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INFLUENCE OF PLASTIC DISSIPATION ON APPARENT FRACTURE ENERGY DETERMINED BY A THREE-POINT BENDING TEST

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Abstract

A three-point bending test used for determining the fracture energy in modus I for wood perpendicular to the grain is studied. If the height of the specimen is varied, the results show a size effect. The specimens used in testing have been analyzed by finite element calculations with an anisotropic elasto-plastic material model in order to determine the influence of plastic dissipation. Analysis has been performed for three sizes of specimens where the height and length have been varied. The computational results are compared with experimental results.

1 Introduction

1.1 General

In the evaluation of the fracture energy of wood from test recordings obtained during a three-point bending test, Figure 1, it is assumed that the material outside the fracture section behaves in a linear or non-linear elastic manner. This assumption may be more or less accurate. Possible plastic, energy dissipating and non-recoverable deformations of the wood during the test may result in an apparent value of the fracture energy greater than the true value. Using an elasto-plastic material model with the yield criterion of Hill [1], the main purpose of the present study is to estimate the magnitude of the plastic dissipation by means of finite element analysis.

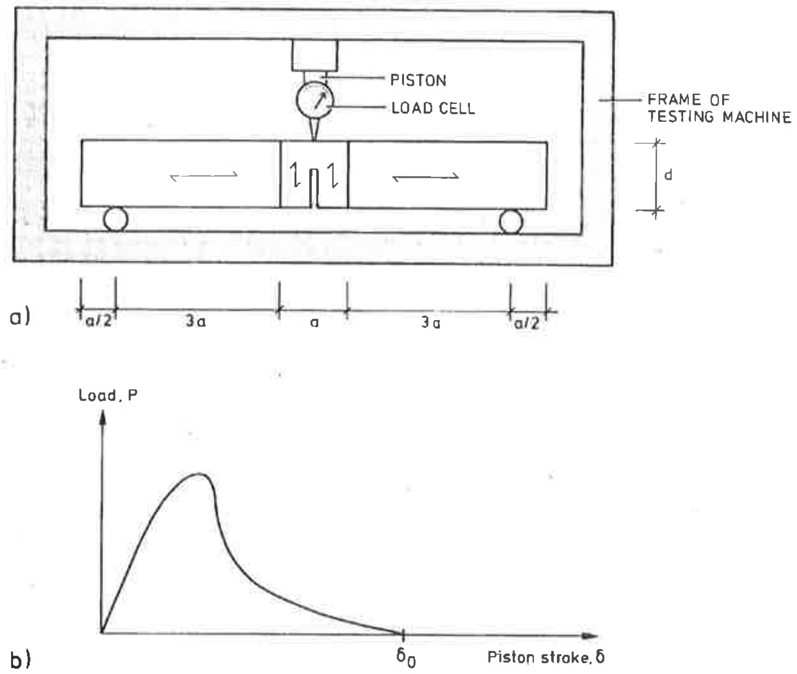

Figure 1: *a) Fracture energy test setup for wood; b) Recorded performance.*

1.2 Fracture energy test method

The test setup used is shown schematically in Figure 1a). The test specimen is composed of three pieces of wood glued together to form a beam. During the last few years, several investigators have used the test method [3,4,5,6,7], and some different geometrical shapes and sizes of the test specimen have also been tried. In many tests beam depth, d, has been made equal to 80 mm, the notch depth has been made equal to $0.5 \cdot 80 = 40mm$, the length measure, a, equal to $80mm$, and beam width, b, equal to $45mm$. These dimensions, together with the dimensions obtained by increasing and decreasing specimen height and length by the factor of 0.5 and 2, respectively, have also been adopted in the present study.

The beam is loaded in three-point bending with a constant rate of piston stroke so that collapse is obtained in about 3 minutes. During the test, load, P, and stroke, δ, are recorded. From the complete and stable $P - \delta$ curve, Figure 1b), the fracture energy G_f, is evaluated as the total work of fracture, W, divided by the area of the ligament, A, which can typically be about $45(80-40) = 1800mm^2$:

$$G_f = \frac{W}{A} \tag{1}$$

W is approximately obtained as

$$W = \int_0^{\delta_0} P\, d\delta + \frac{mg\delta_0}{2} + \frac{mg\delta_0}{2} \tag{2}$$

where the integral corresponds to the work carried out by the piston, which can be evaluated as the area under the $P - \delta$ curve.

