Waveform design of DFRC System for Target detection in Clutter Environment

Jinkun Zhu, Wei Li, Kai-Kit Wong, *Fellow, IEEE*, Tian Jin, Kang An

***Abstract*—Dual-function radar and communication (DFRC) has recently drawn significant attention due to its enormous potential. This letter deals with waveform design of DFRC to improve target detectability embedded in clutter environment while guaranteeing the service quality of communication users. Our design objective is to maximize the output signal-to-clutter-plus-noise ratio (SCNR) of multiple-input multiple-output (MIMO) radar, subject to worst-case received symbol errors at communication users. Coordinate descent (CD) as an efficient iteration algorithm is proposed to solve above optimization problem, which splits high-dimensional problem into multiple one-dimensional problem. Furthermore, we introduce Dinkelbach algorithm (DA) to increase rate of convergence, which is an efficient way to reduce complexity. Finally, simulation results are presented to illustrate the effectiveness of the proposed techniques.**

***Index Terms*—Dual-function radar and communication, Output SCNR, MIMO radar, MU-MIMO communication.**

I. INTRODUCTION

WITH the advancement of electronic systems towards intelligence and miniaturization, dual-function radar and communication (DFRC) systems have become a research hotspot in the academic and industrial fields by sharing hardware, frequency, signal and software resources to achieve dual-functional capability [1]–[3]. The application of Multi-Input Multi-Output (MIMO) technology in the fields of communication and radar has been proven to be a effective approach to improve system performance. Therefore, the waveform design of DFRC under MIMO system has received widespread attentions.

Waveform design of DFRC system generally includes three philosophies, namely, sensing-centric design, communication-centric design and joint design [2]. 1) *Sensing-centric design*: this way primarily guarantees the sensing performance while embedding communication information into sensing waveform. This way will make original radar waveform random since it carries communication information [4]. The work of [5] proposed multiple orthogonal waveforms in tandem with sidelobe control are used to embed information. In [6], communication information was embedded into the emission of MIMO radar using sparse antenna array configurations. A scheme of index modulation exploiting the inherent spatial and spectral randomness to convey information was proposed in [7]. 2) *Communication-centric design*: communication is the primary functionality while implementing sensing functionality over the communication waveform. The work of [8] proposed the optimization problem of achievable sum-rate in multi-user communication with the constraint of radar signal-to-noise ratio (SNR). In [9], Cramér-Rao bound (CRB) was employed as a minimization object while guaranteeing signal-to-interference-plus-noise ratio (SINR) for each user. 3) *Joint design*: this way focuses on conceiving fully novel waveform which can satisfy the specific demand rather than rely on existing waveform. The work of [10] considered the flexible trade-off between radar and communication performance through weighted optimization. In [11], an original optimization framework was proposed and has been proved to achieve its optimality under special situation.

In non-uniform and time-varying geographical environment, there are usually non-uniform and variable strong scattering points, which is called the point clutters. Therefore, this letter considers waveform design in clutter environment, which aims at maximizing output SCNR of MIMO radar while ensuring that received symbol error of communication user is below the given threshold. In addition, the transmit waveform is constrained to be constant envelope to guarantee the efficiency of power amplifiers. To tackle with the proposed non-convex optimization, coordinate descent (CD) iteration algorithm is proposed to solve it, which can split high-dimensional problem into multiple one-dimensional problem. In addition, Dinkelbach iteration algorithm is used to increase the rate of convergence and reduce the complexity. The effectiveness of proposed waveform design scheme is validated via simulations.

Notations. Matrices are denoted by bold uppercase letters (i.e., $\mathbf{A}$), vectors are denoted by bold lowercase letters (i.e., $\mathbf{a}$). $\mathbb{E}$ is the expectation operator. $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product of $\mathbf{A}$ and $\mathbf{B}$. $[\cdot]^*$, $[\cdot]^T$ and $[\cdot]^H$ denote the conjugate, transpose and conjugate-transpose of its argument respectively. $\Re\{\cdot\}$ denotes the real part of its argument. $\|\cdot\|$ denotes the l_2 norm. Finally, $tr(\cdot)$ denotes the trace of a square matrix.

