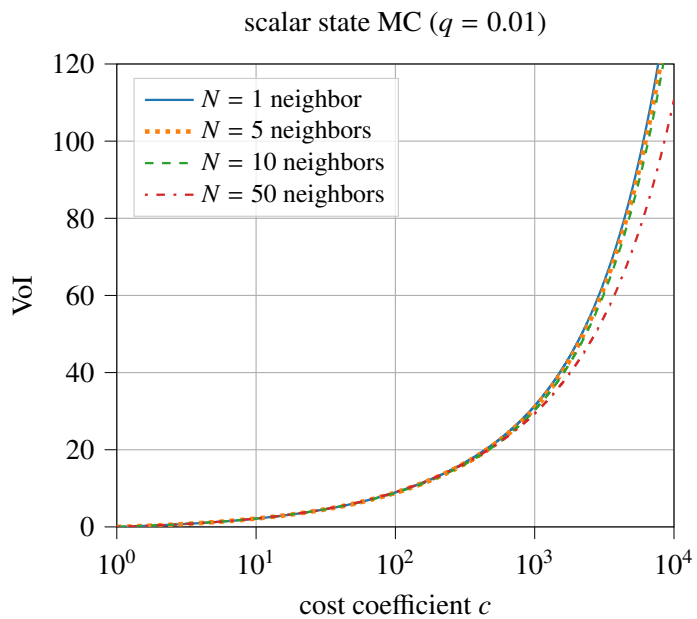


Figure 8: VoI of the scalar state MC with  $q = 0.01$  vs. the cost coefficient  $c$ .

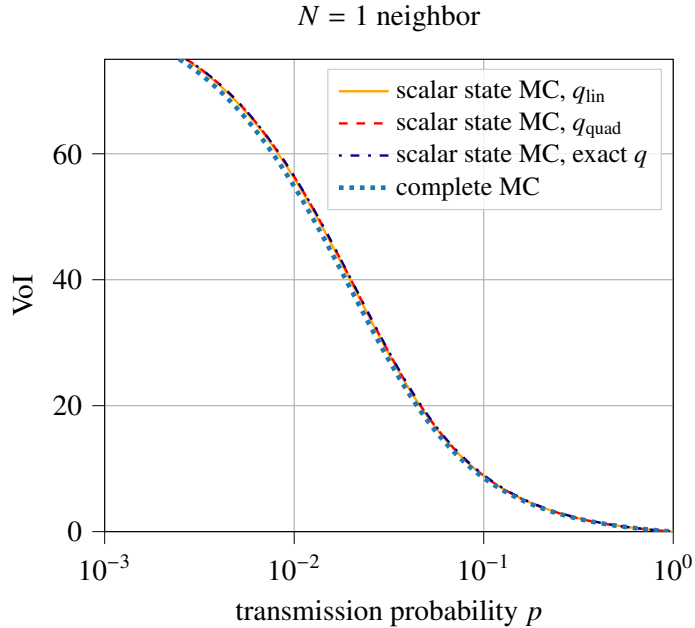


Figure 9: Comparison between complete ( $\alpha = 0.1$ ) and scalar state MCs with  $N = 1$  neighbor, VoI vs. transmission probability  $p$ .

coherently with intuition, for a small  $c$  we have a small VoI, while for a large  $c$  the VoI tends to increase. This reflects the fact that for small values of  $c$  the sensors pay a small price for each transmission and therefore are encouraged to transmit as much as possible. Similar conclusions can be drawn for the scalar MC in Figures 7 and 8.

Then, we can compare the results obtained by the two MCs. In Figures 9, 10 and 11 we show how the average VoI changes based on the transmission probability  $p$ , for the number  $N$  of neighbors set in these figures to 1, 10, and 25, respectively. We used  $\alpha = 0.1$  for the complete MC. For the scalar state MC, we compute  $q$  through 3 different approaches, namely, the linear interpolation  $q_{\text{lin}}$ , the quadratic interpolation  $q_{\text{quad}}$ , and the precise evaluation of (12). As we will remark, the differences in these approaches are barely noticeable.