The first $mg\delta_0/2$-term corresponds to the work carried out by the weight of the beam, mg, when beam deflection increases from zero to δ_0. For $\delta > \delta_0$ the fracture work carried out by the dead weight corresponds to the second $mg\delta_0/2$-term. This term is derived on the assumption that for large δ the final collapse takes place as a rotation around a hinge in the upper edge of the beam. This implies that for large δ the shape of the $P - \delta$ curve is such that $P \sim 1/\delta^2$. The accuracy of the actual consideration to dead weight can be studied in various experimental and computational ways [8,9,10].

1.3 Assumption made in the evaluation of fracture energy

The major assumption made in the evaluation of G_f from the test results is that no less and no more than the external work, W, goes to the fracture surface related energy dissipation, often called the material separation work. This means: (1) that no, or only a negligible part of, possible elastic strain energy at zero external load, e.g. due to shrinkage or growth stress, is assumed to be released during the test; (2) that the material outside the fracture process region is assumed to perform in a linear or non-linear elastic manner, and (3) that no or only negligible work is assumed to be transferred to kinetic energy which then is lost by damping. The energy-quantities corresponding to these three assumptions can be identified in a general energy balance equation [11] for a body with a growing crack.

Not only the wood outside the fracture region is assumed to be elastic but also the testing machine, including beam support and loading arrangements. If non-recoverable deformation of the testing machine may occur, beam deflection must be recorded instead of piston stroke. Significant kinetic energy may arise during a test if the $P - \delta$ performance becomes unstable. While occurrence of energy dissipating plastic deformations may be difficult to detect, instability results in a momentary decrease in the load, P. If assumption (1) made in the evaluation of G_f is not fulfilled, G_f will be under-estimated [10], and if assumption (2) or (3) is violated, G_f will be over-estimated.

1.4 Qualitative estimation of error in evaluated fracture energy versus specimen size

By definition the separation work of the material, C, is equal to the fracture energy of the material, G_f, times the size, A, of the fracture section. Considering specimens of equal shape but different absolute size, the absolute size being quantified by the depth of the beam, d, $A \sim d^2$ and accordingly

$$C = G_f \cdot A \sim G_f \cdot d^2 \tag{3}$$

The plastic dissipation is on the other hand not proportional to the fracture area but instead related to a volume, the volume of the yielding material. Assuming similar development of plastic regions in specimens of different size, the plastic dissipation, D, is

$$D \sim d^3 \tag{4}$$

Denoting the apparent value of the fracture energy as evaluated according to Sections 1.2 and 1.3 by G_f', equations (3) and (4) give

$$G_f' = \frac{C+D}{A} = \frac{G_f \cdot d^2 + c_D \cdot d^3}{d^2} = G_f + c_D \cdot d \tag{5}$$

where c_D is a constant of proportionality. Accordingly, the apparent value of the fracture energy may be expected to increase with the size of the specimen. Moreover, according to the above simplified reasoning, this increase is linear, and by reducing the size of the specimen towards zero it is in theory possible to eliminate the disturbing effect of possible plastic dissipation.

As a consequence of equation (4) being a simplification, equation (5) may be expected only to be a rough approximation. In the following sections, numerical as well as experimental results showing the apparent fracture energy versus absolute size of the specimen are given.

By a reasoning similar to the above, it can be found that possible initial elastic strain energy released during the test, E, is approximately proportional to d^3. Consequently, in the case of both plastic dissipation and release of initial strain energy:

$$G_f' = G_f + (c_D - c_E)d \tag{6}$$

where c_D and c_E are constants greater or equal to zero corresponding to the plastic dissipation and the initial strains, respectively. In this study possible effects of initial strains and stresses are not further discussed.

2 Finite Element Analysis

2.1 Elasto Plastic Material Model

In order to make a proper estimation of the influence of the plastic energy consumption in the three-point bending test, a plasticity material model suitable for wood must be used. Wood is an orthotropic material and therefore the plasticity model must take anisotropy into account. For anisotropic materials there exist a number of yield criteria. The first anisotropic yield criterion was proposed by Hill [1], and is an extended form of the von Mises yield criterion. Hill's criterion implies that the yielding stresses are equal in tension and compression. If different yielding in tension and compression is to be modelled we have the Hoffman yield criterion [2] which is similar to the Hill criterion, but extended with linear terms. If we extend the Hoffman criterion further we get the Tsai-Wu yield criterion [12], in which account is taken of the coupling between the different stress terms.