II. SYSTEM MODEL

Consider a downlink DFRC system in which both radar and communication functionalities are implemented simultaneously. A uniform linear array (ULA) with M antennas is shared by a colocated monostatic MIMO radar system and a multiuser MIMO communication system, simultaneously serving K single-antenna users and detecting radar targets. Assume that there is a point target and P independent point clutters located in the far-field environment.

Jinkun Zhu, Wei Li and Tian Jin are with the College of Electronic Science and Technology, National University of Defense Technology, Changsha 410073, China (e-mail:zhujinkun@hotmail.com; liwei.nudt.cn@gmail.com; tianjin@nudt.edu.cn); Kat-Kit Wong is with the Department of Electronic and Electrical Engineering, University College London, Edinburgh, London WCIE 7JE, U.K. (e-mail:kai-kit.wong@ucl.ac.uk); Kang An are with the Sixty-Third Research Institute, National University of Defense Technology, Nanjing 21007, China (e-mail:ankang89@nudt.edu.cn).

A. Communication Model

With N representing the number of samples, transmitted signal matrix is denoted as $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N] \in \mathbb{C}^{M\times N}$. Let $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_K]^T \in \mathbb{C}^{K\times M}$ denote channel matrix. Received signal matrix $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_K]^T \in \mathbb{C}^{K\times N}$ is given by

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W}. \tag{1}$$

Where $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_N] \in \mathbb{C}^{K\times N}$ denotes noise matrix and $\mathbf{w}_j \sim \mathcal{CN}(0, \sigma_c^2\mathbf{I}), \forall j$. With $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_K]^T \in \mathbb{C}^{K\times N}$ representing the desired constellation symbol matrix for users, the power of error between received signal and desired symbol for k-th user and n-th sample is defined as

$$\mathbf{e}_k(n) = \mathbb{E}(|\mathbf{y}_k(n) - \mathbf{s}_k(n)|^2) \quad = |\mathbf{h}_k\mathbf{x}_n - \mathbf{s}_k(n)|^2 + \sigma_c^2. \tag{2}$$

where σ_c^2 denotes the noise variance and $\mathbf{h}_k^T\mathbf{x}_n - \mathbf{s}_k(n)$ is considered to be interference. The power of interference term has been verified to be directly related to the achievable sum rate [12].

B. Radar Model

The transmit covariance of MIMO radar is given by

$$\mathbf{R} = \frac{1}{N}\mathbf{X}\mathbf{X}^H = \sum_{n=1}^{N} \mathbf{x}_n\mathbf{x}_n^H. \tag{3}$$

The n-th sampled baseband discrete echo $\mathbf{r}_n$ is expressed as

$$\mathbf{r}_n = \alpha_0\mathbf{a}^*(\theta_0)\mathbf{a}^H(\theta_0)\mathbf{x}_n + \mathbf{d}_n + \mathbf{v}_n, n = 1, \ldots, N. \tag{4}$$

where α_0 denotes complex amplitude, $\mathbf{v}_n \in \mathcal{CN}(0, \sigma_v^2\mathbf{I}_M)$ denotes Gaussian white noise. Let $\mathbf{a}(\theta_0) = \frac{1}{\sqrt{M}}\left[1, e^{j2\pi\Delta\sin(\theta_0)}, \ldots, e^{j2\pi(M-1)\Delta\sin(\theta_0)}\right]^{\mathrm{T}} \in \mathbb{C}^{M\times 1}$ denote steering vector in direction θ_0 where Δ is the ratio of gap distance between adjacent antennas to the signal wavelength. $\mathbf{d}_n$ is the superposition of P independent point clutter echoes, given by

$$\mathbf{d}_n = \sum_{p=1}^{P} \alpha_p\mathbf{a}^*(\theta_p)\mathbf{a}^H(\theta_p)\mathbf{x}_n \tag{5}$$

