

Novel generation schemes for stable soliton states in optical microcavities

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Abstract—The excitation of Kerr optical frequency combs (OFC) is frequently non-deterministic and remains a cumbersome problem in many practical situations. While standard techniques to generate Kerr solitons in passive resonators employ a continuous wave pump, recently pulsed pumping has also been proposed. In this study we individuate and classify OFC states in a phase space defined by an experimental set of coordinates and triggered by a general super-Gaussian chirped driving field. Our numerical analysis shows how the soliton drifts caused by the phase modulation of the input field accelerate the dynamics and convergence towards a stable soliton state.

Index Terms—optical frequency comb, solitons, nonlinear optics

I. INTRODUCTION

Since the very first experimental demonstration of OFC [1], researchers have investigated the rich nonlinear dynamics of light-matter interaction in optical microcavities, mainly focusing on the so-called primary combs and solitons states [2]. The interest comes from the wide range of applications, from metrology to optical communications. Soliton states are triggered from the modulation instability of a locally flat background due to the interplay of dispersion and optical nonlinearities in a passive resonator. While typically schemes for soliton state excitation are based on coupling a continuous wave (CW) pump into a nonlinear cavity [3], pulsed pumping have been recently proposed as capable of sustaining and controlling the cavity soliton dynamics [4]–[7]. Solitons drifts are determined by amplitude and phase modulation [8], with interesting applications for controlling the OFC generation [9], the OFC repetition rate [10] or for optical tweezing [11].

In this contribution we map the OFC generation states of interest by sweeping a nonlinear cavity with a ramp of detuning $\Delta(t)$ and considering a super-Gaussian chirped pump with a quadratic phase. We map the dynamics by means of the set of coordinates $\{C, \Delta(t), N\}$, where C is the chirp parameter determining the extent of the quadratic phase, Δ the laser-cavity detuning, and N is the number of intra-cavity field peaks. By means of this analysis, we show how the controllable drift dynamics help a fast convergence towards soliton regimes, resulting in a robust stationary state with a wide locking range.

II. OFC STATES MAPPING

A simple generalization of the Lugiato-Lefever equation for pulsed pumping is given by [8]:

$$\frac{\partial E}{\partial t} = \left[-1 + i(|E|^2 - \Delta) + i \frac{\partial^2}{\partial \tau^2} \right] E + S(\tau), \quad (1)$$

where E and S are the intra-cavity and driving field amplitude, respectively, and Δ the laser-cavity detuning. The two time scale variables represent the *slow* (t) and *fast* (τ) time variables, respectively. The first time scale describes how the mean field amplitude evolves over successive round trips, while the τ -dynamics describes the variation of the intra-cavity field intensity over a single round trip. It has been recently shown how a synchronous Gaussian pumping could sustain a sech-like solution of the LLE model on the edge of a locally quasi-flat intracavity background [7], while a complete description for this dynamics can be provided by considering a driving field modulation of both amplitude and phase [8]. Here we consider a square super-Gaussian input field with a quadratic phase $\propto \tau^2$ tuned by a *chirp parameter* C :

$$S(\tau) = S_0 \exp \left[- \left(\frac{(1 + iC)\tau^2}{2\tau_G^2} \right)^q \right], \quad (2)$$

where S_0 is the driving field amplitude, q the super-Gaussian order and τ_g the variance of the respective Gaussian distribution (i.e. $q = 1$). In our previous work [9] we showed how this special way of pumping completely controls the drifts and the fast time attractors of the cavity soliton. Harnessing the capability of controlling the nonlinear dynamics by just tuning the chirp parameter C , we can accelerate the convergence towards a stable stationary single soliton state. To show this, we numerically map the phase space spanned by the set of coordinates $\{C, \Delta(t), N\}$, where C and Δ are defined in the Eqs.(1,2), while N is the number of intra-cavity field peaks. We perform a set of cavity sweeps for different values of C . Referring to Fig.1, the phase space is reported on the top panel. Each vertical line represents a cavity sweep performed for different value of C . Depending on the values of C , we can trigger multi- or single-soliton states, whose temporal

and frequency dynamics are reported on the central and bottom panels, respectively. For negative C , we generally observe soliton drifts towards the outwards of the quasi-flat super-Gaussian background. This dynamics converges towards unstable sech-like solutions, since the generated solitons tend to escape towards the borders of the plateau, where they cannot be longer sustained by the super-Gaussian driving field. Positive chirps, on the contrary, result in dynamical drifts of the solitons towards the centre of the plateau, allowing a fast convergence towards stable single soliton states.