In a general three-dimensional stress state, the Hill criterion will have 6 yielding constants, the Hoffman criterion 9 yielding constants and the Tsai-Wu criterion will have 27 yielding constants. From this we can see that the amount of experimental work will increase substantially if a more general criterion is applied.

In the present case, yielding is expected to occur only for compression in the direction perpendicular to the grain. The failure in tension is assumed to be brittle without previous plastic deformations. For wood loaded perpendicular to the grain, a compilation of test results [13] suggests that the tensile failure occurs at a lower stress magnitude than the magnitude that is required to produce yielding in compression. Due to this, the yield criterion of Hill is completed with a simple tensile fracture criterion: $\sigma_{11} \leq f_t$ where σ_{11} and f_t are the stress and tensile strength respectively, in the direction perpendicular to the grain. When the tensile criterion has been reached, the material is assumed to reveal a gradual fracture softening, see Section 2.2. When the axes of the 123-coordinate system coincide with the axes of orthotropy, the Hill yield function $f(\sigma)$ is given by

$$\begin{aligned} f(\sigma) \;=\; & A(\sigma_{22}-\sigma_{33})^2 + B(\sigma_{33}-\sigma_{11})^2 + C(\sigma_{11}-\sigma_{22})^2 \\ & +3D\sigma_{23}^2 + 3E\sigma_{13}^2 + 3F\sigma_{12}^2 - \sigma_0^2 = 0 \end{aligned} \tag{7}$$

where the parameters A,B,C,D,E and F are expressed in terms of the yield stresses as

$$A = \frac{\sigma_0^2}{2}\left(\frac{1}{\sigma_{22y}^2} + \frac{1}{\sigma_{33y}^2} - \frac{1}{\sigma_{11y}^2}\right); \quad D = \frac{\sigma_0^2}{3\sigma_{23y}^2}$$
$$B = \frac{\sigma_0^2}{2}\left(\frac{1}{\sigma_{33y}^2} + \frac{1}{\sigma_{11y}^2} - \frac{1}{\sigma_{22y}^2}\right); \quad E = \frac{\sigma_0^2}{3\sigma_{13y}^2}$$
$$C = \frac{\sigma_0^2}{2}\left(\frac{1}{\sigma_{11y}^2} + \frac{1}{\sigma_{22y}^2} - \frac{1}{\sigma_{33y}^2}\right); \quad F = \frac{\sigma_0^2}{3\sigma_{12y}^2}$$

where σ_{ijy} is the yield strengths in the respective orthotropic directions and σ_0 is an arbitrary reference stress. If all the parameters A-F are made equal to unity, we will get the von Mises criterion.

Typical yielding values in compression in a plane strain case for dry wood (Pinus Silvestris) are given below.

Stress direction	σ_{yield} (MPa)	Direction
σ_{11y}	30.0	Parallel to grain
σ_{22y}	5.0	Perp. to grain (tangential)
σ_{33y}	5.0	Perp. to grain (radial)
σ_{12y}	7.0	Parallel/Perp. shear

These values were adopted in the present numerical calculations, without any experimental investigation of the material properties of the tested wood specimens.

2.2 The Finite Element Model

If the crack propagation is assumed to follow the fibres in a pure opening mode it is possible to simulate the propagation of the crack in a fairly easy manner. The specimen is modelled as two symmetric parts which are connected with nonlinear spring elements. Since the shear forces in the assumed crack zone are zero due to symmetry, it is sufficient to model the tensile fracture zone by springs in the direction perpendicular to the assumed crack direction. The finite element mesh used in the calculations is shown in Figure 2.

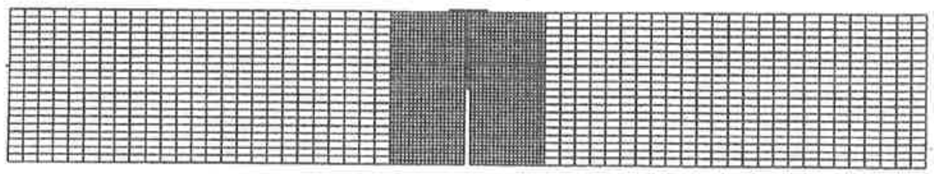

Figure 2: *Element mesh used in the calculations.*

The characteristic of the spring elements, Fig.3, captures the tensile fracture softening behaviour of wood perpendicular to the grain. The spring elements are activated when the tensile strength criterion is reached. The area under the $\sigma - \delta$ curve is the fracture energy for wood, i.e.