Letting $\tau_0 = \mathbb{E}[\|\alpha_0\|^2]$, the power of target echo E_0 can be expressed as

$$E_0 = \frac{1}{N}\sum_{n=1}^{N} \mathbb{E}\left[\|\alpha_0\mathbf{a}(\theta_0)\mathbf{x}_n\|^2\right] = \tau_0^2\mathbf{a}^H(\theta_0)\mathbf{R}\mathbf{a}(\theta_0) \tag{6}$$

Similarly, the power of clutter echoes E_P is given by

$$E_P = \frac{1}{N}\sum_{n=1}^{N} \mathbb{E}\left[\left\|\sum_{p=1}^{P}\alpha_p\mathbf{a}(\theta_p)\mathbf{x}_n\right\|^2\right] = \sum_{p=1}^{P}\tau_p^2\mathbf{a}^H(\theta_p)\mathbf{R}\mathbf{a}(\theta_p) \tag{7}$$

where $\tau_p = \mathbb{E}[\|\alpha_p\|^2]$. The SCNR of radar echoes $\eta(\mathbf{R})$ is expressed as

$$\eta(\mathbf{R}) = \frac{\tau_0^2\mathbf{a}^H(\theta_0)\mathbf{R}\mathbf{a}(\theta_0)}{\sum\limits_{p=1}^{P}\tau_p^2\mathbf{a}^H(\theta_p)\mathbf{R}\mathbf{a}(\theta_p) + \sigma_v^2}. \tag{8}$$

III. PROBLEM FORMULATION AND OPTIMIZATION DESIGN

A. Problem Formulation

In this letter, we aim to maximize the output SCNR of MIMO radar under the constraints of communication quality and constant envelope.

For communication quality, we take the power of error between received signal and desired symbol as the basis of a constraint. The goal of waveform design is to optimize the output SCNR of MIMO radar while guaranteeing that the power of error is below the threshold, yielding the following optimization problem:

$$\max_{\mathbf{x}} \quad \frac{\tau_0^2\mathbf{a}^H(\theta_0)\mathbf{R}\mathbf{a}(\theta_0)}{\sum\limits_{p=1}^{P}\tau_p^2\mathbf{a}^H(\theta_p)\mathbf{R}\mathbf{a}(\theta_p) + \sigma_v^2} \tag{9a}$$

$$s.t. |\mathbf{h}_k\mathbf{x}_n - \mathbf{s}_k(n)|^2 \le \Gamma_k, k = 1, 2, \ldots, K, n = 1, 2, \ldots, N, \tag{9b}$$

$$|x_{m,n}| = \sqrt{u}, m = 1, 2, \ldots, M, n = 1, 2, \ldots, N. \tag{9c}$$

where Γ_k represents the threshold. Constraint (9b) guarantees that interference power is within a certain range. Constraint (9c) ensures the constant envelope characteristics of the integrated waveform. Simplify the numerator of objective function (9a):

$$\tau_0^2\mathbf{a}^H(\theta_0)\mathbf{R}\mathbf{a}(\theta_0) = \frac{\tau_0^2}{N}\left\|\mathrm{vec}\left(\mathbf{a}^H(\theta_0)\mathbf{X}\right)\right\|^2 \tag{10a}$$

$$= \frac{\tau_0^2}{N}\mathbf{x}^H\mathbf{Y}_0\mathbf{x}, \tag{10b}$$

where $\mathbf{x} = \{\mathbf{x}_1^H, \ldots, \mathbf{x}_N^H\}^T = \mathrm{vec}(\mathbf{X}) \in \mathbb{C}^{MN\times 1}$ and $\mathbf{Y}_0 = \mathbf{I}_N \otimes (\mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0))$.