Indeed, we can observe that for  $N = 1$  (Figure 9), the two MCs obtain practically identical results, with the VoI values completely overlapping. The difference is more visible for higher  $N$ , especially considering the range between  $p = 10^{-3}$  and  $p = 10^{-1}$ . More precisely observing Figures 9, 10, 11, the quality of the approximation

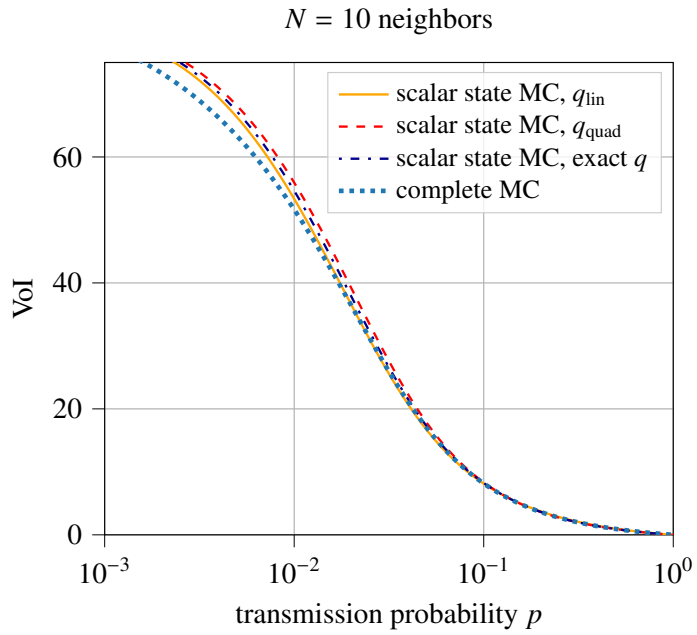


Figure 10: Comparison between complete ( $\alpha = 0.1$ ) and scalar state MCs with  $N = 10$  neighbors, VoI vs. transmission probability  $p$ .

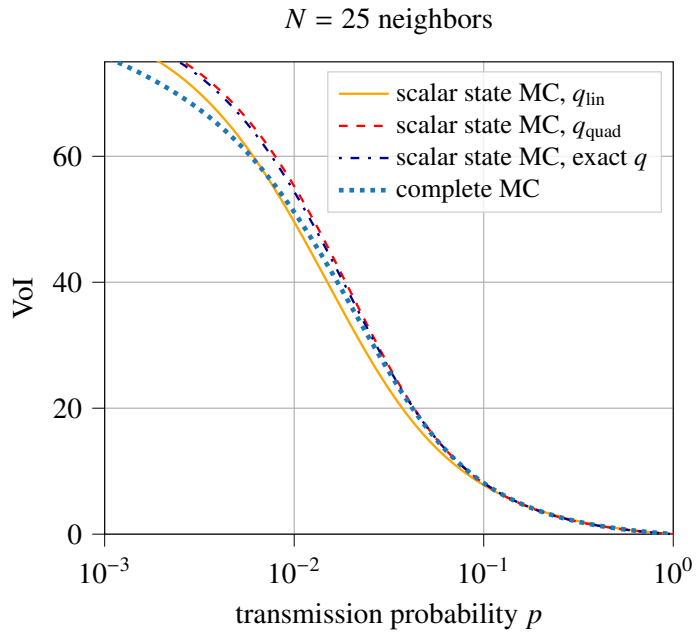


Figure 11: Comparison between complete ( $\alpha = 0.1$ ) and scalar state MCs with  $N = 25$  neighbors, VoI vs. transmission probability  $p$ .