$$G_f = \frac{1}{2} f_t w_c \tag{8}$$

where f_t is the maximum value for σ and w_c is the maximum displacement before complete fracture. The value chosen for the fracture energy of wood perpendicular to the grain is 400 $\mathrm{Nm/m^2}$ and the value for the tensile strength f_t is 3.0 MPa. These values are used throughout this investigation. From these values it is possible

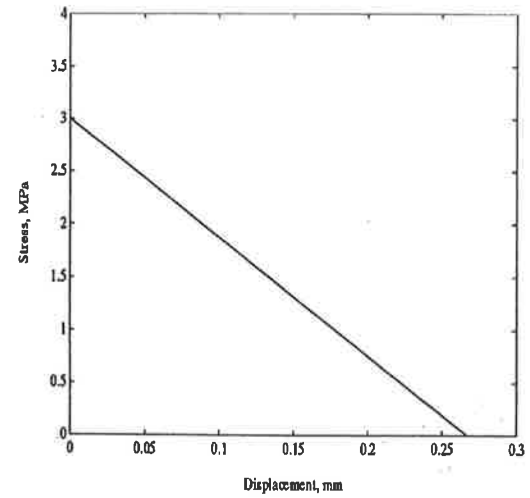

Figure 3: *Characteristics of the spring elements.*

to calculate the corresponding value of w_c: $w_c = 0.267mm$. To simulate the loading, the piston of the testing machine is modelled as a rigid surface. In real testing a piece of soft rubber material is inserted between the piston and the specimen to avoid concentrated forces. This was modelled as an extra layer of elements on the specimen. Special interface elements are introduced between the rigid surface and the rubber material. These elements describe contact forces and friction between the rigid surface and the rubber. Figure 4 shows how the loading of the specimen is modelled.

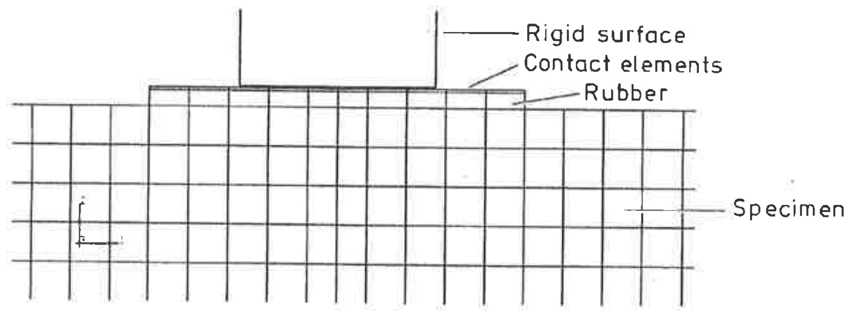

Figure 4: *Finite element model at the area of loading.*

3 Results and Comparison with Experiments

The aim of this investigation was to determine if the size effects in the evaluation of the fracture energy from the three-point bending test can be explained by plastic dissipation only. The finite element mesh that was described in the previous section was therefore used for three different sizes of beam height, 40mm, 80mm and 160mm. The 80mm specimen can be regarded as the reference specimen, while the other two are extracted from the 80mm specimen by scaling all the dimensions up and down with a factor of two, except for the width which is held constant for all specimens. From the calculations it is found that the plastic zone becomes larger if the height of the specimen is larger.

The total external work W obtained from the finite element analysis is calculated in the same manner as for the experiments by considering the displacement δ and reaction force F of the rigid surface, i.e. the piston. Figure 5 shows the $F-\delta$ curves for the three calculations.