In Fig.2 we report different regimes, typical of the LLE dynamics, triggered by the ramp of detuning $\Delta(t)$ reported on the bottom right panel. Initially, the quasi flat background (FBK) is coupled into the cavity. Subsequently we observe the emergence of the modulation instability (a) and chaotic (b) regimes. These three dynamical stages are similar for any C parameter, except for the fact that in the chaotic regime negative chirps tend to push the intra-cavity spikes towards the outwards of the super-Gaussian plateau, eventually resulting in multi-solitons regimes (c); on the other hand, positive C pushes the intra-cavity spikes towards the centre of the plateau, eventually recovering a fast time symmetry (d). Moreover the C magnitude determine the drift speed, and consequently we can accelerate the convergence towards dynamical attractors by setting higher C values. Each of these panels represents a point in the phase space $\{C, \Delta, N\}$. The variable N does not allow for a direct individuation of the single OFC states. Nonetheless, the full chart map can be easily interpreted: in the modulation instability regime, the number of spikes is pretty high ($N \sim 20 \div 25$) and stable. In the chaotic regime N is even larger ($N > 30$) and very unstable, while for the soliton regime N is low, stable and preserved for a wide range of Δ . For all the simulations we consider normalized parameters $S_0 = 2.3$, $q = 4$, and $\tau_g = 40$.

III. CONCLUSIONS

In conclusion, in this work we have presented a theoretical description of different types of OFC states excitations, and we described how a novel method of pumping might be considered to control the dynamics of the comb formation. The key control parameter is the chirp of the pump pulses, which permits the triggering of different OFC states, and can lead to the stability of the single soliton state regime.

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REFERENCES

- [1] P. Del'Haye, A. Schliesser, O. Arcizet, T. Wilken, R. Holzwarth, and T. J. Kippenberg, "Optical frequency comb generation from a monolithic microresonator," *Nature*, vol. 450, no. 7173, pp. 1214–1217, Dec. 2007.
- [2] A. Pasquazi, M. Peccianti, L. Razzari, D. J. Moss, S. Coen, M. Erkintalo, Y. K. Chembo, T. Hansson, S. Wabnitz, P. Del'Haye, X. Xue, A. M. Weiner, and R. Morandotti, "Micro-combs: A novel generation of optical sources," *Physics Reports*, vol. 729, pp. 1–81, Jan. 2018.

- [3] T. Herr, V. Brasch, J. D. Jost, C. Y. Wang, N. M. Kondratiev, M. L. Gorodetsky, and T. J. Kippenberg, "Temporal solitons in optical microresonators," *Nature Photonics*, vol. 8, no. 2, pp. 145–152, Dec. 2013.
- [4] E. Obrzud, S. Lecomte, and T. Herr, "Temporal solitons in microresonators driven by optical pulses," *Nature Photonics*, vol. 11, no. 9, pp. 600–607, Aug. 2017.
- [5] P. Parra-Rivas, D. Gomila, M. A. Matías, and P. Colet, "Dissipative Soliton Excitability Induced by Spatial Inhomogeneities and Drift," *Physical Review Letters*, vol. 110, no. 6, Feb. 2013.
- [6] P. Parra-Rivas, D. Gomila, M. A. Matías, P. Colet, and L. Gelens, "Effects of inhomogeneities and drift on the dynamics of temporal solitons in fiber cavities and microresonators," *Optics Express*, vol. 22, no. 25, p. 30943, Dec. 2014.
- [7] I. Hendry, W. Chen, Y. Wang, B. Garbin, J. Javaloyes, G.-L. Oppo, S. Coen, S. G. Murdoch, and M. Erkintalo, "Spontaneous symmetry breaking and trapping of temporal Kerr cavity solitons by pulsed or amplitude-modulated driving fields," *Physical Review A*, vol. 97, no. 5, May 2018.
- [8] M. Erkintalo, S. G. Murdoch, and S. Coen, "Phase and intensity control of dissipative Kerr cavity solitons," *Journal of the Royal Society of New Zealand*, vol. 52, no. 2, pp. 149–167, Mar. 2021.
- [9] F. R. Talenti, T. Hansson, and S. Wabnitz, "Control of Kerr Cavity Soliton Combs by Chirped Pumping," *OSA Advanced Photonics Congress (AP) 2020 (IPR, NP, NOMA, Networks, PVLED, PSC, SPPCom, SOF)*, 2020.
- [10] J. M. Chavez-Boggio, D. Bodenmüller, A. Baig, S. Ahmed, T. Hansson, F. R. Talenti, S. Wabnitz, and D. Modotto, "Frequency comb generation in silicon nitride resonators with amplitude modulated pump," *Laser Resonators, Microresonators, and Beam Control XXIII*, Mar. 2021.
- [11] J. K. Jang, M. Erkintalo, S. Coen, and S. G. Murdoch, "Temporal tweezing of light through the trapping and manipulation of temporal cavity solitons," *Nature Communications*, vol. 6, no. 1, Jun. 2015.