Similarly, the denominator of the objective function can also be simplified into a similar form. Therefore, problem (9) can be rewritten as

$$\max_{\mathbf{x}} \quad \frac{\mathbf{x}^H\boldsymbol{\Xi}\mathbf{x}}{\mathbf{x}^H\boldsymbol{\Omega}\mathbf{x}} \tag{11a}$$

$$s.t. \quad |\mathbf{h}_k\mathbf{x}_n - \mathbf{s}_k(n)|^2 \le \Gamma_k, k = 1, 2, \ldots, K, n = 1, 2, \ldots, N, \tag{11b}$$

$$|x_{m,n}| = \sqrt{u}, m = 1, 2, \ldots, M, n = 1, 2, \ldots, N, \tag{11c}$$

where

$$\boldsymbol{\Xi} = \frac{\tau_0^2}{N}\mathbf{Y}_0, \tag{12}$$

$$\boldsymbol{\Omega} = \sum_{p=1}^{P}\frac{\tau_p^2}{N}\mathbf{Y}_p + \mathbf{I}_{MN}\frac{\sigma_v^2}{NP}. \tag{13}$$

Problem (11) is non-convex due to the non-convex objective function and constraints and the constant envelope constraint leads that the optimization problem is NP-hard.

B. Proposed Algorithm

It can be found from (11) that each codeword is independent of each other. Therefore, each codeword can be sequentially optimized to monotonically improve output SCNR. Coordinate descent (CD) algorithm is an effective way to solve such problem with low complexity. The core idea of the CD algorithm is to optimize one waveform codeword in each iteration while fixing the remaining codewords, transforming the original high-dimensional problem into multiple easily solvable one-dimensional subproblems. By obtaining the optimal solutions of each subproblem, the original optimization problem is iteratively solved.

In the process of CD algorithm, only one element in vector $\mathbf{x}$ is optimized each time. Let $\mathbf{x}(\alpha)$ represent the α-th element of the vector $\mathbf{x}$. when $\mathbf{x}(\alpha)$ is the optimizing variable, the remaining elements can be treated as constants. Letting $\boldsymbol{\xi}_i$ and $\xi_{j,i}$ represnt i-th column of $\boldsymbol{\Xi}$ and (j,i)-th entry of $\boldsymbol{\Xi}$, respectively, the numerator of the objective function can be rewritten as

$$\begin{aligned}\mathbf{x}^H\boldsymbol{\Xi}\mathbf{x} = |\mathbf{x}(\alpha)|^2\xi_{\alpha,\alpha} + 2\Re\left(\sum_{i=1,i\neq\alpha}^{MN}\mathbf{x}^*(i)\xi_{i,\alpha}\mathbf{x}(\alpha)\right)\\ + \sum_{i=1,i\neq\alpha}^{MN}\sum_{j=1,j\neq\alpha}^{MN}\mathbf{x}^*(j)\xi_{j,i}\mathbf{x}(i).\end{aligned} \tag{14}$$

The same procedure is easily adapted to the denominator of objective function, which is given by

$$\begin{aligned}\mathbf{x}^H\boldsymbol{\Omega}\mathbf{x} = |\mathbf{x}(\alpha)|^2\kappa_{\alpha,\alpha} + 2\Re\left(\sum_{i=1,i\neq\alpha}^{MN}\mathbf{x}^*(i)\kappa_{i,\alpha}\mathbf{x}(\alpha)\right)\\ + \sum_{i=1,i\neq\alpha}^{MN}\sum_{j=1,j\neq\alpha}^{MN}\mathbf{x}^*(j)\kappa_{j,i}\mathbf{x}(i),\end{aligned} \tag{15}$$

where $\kappa_{j,i}$ is the (j,i)-th entry of matrix $\boldsymbol{\Omega}$. Let $\mathbf{x}_n(a)$ denote the a-th element of vector $\mathbf{x}_n$. Assuming the condition $\alpha = (n-1)M+a$ holds, $\mathbf{x}(\alpha)$ and $\mathbf{x}_n(a)$ denote the same element. For the convenience of expressing the formula, we will use $\mathbf{x}_n(a)$ as optimization variable when simplifying constraint (11b). Let us further expand the constraint (11b) as