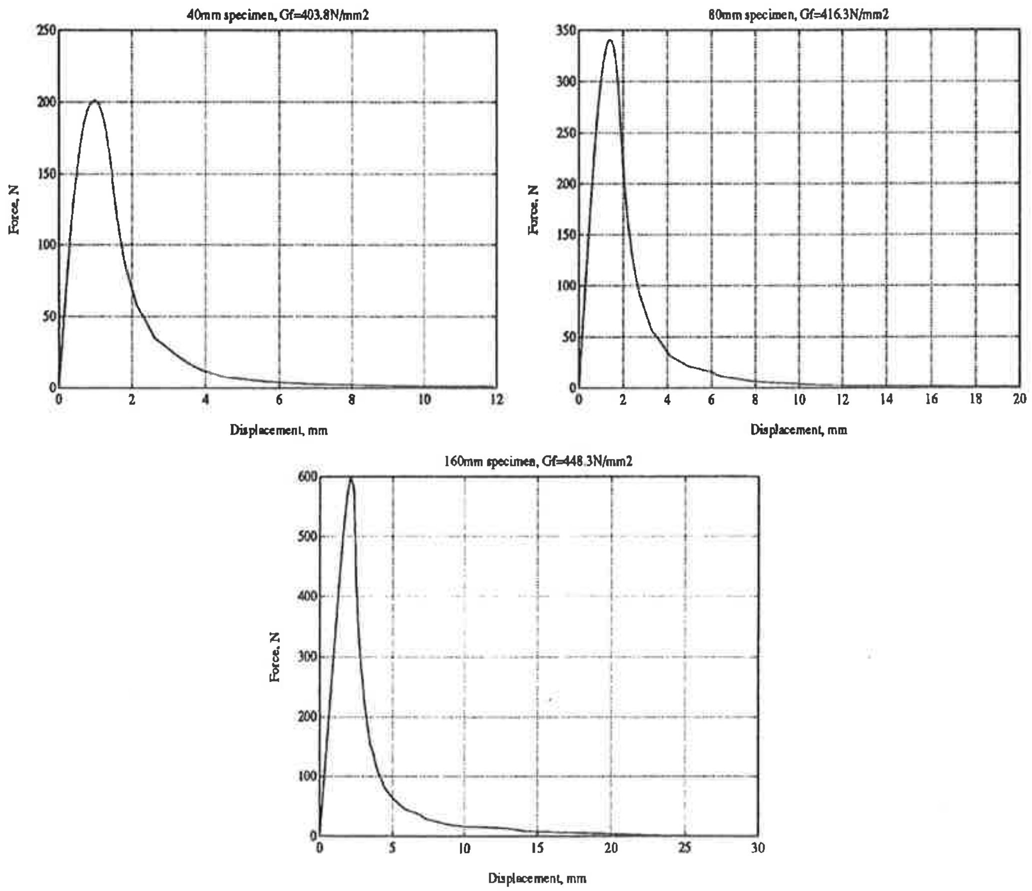

Figure 5: *F-δ curves for the three specimen sizes.*

The comparison between the experimental and calculated results is shown in Figure 6 and the table below. The values are referenced to the 80mm specimen.

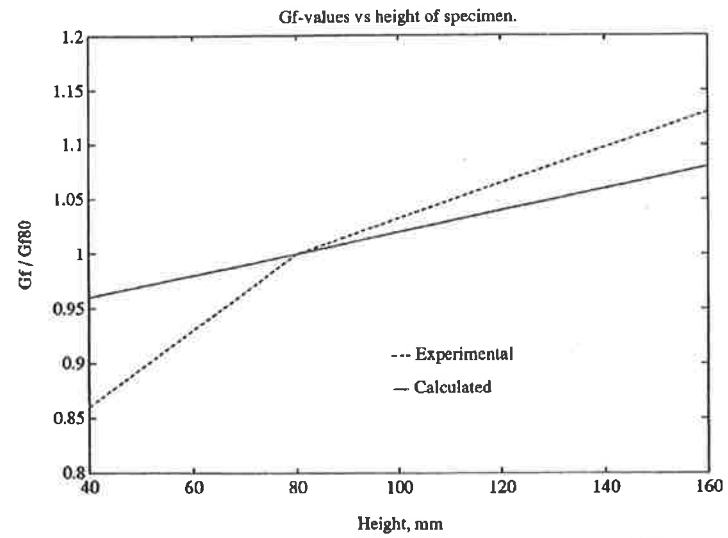

Figure 6: *Calculated and experimental G_f values vs specimen height.*

Height	Experimental results		Calculated results	
(mm)	G_f/G_{f80}	G_f	G_f/G_{f80}	G_f
40	0.86	413	0.96	403.8
80	1.00	483	1.00	416.3
160	1.13	520	1.08	448.3

Figure 6 shows the table above in a diagram.

To verify that the calculation procedure gives correct results, an analysis with linear elastic material outside the cracking zone was also made for the 80mm specimen. This calculation resulted in a G_f-value of 401.1Nm/m^2 which only differs by 0.2% from the correct result of 400.0Nm/m^2.

4 Concluding Remarks

In this study some experimental results have been compared with results from numerical calculations without sufficient knowledge of the material properties. The differences between calculated and experimental results might thus be explained by the fact that the material parameters used in the model have not been experimentally verified. Further, the influence of the size of the finite elements has not been investigated, but if smaller sizes of the finite elements had been choosen, the size effect would probably had been larger. However, the results clearly indicate that yielding due to compression perpendicular to grain should be considered if the beam is high.

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