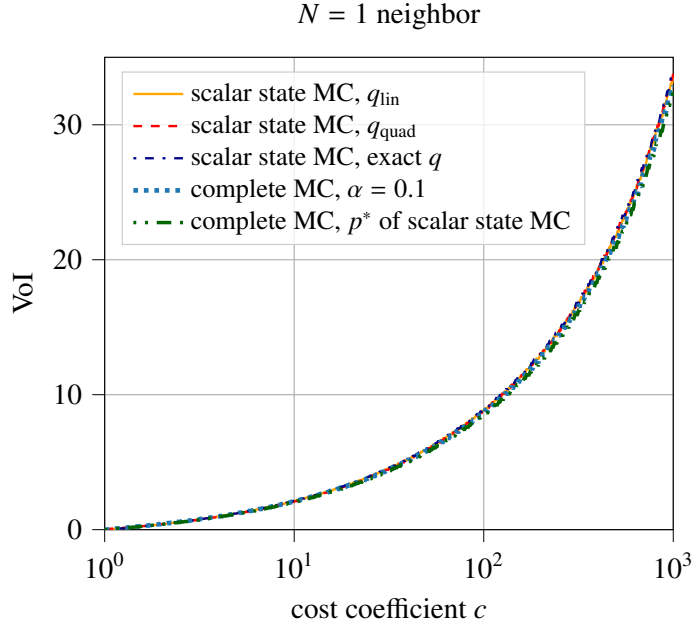


Figure 12: Comparison between complete ( $\alpha = 0.1$ ) and scalar state ( $q = 0.01$ ) MCs with  $N = 1$  neighbor, VoI vs. cost parameter  $c$ . The complete MC with scalar state optimal  $p^*$  is based on the scalar state MC with  $q_{lin}$ .

can be grouped in 3 zones, based on transmission probability  $p$ . For high transmission probability (i.e.  $p > 10^{-1}$ ) we notice that the Scalar MC can behave as the Complete MC, for any number  $N$  of neighbors, even using  $q_{lin}$ . For value of  $p$  between  $10^{-2}$  and  $10^{-1}$  instead,  $q_{quad}$  is a better approximation of the exact  $q$  formula. Finally, for lower transmission probabilities (i.e.  $p < 10^{-3}$ )  $q_{quad}$  still offers a good solution to match the behavior of the 2 MCs, but tends to diverge for high values of  $N$ . However, this divergence is present even if we use an exact  $q$  formula, so it is not related to the approximation but rather to the wide range of analyzed parameters. If the system is expected to perform very sporadic updates, e.g., rarer than 1 every 1000 epochs, it would be convenient to perform a tailored interpolation on this range.



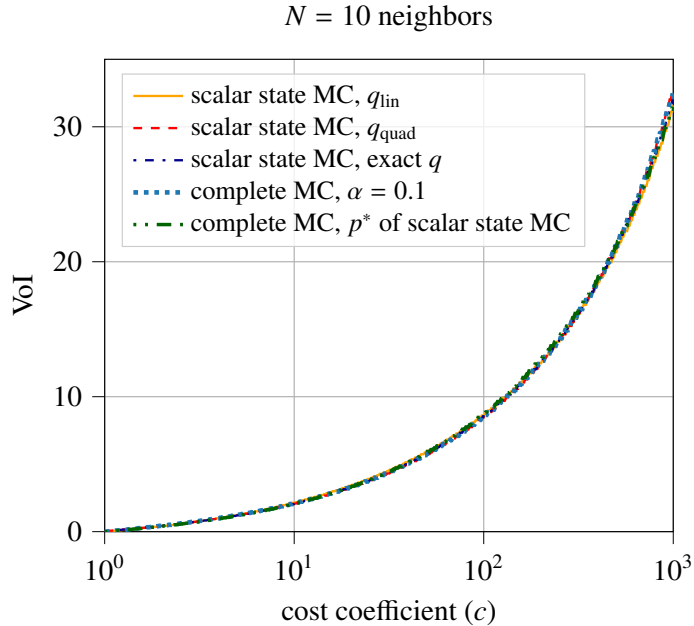


Figure 13: Comparison between complete ( $\alpha = 0.1$ ) and scalar state ( $q = 0.01$ ) MCs with  $N = 10$  neighbors, VoI vs. cost parameter  $c$ . The complete MC with Scalar State optimal  $p^*$  is based on the Scalar State MC with  $q_{lin}$ .

In Figures 12 and 13, we compare the two models to highlight how similar they are in computing the VoI based on the cost coefficient  $c$ . Analogous to Figures 6 and 8, we evaluate the VoI per each  $c$ , by considering the optimal transmission probability  $p^*$  for that specific point. The indirect setup of a VoI value according to  $c$ , through its direct imposition of  $p^*$ , i.e., by controlling the frequency of channel access in a distributed fashion, can be regarded as the main goal of our analysis. Thus, we can claim that even the simpler scalar state MC can be effective for a local implementation onboard the individual sensors.