$$\begin{aligned}&|\mathbf{h}_k^T\mathbf{x}_n - \mathbf{s}_k(n)|^2 = \operatorname{tr}\left(\left(\mathbf{h}_k^T\mathbf{x}_n - \mathbf{s}_k(n)\right)\left(\mathbf{h}_k^T\mathbf{x}_n - \mathbf{s}_k(n)\right)^H\right)\\ &= \operatorname{tr}\left(\mathbf{h}_k^T\mathbf{x}_n\mathbf{x}_n^H\mathbf{h}_k^*\right) - 2\Re\left(\operatorname{tr}\left(\mathbf{s}_k(n)\mathbf{x}_n^H\mathbf{h}_k^*\right)\right) + \operatorname{tr}\left(\mathbf{s}_k(n)\mathbf{s}_k^H(n)\right)\\ &= uM\operatorname{tr}\left(\mathbf{h}_k^T\mathbf{h}_k^*\right) - 2\Re\left(\operatorname{tr}\left(\mathbf{s}_k(n)\mathbf{x}_n^H\mathbf{h}_k^*\right)\right) + \operatorname{tr}\left(\mathbf{s}_k(n)\mathbf{s}_k^H(n)\right).\end{aligned} \tag{16}$$

According to (16), the first and the third items are constant. The second item can be further rewritten as

$$\begin{aligned}2\Re\left(\operatorname{tr}\left(\mathbf{s}_k(n)\mathbf{x}_n^H\mathbf{h}_k^*\right)\right) &= 2\Re\left(\operatorname{tr}\left(\mathbf{x}_n\mathbf{s}_k^H(n)\mathbf{h}_k^T\right)\right)\\ &= 2\Re\left(\sum_{m=1}^{M}\mathbf{x}_n(m)t_{n,m,k}\right),\end{aligned} \tag{17}$$

where $t_{n,m,k}$ denotes the (n,m)-th entry of matrix $\mathbf{T}_k = \mathbf{s}_k^H\mathbf{h}_k^T \in \mathbb{C}^{N\times M}$. According to (16) and (17), when optimizing variable $\mathbf{x}_n(a)$, constraint (11b) can be equivalent to

$$2\Re\left(\mathbf{x}_n(a)t_{n,a,k}\right) \geq \varrho_\alpha - \Gamma_k, k = 1,2,\ldots,K, \tag{18}$$

where

$$\begin{aligned}\varrho_\alpha = uM\operatorname{tr}\left(\mathbf{h}_k\mathbf{h}_k^H\right) + \operatorname{tr}\left(\mathbf{s}_k(n)\mathbf{s}_k^H(n)\right)\\ - 2\Re\left(\sum_{m=1,m\neq a}^{M}\mathbf{x}_n(m)t_{n,m,k}\right).\end{aligned} \tag{19}$$

Let $\mathbf{x}^{(q)} = [\mathbf{x}^{(q)}(1), \mathbf{x}^{(q)}(2), \ldots, \mathbf{x}(\alpha), \ldots, \mathbf{x}^{(q)}(MN)]^T$ denote optimization vector for q-th iteration, in which the α-th element is optimized. In addition, the elements of $\mathbf{x}^{(q)}$ except $\mathbf{x}(\alpha)$ equals to the elements corresponding to $\mathbf{x}^(q-1)$. According to (14), (15) and $|\mathbf{x}(\alpha)|^2 = u$, the problem can be formulated as

$$\begin{aligned}\max_{\mathbf{x}(\alpha)} \quad & \frac{\Re\left(\psi_{1,\alpha}\mathbf{x}(\alpha)\right) + \psi_{2,\alpha}}{\Re\left(\phi_{1,\alpha}\mathbf{x}(\alpha)\right) + \phi_{2,\alpha}}\\ s.t. \quad & 2\Re\left(\mathbf{x}_n(a)t_{n,a,k}\right) \geq \varrho_\alpha - \Gamma_k, k = 1,2,\ldots,K,\\ & |\mathbf{x}(\alpha)| = \sqrt{u},\end{aligned} \tag{20}$$