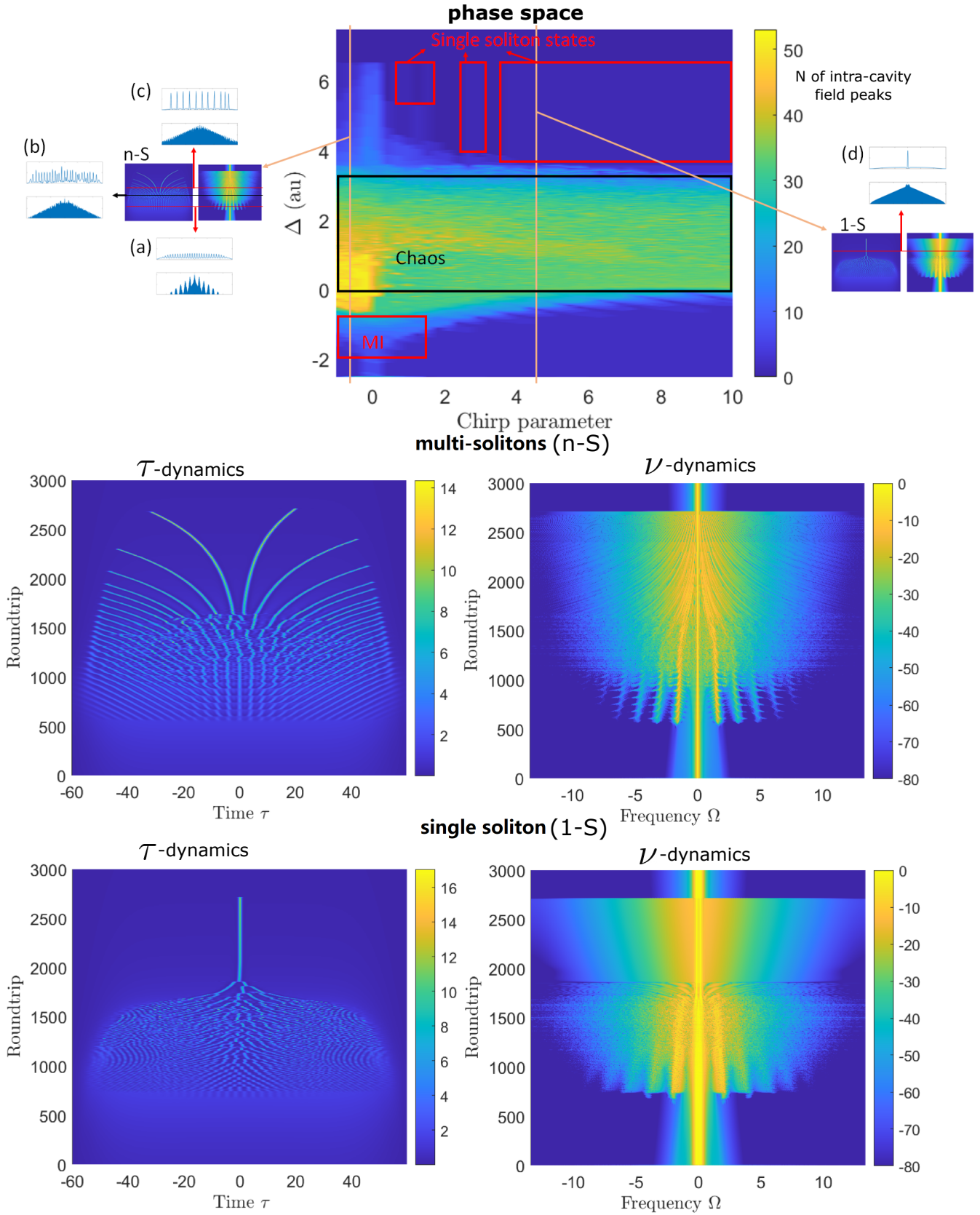


Fig. 1. On top we report the phase space spanned by the set of coordinates $\{C, \Delta, N\}$, where C is the chirp parameter, Δ the detuning and N the number of intra-cavity field peaks. By means of this representation we observe the emergence of solitons states from chaos. Negative chirps result in drifts of the intra-cavity peaks towards the outwards of the super-Gaussian quasi-flat background, which 3-dimensional dynamics is reported in the central panel, for both the temporal and frequency representation. For positive C , on the contrary, the drifts are towards the centre of the plateau, where we observe the coalescence of a single intra-cavity peak (bottom panel of the figure). By properly tuning the ramp of detuning $\Delta(t)$ and the pump phase modulation C we can thus both broaden the locking range and accelerate the convergence towards a stationary single soliton state. The insets (a-d) represent points on the phase space and they are reported fully size, and with the same nomenclature, on Fig.2.