Finally, we compute the load reduction of transmitting nodes with  $N = 5$  neighbors and correlation represented as  $\alpha = 0.1$  as per (18). The results are displayed in Figure 14. We also validate them through numerical simulation, visible in the same figure. It is interesting to note that the reduction is sharper for lower transmission probability. This is especially useful for scenarios where updates are sporadic and correlation is then more relevant. We performed further test with higher numbers of nodes ( $N = 10$ ,  $N = 25$ ); as expected, the reduction in the transmission activity is even higher, which

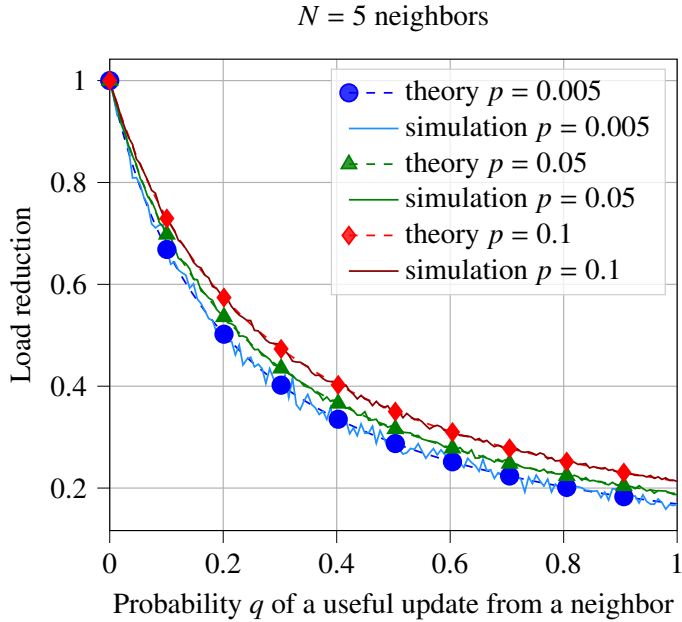


Figure 14: Reduction of transmission activity thanks to the data correlation, versus the probability  $q$  that an update from a neighbor is useful.

is sensible as there is a higher chance that a neighbor node sends a useful update.

## 5. Conclusions

In practical large-scale networks, resource constraints are relevant, and the correlation of multiple sources can be exploited to improve scalability and avoid multiple redundant transmissions. Ultra-reliable and low-latency communication (URLLC) in the IoT [37] may require to characterize the freshness of information even under massive access. Next-generation communications will rely strongly on learning the state of a large number of channels [38]. On top of that, the number of terminals simultaneously active causes a curse of dimensionality that ought to be contained, for example by exploiting data correlation.

The freshness of information in these multi-access scenarios can be addressed through analytical frameworks revolving around the VoI [8, 10, 13]. The case of multiple sources generating correlated information can be captured through discrete time MCs with different complexities. In particular, we presented a model where the im-

pact of correlation is kept into account in the transition probabilities of an MC with a scalar state representing a modified VoI including the freshness of multiple sources with different weights [9]. Such a model is shown to be effective in characterizing basic updating policies with i.i.d. probabilities and can therefore be a suitable solution for large-scale deployments of sensors with limited computational complexity and energy.

Future developments may involve the analysis, in the same spirit, of more advanced updating policies, still with possible simplifications in the system state, to see whether low complexity representations allow for efficient management, and the implementation of this rationale within specific applications for the IoT.

For example, a more detailed representation of the channel conditions and the communication protocol adopted can affect the analysis of the VoI. If the channel is unreliable, or subject to collisions when multiple nodes access it concurrently, it could be beneficial to use a retransmission mechanism such as automatic repeat request (ARQ) [39, 40]. This increases the robustness of the communication but also increases the latency, which in turn affects the AoI and VoI.

Another aspect to consider is that power consumption and generation, e.g., for energy harvesting nodes [28, 34]. Here correlation among sources is present beyond the data, also referring to the battery charging if nodes are powered by similar technological means. All of these problems can be studied through MC-based approaches, therefore our proposed methodology can be applied to them in future investigations.

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