where

$$\psi_{1,\alpha} = 2\sum_{i=1,i\neq\alpha}^{MN}\mathbf{x}^{(q)^*}(i)\xi_{i,\alpha}, \tag{21}$$

$$\psi_{2,\alpha} = u\xi_{\alpha,\alpha} + \sum_{i=1,i\neq\alpha}^{MN}\sum_{j=1,j\neq\alpha}^{MN}\mathbf{x}^{(q)^*}(j)\xi_{j,i}\mathbf{x}^{(q)}(i), \tag{22}$$

$$\phi_{1,\alpha} = 2\sum_{i=1,i\neq\alpha}^{MN}\mathbf{x}^{(q)^*}(i)\kappa_{i,\alpha}, \tag{23}$$

$$\phi_{2,\alpha} = u\kappa_{\alpha,\alpha} + \sum_{i=1,i\neq\alpha}^{MN}\sum_{j=1,j\neq\alpha}^{MN}\mathbf{x}^{(q)^*}(j)\kappa_{j,i}\mathbf{x}^{(q)}(i). \tag{24}$$

Algorithm 1 DA Algorithm for Solving (20)

Input: $\mathbf{x}^{(q)}(i), i = 1,2,\ldots,MN, i \neq \alpha$, $\boldsymbol{\Xi}$, $\boldsymbol{\Omega}$, Γ, $\mathbf{H}$, $\mathbf{S}$;
Output: Global optimal $\mathbf{x}^{(q)}(\alpha)$
1: Initialize $\mathbf{x}_{(0)}(\alpha) = \mathbf{x}^{(q-1)}(\alpha)$ and $l = 0$;
2: Construct ϱ_α, $\psi_{1,\alpha}$, $\psi_{2,\alpha}$, $\phi_{1,\alpha}$ $\phi_{2,\alpha}$ according to (19), (21), (22), (23), (24);
3: $l = l + 1$;
4: Compute $\vartheta_l = \frac{\Re\left(\psi_{1,\alpha}\mathbf{x}_{(l-1)}(\alpha)\right)+\psi_{2,\alpha}}{\Re\left(\phi_{1,\alpha}\mathbf{x}_{(l-1)}(\alpha)\right)+\phi_{2,\alpha}}$;
5: Compute (26) to acquire a solution $\mathbf{x}_{(l)}(\alpha)$;
6: If $|\Re\left(\psi_{1,\alpha}\mathbf{x}(\alpha)\right)+\psi_{2,\alpha}-\vartheta_l(\Re\left(\phi_{1,\alpha}\mathbf{x}(\alpha)\right)+\phi_{2,\alpha})| \leq \lambda$, output $\mathbf{x}^{(n)}(\alpha) = \mathbf{x}_{(l)}(\alpha)$ Otherwise, return to step 3.

Problem (20) is a quadratic fractional programming. The DA algorithm has linear convergence characteristics and can ensure convergence to the global optimal solution [13]. Therefore we exploit the DA iterative algorithm to convert linear fractional programming to linear programming summarized in Algorithm 1. Specifically, the l-th iteration value of objective function ϑ_l is given by

$$\vartheta_l = \frac{\Re\left(\psi_{1,\alpha}\mathbf{x}_{(l-1)}(\alpha)\right) + \psi_{2,\alpha}}{\Re\left(\phi_{1,\alpha}\mathbf{x}_{(l-1)}(\alpha)\right) + \phi_{2,\alpha}} \tag{25}$$

where $\mathbf{x}_{(l-1)}(\alpha)$ denote the solution for $(l-1)$-th iteration. According to the idea of DA algorithm, problem (20) is recast as