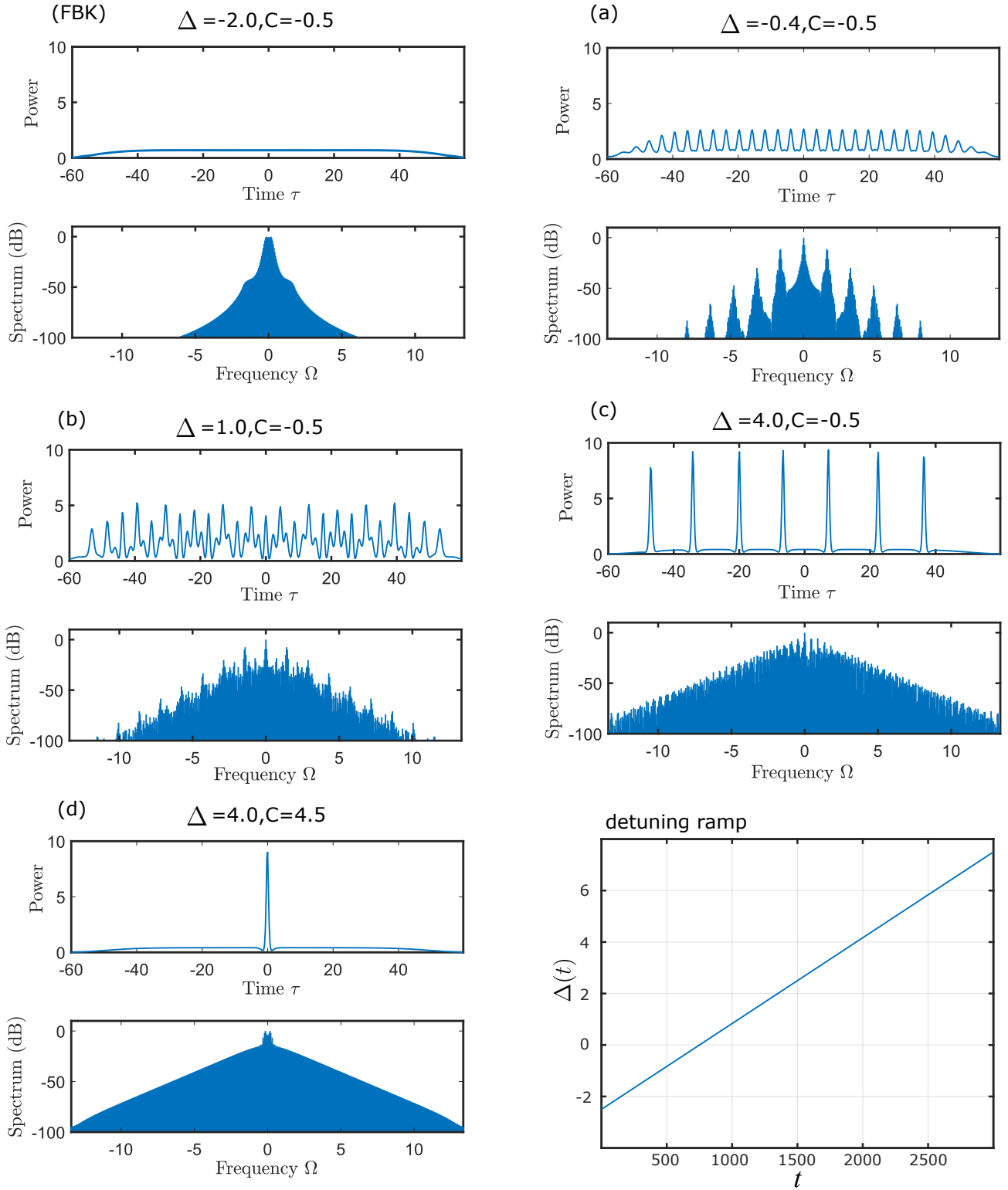


Fig. 2. We report the 1-dimensional temporal and frequency representation of the intra-cavity field for different couples of coordinates $\{\Delta, C\}$. At the beginning of the sweep a fraction of the pump is coupled into the passive resonator and the intra-cavity field shape reflects that of the super-Gaussian pump, resulting in a quasi flat background (FBK). (a) and (b) panels refers to the modulation instability and chaotic regimes, respectively. The convergence towards stationary states arising from chaos depends on the phase of the input field: for negative chirps we observe multi-solitons states (c), while for $C > 0$ single soliton states (d). On the bottom right we report the ramp of detuning used for each cavity sweep. The slow time variable t is reported in units of the number of round trips.