$$\max_{\mathbf{x}(\alpha)} \quad \Re\left(\psi_{1,\alpha}\mathbf{x}(\alpha)\right) + \psi_{2,\alpha} - \vartheta_l\left(\Re\left(\phi_{1,\alpha}\mathbf{x}(\alpha)\right) + \phi_{2,\alpha}\right) \tag{26a}$$

$$s.t. \quad 2\Re\left(\mathbf{x}_n(a)t_{n,a,k}\right) \geq \varrho_\alpha - \Gamma_k, k = 1,2,\ldots,K, \tag{26b}$$

$$|\mathbf{x}(\alpha)| = \sqrt{u}. \tag{26c}$$

Ignoring constant terms, problem (26) can be simplified as

$$\begin{aligned} \max_{\mathbf{x}(\alpha)} \quad & \Re\left(\zeta_\alpha \mathbf{x}(\alpha)\right) \\ s.t. \quad & 2\Re\left(\mathbf{x}_n(a) t_{n,a,k}\right) \geq \varrho_\alpha - \Gamma_k, k = 1, 2, \ldots, K, \\ & |\mathbf{x}(\alpha)| = \sqrt{u}. \end{aligned} \tag{27}$$

where $\zeta_\alpha = \psi_{1,\alpha} - \vartheta_l \phi_{1,\alpha}$. Let ϖ_{ζ_α}, ϖ_α and $\varpi_{t_{n,a,k}}$ denote the phase of ζ_α, $\mathbf{x}(\alpha)$ and $t_{n,a,k}$, respectively. Ignoring constant terms, problem (27) is further recast as

$$\max_{\varpi_\alpha} \quad \cos\left(\varpi_{\zeta_\alpha} + \varpi_\alpha\right) \tag{28a}$$

$$s.t. \quad \cos\left(\varpi_\alpha + \varpi_{t_{n,a,k}}\right) \geq \epsilon. \tag{28b}$$

where ϵ denotes threshold of phase constraint for all codewords. Constraint (28b) can be easily solved to obtain the phase range of ϖ_α, written as

$$\varpi_\alpha \in [\ominus_\alpha, \ominus_\alpha + \beta], \tag{29}$$

where $\ominus_\alpha = -\arccos\epsilon - \arctan\left(\Im\left(\varpi_{t_{n,a,k}}\right) / \Re\left(\varpi_{t_{n,a,k}}\right)\right)$ and $\beta = 2\arccos\epsilon$ .The optimal solution $\bar{\varpi}_\alpha$ of problem (28) is given as

$$\bar{\varpi}_\alpha = -\varpi_{\zeta_\alpha}, -\varpi_{\zeta_\alpha} \in [\ominus_\alpha, \ominus_\alpha + \beta] \tag{30}$$

or

$$\bar{\varpi}_\alpha = \begin{cases} \ominus_\alpha + \beta & \cos\left(\varpi_{\zeta_\alpha} + \ominus_\alpha + \beta\right) \geq \cos\left(\varpi_{\zeta_\alpha} + \ominus_\alpha\right) \\ \ominus_\alpha & \cos\left(\varpi_{\zeta_\alpha} + \ominus_\alpha + \beta\right) < \cos\left(\varpi_{\zeta_\alpha} + \ominus_\alpha\right) \end{cases}. \tag{31}$$

Algorithm 2 CD Algorithm for Solving (11)

Input: $\mathbf{x}^{(0)}$, $\mathbf{\Xi}$, $\mathbf{\Omega}$, Γ, $\mathbf{H}$, $\mathbf{S}$;
Output: An optimized solution $\mathbf{x}$ to problem (11);
1: Initialize $q = 0$ and $j = 0$;
2: $q = q + 1$, $\mathbf{x}^{(q)} = \mathbf{x}^{(q-1)}$;
3: $j = j + 1$;
4: Construct ϱ_j, $\psi_{1,j}$, $\psi_{2,j}$, $\phi_{1,j}$ $\phi_{2,j}$;
5: Update j-th element of $\mathbf{x}^{(q)}$ by finding the optimal solution of (20) using DA algorithm ;
6: If $j = MN$, perform $j = 0$. Otherwise, return to step 3;
7: If $\left|\frac{\mathbf{x}^{(q)H}\mathbf{\Xi}\mathbf{x}^{(q)}}{\mathbf{x}^{(q)H}\mathbf{\Omega}\mathbf{x}^{(q)}} - \frac{\mathbf{x}^{(q-1)H}\mathbf{\Xi}\mathbf{x}^{(q-1)}}{\mathbf{x}^{(q-1)H}\mathbf{\Omega}\mathbf{x}^{(q-1)}}\right| \leq \nu$, output $\mathbf{x} = \mathbf{x}^{(q)}$. Otherwise, return to step 2.

For process of each iteration, problem (20) need to be solved for MN times. The computational complexity to solve (20), which is mainly related to construct ζ_α, is $\mathcal{O}((MN)^2)$. Hence, the total complexity of CD algorithm is $\mathcal{O}((MN)^3)$.

IV. NUMERICAL RESULTS

In this section, we numerically validate the effectiveness of the proposed waveform design scheme and compare the optimization algorithm with sequential quadratic programming (SQP) algorithm [14]. We consider a ULA with $M = 10$ transmitting elements and the number of signal sample is $N = 100$. Target is located at $\theta_0 = 30°$ with power $\tau_0^2 = 0$ dB and there are three point clutters located at $-60°$, $-10°$ and $75°$, respectively, with corresponding power of -10 dB, -10 dB and -10 dB.

We first evaluate the monotonicity and convergence of objective value for Algorithm CD and Algorithm SQP. Fig. 1 shows SCNR versus iteration number with $\epsilon = -0.5, 0$. It can be observed that all SCNR values for the considered algorithm increase with the number of iterations. In addition, CD algorithm has faster rate of convergence than SQP algorithm. The reason is that the DA algorithm greatly improves the convergence rate during the iteration process of each codeword. The CD algorithm can converge within almost 5 iterations, which indicates a significant reduction in complexity.

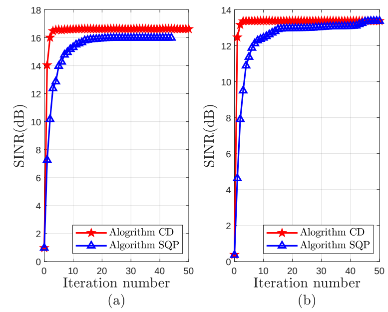

Fig. 1. The SCNR value (dB) versus iteration number , (a) $\epsilon = -0.5$, (b) $\epsilon = 0$.

We then analyze the impact of parameter ϵ on achievable SCNR value, which actually indicates the trade-off between radar performance and communication performance. Fig. 2 depict the achievable SCNR behavior versus ϵ. It is obvious that algorithm CD outperform algorithm SQP in terms of obtained SCNR value. It can be observed that the SCNR values decrease with the increasing ϵ. According to (28b), the value of ϵ is related to the phase range of each codeword. This behavior is reasonable because increasing the value of ϵ will further limit the scope of phase, such that the output SCNR of radar decreases.

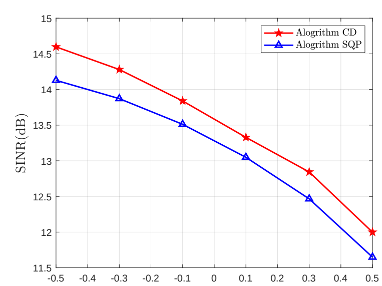

Fig. 2. The SCNR value (dB) versus ϵ value.

V. CONCLUSION

In this letter, we presented a waveform design scheme based on DFRC-MIMO system in clutter environment. We aimed to optimize the output SCNR of MIMO radar to improve the detection probability while guaranteeing the worst-case received symbol error of each user under constant envelope constraint.

Furthermore, we developed CD iteration optimization algorithm and DA algorithm to handle this challenging problem Finally, we evaluated the effectiveness of the proposed scheme by simulations.

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